Determination of the $\pi 1g_{9/2}$ orbit size in ^{88}Sr, ^{90}Zr, and ^{92}Mo from inelastic electron scattering

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A study of the $\pi 1g_{9/2}$ orbit size in ^{88}Sr, ^{90}Zr, and ^{92}Mo is presented. The rms radius for the point-proton density is extracted by studying transitions to 8^+ states in these nuclei. The radii are consistently larger than a value determined in a magnetic electron scattering experiment on ^{93}Nb. A qualitative discussion of the ground state occupation of the $\pi 1g_{9/2}$ orbit based on the transition amplitudes to the 8^+ states is given.

A knowledge of single-particle orbitals is fundamental to the traditional microscopic description of nuclei. In general, however, not much is known about these orbitals. It is difficult to find an experimental quantity which involves an individual single-particle orbit in an unambiguous way. Strongly interacting probes have yielded much qualitative information, but these experiments cannot be interpreted with the same precision as those involving electromagnetic probes. Studies on the differences of ground state charge densities have yielded some information but are influenced by core polarization. Magnetic electron scattering and inelastic electron scattering to single-particle states are our main sources of information at the present.

We have conducted a systematic study of the $\pi 1g_{9/2}$ orbit size in ^{88}Sr, ^{90}Zr, and ^{92}Mo. For this study we have used inelastic electron scattering data on transitions to 8^+ states with dominant configuration $(\pi 1g_{9/2})^2_{8^+}$. These states are found at excitation energies of 5.65 MeV (^{88}Sr), 3.59 MeV (^{90}Zr), and 2.76 MeV (^{92}Mo). These data enable us to determine small changes in the orbit size from one nucleus to the next. By comparing the spectroscopic amplitudes for these transitions with model predictions the occupation of the $\pi 1g_{9/2}$ orbit in the ground state can be studied.

The data on all three nuclei were collected at the MIT-Bates Linear Accelerator using the high-resolution spectrometer facility. These facilities have been described elsewhere, as have the experimental details for the ^{88}Sr and ^{90}Zr experiments.[1–5] A summary of the ^{92}Mo experiment is given here and is representative of the previous two experiments. The insert in Fig. 1 is a typical spectrum for ^{92}Mo showing the 8^+ state clearly resolved from the neighboring states. Spectra were measured at forward angles between 40 and 105 deg, and with incident energies between 100 and 370 MeV. A total of 24 q points were taken covering the momentum transfer range of 0.45—3.0 fm^{-1}. Typical resolutions obtained were better than 20 keV due to improved focal-plane instrumentation.[6] Two

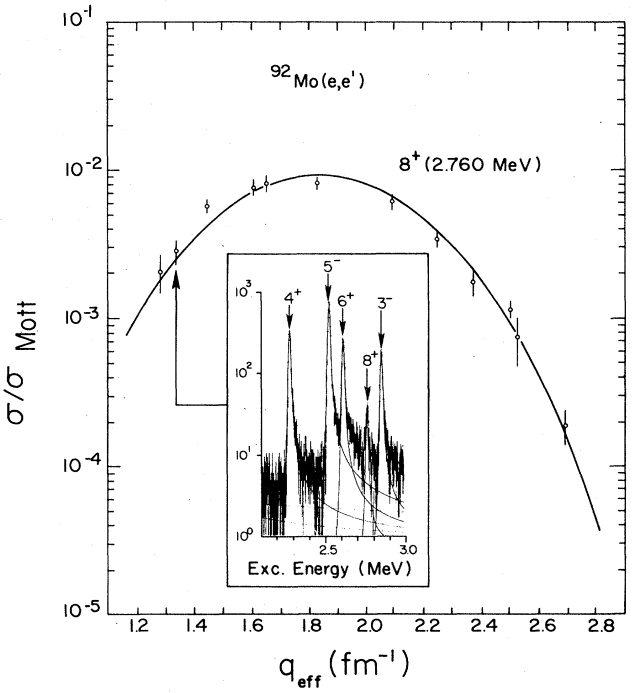

FIG. 1. The insert shows a typical spectrum (semilog) from the ^{92}Mo experiment taken at incident electron energy 190 MeV and 88.0 deg scattering angle. Also shown is the fit to the data for the 8^+ state in ^{92}Mo described in the text.

molybdenum targets were used, both enriched to 97.4% ^{92}Mo, with thicknesses of 5.8 mg/cm^2 and 19.4 mg/cm^2. Incident beam energy calibrations were done by scattering from BeO and BeAl targets as well as by measuring isotopic recoil from impurities in the Mo targets. The low-q data were normalized to agree with the results of the existing elastic scattering data of Dreher,[7] Singhal *et al.*,[8] and Phan *et al.*[9] The high-q data at 370 and 280 MeV were normalized by including low-q points in each angular distribution. We estimate that this normalization procedure in all cases contributes less than 5% uncertainty to the measured cross sections. A systematic error of 5—10 % has been added in quadrature to the statistical errors. For the 8^+ states the systematic uncertainty is usually small compared to the statistical one.

In the shell model picture these isotones are spherical nuclei with closed neutron shells at $N=50$. The low lying excited states are dominated by proton excitations within the $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbits. The 8^+ states fall within this model as part of a band of states based on the configuration $(\pi 1g_{9/2})^2_{0^+,2^+,4^+,6^+,8^+}$. If the $(\pi 1g_{9/2})^2_{8^+}$ shell model configuration is assumed for the 8^+ states the $\pi 1g_{9/2}$ radial wave function can be determined in a straightforward manner. The 8^+ state will be excited in electron scattering only through the $(\pi 1g_{9/2})^2_{0^+}$ configurations present in the ground state. The transition involves only the recoupling of a proton pair, so the scattering will be purely longitudinal and involve only a single nuclear density, $\rho^{\rm tr}_\lambda(r)$. In this case the density is given by (see Ref. 10)

$$\rho^{\rm tr}_\lambda(r)=S^{if}_\lambda C_\lambda u^2_{\pi 1g_{9/2}}(r)\ , \tag{1}$$

where

$$C_\lambda=(-)^\lambda\frac{10}{\sqrt{4\pi}}(\tfrac{9}{2}\tfrac{1}{2}\tfrac{9}{2}-\tfrac{1}{2}\,|\,\lambda 0)\ . \tag{2}$$

Here $u(r)$ is the radial wave function of the $\pi 1g_{9/2}$ orbit, S^{if}_λ is the spectroscopic amplitude, and $\lambda=8$ for our case. The most important higher order term due to a relativistic spin-orbit density[10] has been included in the analysis. The calculations and fits have been done in the distorted-wave Born approximation (DWBA), but for a qualitative description it is more instructive to look at the plane-wave Born approximation (PWBA) form factor,

$$F^C_\lambda(q)=\frac{\hat{J}_i}{\hat{J}_f}\int_0^\infty \rho^{\rm tr}_\lambda(r)j_\lambda(qr)r^2dr\ . \tag{3}$$

In this approximation the shape of the experimental form factor is determined by the Fourier-Bessel transform of the square of the radial wave function. The shape of the radial wave function is therefore directly determined by the experimental data. This qualitative feature of the PWBA carries over into the DWBA.

It is known that model calculations in this region which do not include strong core polarization effects generally fail to reproduce electron scattering transition densities.[5] Both the strength and radial shape of the transition densities can be influenced by core polarization. Core polarization can therefore be viewed as a source of error in determining the shape of the $\pi 1g_{9/2}$ orbit. A recent calculation of this effect for the 2^+ to 8^+ multiplet in ^{90}Zr (Ref. 11) shows a strong spin dependence. The effect on the low-spin states is very strong, gradually decreasing with increasing spin, finally becoming negligible for the 8^+ state. Even for calculations that predict a larger core polarization, the effect on the extracted radius is small. For example, in a broken-pair calculation by Allaart and Akkermans[12] for ^{90}Zr the 8^+ density is enhanced by 10% because of core polarization, yet the rms radius of the $\pi 1g_{9/2}$ orbit is changed by only 0.1%. This is the case because the dominant component of the core polarization is $\pi(1g_{7/2},1g^{-1}_{9/2})$, which has a shape very smililar to $\pi(1g_{9/2},1g^{-1}_{9/2})$. In addition, the electron probe is not sensitive to the neutron contributions in the core polarization. Based on the above discussion we feel the core polarization is a small effect for the 8^+ states and more importantly has little effect on the extracted radius.

Figure 1 shows the fit to the 8^+ data in DWBA for ^{92}Mo. For this least-square fit the radial wave functions were generated from a Woods-Saxon well with diffuseness 0.7 fm, spin-orbit strength 7.5 MeV, and a well depth adjusted to give a separation energy of 6.05 MeV. These parameters were kept fixed for all three isotones, whereas the spectroscopic factor and well radius were fitted. Variations in the fixed parameters had only a small effect on the fitted rms radius of the orbit. The uncertainties associated with small variations of the fixed parameters (10% in diffuseness, spin-orbit strength, and separation energy) have been included in the uncertainties shown in Fig. 2. This contribution is much smaller than the statistical uncertainty indicating that the radius is well determined by the data.

The orbit size obtained from these fits are shown in Fig. 2. The experimental charge density is a convolution of the proton point density with the charge distribution of the proton. This convolution is treated in the Fourier-Bessel expansion[5] using the proton form factor of Simon *et al.*[13] This convolution increases the radius of the density by only 2%. Thus even a 10% error in the proton radius contributes negligibly to the error in the extracted radius. The size of the $\pi 1g_{9/2}$ orbit in Fig. 2 is expressed as

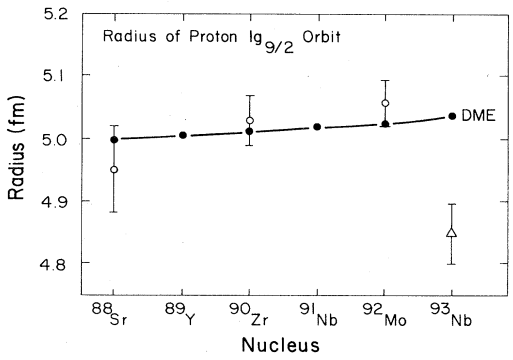

FIG. 2. Fits for the $\pi 1g_{9/2}$ orbit radius are shown as the open circles. The HF-DME results are shown as the solid circles and the open triangle is the value extracted from a magnetic scattering experiment on ^{93}Nb.

the rms radius of the radial point density. It can be seen that the $\pi 1g_{9/2}$ radius is gradually increasing with A for these three isotones. Also shown in Fig. 2 are values obtained from a Hartree-Fock (HF) calculation using the density matrix expansion (DME) interaction of Negele and Vautherin.[14]

A direct comparison of the experimental results with HF results is difficult. The HF wave functions are not given in the center-of-mass (c.m.) system of the calculated nucleus whereas the point proton density extracted here is. There are two ways to handle this disparity. One can either remove the c.m. motion from the HF results or fold the c.m. motion into the experimental results. We have chosen the first procedure. For the DME calculation the residual c.m. motion of the HF nucleus has been unfolded using the procedure for harmonic oscillator wave functions.[15,5] This procedure is not unique, but it introduces only a negligible ambiguity in the radius. Another difficulty is that the HF binding energies do not agree with the experimental values. The experimental value is taken to be the ground state separation energy for a proton in the $\pi 1g_{9/2}$ orbit reduced by the 8^+ excitation energy. The Woods-Saxon fit to the data does show a dependence on the separation energy, but the exact effect for a self-consistent HF treatment is not clear. Particularly with these cautions in mind the agreement with the data is good, and the gradual increase of the rms radius with A is well predicted by the HF theory.

As reported previously[5] these orbit sizes are especially interesting in light of a recent magnetic electron scattering experiment on ^{93}Nb.[16] The results of that experiment are included in Fig. 2. It should be noted that for the results quoted in Ref. 16 the c.m. motion of the calculated nucleus was treated differently than is presented here. In that reference the c.m. motion was folded into the experimental results rather than removed from the theoretical calculation. The ^{93}Nb result shown has been adjusted appropriately. Using a fit which is linear in $A^{1/3}$ to our three data points we predict a rms radius of 5.096 fm for ^{93}Nb, roughly 4% higher than the measured value. The smooth behavior of the HF predictions supports making such an extrapolation but of course does not include any odd-even staggering effects which have been observed for ground state charge densities.[17] Only because of the precision of both measurements can this small difference be viewed as significant.

There are several possible reasons for this discrepancy. One may speculate that the radius of the $\pi 1g_{9/2}$ orbit is different for excited states and hence the two experiments are not really measuring the same quantity. In addition the ^{93}Nb measurement is sensitive to the transition current density while scattering from the 8^+ states involves the transition charge density only. The difference between the two measurements may point to problems with the form of the current operator. Meson exchange current (MEC) corrections applied to the current operator increased the size of the orbit extracted from the magnetic scattering results. This increase, however, is only on the order of 1% and it is unlikely that the entire discrepancy is due to MEC. Another possibility is that medium modifications (by which we mean core polarization, effective mass, and various other corrections applied to one-body operators which in part account for many-body degrees of freedom) differ for the current-density operator and the charge-density operator. Interactions such as the DME are adjusted to fit charge radii and thus implicitly include some of these corrections for the charge operator in the wave function. As a result they may successfully predict the observables based on the charge operator but fail for the current operator where the corrections are different.

In addition to the rms radius the fits yield values for S_λ^{if} as defined in Eq. (1). Since the 8^+ state is excited only from the $(\pi 1g_{9/2})^2$ component in the ground state, this amplitude reflects the ground state occupation of the $\pi 1g_{9/2}$ orbit. However this relation is subject to uncertainties due to core polarization and fragmentation of the 8^+ strength. No attempt has been made to account for these uncertainties. Instead, we show a direct comparison of the fitted values of S_8^{if} with model predictions.

The results of this comparison are summarized in Fig. 3. Errors for the three 8^+ spectroscopic amplitudes include statistical contributions only. Also shown are two shell model calculations, one using the matrix elements and single particle energies of Haxton,[18] and the other of Heisenberg and Dawson.[11] Both calculations were performed using a valence space of the proton $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbitals and both give relatively good agreement for excited state energies. The third calculation is a one broken-pair calculation by Allaart and Akkermans.[12] The model space for this calculation includes the $1f_{7/2}$ through $1h_{11/2}$ orbits. Within this space, proton two-quasiparticle excitations with respect to a number-projected Bardeen-Cooper-Schrieffer (BCS) ground state and neutron 1p-1h excitations were taken

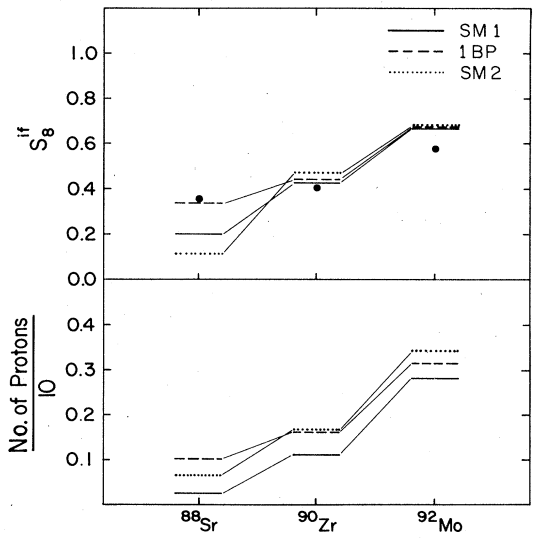

FIG. 3. Spectroscopic amplitudes for ^{88}Sr, ^{90}Zr, and ^{92}Mo. Also shown are three model calculations for the amplitudes and the resulting ground state occupations. The calculation labeled SM1 is from a private communication with Haxton (see also Ref. 17). The calculation labeled 1BP is the one broken-pair calculation of Allaart and Akkermans (Ref. 12). The calculation SM2 is by Heisenberg and Dawson (Ref. 11).

into account. The interaction used was fixed for all three nuclei, but the single-particle energies for the ^{88}Sr calculation were adjusted to improve energy level agreement.

A comparison of the results for the different nuclei illustrates an interesting feature. The amplitude for ^{88}Sr is underpredicted (except for the 1BP calculation) and the amplitude for ^{92}Mo is overpredicted. This general feature indicates that the ground state occupation of the $\pi 1g_{9/2}$ orbit is underpredicted in ^{88}Sr and overpredicted in ^{92}Mo. This shows the softening of the Fermi level is much more pronounced than what is predicted by the shell model calculations. The relatively good agreement of the broken-pair calculation for the ^{88}Sr amplitude points to the importance of the $\pi 1f_{7/2}$ orbit in providing occupation of the $\pi 1g_{9/2}$ orbit. The failure of the broken pair to correctly predict the amplitude in ^{92}Mo is not understood.

In summary the simple structure of the 8^+ states allows us to study a single orbital. The $\pi 1g_{9/2}$ radii extracted from studying these 8^+ transitions are larger than those observed for ^{93}Nb in a magnetic electron scattering experiment. This result has several possible interpretations and deserves further investigation. The spectroscopic amplitudes observed indicate a softening of the Fermi level beyond the model predictions.

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