PHYSICAL REVIEW C VOLUME 32, NUMBER 2 AUGUST 1985

Parametrizing the equation of state of cold and dense nuclear matter

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(Received 29 April 1985)

Various functional parametrizations of the equation of state of cold nuclear matter are considered with regard to both their degree of stiffness and their causal behavior at high density. We show that (i) the inferred nuclear compressibility K is strongly parametrization dependent when the measured pion multiplicity in central heavy ion collisions is fitted, and (ii) most of the commonly used equations of state are acausal at high density.

The primary goal of colliding heavy nuclei at high energy is to obtain information on dense nuclear matter. Pion multiplicity in central collisions may be a signature of the compressional stage of the collision.[1,2] This conjecture was first put on a concrete basis in the context of the standard intranuclear cascade (INC) models. Such models had previously served to accurately predict the pion yield in proton-nucleus collisions, pion-nucleus scattering, and pion absorption on nuclei. However, in near central collisions of Ar+KCl at 0.5–1.8 GeV per nucleon the INC overestimates the pion yield by a factor of 1.5–2.5. This overestimation has been attributed to a factor present in nucleus-nucleus collisions but absent in proton- and pion-nucleus collisions: a density increase, or compression, of the nuclear matter. During the high density stage of a nucleus-nucleus collision some fraction of the incident kinetic energy is transformed into potential, or compressional energy and is therefore unavailable for pion production. An analysis of INC calculations and data results in the compressional energy per nucleon $E_0(n)$ as a function of density n as shown in Fig. 1 (dots). In this Brief Report we do not wish to question the foundations of this analysis; rather, we shall use it as motivation to investigate various functional parametrizations of the equation of state of cold and dense nuclear matter.

One popular parametrization is simply a parabola about the ground state.

Case I:

$$E_0(n)=\frac{K}{18}\left(\frac{n}{n_0}-1\right)^2+E_0(n_0) \quad .$$

Here K is the compressibility at normal density n_0. A second popular parametrization is linear in the density at high density.

Case II:

$$E_0(n)=\tfrac{2}{9}K[(n/n_0)^{1/2}-1]^2+E_0(n_0) \quad .$$

The above $E_0(n)$ have as their only parameter K [$E_0(n_0)=-16$ MeV is always held fixed]. Of course this need not be so as illustrated by the following simple generalization of the latter.

Case III:

$$E_0(n)=\tfrac{2}{9}K[(n/n_0)^{1/2}-1]^2+a[(n/n_0)^{1/3}-1]^3+E_0(n_0) \quad .$$

These three functional forms shall suffice to illustrate several points.

First we adjust the parameters in each case to give the best visual fit to the dots in Fig. 1.

Case I: $K=275$ MeV ,

Case II: $K=550$ MeV ,

Case III: $K=200$ MeV, $a=400$ MeV .

Up to $n=4n_0$ there is hardly any difference at all among these three curves. Thus, point one is that the inferred nuclear compressibility K is strongly parametrization dependent when the measured pion multiplicity in central heavy ion collisions is fit.

Next we calculate the speed of sound in nuclear matter.

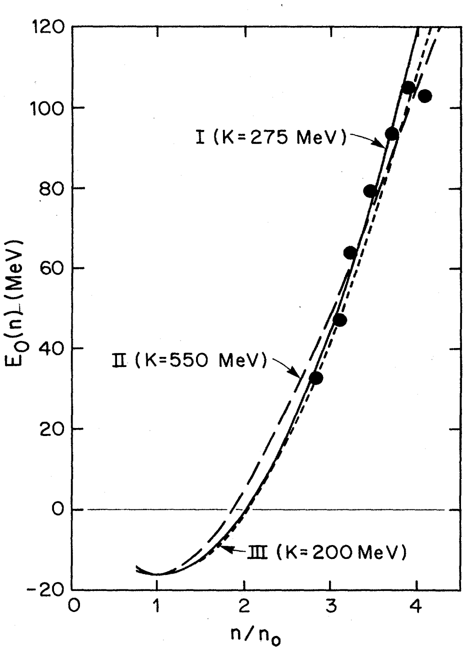

FIG. 1. The compressional energy per nucleon as a function of compression. The curves labeled I, II, and III correspond to the parametrizations given in the text. The points are inferred from measured pion multiplicity in central collisions of Ar+KCl and from intranuclear cascade calculations and are taken from Ref. 1.

It is given by

$$v_s^2 = \frac{dP_0(n)}{d\epsilon_0(n)} \ ,$$

where the pressure is

$$P_0(n) = n^2 \frac{dE_0(n)}{dn} \ ,$$

and the total energy density is

$$\epsilon_0(n) = n[m + E_0(n)] \ .$$

The square of the speed of sound as a function of density is plotted in Fig. 2 for the three different equations of state. Notice that the speed of sound exceeds the speed of light when $n > 4.5n_0$ in the case of parametrization I, whereas it happens when $n > 7n_0$ in the case of III. Even in the case of II, $v_s > 1$ when $n > 32.5n_0$. The acausality present in case I is well known but in cases II and III it is rather surprising. Consider, for example, the analysis suggested in Ref. 3. At high density let

$$E_0(n) = bn^\beta \ ,$$

where $b > 0$ and $\beta > 0$. Then

$$v_s^2 = \frac{b\beta(\beta+1)n^\beta}{m + b(\beta+1)n^\beta} \underset{n\to\infty}{\longrightarrow} \beta \ .$$

Thus, causality requires $\beta \leq 1$. The sources of acausality in cases II and III are the subleading terms in n/n_0.

Actually it is still a rather unsettled question as to whether $v_s > 1$ implies causality. It could be that the nucleons cannot respond rapidly enough to density perturbations which want to propagate with a speed near that of light. In that case density perturbations would be overdamped. An argument against overdamping is that sound waves in air propagate with a speed nearly equal to the mean molecular speed, so that sound waves with $v_s \sim 1$ may very well propagate in a system of interacting relativistic nucleons. To study this question further requires a microscopic Hamiltonian for nuclear matter and a detailed analysis using kinetic theory or linear response theory. In any case, a practical consequence is that the use of any equation of state which exhibits $dP_0/d\epsilon_0 > 1$ for some range of density in the relativistic hydrodynamic equations of motion will result in either an unphysical solution or no solution at all. Finally, we point out that nuclear matter should undergo a phase transition to quark matter at sufficiently high density ($n \sim 10n_0$ at zero temperature?). Thus, acausal behavior in the nuclear matter equation of state may be irrelevant if it occurs well above the transition density.

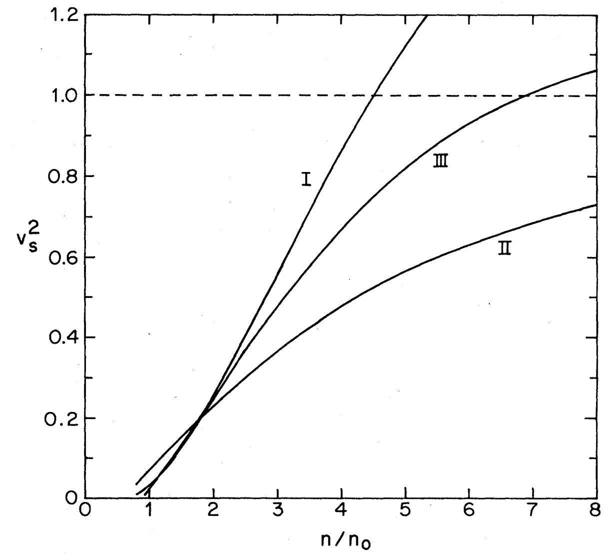

FIG. 2. The square of the speed of sound as a function of compression for the three parametrizations of the equation of state given in the text.

In summary, we have made two points in this Brief Report. The first point is that the nuclear compressibility K is strongly parametrization dependent when inferred from the measured pion multiplicity in central high energy heavy ion collisions. Conversely, knowledge of the behavior of nuclear matter near the ground state, such as the compressibility, is not sufficient to extrapolate to high density. This gives added importance to the study of high energy heavy ion collisions. The second point is that most of the commonly used analytic parametrizations of the compressional energy $E_0(n)$ yield a speed of sound which exceeds the speed of light at sufficiently high density. This may or may not have physical consequences depending on whether the supposed sound waves propagate or are overdamped, and on whether such occurs before or after a phase transition to quark matter. An acausal equation of state cannot be used in a relativistic hydrodynamic calculation. It is worth remarking that relativistic mean field models of nuclear matter based on Lagrangian field theory do not exhibit acausal behavior at any density.[4,5]

This work was supported by the U.S. Department of Energy under Contract No. DOE/DE-AC02-79ER-10364.

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