Sensitivity of the triton binding energy to the D-state probability in the deuteron

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We report on the sensitivity of the triton binding energy to the D-state probability P_D in the deuteron. Based on a recently proposed energy-dependent separable potential, the $^3S_1$-$^3D_1$ interaction is constructed in such a way that while varying P_D, the off-shell and on-shell behavior of the nucleon-nucleon forces employed (phase shifts are taken directly from experiment) are independent of the choice of P_D. The increase of P_D within the experimental bounds from 4% to 7% decreases the triton binding energy by ≈ 0.55–0.60 MeV. These values are in very good agreement with the results obtained for the Reid potential by Harms *et al.*, which differ strongly from the results by Brady *et al.*

There have been two reports[1,2] in the past on the sensitivity of the triton binding energy E_B to the deuteron percentage D state P_D. Brady *et al.*[1] have calculated E_B vs P_D for separable potentials of Yamaguchi form. For the difference $\Delta E = E_B(P=4\%) - E_B(P=7\%)$, they obtain 1.1 MeV. Harms *et al.*[2] have reported on similar studies using a different order of unitary pole expansion of the Reid potential and obtain for $\Delta E = 0.549$–0.567 MeV (for unitary pole approximation $\Delta E = 0.60$ MeV), which is smaller by a factor of 2 compared with the result by Brady *et al.*

In both calculations the variation of P_D entails variations of the off-shell behavior and of the phase shifts at medium energies of the pertinent interaction model. In view of the fact that we are comparing differences with one another, and that the discrepancy by a factor of 2 appears to be large, it seems to be of some importance to assess the model dependence on ΔE and, in particular, the influence of the changes of the off-shell behavior and of the phase shifts at medium energies present in the calculations of Refs. 1 and 2.

Although the D-state probability P_D is not a directly measurable quantity, it can be obtained from experiment provided that other quantities are known. Information on P_D can be obtained from (i) deuteron tensor polarization T_{22} at 22.7 MeV, (ii) $\sigma(\theta)$ at 130°; this minimum is mainly filled by the tensor force, (iii) measurement of the deuteron magnetic form factor, (iv) ed tensor polarization and deuteron photodisintegration, and (v) breakup n-d iT_{11} and Ay measurements. For these quantities the integral measure of the D state is relevant. Moreover, P_D is an important constraint on the construction of the NN force models. It would help solve the problem that a strong tensor force is needed at the bound state pole for a reasonable description of the deuteron, while the low energy mixing parameter requires rather a weak tensor force. The solution of this problem would also facilitate a more precise extraction of neutron-neutron parameters from quasifree scattering data. In addition, the knowledge of the range of the D wave could constrain P_D within a range of one percent and vice versa.

In this work we report on exact triton bound state calculations using an energy-dependent separable force model. The $^3S_1$-$^3D_1$ interaction is constructed in such a way that while changing P_D the off-shell and on-shell behavior of the pertinent forces are kept fixed. Moreover, the employed potentials reproduce exactly the experimental phase shifts. The use of these potential enables us, in turn, to obtain a direct—off-shell and phase shift unambiguous—relation between D-state probability and triton binding energy. We wish also to recall that the assessment of the quantitative influence of P_D by itself on E_B, aside from contributions from three-body forces and from the "improper" off-shell behavior of the nucleon, is important in resolving the notorious discrepancy between theoretical values for E_B and the experiment.

We use an energy-dependent potential of the following form:[3]

$$V(E,p,p') = \lambda(E)\bar{V}(p,p') \ ,$$

where $\bar{V}(p,p')$ is a Hermitian potential and the construction of $\lambda(E)$ at positive and negative energies along the lines of Ref. 4 is described below. It has been shown[3,4] that the method employed for the construction of $\lambda(E)$ at negative energies reproduces in a qualitative way the constraints imposed by the conventional Hermitian potential at negative energies on the pertinent t matrix, and fulfills the required unitarity and analyticity conditions.[4] An advantageous feature of this model is that the same behavior of a t matrix at negative and positive energies can be used for any reasonable off-shell extension. Thus, the off-shell behavior is decoupled in a clear way from the on-shell behavior at positive energies and from the behavior of the corresponding t matrix at negative energies as, required by the structure of the Faddeev equations.

For our subsequent calculations we use the following energy-dependent separable forces: For $^1S_0$ interaction $\lambda_S(E)g_S(p)g_S(p')$ and for $^3S_1$-$^3D_1$ interaction also Yamaguchi-type form, where

$$g(p) = g_1(p) + \frac{1}{\sqrt{8}}S_{ij}(\hat{\mathbf{p}})g_2(p),$$

g_S and g_1 are of the same form $g_0(p) = (p^2+\beta_0^2)^{-1}$ and $g_2(p) = -t(P_D)p^2(p^2+\beta_2^2)^{-2}$.

$$S_{ij}(\hat{\mathbf{p}}) = 3\boldsymbol{\sigma}_j\cdot\hat{\mathbf{p}}\boldsymbol{\sigma}_i\cdot\hat{\mathbf{p}} - \boldsymbol{\sigma}_i\boldsymbol{\sigma}_j \ ,$$

where $\mathbf{p} = \mathbf{p}_{ij}$ and $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. For given inverse force ranges, given D-state probability P_D, and given $^3S_1$ phase shifts [by the effective range quantity $p\cot\delta_{\exp}^t(p)$], the coupling strength $t(P_D)$ and the potential strength $\lambda(E=p^2/m, P_D)$

are given by

$$t(P_D)=\left[\frac{P_D}{1-P_D}\frac{8\beta_2(\alpha+\beta_2)^5}{\alpha\beta_1(\alpha+\beta_1)^3(5\alpha+\beta_2)}\right]^{1/2} \tag{1}$$

and

$$\lambda(p^2/m,P_D)=\frac{1}{2\pi^2}\left[\frac{\beta_1^2-p^2}{2\beta_1(\beta_1^2+p^2)}+\frac{t^2(P_D)(\beta_2^6+5\beta_2^4p^2+15\beta_2^2p^4-5p^6)}{16\beta_2(\beta_2^2+p^2)^4}+\left[\frac{1}{(\beta_1^2+p^2)}+\frac{t^2(P_D)p^4}{(\beta_2^2+p^2)^4}\right]p\cot\delta^t_{\rm exp}(p)\right]^{-1}, \tag{2}$$

where $\alpha=0.2316$ corresponds to the deuteron bound state pole. The singlet potential strength is given by

$$\lambda(p^2/m)=\frac{\frac{1}{2\pi^2}(p^2+\beta_s^2)^2}{[p\cot\delta^s_{\rm exp}(p)-(p^2-\beta_s^2)/(2\beta_s)]}\ . \tag{3}$$

The construction of the energy-dependent model[4] from energy-independent conventional potentials either nonlocal (separable potentials) or local (square well potentials) has yielded an energy-dependent $\lambda(E)$, which turned out to be symmetric with respect to $p^2/m=0$. This symmetry defines then $\lambda(E)$ at negative energies. The nature of basic physics underlying this finding will be discussed elsewhere. Obviously, $\lambda(E)$ is determined in such a way as to compensate the deficiencies of $\bar{V}(p,p')$ at short distances. In any case, this point is of no importance to the present investigation, as demonstrated already in Refs. 4–6, and because we are comparing results with one another for the same force model changing only P_D while keeping all other on-shell and off-shell quantities fixed. Therefore, the model is adequate to investigate small differences in binding energy as a function of small differences in P_D.

By the above choice of parameters, the deuteron bound-state pole, the singlet $\delta^s_{\rm exp}$ and triplet $\delta^t_{\rm exp}$ phase shifts are reproduced exactly up to 1000 MeV (Ref. 7); the D-wave phase shifts are chosen in the Yamaguchi tensor approximation,[8] which is reasonable especially at low and medium energies and has the advantage that it is independent of all the above parameters and thus remains unaltered for any choice of P_D. This potential enables us for the first time to test the sensibility of the triton binding energy with respect to P_D while keeping all other two-nucleon off-shell and on-shell quantities unchanged.

With the aforementioned potential model, we are solving a Faddeev equation of the following type:

$$A_i(p)=\tau_i(K^2,p)\sum_j\chi_{ij}\int_0^\infty\left[\int_{-1}^{+1}F_{ij}(K^2,p,p',x)C_{ij}(p,p',x)\,dx\right]p'^2A_j(p')\,dp'\ , \tag{4}$$

where $A_i(p)$ denote the spectator functions, τ_i the reduced t matrices for separable potentials, χ_{ij} describe the spin coupling coefficients, $F_{ij}(K^2,p,p',x)$ are the Faddeev kernels, and $C_{ij}(p,p',x)$ describe the angular coupling between different partial waves. K^2/m is the negative triton binding energy, m being the nucleon mass.

The solution of this equation using experimental phase shifts and low energy parameters $r^t_{\rm np}=1.77$ fm, $a^t_{\rm np}=5.42$ fm, $r^s_{\rm np}=2.74$ fm, $a^s_{\rm np}=-23.71$ fm, and $P_D=0.042$ using the usual ranges of the Yamaguchi tensor force,[8,9] yields for the triton binding energy -7.81 MeV. Using triplet ranges employed, for example, in Ref. 9 we obtain values between -7.81 and -7.20 MeV for E_B.

This shows that the results obtained with our force model differ strongly from the results obtained with conventional separable forces, but are in close agreement with the predictions by realistic local potentials, and therefore do not lead to the customary overbinding of the triton obtained for separable potentials.

In Fig. 1 the triton binding energy for fixed on-shell and off-shell behavior of the two-nucleon force is displayed as a function of D-state probability P_D in the deuteron. With the increase of P_D within the experimental bounds of uncertainty from 0.04 to 0.07, the triton binding energy decreases considerably by 0.60 MeV. We have evaluated this difference for other choices of the potential parameter and find ΔE to be always in the range 0.55–0.60 MeV. We have checked the consistency of our model by calculating the triton binding energy for $P_D=0$ for comparison with independent triton calculation using s-wave interactions only with the same force ranges, and by reproducing the experimental phase shifts and obtain the same binding energy. It can be seen that our results obtained with the energy-dependent separable model reproduce almost exactly the results obtained by Harms *et al.*[2] for the Reid potential and differs strongly from the results by Brady *et al.*,[1] who have used the same Yamaguchi form factors as employed in the present investigation. The agreement of our results, obtained in an off-shell and on-shell unambiguous model, with those by Harms *et al.* indicates that the relative difference $\Delta E/E_B(P_D=4\%)=0.07$–$0.077$ and also the absolute different $\Delta E=0.55$–0.60 MeV are model independent to

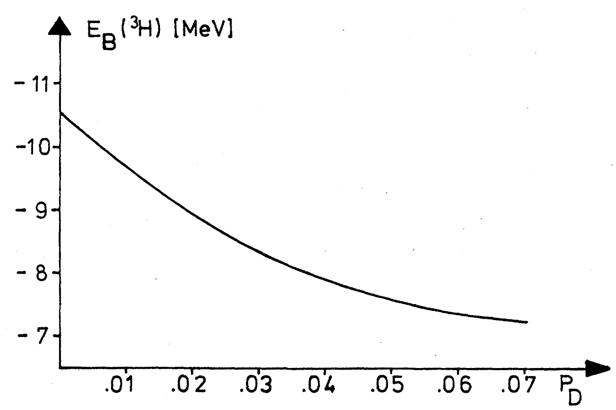

FIG. 1. Triton binding energy $E_B(^3H)$ as a function of D-state probability P_D for fixed on- and off-shell behavior of the two-nucleon force.

within 8%, once all quantities are described reasonably, i.e., $0.50 < \Delta E < 0.64$. It can be argued that ΔE will change if the experimental triton binding energy E_B is predicted exactly. The change of E_B can be due to the $^1S_0$ force (which does not affect P_D) and due to the $^3S_1$-$^3D_1$ force. We have investigated the first possibility by changing artificially the strength or range of the $^1S_0$ force such as to obtain 8.48 MeV and some binding energies in the vicinity of this value. For different $^1S_0$ forces we have calculated ΔE and obtain always 0.60 MeV; the corresponding E_B curves are merely shifted with respect to each other. As to the second possibility, we have to note that the only quantity by which P_D enters the triton bound state calculation is the coupling strength t as given by Eq. (1). The quantity t, in turn, enters the three-body calculation linearly via the Faddeev kernels and also as t^2 in one Faddeev kernel and via the τ-matrix [via Eq. (2)] pertinent to the deuteron interaction. The analysis of the Faddeev kernels shows that the contribution of terms (coupling kernels) in which t is linear is dominant.

From Eq. (1) it can be shown that the relative change in t:

$$\frac{1}{t(P_D)}\frac{dt}{dP_D}=\frac{1}{2}\frac{1}{P_D-P_D^2}\ , \tag{5}$$

is independent of the details of the separable force $(\beta_1, \beta_2, \alpha)$. Equation (5) explains why the relative change in the binding energy is independent of the shape of the form factors of the separable potential. It also explains the diminishing slope of the curve in Fig. 1 with larger P_D. From this, it follows that also the absolute difference ΔE is model independent (within the separable potential models) to a good approximation once E_B is not too far from the experimental value. In our case the difference between the binding energy of 7.81 MeV and $E_{\rm exp}=8.48$ MeV leads to $\Delta E \approx 0.59$–0.64 MeV. In addition, it is found that for separable potentials of Yamaguchi type for different exponents, the factor in Eq. (1) and containing the potential parameters is independent of the parameter variation to a good approximation, once the deuteron properties are described satisfactorily. The agreement between our results and those by Harms *et al.*, who used local potentials, indicates that very probably our finding is not only valid for separable potentials, but also for local potentials, because the coupling mechanism between the $^3S_1$ and $^3D_1$ states is essentially the same.

Therefore, from our analysis and from the comparison with the results for local potentials, it can be inferred that our result is model independent to a good approximation.

Since the absolute values for E_B are also very close to each other for the local and energy-dependent separable model, these calculations indicate that the variation of the off-shell and medium energy phase shift behavior present in the calculations by Harms *et al.* do not affect ΔE significantly. Our calculations show also that the sensitivity of E_B with respect to P_D is decreasing with increasing P_D, in agreement with Eq. (5). The sensitivity of E_B with respect to P_D in the vicinity of $P_D = 7\%$ is smaller by a factor of 5 compared with the sensitivity in the vicinity of $P_D = 4\%$.

Since there are some indications that P_D should be closer to 4% rather than 7%, our results indicate that the binding energy can serve as a sensitive indicator for this quantity.

In conclusion, we have presented a direct off-shell unambiguous relation between the D-state probability and triton binding energy, and have found in a realistic triton calculation that the experimental uncertainty in P_D of 0.03 accounts for 0.55–0.60 MeV in the triton energy, in complete agreement with the results by Harms *et al.*[2] for the Reid potential.

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