Microscopic description of the (p,π^-) continuum at intermediate proton energy

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The continuum spectra of the (p,π^-) reaction on nuclei in the medium and heavy mass region at $E_p \sim 200$ MeV are studied in terms of the standard pion-absorption-production model. The threshold energy and the linear rise of the pion cross section with the excitation energy are reproduced. By including the medium effect on the rescattering pion and rho meson fields we can also account for the absolute value of the cross section.

Using the Indiana University Cyclotron Facility (IUCF), Vigdor *et al.* have performed a series of (p,π^-) experiments at $E_p \sim 200$ MeV on various target nuclei.[1] The striking feature of the pion spectra is the appearance of a few sharp peaks, the origin of which has been interpreted within the single j shell model as high spin stretched states by Brown, Scholten, and Toki.[2] Another interesting feature is the continuum, which seems to increase linearly with the excitation energy from some threshold energy. In this Brief Report we will present a microscopic study of the (p,π^-) continuum as a step towards a full understanding of the proton induced pion production.

The standard model we shall use is based on the s-wave and p-wave pion-absorption-production mechanisms, which have been used extensively in the description of pionic disintegration of deuteron,[3] pion optical potential,[4] pion scattering,[5] and free space pion production.[6] We show in Fig. 1 the Feynman diagrams of the pion production reaction, where the incoming proton is denoted by a solid line with momentum p and the outgoing pion by a dotted line with k. The blob corresponds to the s- and p-wave vertices, the details of which are given below. The difference between the two processes [Figs. 1(a) and 1(b)] is mainly the energy carried by the interaction line (wavy line). In the process (a) the energy transfer is essentially zero, while in the process (b) it is about the incident proton energy. The intermediate boson line in the process (b) is far off shell; $q^0 \sim 200$ MeV and $|\mathbf{q}| \sim 600$ MeV, and hence the interaction has to be very short ranged. In addition, this high energy object going through the nuclear medium is subject to a strong absorption due to the large available phase space before it produces a pion. Furthermore, a delta or nucleon, especially a delta isobar, after absorbing the energy and momentum carried by the short range interaction has two decay modes; one is pion production and the other inelastic collision with another nucleon ($\Delta + N \rightarrow N + N$). The decay width of a delta by the inelastic process is much larger than that of the pionic decay at this energy known from the analysis of pion optical potential.[7] Hence, the medium polarization effect on the interaction line in the process (b) acts to cut down the pion production process as opposed to the enhancement caused by the medium polarization effect in the process (a), which will be discussed later. Although we can calculate these absorption effects in process (b) in principle, in this Brief Report we drop the contribution entirely and concentrate on showing the results with only process (a).

Using the standard Feynman technique,[8] we can easily work out the T matrix for the s- and p-wave mechanisms and give here only the final results for the cross sections:

$$\sigma_s = \frac{1}{v}\frac{d^3k}{(2\pi)^3}\frac{1}{2k^0}\left(4\pi\frac{f}{\mu}\right)^2 4\int\frac{d^4q}{(2\pi)^4}u(\mathbf{p}-\mathbf{q}-\mathbf{k})2\pi\delta[\epsilon(\mathbf{p})-\epsilon(\mathbf{p}-\mathbf{q}-\mathbf{k})-q^0-k^0] \times\left[\left(\frac{2\lambda_1}{\mu}\right)^2+\left(\frac{\lambda_2}{\mu^2}\right)^2(k^0-q^0)^2\right]|D(q)|^2q^2U(q,q^0) \tag{1}$$

for the s-wave process and

$$\sigma_p = \frac{1}{v}\frac{d^3k}{(2\pi)^3}\frac{k^2}{2k^0}\left(\frac{f^*}{f}\right)^4\left(\frac{f}{\mu}\right)^2\left(\frac{2}{9}\right)^2\int\frac{d^4q}{(2\pi)^4}u(\mathbf{p}-\mathbf{q}-\mathbf{k})2\pi\delta[\epsilon(\mathbf{p})-\epsilon(\mathbf{p}-\mathbf{q}-\mathbf{k})-q^0-k^0]|G_\Delta(p_\Delta)|^2 \times[(1+3\cos^2\theta)|V^L(q)|^2+(5-3\cos^2\theta)|V^T(q)|^2]U(q,q^0) \tag{2}$$

for the p-wave process with the Δ isobar intermediate state. v is the relative velocity between the incoming proton and the target nucleus, k is the outgoing pion four momentum, and μ the pion mass. q is the four momentum carried by the interaction. The $\pi N\Delta$ coupling constant f^* is twice the πNN coupling constant f with $f=1$. The isoscalar and isovector s-wave coupling constants are $\lambda_1 = 0.0065$ and $\lambda_2 = 0.046$. $\epsilon(\mathbf{p})$ is the kinetic energy of nucleon $\epsilon(\mathbf{p}) = p^2/2m$. $D(q)$ and $G_\Delta(p_\Delta)$ are the pion and the Δ isobar propagators. $u(\mathbf{p}) = \theta(|\mathbf{p}|-p_F)$ with p_F being the Fermi momentum and $v(\mathbf{p}) = \theta(p_F-|\mathbf{p}|) = 1-u(\mathbf{p})$. The angle θ is the angle between $\mathbf{k}$ and $\mathbf{q}$. The longitudinal and the

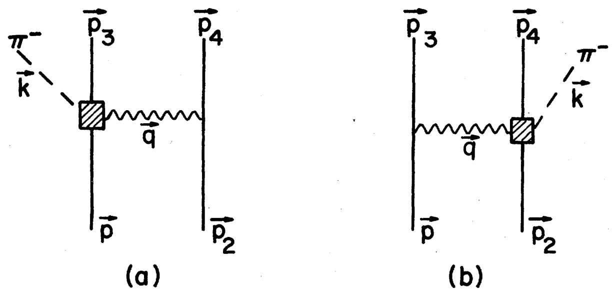

FIG. 1. Feynman diagrams of the (p,π^-) reaction. The incoming proton is represented by a solid line with momentum p, while the outgoing pion is denoted by the dotted line with momentum k. The bound nucleon has momentum p_2 and the outgoing ones p_3 and p_4. The intermediate field carries momentum q and is denoted by the wavy line.

transverse interactions are

$$V^L(q)=\frac{f^2}{\mu^2}\Gamma_\pi(q)\left[\frac{q^2}{q^{02}-q^2-\mu^2}+g'\right] , \quad (3a)$$

$$V^T(q)=\frac{f^2}{\mu^2}\left[\Gamma_\rho(q)C_\rho\frac{q^2}{q^{02}-q^2-\mu_\rho^2}+\Gamma_\pi(q)g'\right] . \quad (3b)$$

The form factor

$$\Gamma_\pi(q)=\left[\frac{\Lambda_\pi^2-\mu^2}{\Lambda_\pi^2-q^{02}+q^2}\right]^2$$

is calculated with $\Lambda_\pi=1.2$ GeV and the same form is used for $\Gamma_\rho(q)$ with $\Lambda_\rho=2$ GeV. The short range correlation parameter g' is taken $g'=0.6$. Finally, $U(q)$ is the response function

$$U(q,q^0)=\int\frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}u(\mathbf{p}')v(\mathbf{p})(2\pi)^4\delta^4(p'-p-q) . \quad (4)$$

With the use of the infinite matter response function, as given in Eq. (4), the cross section is normalized to unity per unit volume. Since both the incoming proton and the outgoing pion are strongly absorbed in the nuclear medium the reaction takes place predominantly at the nuclear surface. Therefore only a few nucleons participate in the reaction; and for this reason we introduce the concept of effective number of neutrons ($N_{\rm eff}^\nu$), which can be calculated using the Glauber approach as has been done in Ref. 9. The response for the nucleus can now thus be written as

$$U(q,q^0)\rightarrow U(q,q^0)\Big/\left[\frac{4\pi}{3}p_F^3\right](2\pi)^3\cdot N_{\rm eff}^\nu . \quad (5)$$

The analytic expression for $U(q,q^0)$ has been provided in Ref. 9.

We should consider further the effects of medium corrections on both the incoming proton and the outgoing pion and on the propagator for the intermediate interaction. In the interaction region, the incoming proton feels the nuclear as well as the Coulomb potentials. Hence, we shall have to modify the energy

$$E_p\rightarrow E_p+|V_n|-|V_C|\sim E_p+35\text{ MeV} \quad (6a)$$

and for the outgoing pion

$$k^0\rightarrow k^0+|V_N'|+|V_C'|\sim k^0+20\text{ MeV} \quad (6b)$$

and accordingly the corresponding momenta; i.e., the nucleon and the pion are on shell.

The other modification is related to the renormalization of the longitudinal (pion) field and the transverse field. Such a medium polarization effect has been extensively discussed in the literature[10] and we only state here the necessary modification;

$$V_L(q)\rightarrow V_L(q)/[1-V_L(q)\Pi(q)] , \quad (7a)$$

$$V_T(q)\rightarrow V_T(q)/[1-V_T(q)\Pi(q)] . \quad (7b)$$

Here $\Pi(q)$ the Lindhard function due to nuclear particle-hole and isobar-hole excitations. The explicit expressions are provided in Ref. 8.

As a first application, we calculated the continuum for ^{42}Ca. The effective number of nucleons, was calculated using the method given in Ref. 9, yielding $N_{\rm eff}=9.8$. Since in ^{42}Ca there are almost as many protons as neutrons, the effective number of neutrons is calculated as $N_{\rm eff}^\nu=N_{\rm eff}/2=4.9$. The Fermi energy has been taken equal to 35 MeV, corresponding to a separation energy of 10 MeV. The result of the calculation is compared with experiment in Fig. 2. Not only the structure, but also the absolute magnitude of the cross section is reproduced. The slow increase of the continuum at low excitation energies, $E_x<5$ MeV, can be understood from the fact that in this energy regime the response function $U(q,q_0)$ is proportional to q^0. Since the cross section is obtained by integrating $U(q,q^0)$ over q^0 from 0 to E_x, this gives rise to a cross section that depends quadratically on excitation energy. At higher excitation energies (5 MeV $<E_x<$ 20 MeV) the response is essentially independent of q^0, which gives rise to a linearly rising cross section with excitation energy. At even higher excitation energies the available phase space starts to play the dominant role. The dominant contribution to the background, as given in Fig. 2, arises from the longitudinal component of the pion field. The transverse and the s-wave components contribute only about 5% each, and increase in importance with excitation energy. The effect of medium polarization on the longitudinal component is, however, extremely important. It gives rise to an enhancement factor of

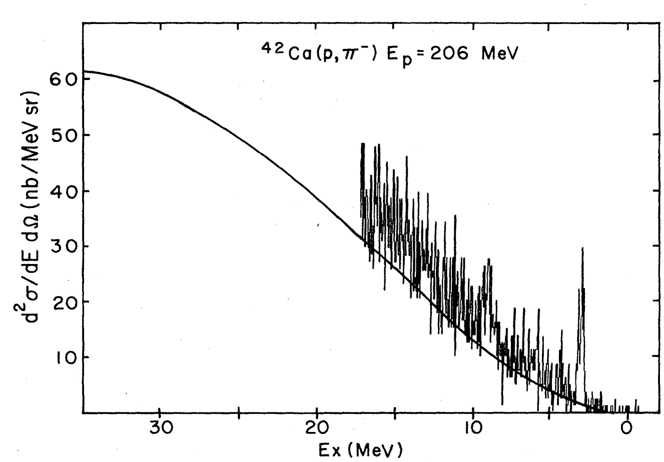

FIG. 2. The continuum spectra of ^{42}Ca(p,π^-) at the incoming proton energy $E_p=206$ MeV as a function of the excitation energy. The experimental spectrum (Ref. 1) is measured at an angle $\theta_L=30°$.

the order of 5. At higher excitation energies the medium polarization effect is slightly diminished.

Experimentally it has been found[1] that the cross section of the continuum for ^{48}Ca is about a factor of 2 larger than for ^{42}Ca. The effective number of nucleons ($N_{\rm eff}$) for ^{48}Ca using the method given in Ref. 9 is, however, only a fraction larger than for ^{42}Ca (10 vs 9.8), which cannot explain the difference in cross section. Hartree-Fock calculations[11] indicate, however, that in ^{48}Ca the eight additional neutrons are predominantly concentrated at the surface. The effective number of neutrons ($N^{\nu}_{\rm eff}$), which is the important quantity for the present calculations, is therefore not $N^{\nu}_{\rm eff} = N_{\rm eff}/2$ like for ^{40}Ca but rather $N^{\nu}_{\rm eff} = 8 + (N_{\rm eff} - 8)/2 = 9$. This increase in $N^{\nu}_{\rm eff}$ can explain the strong increase in the continuum cross section. In a forthcoming, more extensive, paper[12] this point will be investigated in more detail.

Another nucleus for which the continuum cross section has been measured[1] is ^{90}Zr. The shape of the continuum closely resembles those of ^{42}Ca and ^{48}Ca with a cross section that is a factor 1.5 larger than in ^{42}Ca. The effective number of nucleons calculated for ^{90}Zr is $N_{\rm eff} = 12.9$, which would imply a cross section that is a factor 1.3 larger than in ^{42}Ca if we assume equal numbers of neutrons and protons on the surface of ^{90}Zr. The Hartree-Fock calculations,[11] however, also indicate for this nucleus an excess of neutrons at the surface, which would give a larger increase in the cross section than the calculated factor 1.3.

We show in Fig. 3 the energy dependence of the (p,π^-) cross section. The effective number of neutrons $N_{\rm eff}$ is set equal to one. Since $N_{\rm eff}$ is a function of both energy and the target nucleus, we have to multiply an appropriate value to the curves in order to compare with experiment. Up to energies of about 400 MeV the structure of the spectrum essentially remains the same, but for higher proton energies the cross section at low excitation energies ($E_x < 20$ MeV) decreases. This occurs when the momentum transfer is larger than can be absorbed by the nucleons without introducing an appreciable excitation energy. At zero excitation energy the maximum momentum that can be absorbed by the three bound nucleons is $3k_F$. Since the momentum transfer at zero excitation energy equals $[2M_{\rm N}T_{\rm p} - (T_{\rm p} - M_\pi)2M_\pi]^{1/2}$,

$$T^c_{\rm p}\left(9\epsilon_F - \frac{M_\pi^2}{M_{\rm N}}\right)\frac{M_{\rm N}}{M_{\rm N} - M_\pi} \simeq 350 \text{ MeV}$$

for $\epsilon_F = 35$ MeV. At energies higher than this critical energy $T^c_{\rm p}$ an increasing part of the low excitation energy spectrum becomes kinematically forbidden in a Fermi gas model. This implies that a measurement of the (p,π^-) continuum at medium high proton energies could provide a clear probe of the structure of the nuclear surface.

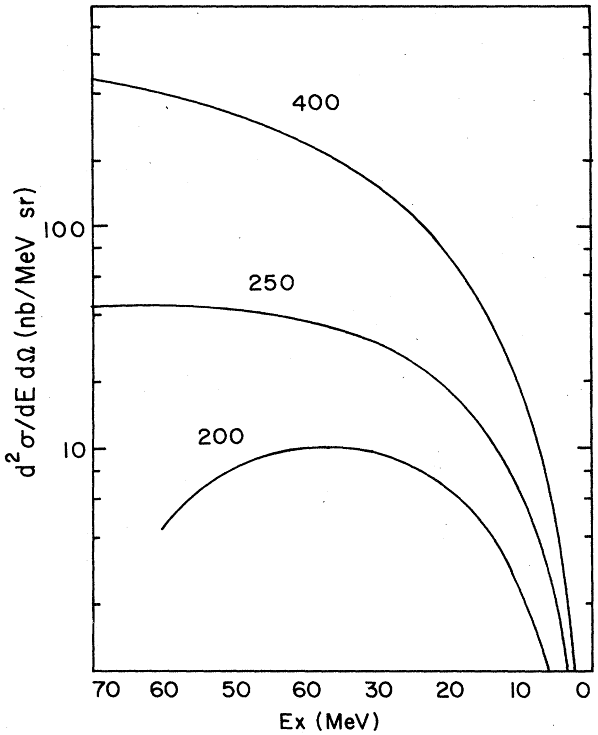

FIG. 3. The continuum spectra of the (p,π^-) reaction at different incident energies.

In conclusion, we have shown that the standard pion-absorption-production model with the appropriate medium effect can account for the behavior of the continuum cross sections in the (p,π^-) reaction on the medium mass nuclei. We have also calculated the continuum spectra at higher energies than the presently available proton energy of $E_p \sim 200$ MeV, and discussed the change of the threshold energy. Since the (p,π^-) reaction is sensitive to the surface behavior of nuclei through the distortions of incoming proton and outgoing pion, medium polarization effects of the pionic and the rhomesic fields, the effective number of neutrons, and the Pauli blocking, this reaction is an additional tool to investigate the surface properties of nuclei.

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