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Backward cross section in the generalized exciton model

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The double differential cross sections of the (p,p′) process are calculated for ^{54}Fe, ^{120}Sn, ^{197}Au, and ^{209}Bi targets at $E_p = 62$ MeV based on the generalized exciton model. Finite nuclear size effects are treated for the scattering kernel. Our results indicate that even at backward angles the angular distribution of the preequilibrium reaction can be well reproduced within the framework of the generalized exciton model.

The generalized exciton model[1] has been extensively applied to give the preequilibrium angular distributions with the help of a useful mathematical reformulation[2,3] which makes handling of it easier. Although the model was able to reproduce the global characteristics of preequilibrium angular distributions at the incident energies of several tens of MeV, it greatly underestimates the angular distribution at backward angles.[4,5] Since the generalized exciton model is widely used to analyze the extensive experimental data of preequilibrium reaction, it is very important to investigate whether the difficulty at the backward angles can be removed by some modifications of the model or not. Former approaches based on it used the Fermi gas model for target nucleons in evaluating the single nucleon-nucleon scattering kernel $G(\Omega,\Omega')$ (SSKG) between the fast nucleon and a target one that describes the angular distribution of the fast nucleon after a collision.[4–6] It is this scattering kernel which mainly dominates the angular distribution of the emitted nucleon in the early stages of the reaction. We expect that the treatment of finite nuclear size effects in the evaluation of the SSKG plays a crucial role in explaining the angular distribution at backward angles.

In the present work, we report briefly on a study of preequilibrium angular distribution using the scattering kernel along this line in the framework of the generalized exciton model. We use the SSKG in describing the scattering process between the fast nucleon N and a bound target nucleon N_t moving in the harmonic oscillator potential of the oscillator parameter ω. To simplify the model we assume that (i) two nucleons N and N_t interact with each other via the interaction $V_0\delta(\mathbf{r}-\mathbf{r}_t)$ and (ii) the scattering can be treated in the plane-wave Born approximation. The strength V_0 will turn out to be canceled in the expression of SSKG in Eq. (5). The effect of the refraction seems to be small at energies of several tens of MeV.[5] Under such assumptions the differential cross section for the inelastic scattering of the fast nucleon N by target nucleon N_t moving in the lowest bound state is obtained analytically.[7] We need to extend such a calculation to all target nucleons which lie below the Fermi level. Performing the summation over all target nucleons moving in the $\tilde{n}$-fold degenerate states and over all excited states of n-fold degeneracy, we can obtain the differential cross section in the analytic form

$$\frac{d\sigma(\tilde{n}\to n)}{d\Omega'} = \frac{\sigma_0}{\pi}\frac{p'}{p}\sum_{i=-\tilde{n}}^{\tilde{n}} C_{n+i}^{\tilde{n}} f_{n+i}(q) \ , \tag{1}$$

where σ_0 is the free nucleon-nucleon scattering cross section, p and p' are the momentum of the fast nucleon before and after the collision, and $\hbar q$ is the momentum transfer. The function $f_n(q)$ is defined by

$$f_n(q) = (q^2/2\alpha^2)^n/n! \ , \tag{2}$$

with $\alpha=\sqrt{m\omega/\hbar}$, and the coefficients $C_{n+i}^{\tilde{n}}$ $(i=-\tilde{n}, -\tilde{n}+1, \ldots, \tilde{n})$ are simple polynomials in n of degree $\tilde{n}$. If we construct the differential cross section that is a sum of Eq. (1) over all final subshells n and an average over all occupied ones $\tilde{n}$, we can get the energy averaged SSKG[6] after doing its normalization over the angle. Since making the average over all final nucleon energies discards the important angle-energy correlation,[4] we should construct the differential cross section averaged only within the small region Γ around its expectation value E' when we treat scattering such that the fast nucleon N has energy E and E' before and after the collision, respectively. Such a smearing of the final nucleon energy after a collision in the m-exciton state is actually expected due to the finite time interval in which collision occurs equivalent to the life $\hbar/\Gamma_m$ of the m-exciton state (uncertainty principle). We can make such a cross section by introducing the following weight factor into the sum over $\tilde{n}$ and n such as

$$w(\tilde{n},n;\Gamma) = \begin{cases} 1 \text{ for } \Delta E-\Gamma/2 \le \Delta E(\tilde{n},n) \le \Delta E+\Gamma/2 \\ 0 \text{ for otherwise} \end{cases} : \text{ clear cut}$$

$$= \frac{\Gamma}{2\pi}\frac{1}{[\Delta E(\tilde{n},n)-\Delta E]^2+(\Gamma/2)^2} : \text{ Lorenzian}$$

$$= \frac{2}{\sqrt{\pi}\Gamma}\exp\{-[\Delta E(\tilde{n},n)-\Delta E]^2/(\Gamma/2)^2\} : \text{ Gaussian} \ , \tag{3}$$

where $\Delta E = E - E'$ is the energy loss of the fast nucleon N, and $\Delta E(\tilde{n},n)$ is the corresponding excitation energy of the target nucleon N_t. It is then written as

$$\frac{d\sigma}{d\Omega'}=\sum_{\tilde{n}=0}^{n_F}\sum_{n>n_F}^{n_{\max}(\tilde{n})}\frac{d\sigma(\tilde{n}\to n)}{d\Omega'}\times w(\tilde{n},n;\Gamma)/g \ , \tag{4}$$

where g is the number of all occupied single particle states; the sum is taken over all occupied subshells $\tilde{n}$, and over ones n above the Fermi level n_F, and below $n_{\max}(\tilde{n})$. To determine the maximum subshell quantum number $n_{\max}(\tilde{n})$ for each $\tilde{n}$, we demand that the energy E' is to be positive. Finally we obtain the SSKG by

$$G(\Omega,\Omega')=\frac{d\sigma}{d\Omega'}\Big/\int\frac{d\sigma}{d\Omega'}d\Omega' \ , \tag{5}$$

which now implicitly depends on E' as well as on E. The present SSKG has a tractable form utilizing the analytic expressions of Eq. (1), which is suitable for easier application of the generalized exciton model.

Double differential cross section for the emitted nucleon in the early stages of the reaction can be calculated if we put the eigenvalues μ_l $(l=0,1,2,\ldots)$ of the SSKG of Eq. (5) into the simple closed-form expression in Ref. 3. Considering that our μ_l are dependent on energies of the fast nucleon before and after the collision in the m-exciton state, we make the following replacement:

$$\mu_l^{(m+2-m_0)/2}\to\mu_l(E_{\rm in},E_1)\times\mu_l(E_1,E_2)\times\cdots\times\mu_l(E_{m-m_0/2},E_{\rm out}) \tag{6}$$

in the expansion coefficients $\zeta_l(m)$. In Eq. (6), $E_{\rm in}$ and $E_{\rm out}$ are the incident and the outgoing energies, respectively, of the fast nucleon, and $E_1,E_2,\ldots$ are the mean intermediate energies of the fast nucleon after a single collision, after a double collision, etc. This substitution corresponds to approximately replacing the convolution-type integral of Eq. (14) in Ref. 4 with mean intermediate energies E_i $[i=1\sim(m-m_0)/2]$ in the small energy interval Γ for every intermediate energy integral. (Note that our μ_l have no dimension in contrast with their eigenvalues of MeV^{-1}.) In a simple case that the emitted nucleon comes from the m-exciton state, after having lost its initial energy equally in every step of the total $(m-m_0+2)/2$ times collisions, the energies E_i take the form

$$E_i=E_{\rm in}-i\times(E_{\rm in}-E_{\rm out})\times\frac{2}{m-m_0+2} \ . \tag{7}$$

We have analyzed the angular distributions for several (p,p′) reactions with incident energies of several tens of MeV. In the calculation, we always put the initial exciton number $m_0=3$, and other parameters peculiar to the exciton model are the same as in Refs. 8 and 9. We used the SSKG based on Eqs. (4) and (5) with $\hbar\omega=41A^{-1/3}$, the weight factor of the Gaussian type, or the clear cut one and fixed the smearing width Γ of Eq. (3) to be 10 MeV. Our setting of $\Gamma=10$ MeV is consistent with the value of the width Γ_m of every m-exciton state in the early stages in the usual exciton model for the present incident energy of the proton. Calculations with various choices of Γ show that the SSKG is insensitive to its value for Γ less than 10 MeV. In Fig. 1 we compare the calculated angular distributions using Eq. (7) with the experimental data[10] for the reaction ^{120}Sn(p,p′) with $E_p=62$ MeV for three different outgoing energies. In

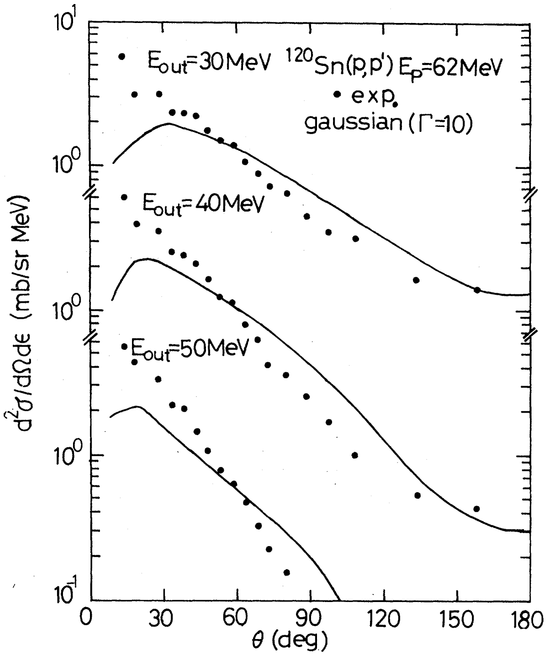

FIG. 1. Comparison of the calculated and experimental angular distributions for the reaction ^{120}Sn(p,p′) at 62 MeV for outgoing energies 30, 40, and 50 MeV. The solid curves show the calculated results based on Eq. (7). The experimental data from Ref. 10 are denoted by the solid circles. Gaussian type weight factor and the value of $\Gamma=10$ MeV are used for the SSKG in Eqs. (4) and (5).

the present calculation we used the weight factor of Gaussian type. Another choice of the clear cut one gives the similar result for this system. As is clearly seen from the figure, the agreement is very good except at small angles. Especially, the agreement in the backward angles is remarkably improved to the extent of reproducing the absolute value of the cross section. The improvement of the backward cross section originates from that of the SSKG due to the use of the momentum wave function including finite nuclear size effects over the former one based on the Fermi gas model.[4] In fact, the calculated SSKG turned out to not give a little backward contribution, where the transitions $\tilde{n}\to n$ of $\tilde{n}=n_F$ give the largest contribution in Eqs. (4) and (5), increasing with the energy loss of the fast nucleon in a collision. The discrepancy at small angles, which is usually the case for every model, is discussed in Refs. 4 and 11. To check the sensitivity of the result to Eq. (7), we have also made the calculation using the assumption that the fast nucleon loses its energy randomly in every step of the total $(m-m_0+2)/2$ times successive collisions instead of Eq. (7). This turned out to bring forth only a slight change (within 20%) of Fig. 1. Our calculated results for other targets such as ^{54}Fe, ^{197}Au, and ^{209}Bi $[\hbar\omega=55A^{-1/3}$ is used for heavy targets[12,13] ^{197}Au and ^{209}Bi consistent with our result that target nucleons lying at the Fermi surface largely contribute to the whole transitions in Eq. (4)] show nearly the same fitting with the experimental data, although the overestimation of the calculated value becomes a little more than the case of ^{120}Sn target (within a factor of 2 or 3). The results shown so far for various targets suggest the importance of treating finite size effects in order to explain the

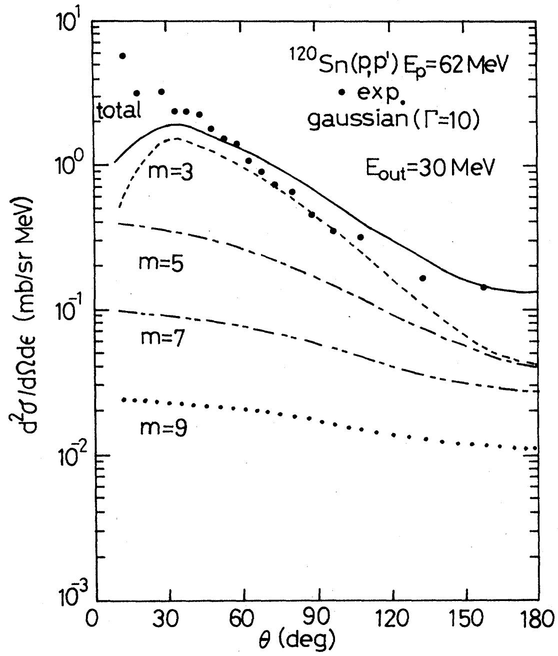

FIG. 2. Decomposition of the calculated cross section which is given in Fig. 1 into contributions from the various exciton states only for outgoing energy 30 MeV. The various broken curves labeled by m denote the cross sections coming from the m-exciton state. The solid curve is the total cross section corresponding to the curve of Fig. 1 for $E_{out} = 30$ MeV.

preequilibrium angular distribution at backward angles in the framework of the generalized exciton model. A little overestimation of the absolute values of the cross section would be reduced in the more detailed estimation of SSKG than our simple model. We show in Fig. 2 the decomposition of the cross section at $E_{out} = 30$ MeV shown in Fig. 1 into contributions from various exciton states. It is noticed that at every angle there is the large contribution from the $m = 3$ exciton state (one-step process) in contrast with the rapid falling off of its contribution at backward angles in the results of the Fermi gas model.[4,6] Figure 2 shows that at backward angles both the one- and two-step processes account for about 30% of the total cross section, respectively, and the remaining 40% comes from the processes more than two step ($m > 5$). The multistep direct reaction approach by Tamura *et al.*,[11] which made quantum-mechanical treatment of the finite nucleus, is known to reproduce the preequilibrium angular distributions very well. In their calculation the contribution of the two-step cross section accounts for about 70% (30% one-step contribution) of the total one at backward angles for ^{27}Al target, or about 50% (50% one-step contribution) for ^{209}Bi at this outgoing energy. Comparing the present approach based on the generalized exciton model with those, we see that the basic physical picture looks quite different. From this point of view, further investigation is necessary to get much better understanding of the preequilibrium particle emission process.

In conclusion, the use of the scattering kernel which includes finite nuclear size effects, as well as the influence of Fermi motion and the Pauli principle, greatly improves the backward cross section of preequilibrium nucleon emission in the framework of the generalized exciton model. In the calculation, one- and two-step processes dominate even at backward angles, but those of more than two steps do not give a little contribution. Further study to refine the present model and to apply it to the light composite particle emission is desired.

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