F-spin multiplets in collective nuclei

H. Harter, P. von Brentano, and A. Gelberg
Institut für Kernphysik, Universität zu Köln, 5 Köln 41, West Germany

R. F. Casten
Brookhaven National Laboratory, Upton, New York 11973 and Institut für Kernphysik, Universität zu Köln, 5 Köln 41, West Germany

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The excitation energies of low lying levels in F-spin multiplets are compared. An overall constancy of these energies is found, which indicates that the interacting boson model Hamiltonian has rather constant parameters in the multiplet.

In recent years it has been shown that the low lying levels in collective nuclei can be described by the interacting boson model (IBM-2) as a system of proton and neutron bosons. In formal analogy to the existence of an SU(2) proton-neutron isospin symmetry in fermion systems, an SU(2) proton-neutron F-spin symmetry has been introduced by Arima, Otsuka, Iachello, and Talmi[1] and has been further discussed in Refs. 2–7. Thus F spin plays the same role for proton-neutron boson systems as isospin does for proton-neutron fermion systems. From the widespread success of IBA-1 calculations, which clearly deal with states of one kind of boson only (i.e., with good F spin), one suspects that F spin is in fact a rather good quantum number, for low lying states. The idea of F spin has recently taken on new importance because of the results of a recent experiment by Bohle *et al.*[8] who observed a number of excited 1^+ states in rare earth nuclei. These have been interpreted[4] as states of so-called mixed symmetry in which the F-spin quantum number differs from the ground state F spin by one unit. It would therefore be highly interesting to have a reliable estimate of how good the F-spin quantum number is in nuclei, and in particular for the low lying states. A rather direct and empirical approach to this question is to compare the level schemes of nuclei with the same and with different F-spin values. Such comparisons will be the principal focus of this paper.

In the following we will discuss various F-spin properties of nuclei. Although our conclusions will be valid for the general IBM-2 Hamiltonian, for simplicity we will consider the following Hamiltonian which has been used by Dieperink[9] to describe excitation energies.

$$H=\epsilon_p n_{dp}+\epsilon_n n_{dn}+k[Q_p(\chi_p)+Q_n(\chi_n)]^2 + k'Q_p(\chi_p)\cdot Q_n(\chi_n)-\lambda F^2 , \quad (1)$$

where n_{dp}, n_{sp} and n_{dn}, n_{sn} are the number operators for the d and s proton and neutron bosons, respectively, and where the last term is a form of the so-called Majorana operator written in terms of the F spin.[6] The parameters in Eq. (1) are functions of the total boson number operators $N_p=n_{dp}+n_{sp}$ and $N_n=n_{dn}+n_{sn}$ for protons and neutrons, respectively. In the following, however, it will be more convenient to consider them as functions of the total boson number operator $N=N_p+N_n$ and the zero component of the F-spin operator $F_0=\frac{1}{2}(N_p-N_n)$, respectively. Thus we will have, for example, $\chi_p=\chi_p(N,F_0)$. The SU(2) F-spin generators $F_\pm$ and F_0 have been given by Otsuka *et al.*[3] as $F_+ =\sum_k d^\dagger_{pk}d_{nk}+s^\dagger_p s_n$ and

$$F_0=\tfrac{1}{2}\sum_k(d^\dagger_{pk}d_{pk}-d^\dagger_{nk}d_{nk})+s^\dagger_p s_p-s^\dagger_n s_n ,$$

where $d^\dagger_{pk}s^\dagger_p, d^\dagger_{nk}$ and $s^\dagger_n$ create a proton and a neutron boson with projection quantum number k, respectively, and where $F_-=F^\dagger_+$. Since both N_p and N_n have fixed values in a given nucleus, $F_0=\frac{1}{2}(N_p-N_n)$ trivially commutes with H.

We will first discuss the strongest F-spin symmetry, namely, the full F-spin invariance of the Hamiltonian. In this case we have $[H,F_\pm]=0$ and $[H,F_0]=0$. An immediate consequence of F-spin invariance is the existence of F-spin multiplets with constant excitation energies. An F-spin multiplet consists of a group of levels occurring in a sequence of nuclei that have a constant value of F and varying values of F_0. The nuclei in the F-spin multiplet have N_p proton bosons of a definite character (particle or hole) and N_n neutron bosons of a definite character (particle or hole), and they have a constant boson number $N=N_p+N_n$. Because the character of the boson changes from particle to hole in the middle of a major shell, an F-spin multiplet with F-spin F will often contain fewer than $(2F+1)$ nuclei. This is different from the case of isospin multiplets. The F-spin invariance of H implies further that H is an F scalar and that the parameters in (1) depend only on the total boson number N and not on F_0 or N_p or N_n. Thus we have $\epsilon_p=\epsilon_n=\epsilon(N)$, $\chi_p=\chi_n=\chi(N)$, and $k'=0$.

As F-spin invariance is a very strong condition, we want to consider also a weaker condition on H, that F spin is a rigorously good quantum number for all levels, viz., $[H,F^2]=0$. This implies again that the Hamiltonian contains only scalar parts, but the coefficients such as $\chi=\chi(N,F_0)=\chi(N_n,N_p)$ can now be different for each nucleus. The assumption of a pure F-scalar Hamiltonian is still a very strong one and it is at variance with the common use of a $Q_p\cdot Q_n$ force in IBM-2 calculations. Thus we will consider finally the even weaker condition of approximately good F spin for the low lying states with H containing also F-vector and F-tensor parts. This situation is realized in typical IBM-2 calculations, in which a large value of the Majorana term is used to separate the levels of different F spin in energy. In this case we still have F-spin multiplets, but the excitation energies of the states within the multiplet are functions of F_0—in analogy to the case of isospin multiplets.

We will now discuss empirical evidence for F-spin multiplets. We remark that, whereas an isospin multiplet always consists only of isobaric nuclei, an F-spin multiplet can occur either in a series of isobaric nuclei defined by $(A,Z), (A,Z+2), (A,Z+4), \ldots$ (type I), when neutrons and protons are either both particles at the beginning of major shells or both holes at the end of major shells, or in the series of nuclei $(A,Z), (A+4,Z+2), (A+8,Z+4), \ldots$ (type II) when one kind of boson is of particle and the other is of hole character. We give two examples for F-spin multiplets with $N=7$ and $N=6$ for case II in the upper part of Fig. 1, which shows nuclei stretching from ^{122}Te to ^{142}Sm. We have shown all nuclei of these multiplets, with the exception of the magic nuclei with $Z=50$ or $N=82$, which are outside the IBM. In Fig. 2, we give an example of case I, by showing an incomplete F multiplet with $N=11$ in the Os-Pt region. In these figures we show all known positive parity levels up to an excitation energy around 2 MeV for the Xe region and around 1 MeV for the Os region and we show the ground and gamma bands up to the energy of the 8_1^+ level. We note that the higher lying levels of these nuclei are not known equally well. This is particularly true for the nuclei further from stability.

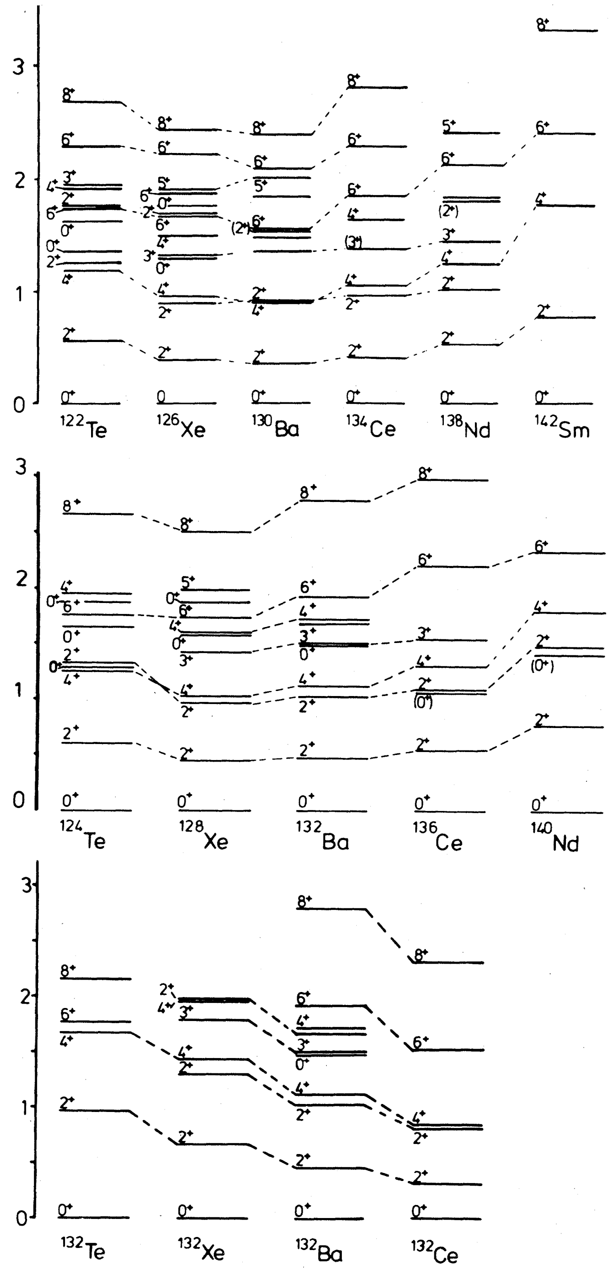

FIG. 1. The figure shows F-spin multiplets for ^{124}Te-^{140}Nd (type II) with $N=6$ (middle part) and for ^{122}Te-^{142}Sm with $N=7$ (upper part). We have shown all nuclei in the multiplet with the exception of the magic nuclei with $Z=50$ or $N=82$. We also show the isobaric nuclei ^{132}Te-^{132}Ce for comparison (lower part). Note that the boson number varies from $N=2$ to $N=8$ for the ^{132}Te-^{132}Ce nuclei, whereas it is constant in the two upper figures. We give all known positive parity levels up $E_x \simeq 2$ MeV and the ground and gamma bands up to the energy of the 8_1^+ level. The data are from Refs. 10–19. The units are in MeV.

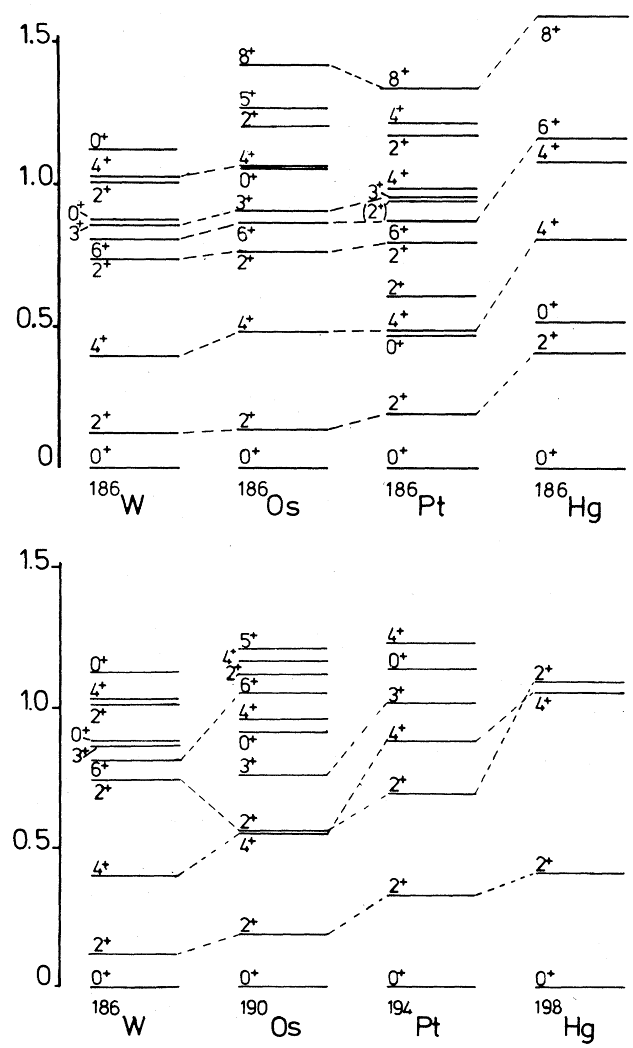

FIG. 2. The figure shows an F-spin multiplet for ^{186}W-^{186}Hg (type I) with a constant boson number $N=11$ (upper part). Also shown for comparison are the nuclei ^{186}W-^{198}Hg with $N=11$-5. We give all known positive parity levels up to $E_x \simeq 1$ MeV and the ground and gamma bands up to the energy of the 8_1^+ level. The data are from Refs. 20 and 21. The units are in MeV.

By comparing the excitation energies of corresponding levels, such as the ground band and the quasi-gamma band, in nuclei with different values of F_0 we find, indeed, that the excitation energies of the members of the ground and gamma bands in the various nuclei vary smoothly with F_0 and seem to display the properties of an F-spin multiplet.

In order to get a more quantitative feeling for the dependence of the energies on F_0 in F-spin multiplets (with constant N), we show in the lower parts of Figs. 1 and 2 a series of nuclei with monotonically varying N. These latter nuclei are not an F-spin multiplet. Clearly, the overall picture is that the excitation energies in the F-spin multiplets are only weakly dependent on F_0, whereas they vary strongly with N. Indeed, it is rather remarkable that nuclei spread over such a large mass region appear so similar when exhibited as part of a multiplet. Hence it is apparent that the concept of such an F-spin multiplet and the approximate validity of the F-spin quantum number receive empirical confirmation in this way.

It is interesting to note that there are a number of intruder states in the spectra, which do not belong to the boson space. We have not attempted to delete such intruders, because we could not do this in a consistent fashion. One example of such intruders is probably given by the quasibeta band in ^{186}Pt. This example indicates that such systematics of nuclear properties for nuclei with the same N might be a more powerful tool to detect intruder states than the conventional plots against A and Z.

A possible interpretation of the near constancy of the energies is based on the relation between IBM-2 and IBM-1. By projecting the operators of IBM-2 on to those of IBM-1, relations can be obtained between the parameters of the two models.[22,23] In the case of states with nearly pure $F=F_{max}$, the eigenvalues of the original IBM-2 Hamiltonian and those of the projected one are nearly equal. The projected IBM-1 parameters depend on F_0 through $N_p N_n / N(N-1)$. Therefore, if the IBM-2 parameters are F_0 independent and $N_p N_n$ is constant within a multiplet, the eigenvalues of the projected Hamiltonian will not depend on F_0. The quantity $N_p N_n$ varies little in the considered multiplets, except at the limits.

Summing up, we have shown that there are F-spin multiplets in nuclei with rather constant excitation energies overall, if we exclude the magic nuclei. This fact implies that F spin is a useful label for low lying levels, and it appears to indicate that the IBM-2 parameters are rather constant within a multiplet. This reinforces other, indirect, arguments for good F spin, such as the general success of IBA-1 calculations which, manifestly, yield states with good F spin, and the small values of experimental $B(M1)$ values between the low lying collective states. The consideration of F-spin multiplets seems to be a useful tool in nuclear structure studies that warrants further investigation.

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