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Relation between the interacting boson-fermion approximation model and dynamical boson-fermion symmetries

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The relation between the phenomenological Hamiltonian as used in dynamical boson-fermion symmetries and the semimicroscopic Hamiltonian of the interacting boson-fermion approximation, both used in the description of odd-mass nuclei, is studied in detail. Predictions are obtained for the parameters of the boson fermion symmetry Hamiltonian. From the microscopic picture behind the interacting boson-fermion approximation model, predictions are also obtained for the regions in the table of isotopes where the boson-fermion symmetries can be expected to occur. In addition we derive the structure of an additional exchange force, which results from the interaction between like particles.

I. INTRODUCTION

In the interacting boson model (IBM) the properties of low lying collective states in medium mass even-even nuclei are described in terms of a system of interacting s and d bosons. Different aspects of the Hamiltonian have been investigated. First, in a study of the algebraic properties of the Hamiltonian the importance of dynamical symmetries has been emphasized.[1–3] In this approach the structure of the Hamiltonian is based entirely on group theoretical arguments. Many examples of the three different possible dynamical symmetries, the U(5), the SU(3), and the SO(6) limit have been discussed in the literature. Second, in a more microscopic approach in which the s and d bosons are interpreted as correlated pairs of identical nucleons with angular momentum $J=0$ and $J=2$, the structure of the IBM Hamiltonian has been related to the underlying shell-model Hamiltonian.[4–6] The resulting IBM-2 Hamiltonian has been used extensively in phenomenological studies of collective properties in a large variety of even-even nuclei. Both basically different approaches have been very successful and the relation between the two formulations is well understood.

In the extension of IBM to odd mass nuclei by coupling the single particle (fermion) degrees of freedom to the collective (bosons) degrees of freedom of the core nucleus, a similar situation exists.[7,8] In one approach, limiting situations of the interacting boson-fermion model (IBFM) for which energy eigenvalues can be obtained in closed form have been studied.[9–17] The structure of the Hamiltonian in these dynamical boson-fermion symmetries is determined solely by group theoretical arguments. Several examples of these symmetries have been found in the spectra of odd-even nuclei. In Sec. II we present a short review of dynamical boson-fermion symmetries. The concept of dynamical boson-fermion symmetries has been extended even further which has led to the introduction of dynamical supersymmetries (SuSy) in nuclear physics.[18,19]

In a different approach, hereafter called the interacting boson fermion approximation (IBFA), the structure of the boson-fermion (BF) interaction is derived using a semimicroscopic theory.[8,20] In analogy with even-even nuclei it is assumed in Refs. 8 and 20 that for odd-even nuclei the most important terms in the boson-fermion interaction also arise from a shell model quadrupole force between protons and neutrons. As a result the BF interaction contains a direct quadrupole and an exchange force (Sec. III). The latter arises from the two-particle nature of the bosons. This version of the model has been applied successfully to a large variety of odd-even nuclei.[21–24] In Sec. IV we will show that a quadrupole pairing interaction between identical nucleons in the shell model space introduces additional terms in the BF interaction. These extra terms will be used in the discussion of Sec. V.

The question now arises how the IBFA and the symmetry approach to describe the structure of odd mass nuclei are related to each other. In Sec. V this relation is worked out in detail for a specific example, the SO(6)$\otimes$U(2) limit of the IBFM. We obtain a set of equations that relates the parameters in both Hamiltonians. From these we can derive under what conditions dynamical boson-fermion symmetries can be expected to occur in real nuclei, provided the IBFA model is a good description of odd-even nuclei.

II. DYNAMICAL BOSON-FERMI SYMMETRIES

In this section, a short overview will be given of the concepts of dynamical symmetries in nuclei, insofar as needed in the rest of this paper. A more detailed treatment can be found in the literature.[1–3,9–17]

For even-even nuclei the importance of symmetries in the IBM model is well established. Since the group structure of the general IBM Hamiltonian is U(6), the most general one- and two-body Hamiltonian can be expressed as a sum of terms which are at most quadratic in the generators of the group U(6). In general, no further symmetry is present (apart from rotational invariance) and the Hamiltonian must be diagonalized numerically. However, when this sum reduces to a sum of Casimir invariants of a chain of subgroups of U(6), the Hamiltonian exhibits a dynamical symmetry which is conventionally labeled by the largest subgroup. There are three possible symmetries, U(5), SU(3), and SO(6), each corresponding to a different geometrical picture of a spherical, axially symmetric and gamma unstable nucleus, respectively,

$$\mathrm{U}(6)\supset\begin{cases}\mathrm{U}(5)\supset\mathrm{SO}(5)\supset\mathrm{SO}(3), & (2.1\mathrm{a})\\ \mathrm{SU}(3)\supset\mathrm{SO}(3), & (2.1\mathrm{b})\\ \mathrm{SO}(6)\supset\mathrm{SO}(5)\supset\mathrm{SO}(3). & (2.1\mathrm{c})\end{cases}$$

Whenever the Hamiltonian possesses one of the symmetries, a set of closed analytic expressions can be obtained for excitation energies, electromagnetic transition rates, and other nuclear properties of interest, thus providing a simple scheme for interpreting and classifying experimental data.

For odd mass nuclei the situation is similar. In this case the group structure of the Hamiltonian is more complicated, since we are now dealing with a mixed system of boson (collective) and fermion (single-particle) degrees of freedom. In this paper we will discuss the case in which the single particle levels that can be occupied by the odd nucleon are limited to orbits with $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$. This situation is of considerable physical interest since this situation approximately occurs in the low lying negative parity states in the odd mass W, Pt, and Hg isotopes as well as in the odd mass Kr and Rh isotopes. The group structure in this case is $G=\mathrm{U}^{(B)}(6)\otimes\mathrm{U}^{(F)}(12)$ where we have added the superscripts B and F to distinguish between the boson and the fermion groups. The symmetry group G can be reduced to the angular momentum group in several different ways. We will focus our attention to the reduction that has proven to be the most appropriate one in applications to real nuclei. First, the fermion angular momenta $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ are split into a pseudo-orbital part, $k=0$, and 2, and a pseudo-spin part, $s=\frac{1}{2}$. This corresponds to the decomposition of the fermion group $\mathrm{U}^{(F)}(12)$ into the direct product of a pseudo-orbital group $\mathrm{U}_k^{(F)}(6)$ and a pseudo-spin group $\mathrm{U}_s^{(F)}(2)$,

$$\mathrm{U}^{(F)}(12)\supset\mathrm{U}_k^{(F)}(6)\otimes\mathrm{U}_s^{(F)}(2)\ . \tag{2.2}$$

The subscripts k and s refer to the pseudo-orbital and spin part, respectively. Next, the boson and fermion groups can be combined at the level of U(6) into a common boson-fermion group $\mathrm{U}^{(\mathrm{BF})}(6)$ which can then be further decomposed into three different chains of subgroups,

$$\mathrm{U}^{(B)}(6)\otimes\mathrm{U}^{(F)}(12)\supset\mathrm{U}^{(B)}(6)\otimes\mathrm{U}_k^{(F)}(6)\otimes\mathrm{U}_s^{(F)}(2)\supset\mathrm{U}^{(\mathrm{BF})}(6)\otimes\mathrm{U}_s^{(F)}(2)$$

$$\supset\begin{cases}\mathrm{U}^{(\mathrm{BF})}(5)\otimes\mathrm{U}_s^{(F)}(2)\supset\mathrm{SO}^{(\mathrm{BF})}(5)\otimes\mathrm{U}_s^{(F)}(2)\supset\mathrm{SO}^{(\mathrm{BF})}(3)\otimes\mathrm{SU}_s^{(F)}(2)\supset\mathrm{Spin}(3), & (2.3\mathrm{a})\\ \mathrm{SU}^{(\mathrm{BF})}(3)\otimes\mathrm{U}_s^{(F)}(2)\supset\mathrm{SO}^{(\mathrm{BF})}(3)\otimes\mathrm{SU}_s^{(F)}(2)\supset\mathrm{Spin}(3), & (2.3\mathrm{b})\\ \mathrm{SO}^{(\mathrm{BF})}(6)\otimes\mathrm{U}_s^{(F)}(2)\supset\mathrm{SO}^{(\mathrm{BF})}(5)\otimes\mathrm{U}_s^{(F)}(2)\supset\mathrm{SO}^{(\mathrm{BF})}(3)\otimes\mathrm{SU}_s^{(F)}(2)\supset\mathrm{Spin}(3). & (2.3\mathrm{c})\end{cases}$$

We note that Eqs. (2.3a)—(2.3c) are analogous to the three group chains of the IBM model for even-even nuclei [see Eqs. (2.1a)—(2.1c)]. When the Hamiltonian is written as a sum of Casimir invariants of the subgroups of one group chain, analytic formulas can be obtained for both excitation energies and electromagnetic transition rates. This allows for a simple interpretation of the otherwise very complicated spectrum of odd mass nuclei. In the following we will limit ourselves to the group chain Eq. (2.3c).

In general the IBFM Hamiltonian can be written as

$$H=H_B+H_F+V_{\mathrm{BF}}\ , \tag{2.4}$$

where H_B is the IBM Hamiltonian which describes the collective degrees of freedom in the even-even core nuclei, H_F represents the fermion Hamiltonian,

$$H_F=\sum_{jm}\epsilon_j a_{jm}^{\dagger}a_{jm}\ , \tag{2.5}$$

and V_{BF} is the boson-fermion interaction.

The Hamiltonian of the boson-fermion symmetry associated with group chain (2.3c), which will be referred to as the $\mathrm{SO}(6)\otimes\mathrm{U}(2)$ limit of the IBFM, can be expressed in terms of the linear, C_{1g} and the quadratic, C_{2g} Casimir operators of the groups appearing in Eq. (2.3c). Omitting the terms that only contribute to binding energies the symmetry Hamiltonian is given by

$$H_{\mathrm{sym}}=\zeta C_{2\mathrm{U}^{(\mathrm{BF})}(6)}+\eta C_{2\mathrm{SO}^{(\mathrm{BF})}(6)}+\beta C_{2\mathrm{SO}^{(\mathrm{BF})}(5)}+\gamma C_{2\mathrm{SO}^{(\mathrm{BF})}(3)}+\gamma' C_{2\mathrm{Spin}(3)}\ . \tag{2.6}$$

The energy spectrum can be readily constructed,

$$\begin{aligned}E=&\zeta[N_1(N_1+5)+N_2(N_2+3)]\\ &+2\eta[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)+\sigma_3^2]\\ &+2\beta[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]\\ &+2\gamma L(L+1)+2\gamma' J(J+1)\ .\end{aligned} \tag{2.7}$$

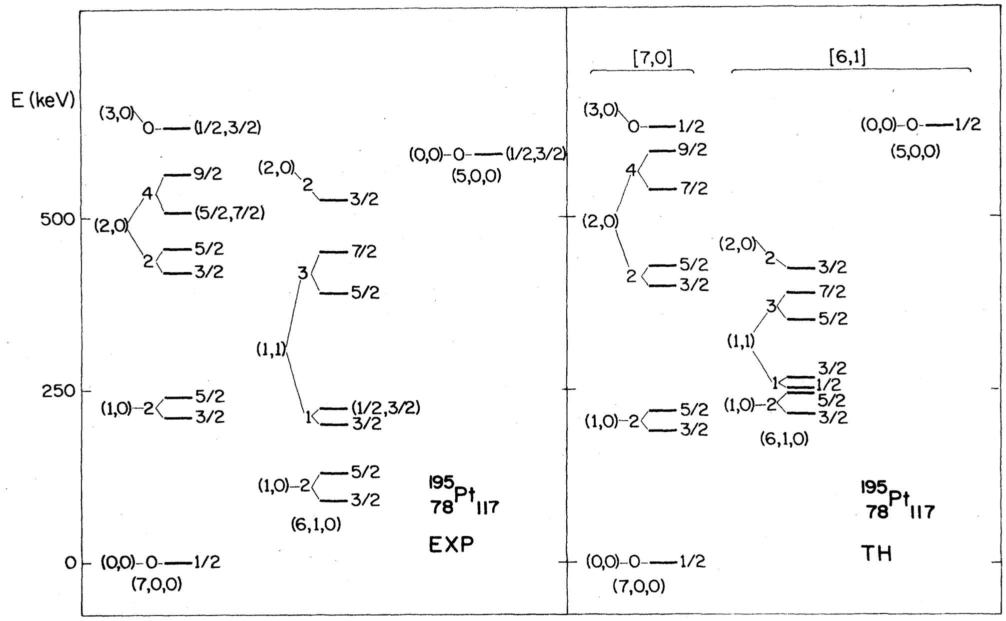

FIG. 1. A comparison between the experimental low-lying spectrum of ^{195}Pt (Refs. 25–27) and that obtained using Eq. (2.5) with $\zeta=31.6$ keV, $\eta=-16.75$ keV, $\beta=17.5$ keV, $\gamma=2.5$ keV, and $\gamma'=3$ keV. The number of bosons is $N=6$.

This expression for the excitation energies has been applied to the negative parity states in ^{195}Pt.[12,14,16,17] In Fig. 1 we show a comparison between the experimental and theoretical energy spectra. For a discussion of the quantum numbers, etc., we refer to the extensive literature on boson-fermion symmetries.[11,14,16,17]

For the purpose of the present paper it is convenient to rewrite the Hamiltonian $H_{\rm sym}$ explicitly in terms of the boson and fermion generators. The fermion and boson-fermion parts of $H_{\rm sym}$ can be expressed as

$$H_{F,\rm sym}=(2\zeta N+6\zeta+10\eta)K^{(0)}(0,0)+(6\zeta+10\eta+8\beta+12\gamma)\sqrt{5}K^{(0)}(2,2) \tag{2.8}$$

and

$$\begin{aligned}V_{\rm BF,sym}= &-2\sqrt{5}\zeta(d^\dagger\tilde{d})^{(0)}\cdot K^{(0)}(0,0)+2\zeta(d^\dagger\tilde{d})^{(0)}\cdot K^{(0)}(2,2)+(2\zeta+8\eta+8\beta+40\gamma)(d^\dagger\tilde{d})^{(1)}\cdot K^{(1)}(2,2)\\ &+\phi 4\eta(s^\dagger\tilde{d}+d^\dagger s)^{(2)}\cdot[K^{(2)}(0,2)+K^{(2)}(2,0)]+\phi 2\zeta[(s^\dagger\tilde{d})^{(2)}\cdot K^{(2)}(2,0)+(d^\dagger s)^{(2)}\cdot K^{(2)}(0,2)]\\ &+2\zeta(d^\dagger\tilde{d})^{(2)}\cdot K^{(2)}(2,2)+(2\zeta+8\eta+8\beta)(d^\dagger\tilde{d})^{(3)}\cdot K^{(3)}(2,2)+2\zeta(d^\dagger\tilde{d})^{(4)}\cdot K^{(4)}(2,2)\ .\end{aligned} \tag{2.9}$$

The operators $K^{(\lambda)}(l,l')$ in Eq. (2.9) represent the pseudo-orbital generators of the $U_k^{(F)}(6)$ group and are defined in the Appendix. In Eqs. (2.8) and (2.9) we have assumed that $H_{\rm sym}$ is invariant under the $SU_s^{(F)}(2)$ pseudo-spin group ($\gamma'=0$). Furthermore, we note that $H_{\rm sym}$ is invariant under the phase ϕ (see the Appendix).

III. THE IBFA MODEL

In the IBFA model the boson-fermion interaction is constructed on the basis of semimicroscopic arguments from the underlying shell model (SM) Hamiltonian $H_{\rm SM}$. Usually it is assumed that the most important term in $H_{\rm SM}$ is a quadrupole-quadrupole interaction between neutrons and protons. In Sec. IV we will also consider the terms arising from the quadrupole pairing interaction between like particles. In this section we will give the "standard" formulation of the boson-fermion interaction in the IBFA model with a short explanation of the physical significance of the different terms.

The IBFA Hamiltonian has been derived from the shell model Hamiltonian using several different approaches. The general result can be written as

$$H_F=\sum_{jm}E_j a_{jm}^\dagger a_{jm}\ , \tag{3.1a}$$

$$\begin{aligned}V_{\rm BF}=&\sum_{jj'}\Gamma_{jj'}[Q_B^{(2)}(a_j^\dagger\tilde{a}_{j'})^{(2)}]^{(0)}\\ &+\sum_{jj'j''}\Lambda_{jj'}^{j''}:[(a_j^\dagger\tilde{d})^{(j'')}(d^\dagger\tilde{a}_{j'})^{(j'')}]^{(0)}:\\ &-\sum_j A_j\sqrt{5(2j+1)}[(d^\dagger\tilde{d})^{(0)}(a_j^\dagger\tilde{a}_j)^{(0)}]^{(0)}\ ,\end{aligned} \tag{3.1b}$$

where

$$Q^{(2)}_{B,m}=(s^\dagger\tilde{d}+d^\dagger s)^{(2)}_m+\chi(d^\dagger\tilde{d})^{(2)}_m\ , \tag{3.2}$$

and $a_j^\dagger$ is the creation operator of the odd particle degree of freedom. The parameters in (3.1b) can be related to the occupancies v_j^2 of the spherical shell model orbits,

$$\Gamma_{jj'}=\Gamma_0 Q_{jj'}(u_j u_{j'}-v_j v_{j'})\sqrt{5}\ , \tag{3.3a}$$

$$\Lambda^{j''}_{jj'}=-\sqrt{5}\Lambda_0[Q_{j'j''}(u_{j'}v_{j''}+v_{j'}u_{j''})\beta_{j''j}+\beta_{j'j''}(u_{j''}v_j+v_{j''}u_j)Q_{j''j}]\times\frac{1}{\sqrt{(2j''+1)}}\ , \tag{3.3b}$$

where $Q_{jj'}$ are the single particle matrix elements of the quadrupole operator. Assuming that for orbits in the same major shell the radial overlaps can be approximated by a constant, which can subsequently be absorbed in the strength parameters Γ_0 and Λ_0, $Q_{jj'}$ can be written as

$$Q_{jj'}=\langle j||Y^{(2)}||j'\rangle\ . \tag{3.4}$$

The coefficients $\beta_{jj'}$ in Eq. (3.3) are related to the internal structure of the D-fermion pair state, which is the shell-model equivalence of the d boson. Assuming that the D-pair state exhausts the valence $E2$ strength, these coefficients can be written as

$$\beta_{jj'}=Q_{jj'}(u_j v_{j'}+v_j u_{j'})\ . \tag{3.5}$$

The normalization constant K_β

$$K_\beta=\left[\sum_{jj'}(\beta_{jj'})^2\right]^{1/2} \tag{3.6}$$

can also be absorbed in the strength parameters Γ_0 and Λ_0. In a more sophisticated approach,[21] an energy denominator has been introduced in Eq. (3.5) to take into account the nondegeneracy of the single-particle orbits. Since this extension has little effect on the conclusions of the present paper, we will use the simpler form of Eq. (3.5) in the following.

The first two terms in Eq. (3.1) have a specific microscopic interpretation. The first term is the direct quadrupole-quadrupole interaction analogous to the particle-phonon interaction in the particle-vibration model.[28] The second term represents the exchange force. It results completely from the action of the Pauli principle between the odd nucleon and the bosons, which are interpreted as collective fermion pair states and can therefore occupy the same shell model orbits as the odd nucleon. The exchange force vanishes whenever the orbits considered for the odd particle are filled ($v_j^2=1$) or empty ($v_j^2=0$) and is important when the orbits are partially filled. A similar term has been introduced in the particle-vibration model.[29,30] Also the strength of the direct quadrupole interaction is affected by the Pauli principle as can be seen from (3.3a). Contributions to the last term in Eq. (3.1b), the monopole force, can have various different physical origins.

A complete microscopic derivation of the IBFA model also gives an explicit relation between the interaction strengths in the shell-model and the IBA-IBFA model Hamiltonians. However, to make a quantitative analysis of energies in the boson model, the renormalization effects arising from the truncation of the shell-model space to the collective S and D pair subspace should be taken into account. In phenomenological calculations therefore, the strengths Γ_0, Λ_0, and A_j are considered as free parameters, which can be adjusted to give a best fit to the excitation energies. It has been shown[8,21–24] that with the parametrization equations (3.1)–(3.5) of the boson fermion interaction it is possible to describe the main features of a large variety of odd-mass nuclei.

IV. THE QUADRUPOLE PAIRING INTERACTION

The boson-fermion interaction in the IBFA model, Eq. (3.1), is derived from the shell model neutron proton quadrupole interaction. In Refs. 30 and 31 it has been shown that the interaction between the like particles also gives a contribution to the exchange force. The two terms that are most important in the like particle interaction are the monopole and quadrupole pairing interaction.[31] The monopole pairing interaction contributes to the monopole term in Eq. (3.1b) and is responsible for replacing the single particle energies by quasiparticle energies in the fermion Hamiltonian equation (3.1a). In the following we will study the boson-fermion image of the quadrupole pairing interaction:

$$V=V_2(A^{\dagger(2)}\tilde{A}^{(2)})^{(0)}\ , \tag{4.1}$$

where

$$A_m^{\dagger(2)}=\sum_{jj'}b_{jj'}\frac{1}{\sqrt{2}}(c_j^\dagger c_{j'}^\dagger)^{(2)}_m\ . \tag{4.2}$$

In Eq. (4.2) $c_j^\dagger$ are the shell model single nucleon creation operators. The coefficients $b_{jj'}$ in Eq. (4.2) are normalized such that

$$\langle 0|(A_m^{\dagger(2)})^\dagger A_m^{\dagger(2)}|0\rangle=1\ , \tag{4.3}$$

leading to

$$\sum_{jj'}(b_{jj'})^2=1\ . \tag{4.4}$$

In order to construct the boson-fermion image of the quadrupole pairing interaction, we will make use of the pseudo-particle creation operator $\check{c}_j^\dagger$ as was introduced in Refs. 8 and 20,

$$\check{c}^\dagger_{jm}=u_j a^\dagger_{jm}+\frac{v_j}{\sqrt{N}}(s^\dagger a_j)^{(j)}_m+\sum_{j'}u_j\bar{\beta}_{j'j}\left[\frac{10}{2j+1}\right]^{1/2}(d^\dagger\tilde{a}_{j'})^{(j)}_m-\sum_{j'}\frac{v_j}{\sqrt{N}}\bar{\beta}_{j'j}\left[\frac{10}{2j+1}\right]^{1/2}(s^\dagger\tilde{d}a^\dagger_{j'})^{(j)}_m\ . \tag{4.5}$$

The coefficients $\bar{\beta}_{jj'}$ are the normalized d-boson structure coefficients, which are related to those introduced in Eqs. (3.5) and (3.6) by,

$$\bar{\beta}_{jj'}=\beta_{jj'}/K_\beta\ . \tag{4.6}$$

We note that in the special case of only S pairs, Eq. (4.5) reduces to the Bogoliubov-Valatin quasi-particle transformation when we make the approximation $s^\dagger/\sqrt{N} \simeq 1$, which is valid for low-seniority states. The pseudo-particle operator $\check{c}_j^\dagger$ is a first-order approximation in the d-boson operators to the boson-fermion image of the shell model single particle creation operator $c_j^\dagger$. The image of the two-nucleon creation operator (4.2) can now be constructed by replacing the shell-model creation operators $c_j^\dagger$ by the psuedo-particle creation operators $\check{c}_j^\dagger$.

Keeping only the terms that act within the one quasi-particle space of the IBFA model and are of first order in the d-boson operators, we obtain

$$\check{A}_m^\dagger(2) = \sum_{jj'} b_{jj'} u_j u_{j'} \bar{\beta}_{jj'} d_m^\dagger - \sum_{jj'} b_{jj'} v_j v_{j'} \frac{1}{N} \bar{\beta}_{jj'} s^\dagger s^\dagger \tilde{d}_m + \sqrt{2} \sum_{jj'} b_{jj'} u_j v_{j'} \frac{1}{\sqrt{N}} s^\dagger (a_j^\dagger \tilde{a}_{j'})_m^{(2)}$$
$$+ \sqrt{2} \sum_{jj'j''} b_{jj'} u_j u_{j'} \bar{\beta}_{j''j'} \left[\frac{10}{2j'+1} \right]^{1/2} [a_j^\dagger (d^\dagger \tilde{a}_{j''})^{(j')}]_m^{(2)}$$
$$- \sqrt{2} \sum_{jj'j''} b_{jj'} v_j v_{j'} \frac{1}{N} \bar{\beta}_{j''j} \left[\frac{10}{2j+1} \right]^{1/2} s^\dagger s^\dagger [(\tilde{d} a_{j''}^\dagger)^{(j)} \tilde{a}_{j'}]_m^{(2)} . \tag{4.7}$$

It should be noted that a more rigorous derivation also introduces terms of higher order in the d-boson operators, such as $s^\dagger(d^\dagger \tilde{d})^{(2)}$, in the two-nucleon transfer operator. However, to calculate these terms consistently a knowledge of the terms that are quadratic in the d-boson operators in the pseudo-particle operator (4.5) would also be required. For the present purpose these higher-order terms do not play an important role and are neglected.

To evaluate the contribution of the quadrupole pairing interaction to the boson-fermion interaction of the IBFA model, we substitute Eq. (4.7) in Eq. (4.1). Again retaining only the terms that are of first order in the d-boson operators, we obtain

$$V'_{\rm BF} = \sum_{jj'} \{ \Lambda'_{jj'} [(s^\dagger \tilde{d})^{(2)} (a_j^\dagger \tilde{a}_{j'})^{(2)}]^{(0)} + \text{H.c.} \}$$
$$+ \sum_{jj''} \{ \Lambda''_{jj'} [(d^\dagger s)^{(2)} (a_j^\dagger \tilde{a}_{j'})^{(2)}]^{(0)} + \text{H.c.} \} \tag{4.8}$$

with

$$\Lambda'_{jj'} = \Lambda'_0 u_j v_{j'} Q_{jj'} \sqrt{5} , \tag{4.9a}$$

$$\Lambda''_{jj'} = \Lambda''_0 u_j v_{j'} Q_{jj'} \sqrt{5} , \tag{4.9b}$$

and

$$\Lambda'_0 = V_2 \sqrt{2/N} z^2 \sum_{jj'} Q_{jj'} u_j u_{j'} \bar{\beta}_{jj'} , \tag{4.10a}$$

$$\Lambda''_0 = - V_2 \sqrt{2/N} z^2 \sum_{jj'} Q_{jj'} v_j v_{j'} \bar{\beta}_{jj'} . \tag{4.10b}$$

In Eqs. (4.9) and (4.10) we have assumed that the coefficients $b_{jj'}$ are proportional to the single-particle matrix elements of the quadrupole operator $Q_{jj'}$ equation (3.4) (see, i.e., Ref. 32)

$$b_{jj'} = z Q_{jj'} . \tag{4.11}$$

It can be checked easily that the terms of second order in the d-boson operators are similar in structure to the exchange force that is normally considered in the IBFA model. The contribution to the boson fermion interaction given by (4.8) represent, however, terms that so far have not been taken into account in the standard IBFA model. In the special case in which the odd nucleon occupies only a single j orbit or when the coefficients Λ'_0 and Λ''_0 are equal, this contribution is symmetric in the boson part and can to a large extent be absorbed in the quadrupole interaction of Eqs. (3.1) and (3.2). In general, however, this is not the case and $V'_{\rm BF}$ gives an extra contribution to the boson-fermion interaction. As will be shown in the next section, an interaction term of this kind is required to reproduce the dynamical symmetries of the IBFM for odd mass nuclei.

The fact that $V'_{\rm BF}$ is not symmetric in the boson operators may look surprising. The reason for this can be seen by writing the simplest boson-fermi image of the two nucleon creation operator. Omitting all terms which are higher than first order in the boson operators, this image has the form of

$$A^{\dagger(2)} \sim x_{jj'} s^\dagger (a_j^\dagger \tilde{a}_{j'})^{(2)} + y d^\dagger .$$

It can be seen that this represents the subset of terms in Eq. (4.7) that contributes to Λ'. Up to this order there is no contribution to Λ''_0, resulting in a boson image of the quadrupole pairing interaction which is nonsymmetric to a maximal extent.

V. BOSON FERMION SYMMETRIES AND THE IBFA MODEL

The properties of several odd mass nuclei have been interpreted in terms of a dynamical boson-fermion symmetry. To obtain a better physical insight as to why these nuclei exhibit a dynamical symmetry it is important to study the relation between the symmetry Hamiltonian and the boson-fermion interaction of the more microscopic IBFA model.

A first step in understanding this relation is to note the occurrence of an SU(2) pseudo-spin symmetry in $V_{\rm BF}$, Eq. (3.1).[33] When we consider the case of two shell model orbits coupled to a core nucleus, with $j_1 = j$ and $j_2 = j-1$, the pseudo-spin symmetry occurs whenever these two orbits are degenerate, which means equal single-particle energies and equal occupancies. The physical reason for the occurrence of this SU(2) symmetry is that the single particle matrix elements of the quadrupole operator equation (3.4) depend only on the quasi-orbital angular momentum, $k=(j_1+j_2)/2$, but not on the orientation of the spin, $s=\frac{1}{2}$. The concept of this pseudo-spin symmetry can easily be extended to a more general case, in which the odd nucleon can occupy more than two orbits.

Suppose in this case that the single-particle angular momenta $j=j_1,j_2,\ldots,$ can be decomposed into a pseudo-orbital part, $k=k_1,k_2,\ldots,$ and a pseudo-spin part, $s=\frac{1}{2}$. The pseudo-spin symmetry now occurs whenever the single-particle orbits with the same value of the pseudo-orbital angular momentum (pseudo spin-orbit partners) are degenerate. We note that this property does not depend on the strengths of the quadrupole and exchange terms Γ_0 and Λ_0.

In the construction of the boson-fermion symmetry associated with the group chains in Eq. (2.3), a similar procedure is followed. The $SU_s^{(F)}(2)$ symmetry group of Eq. (2.3) is equivalent to the SU(2) pseudo-spin symmetry group discussed above. Since the pseudo-spin symmetry in the IBFA Hamiltonian only holds for $s=\frac{1}{2}$, the dynamical boson-fermion symmetries in which the pseudo-spin is different from $s=\frac{1}{2}$ do not have a counterpart in the IBFA model. In the symmetry Hamiltonian H_{sym} of Eq. (2.6) the pseudo-spin symmetry is only broken by the last term which is proportional to the total angular momentum. The excitation energies of the negative parity states in ^{195}Pt, shown in Fig. 1, indicate that the energy spectrum is characterized by a series of doublets of states. This feature implies that in this case the pseudo spin-orbit force is small and therefore $\gamma'\simeq 0$ in H_{sym}. For simplicity in the following we will assume that H_{sym} is pseudo-spin invariant ($\gamma'=0$). The presence of a pseudo-spin symmetry in the IBFA model implies that the $3p_{3/2}$ and $2f_{5/2}$ s.p. orbits are degenerate and form a $k=2$ doublet, $E_{1/2}=E_0$, $E_{3/2}=E_{5/2}=E_2$, $v_{1/2}=v_0$, $v_{3/2}=v_{5/2}=v_2$. If in addition we take $A_{1/2}=A_0$, $A_{3/2}=A_{5/2}=A_2$, the total IBFA Hamiltonian equation (3.1) is pseudo-spin invariant and can be expressed in terms only of the pseudo-orbital operators $K^{(\lambda)}(k,k')$ defined in Eq. (A2). This will enable us to make a term by term comparison between the symmetry Hamiltonian H_{sym} and the IBFA Hamiltonian. The fermion Hamiltonian of Eq. (3.1a) can now be rewritten as

$$H_F=\sum_{k=0,2} E_k\sqrt{2k+1}K^{(0)}(k,k)\ . \tag{5.1}$$

The quadrupole term in Eq. (3.1b) gives

$$V_Q=\sum_{k,k'=0,2} q_{kk'}[(s^\dagger\tilde{d}+d^\dagger\tilde{s})^{(2)}+\chi(d^\dagger\tilde{d})^{(2)}]\cdot K^{(2)}(k,k')\ , \tag{5.2}$$

with

$$q_{02}=q_{20}=-\frac{1}{2}\left[\frac{5}{\pi}\right]^{1/2}\Gamma_0(u_0u_2-v_0v_2)\ ,$$
$$q_{22}=5\left[\frac{1}{14\pi}\right]^{1/2}\Gamma_0(u_2^2-v_2^2)\ . \tag{5.3}$$

Similarly the exchange term in Eq. (3.1b) can be rewritten in terms of boson and pseudo-orbital generators as

$$V_E=\sum_{\lambda}\sum_{k,k'=0,2} e_{kk'}^{(\lambda)}(d^\dagger\tilde{d})^{(\lambda)}\cdot K^{(\lambda)}(k,k')\ . \tag{5.4}$$

with

$$e_{00}^{(0)}=\frac{5}{2\pi}\Lambda_0(u_0v_2+v_0u_2)^2\ ,$$
$$e_{02}^{(2)}=e_{20}^{(2)}=-\frac{5}{\pi}(\tfrac{2}{7})^{1/2}\Lambda_0u_2v_2(u_0v_2+v_0u_2)\ , \tag{5.5}$$
$$e_{22}^{(\lambda)}=\frac{5^{1/2}}{2\pi}\Lambda_0\left[(u_0v_2+v_0u_2)^2+(-1)^\lambda\tfrac{200}{7}u_2^2v_2^2\begin{Bmatrix}2&2&2\\2&\lambda&2\end{Bmatrix}\right]\ .$$

Finally, the monopole term can be expressed as

$$V_M=\sum_{k=0,2} x_k(d^\dagger\tilde{d})^{(0)}\cdot K^{(0)}(k,k)\ , \tag{5.6}$$

with

$$x_k=\sqrt{5(2k+1)}A_k;\quad k=0,2\ . \tag{5.7}$$

In the special case, in which $v_2=0$ or 1, the exchange force V_E in Eq. (5.4) reduces to a simple form. The coefficients $e_{02}^{(2)}$ and $e_{20}^{(2)}$ vanish, $e_{22}^{(\lambda)}$ becomes independent of λ, and the term, proportional to $e_{00}^{(0)}$, can be combined with the monopole term x_0. The remaining terms in V_E correspond to the part of the $U^{(BF)}(6)$ Casimir operator in Eq. (2.7) that is of second order in the d operators and was considered in Ref. 34. In the following we will consider the more general case with arbitrary values for the occupation probabilities.

When we compare the IBFA and the symmetry Hamiltonian there are two terms which cannot be accounted for. The first one arises from V_Q,

$$(s^\dagger\tilde{d}+d^\dagger s)^{(2)}\cdot K^{(2)}(2,2)$$

and the second one from H_{sym},

$$(s^\dagger\tilde{d})^{(2)}\cdot K^{(2)}(2,0)+\text{H.c.}$$

The second term which is not symmetric in the boson part is not present in the standard form of the IBFA Hamiltonian. However, as we have shown in Sec. IV, quadrupole pairing interaction between like particles gives a contribution to the boson-fermion interaction that has precisely this structure. Rewriting the interaction of Eq. (4.8) in terms of the boson and pseudo-orbital generators gives

$$V_E'=\sum_{k,k'=0,2} e_{kk'}'[(s^\dagger\tilde{d})^{(2)}\cdot K^{(2)}(k,k')+\text{H.c.}]\ , \tag{5.8}$$

with

$$e_{02}'=-\frac{1}{2}\left[\frac{5}{\pi}\right]^{1/2}(u_0v_2\Lambda_0'+v_0u_2\Lambda_0'')\ ,$$
$$e_{20}'=-\frac{1}{2}\left[\frac{5}{\pi}\right]^{1/2}(u_2v_0\Lambda_0'+v_2u_0\Lambda_0'')\ , \tag{5.9}$$
$$e_{22}'=\frac{5}{\sqrt{14\pi}}(\Lambda_0'+\Lambda_0'')u_2v_2\ .$$

By comparing the coefficients of the corresponding terms in the multipole expansion of the IBFA Hamiltonian, $H_F+V_Q+V_E+V_M+V_E'$ defined in Eqs. (5.1)–(5.9), and the Hamiltonian in the SO(6)⊗U(2) limit, $H_{F,\mathrm{sym}}+V_{\mathrm{BF,sym}}$, Eqs. (2.8) and (2.9), we obtain a set of equations that relates for the parameters in the two Hamiltonians.

First we will investigate under which conditions the symmetry can arise from the IBFA Hamiltonian. The IBFA parameters then have to fulfill the following equations:

$$u_2v_2(u_0u_2-v_0v_2)=-(u_2^2-v_2^2)(u_0v_2+v_0u_2)\ , \qquad (5.10a)$$

$$\chi\Gamma_0(u_2^2-v_2^2)=2\left[\frac{10}{7\pi}\right]^{1/2}u_2^2v_2^2\Lambda_0\ , \qquad (5.10b)$$

$$(\Lambda_0'+\Lambda_0'')\chi=-2\left[\frac{10}{7\pi}\right]^{1/2}u_2v_2\Lambda_0\ , \qquad (5.10c)$$

$$\phi(\Lambda_0'-\Lambda_0'')(u_0v_2-v_0u_2)=\left[\frac{1}{\pi}\right]^{1/2}\Lambda_0[(u_0v_2+v_0u_2)^2+\tfrac{80}{49}u_2^2v_2^2]\ , \qquad (5.10d)$$

$$x_0=-\frac{5}{\pi}\Lambda_0[(u_0v_2+v_0u_2)^2+\tfrac{40}{49}u_2^2v_2^2]\ . \qquad (5.10e)$$

$$x_2=-\frac{100}{49\pi}5^{1/2}\Lambda_0u_2^2v_2^2\ , \qquad (5.10f)$$

$$E_0=(N+3)\frac{5^{1/2}}{2\pi}\Lambda_0[(u_0v_2+v_0u_2)^2+\tfrac{80}{49}u_2^2v_2^2]-\phi\frac{5}{4}\left[\frac{5}{\pi}\right]^{1/2}[\Gamma_0(u_0u_2-v_0v_2)+\Lambda_0''v_0u_2+\Lambda_0'u_0v_2]\ , \qquad (5.10g)$$

$$E_2=\frac{3}{2\pi}5^{1/2}\Lambda_0[(u_0v_2+v_0u_2)^2+\tfrac{30}{49}u_2^2v_2^2]-\phi\frac{1}{4}\left[\frac{5}{\pi}\right]^{1/2}[\Gamma_0(u_0u_2-v_0v_2)+\Lambda_0''v_0u_2+\Lambda_0'u_0v_2]\ . \qquad (5.10h)$$

The phase factor, ϕ, in Eqs. (5.10d) and (5.10g) is the same one as in the terms in the Hamiltonian of Eq. (2.9) that change the number of d bosons. The phase can be chosen such that $(\Lambda_0'-\Lambda_0'')\geq 0$. With this choice the $\mathrm{SO^{(BF)}(6)}$ term in H_{sym}, which is proportional to η, becomes attractive. This is necessary for a correct ordering of the various bands in the boson-fermion symmetry. This phase factor is related to a particle-hole conjugation and changes sign in the middle of the shell ($v_0^2=v_2^2=0.5$). The allowed values of v_0^2 and v_2^2 are given by Eq. (5.10a), $0\leq v_0^2\leq 1$ and $\frac{1}{3}\leq v_2^2\leq\frac{2}{3}$. In Fig. 2 the calculated values for v_0^2 and the ratios of the IBFA parameters are given as a function of v_2^2 for three different values of χ, $|\chi|=0.4$, 0.8, and 1.2. The sign of χ is determined by the fact that both Γ_0 and Λ_0 have to be positive [see Eq. (5.10b)] and

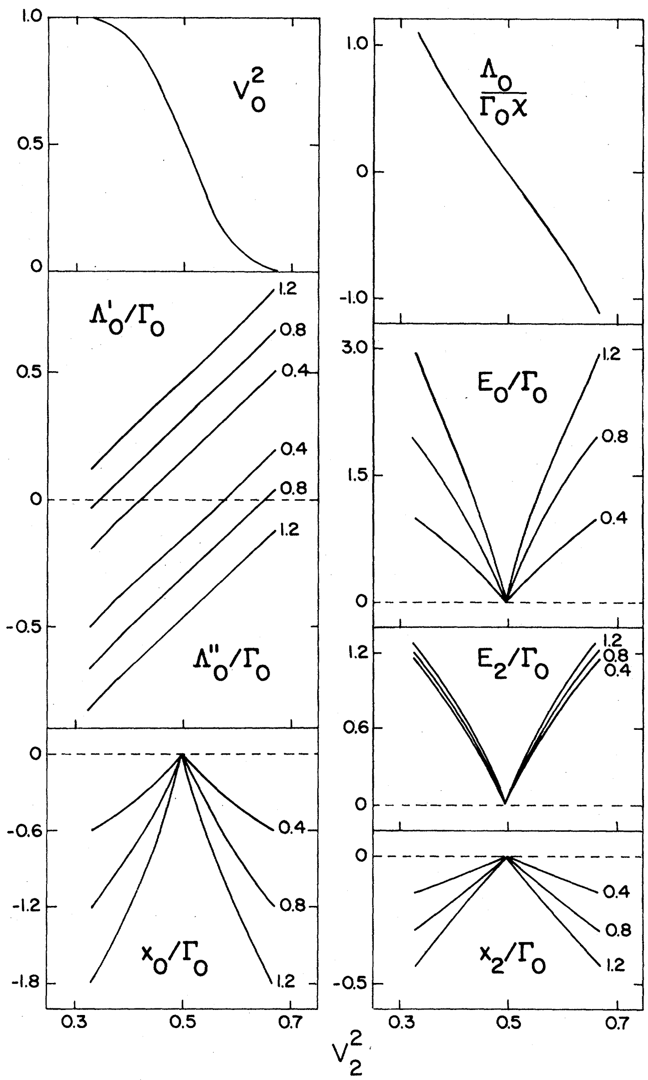

FIG. 2. Ratios of IBFA parameters as a function of v_2^2 for three different values of $|\chi|=0.4$, 0.8, and 1.2.

changes at $v_2^2=0.5$. For $v_0^2=v_2^2=0.5$ there is a singular solution, since at this point the contribution of the quadrupole interaction vanishes.

By requiring that the IBFA parameters fall in a "physically allowed" region, Eqs. (5.10a)–(5.10h) can be used to determine where spectra with SO(6)⊗U(2) symmetry can be expected to occur. Phenomenological calculations for the odd Eu (Ref. 22), Pm (Ref. 24), Xe (Ref. 23), and Ir, Pt, Au (Ref. 21) indicate that $\Gamma_0/\Lambda_0\leq 1$. From the phenomenology of the IBA-2 model it is known that in essentially all cases $|\chi|\leq 1.3$. From Fig. 2 it can be seen that this implies that $\frac{1}{3}<v_2^2\lesssim 0.4$ or $0.6\lesssim v_2^2<\frac{2}{3}$. Furthermore, the two coefficients Λ_0' and Λ_0'' [see Eq. (4.10)] should have opposite signs. All these conditions together limit the region where the SO(6)⊗U(2) symmetry can be expected to occur to (i) $\chi\simeq-1.2$, $v_2^2\simeq 0.6$, and $v_0^2\simeq 0.1$ or (ii) $\chi\simeq 1.2$, $v_2^2\simeq 0.4$, and $v_0^2\simeq 0.9$. For smaller values of χ or $|v_2^2-0.5|$ the ratio Λ_0/Γ_0 is considerably smaller than 1, while for values of v_2^2 closer to $\frac{1}{3}$ or $\frac{2}{3}$ the signs

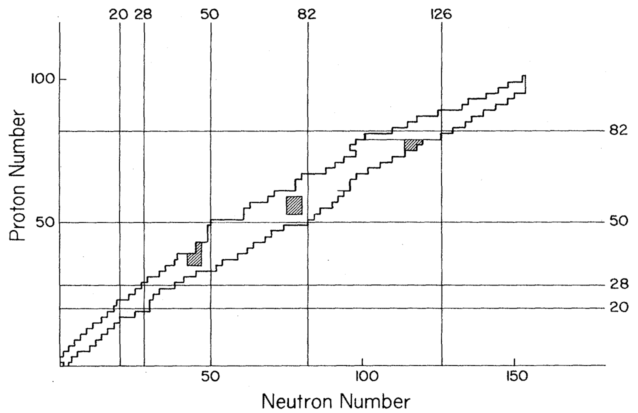

FIG. 3. Regions where a description in terms of the SO(6) limit may be appropriate (dashed areas). The even-even nuclei shown in the chart are those for which the first excited 2^+ state is known.

of Λ_0' and Λ_0'' will become equal. In addition to the conditions (i) or (ii) mentioned above the core nucleus should have SO(6) symmetry and the single-particle levels with spin $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ should be well separated from the other shell model orbits. There are three regions in the nuclear mass table where the SO(6) symmetry has been applied, namely the Pt-Os region,[3,35] the Xe-Ba region,[3] and the Kr-Sr region[36] (see Fig. 3). Inspection of the single-particle level scheme (Fig. 4) shows that the SO(6)⊗U(2) symmetry could occur for

(a) the odd-neutron nuclei in the Pt-Os region with the odd neutron occupying the $3p_{1/2}$, $3p_{3/2}$, and $2f_{5/2}$ orbits; or

(b) the odd-neutron nuclei in the Kr-Sr region with the odd neutron occupying the $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbits.

The fact that the symmetry is expected to occur only for negative parity states (the $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ orbits), instead of for positive states (the $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ orbits) is no accident. For positive parity states the condition that the $j=\frac{3}{2}$ and $\frac{5}{2}$ levels are almost degenerate is never realized because of the effects of the spin-orbit splitting of the s.p. levels. The spin-orbit force splits the p levels such that the $p_{3/2}$ orbit lies below the $p_{1/2}$. The occupancy of the $p_{1/2}$ is therefore always smaller than that of the $p_{3/2}$, which reduces region for the SO(6)⊗U(2) symmetry to

$$\chi \simeq -1.2,\quad v_2^2 \simeq 0.6,\quad v_0^2 \simeq 0.1\ . \tag{5.11}$$

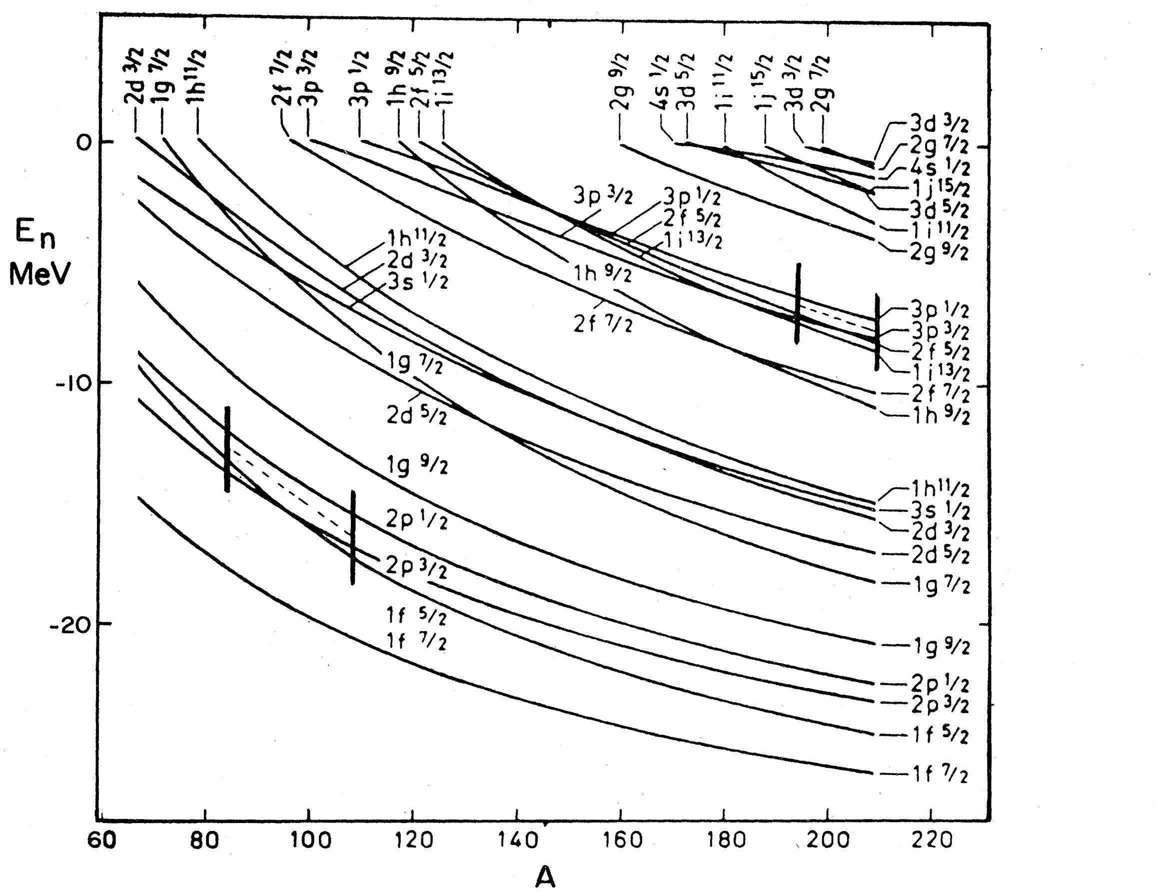

FIG. 4. Single neutron energies (Ref. 28) as a function of mass number. The regions where the single-particle levels with $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ are well separated from the other shell model orbits have been indicated in the figure.

TABLE I. Quasiparticle energies and occupation probabilities of the single-particle orbits in the neutron 82-126 major shell for ^{195}Pt, calculated by solving the BCS gap equations using $\Delta=135/A$.

	$2f_{7/2}$	$1h_{9/2}$	$1i_{13/2}$	$3p_{3/2}$	$2f_{5/2}$	$3p_{1/2}$
ϵ_j (MeV)[a]	1.07	0.00	1.97	2.57	2.92	3.51
E_j (MeV)	1.88	2.93	1.10	0.73	0.69	0.95
v_j^2	0.96	0.99	0.89	0.67	0.46	0.16

[a]From Ref. 37.

In the following we will investigate whether this condition is satisfied in the case of ^{195}Pt, for which it has been suggested[11,12,14,16,17] that the negative parity states provide a good example of the SO(6)⊗U(2) symmetry (see also Fig. 1).

The results of a Bardeen-Cooper-Schrieffer (BCS) calculation for the quasi-particle energies and the occupation probabilities of the orbits in the neutron 82-126 major shell for ^{195}Pt is given in Table I. The occupancy for the $3p_{1/2}$ shell is small, while that of the $3p_{3/2}$ and $2f_{5/2}$ orbits is of the order of 0.5. We note that these values depend strongly on the position of the $1i_{13/2}$ orbit. Since the BF quadrupole interaction arises mainly from the neutron-proton quadrupole force, for the odd-neutron nuclei the values of the coefficient χ should be taken equal to that in the proton boson quadrupole operator. The value of χ_π used in phenomenological calculations for the even-even Pt isotopes was $\chi_\pi=-0.80$.[38] These values for the occupancies and χ are in agreement with the conditions (5.11) and this explains why, in first approximation, the negative parity states in ^{195}Pt provide a good example of the SO(6)⊗U(2) limit of the IBFM. The larger breaking of this symmetry observed in the neighboring nuclei ^{193}Pt and ^{197,199}Pt (Refs. 39 and 40) can then be understood by noting that the occupancies of the $3p_{3/2}$, $2f_{5/2}$, and that of the $3p_{1/2}$ orbits are different from the ones in ^{195}Pt. Therefore one should rather study the $N=117$ isotones in this mass region to find more experimental examples of this type of symmetry, since for these nuclei the single particle occupancies will remain essentially unchanged. However, the corresponding core nuclei, ^{193}Os and especially ^{191}W, do not exhibit an SO(6) symmetry.

It is also possible to take a different approach in which one uses the relation between the SO(6)⊗U(2) limit and the IBFA model to calculate a "physical" region for the BF symmetry parameters. The results of such a comparison are

$$\zeta=\frac{5^{1/2}}{4\pi}\Lambda_0[(u_0v_2+v_0u_2)^2+\tfrac{80}{49}u_2^2v_2^2]\ ,$$

$$\eta=-\phi\frac{1}{8}\left[\frac{5}{\pi}\right]^{1/2}[\Gamma_0(u_0u_2-v_0v_2)+\Lambda_0''v_0u_2+\Lambda_0'u_0v_2]\ , \qquad (5.12)$$

$$\beta=-\frac{15}{49\pi}5^{1/2}\Lambda_0u_2^2v_2^2-\eta\ ,$$

$$\gamma=-\tfrac{1}{4}(\beta+\eta)\ .$$

The phase factor ϕ in Eq. (5.11) again is the same as in Eq. (2.7) and has been chosen such that the coefficient η always has a negative value as explained before. The ratios of the symmetry parameters are plotted in Fig. 5 as a function of v_2^2 and $|\chi|$. In the analysis of the spectrum of ^{195}Pt in terms of the SO(6)⊗U(2) symmetry, the values obtained for the ratios $\beta/\eta=-1.05$ and $\gamma/\zeta=-0.08$ are in good agreement with the values given in Fig. 5 for $|\chi|=0.80$. The value for $\zeta/\eta=-1.9$, however, is about twice as large as that extracted from Fig. 5.

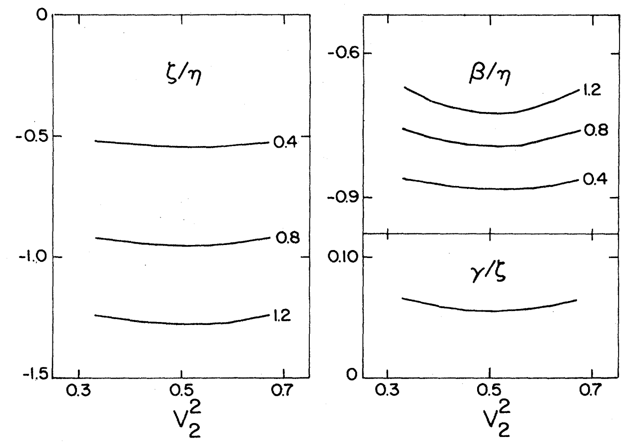

FIG. 5. Ratio of parameters in the SO(6)⊗U(2) limit as a function of v_2^2 for three different values of $|\chi|=0.4$, 0.8, and 1.2.

Finally in Fig. 6 the ratio η/Γ_0 is plotted. Using the value for η as found in the analysis of ^{195}Pt we obtain $\Gamma_0\simeq0.1$ MeV for $|\chi|=0.80$ and $v_2^2=0.65$. This value is considerably smaller than the one used in single-j shell calculations in the Pt region in Ref. 21. However, in these calculations a limited single particle space was used which gives rise to a larger strength of the effective quadrupole-quadrupole interaction.

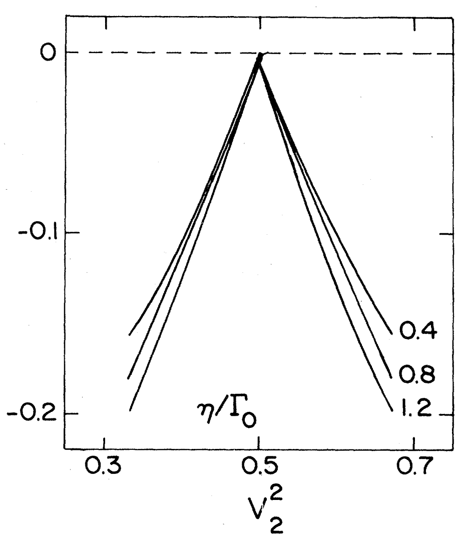

FIG. 6. Ratio η/Γ_0 as a function of v_2^2 for three different values of $|\chi|=0.4$, 0.8, and 1.2

VI. CONCLUSIONS

In this paper we have investigated the relation between a description of odd-A nuclei in terms of boson-fermion symmetries and that in terms of the IBFA model. As an example we studied the conditions under which the SO(6)$\otimes$U(2) symmetry arises from the IBFA model Hamiltonian. The same procedure can be used for other examples of dynamical boson-fermion symmetries. In a future publication, a detailed investigation will be presented of the other limits given in Eq. (2.3), the $\mathrm{U^{BF}}(5)$ and $\mathrm{SU^{BF}}(3)$ limits. It was shown that in order to obtain an exact correspondence between the two Hamiltonians an additional term in the boson-fermion interaction of the IBFA model had to be introduced. This extra term, which can be derived from a quadrupole pairing interaction between identical particles, has up to now been neglected in the IBFA Hamiltonian.

The close relation between the IBFA model and the multi-j boson-fermion symmetries is based on the fact that in both descriptions the Hamiltonian is invariant under an SU(2) pseudo-spin $s=\frac{1}{2}$ symmetry. It was found that the symmetries correspond to specific values of the occupation probabilities of the single-particle orbits in the IBFA Hamiltonian. This explains why only a few good examples of these symmetries are found in odd-even nuclei. In ^{195}Pt the "microscopic conditions" are indeed satisfied.

It should be realized that also the IBFA model offers only a simplified semimicroscopic description of odd-mass nuclei. If more terms are added to the IBFA Hamiltonian, such as for example an explicit hexadecapole force, symmetries can occur for a wider range of parameters. The good agreement for the values of the symmetry parameters obtained from the predictions, and those obtained from a phenomenological fit to ^{195}Pt, can be seen as an indication that the effective BF interaction used in the IBFA model is indeed complete.

The present investigation of the relation between the BF symmetry Hamiltonian and the IBFA model Hamiltonian can be seen as a first step towards a real microscopic understanding of BF symmetries in nuclei. In the IBFA model only the structure of the Hamiltonian is taken from microscopic considerations. The interaction strengths in the Hamiltonian are treated as adjustable parameters in phenomenological applications. This is similar in spirit to the approach taken for even-even nuclei where also the structure of the Hamiltonian is determined qualitatively using microscopic arguments but where a quantitative calculation of the model parameters has proven to be complicated.

In conclusion, we have shown that dynamical boson-fermion symmetries correspond to special cases of the "semimicroscopic" IBFA model Hamiltonian. An attractive feature of dynamical symmetries, both in even-even and odd-even nuclei, is that they provide a simple framework to classify and analyze the experimental data, which can then be used as a starting point for a more detailed description of nuclear properties by adding symmetry breaking terms in the IBFA.

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APPENDIX

The generators $G^{(\lambda)}_\mu(l,l')$ of the boson-fermion group $\mathrm{U^{(BF)}}(6)$ can be obtained by combining those of the boson group $\mathrm{U}^{(B)}(6)$,

$$B^{(\lambda)}_\mu(l,l')=(b_l^\dagger\tilde{b}_{l'})^{(\lambda)}_\mu ,\qquad b^\dagger_{00}=s^\dagger;\quad b^\dagger_{2m}=d^\dagger_m , \tag{A1}$$

with the corresponding ones of the pseudo-orbital group $\mathrm{U}^{(F)}_k(6)$,

$$K^{(\lambda)}_\mu(k,k')=-\sum_{jj'}[(2j+1)(2j'+1)]^{1/2}(-1)^{j'+\lambda+1/2}\times\begin{Bmatrix} j & j' & \lambda\\ k' & k & \frac{1}{2}\end{Bmatrix}(a_j^\dagger\tilde{a}_{j'})^{(\lambda)}_\mu \tag{A2}$$

into

$$\begin{aligned}G^{(\lambda)}_\mu(l,l)&=B^{(\lambda)}_\mu(l,l)+K^{(\lambda)}_\mu(l,l) ,\\ G^{(2)}_\mu(0,2)&=B^{(2)}_\mu(0,2)+\phi K^{(2)}_\mu(0,2) ,\\ G^{(2)}_\mu(2,0)&=B^{(2)}_\mu(2,0)+\phi K^{(2)}_\mu(2,0) ,\end{aligned} \tag{A3}$$

with $l=0,2$ and $\lambda=0,\ldots,4$. The generators $G^{(\lambda)}_\mu(l,l')$ in Eq. (A3) have been determined by requiring that they form a closed set under the commutation relations of the group $\mathrm{U^{(BF)}}(6)$. This, however, leaves a freedom of sign, ϕ, in $G^{(2)}_\mu(0,2)$ and $G^{(2)}_\mu(2,0)$. We note that the eigenvalues, as well as the eigenvectors of the Hamiltonian H_{sym} in Eq. (2.6), are invariant under this phase. The generators of the subgroups of $\mathrm{U^{(BF)}}(6)$ appearing in Eq. (2.3c) are

$$\begin{aligned}&\mathrm{SO^{(BF)}}(6):\ \{G^{(1)}_\mu(2,2),G^{(2)}_\mu(0,2)+G^{(2)}_\mu(2,0) ,\ G^{(3)}_\mu(2,2)\} ,\\ &\mathrm{SO^{(BF)}}(5):\ \{G^{(1)}_\mu(2,2),G^{(3)}_\mu(2,2)\} ,\\ &\mathrm{SO^{(BF)}}(3):\ \{G^{(1)}_\mu(2,2)\} .\end{aligned} \tag{A4}$$

Finally the generators of the spinor group Spin(3) are proportional to the total angular momentum operator

$$J^{(1)}_\mu=\sqrt{10}(d^\dagger\tilde{d})^{(1)}_\mu-\sum_j[\tfrac{1}{3}j(j+1)(2j+1)]^{1/2}(a_j^\dagger\tilde{a}_j)^{(1)}_\mu . \tag{A5}$$

The Casimir operators appearing in H_{sym}, Eq. (2.6) can be expressed in terms of the generators in Eqs. (A3)—(A5) as

$$C_{2\mathrm{U}^{(\mathrm{BF})}(6)} = G^{(0)}(0,0)\cdot G^{(0)}(0,0) + G^{(2)}(0,2)\cdot G^{(2)}(2,0) + G^{(2)}(2,0)\cdot G^{(2)}(0,2) + \sum_{\lambda=0}^{4} G^{(\lambda)}(2,2)\cdot G^{(\lambda)}(2,2) ,$$

$$C_{2\mathrm{SO}^{(\mathrm{BF})}(6)} = 2[G^{(2)}(0,2)+G^{(2)}(2,0)]\cdot[G^{(2)}(0,2)+G^{(2)}(2,0)] + 4\sum_{\lambda=1,3} G^{(\lambda)}(2,2)\cdot G^{(\lambda)}(2,2) ,$$

$$C_{2\mathrm{SO}^{(\mathrm{BF})}(5)} = 4\sum_{\lambda=1,3} G^{(\lambda)}(2,2)\cdot G^{(\lambda)}(2,2) ,$$

$$C_{2\mathrm{SO}^{(\mathrm{BF})}(3)} = 20G^{(1)}(2,2)\cdot G^{(1)}(2,2) ,$$

$$C_{2\mathrm{Spin}(3)} = 2J^{(1)}\cdot J^{(1)} .$$

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