Theory of mesonic and dibaryonic excitations in the πNN system: Derivation of πNN scattering equations

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A πNN theory, incorporating mesonic and dibaryonic excitation mechanisms, is introduced to give a unified description of NN and πd reactions. The mesonic mechanism is built into the theory by extending the conventional meson theory of nuclear force to include the isobar Δ excitation. The dibaryonic excitation at short distance is introduced according to current understanding of six-quark dynamics. The theory is free of the nucleon mass renormalization problem and is therefore tractable in practice. The model Hamiltonian consists of (a) V_{BB} for two-baryon interactions between NN, NΔ, and $\Delta\Delta$ states; (b) $h_{\pi N\leftrightarrow\Delta}$ for Δ excitation; (c) $v_{\pi N}$ for πN two-body interaction in nonresonant channels; (d) $F_{\pi NN\leftrightarrow NN}$ for nonresonant pion production; and (e) $H_{D\leftrightarrow BB}$ for the formation of a dibaryon state D. Dynamical equations for NN and πd scattering are derived by making the assumption that all NN and πd processes can be described in a subspace spanned by NN, NΔ, $\Delta\Delta$, πNN, and the dibaryon state D. The resulting scattering theory satisfies the essential two-body (NN) and three-body (πNN) unitarity relations. The projection technique is applied to cast the theory into a form such that all NN and πd reaction transition matrix elements can be calculated by solving, separately, a two-body integral equation and a Faddeev-type three-body equation. Both can be solved by well-established numerical methods. This makes the calculation based on the most sophisticated meson theory of nuclear force possible. Explicit formalisms have also been developed for exploring the question of the excitation of a dibaryon resonance during NN and πd scattering from the point of view of six-quark dynamics. The numerical results obtained from the theory are presented in a separate paper.

I. INTRODUCTION

The main feature of intermediate energy nuclear reaction, induced by pion, nucleon, electron, photon, or heavy ion, is the production or absorption of *on-mass-shell* pions. Therefore, a microscopic approach to the problem should start from a theory of the coupled NN+πNN system (called the πNN system from now on). An acceptable πNN theory should describe simultaneously all of the following processes:

$$\pi N \rightarrow \pi N(E_{lab} < 300\ \text{MeV}) , \tag{1.1a}$$

$$NN \rightarrow NN(E_{lab} < 1000\ \text{MeV}) , \tag{1.1b}$$

$$\rightarrow \pi NN , \tag{1.1c}$$

$$\pi d \rightarrow \pi d(E_{lab} < 300\ \text{MeV}) , \tag{1.1d}$$

$$\rightarrow \pi NN , \tag{1.1e}$$

$$\rightarrow NN . \tag{1.1f}$$

In addition, we must face the fact that at this higher energy two colliding hadrons are more likely to overlap and the effect of their internal quark structure could become important. To describe all the processes listed in Eq. (1.1), it may not be possible to parametrize the baryon-baryon interaction at short distance solely in terms of the conventional meson-baryon-baryon form factors or a phenomenological repulsive core.

Theoretical investigation in these two directions have been active in the past few years. The unitary πNN models[1-7] have succeeded in describing extensive spin-averaged πd data, but could not reproduce many features of the spin observables. Except for the phenomenological model of Ref. 4, these unitary models and several less ambitious approaches[8-12] so far have not been able to give a satisfactory description of NN scattering phase shifts. Another important development is the extension of the conventional meson theory of the NN potential to include the excitation of the Δ resonance. This approach[13-18] has achieved reasonable successes in describing NN scattering up[16] to 2 GeV and some πd scattering data.[18] However, difficulties are also encountered in describing NN spin observables. The investigation of NN short-range interaction[19-25] based on a quark mechanism is still in the developing stage. The existing models have only made very qualitative contact with the πNN data. In particular, pion production channels have not been considered in a realistic and unitary way.

All of these theoretical efforts, in particular the attempt to understand the energy dependences of NN and πd spin observables, have clearly indicated that none of these approaches can succeed without going beyond their present scope. The main purpose of this work is to unify these theoretical efforts by extending the unitary πNN theory[1–7] to account for six-quark dynamics at short distances and show how the theory can be applied in the study of extensive NN and πd data. Because of the complexity of the problem, we will report our results in three separate publications. In this paper we present our theory and focus on the derivations of basic scattering equations. In future work, we hope to discuss our numerical strategy and present our results on NN and πd scattering.

In Sec. II we discuss the basic πNN mechanisms based on the meson theory of nuclear force and the current understanding of six quark dynamics. We then postulate a model Hamiltonian of the πNN system which contains all of the essential physics and can be used to develop a mathematically rigorous and also *manageable* πNN scattering theory. To relate this work to previous unitary πNN theories, we derive in Sec. III all πNN scattering equations governed only by the mesonic mechanisms. In Sec. IV, we develop formalisms which include the excitation of a dibaryon state D in the study of NN and πd scattering. In Sec. V, we summarize our results. All numerical results and comparisons with the data are presented in separate papers.

II. MODEL HAMILTONIAN

To motivate our approach, let us first discuss qualitatively the mechanisms which should be considered in defining the πNN interactions. The interaction at long distance is due to the one-pion exchange, which is conventionally taken to describe the high partial-wave NN phase shifts. This pionic force is also responsible for exciting the nucleon to the Δ isobar state, which then decays into the πN state asymptotically if the collision energy is above the pion production threshold. In the current πNN models and the model we are going to consider, this pionic excitation is described by constructing an isobar model with a $\pi B' \leftrightarrow B$ vertex (B and B' could be N or Δ) to fit the πN scattering phase shift. Its relation to quark dynamics can be established through the chiral (cloudy) bag model,[26–28] although much work remains to be done in this direction.

The NN low partial-wave phase shifts clearly indicate that other mechanisms are at work at shorter distances. Here, we face an interesting and still unclear situation. First, it is undeniable that the heavy mesons, such as ω and ρ, are observed experimentally and they must play some role in determining the NN force. However, the exchange of heavy mesons is unrealistic if the size of a baryon is comparable to or larger than their Compton wavelength. This seems to be the case, as suggested by bag model study of nucleon structure. Therefore, the most probable mesonic process other than the one-pion exchange is the "sequential" exchange of two-pions. The most detailed analysis of the two-pion-exchange mechanism is the nonperturbative approach of the Paris group.[29] It is, therefore, advantageous to develop a theoretical framework in which the Paris potential (as well as other phenomenological or meson-exchange potentials) can be taken as the starting point to define the baryon-baryon interaction. An important step to describe pion production is to also define interactions which can couple the NN channel to NΔ and $\Delta\Delta$ states. The one-pion-exchange component of these coupling interactions can be generated by the vertex interaction $\pi B \leftrightarrow B'$. However, no two-pion-exchange model has been developed in a nonperturbative approach[29] to define the interactions between channels involving at least one Δ. It might be reasonable to follow the conventional approach[30] by including the exchange of a ρ meson to approximately describe this process. (Note that the coupling interaction NN$\to$NΔ or NN$\leftrightarrow\Delta\Delta$ can generate an effective NN interaction. Consequently, if these transition interactions are treated explicitly, we need to introduce a procedure to remove from the Paris potential the uncorrelated two-pion exchange with intermediate Δ excitation. This procedure has been introduced in Ref. 16.)

The short range part of the NN force is usually treated phenomenologically. At the present time, much experimental evidence, in particular the NN and πd spin observables, have pointed to the need for a more microscopic approach in defining baryon-baryon (BB) interactions at very short distances. Especially, the possible existence of the dibaryon states[31–33] can be better resolved if we relate the short range BB force to six-quark dynamics. We discuss this nontrivial connection to quark dynamics based on the bag model[34–36] and the resonating group method[19–21] of calculating baryon-baryon scattering.

The bag model calculation (for example, the calculation by Mulders *et al.*[34]) has predicted the masses of confined q^6 states in each $B=2$ color-singlet eigenchannel. It has been suggested that these confined q^6 states are the so-called dibaryon (one-body) states which could be excited at a very short distance during NN or πd scattering. Through their coupling to NN, NΔ, or πNN channels, some of these q^6 states may be responsible for the strong energy dependences seen in NN and πd spin observables. This interpretation implies that the baryon-baryon interaction has two entirely different mechanisms, just like the situation in the study of low energy nuclear reactions. The first mechanism is the fast direct process which can be described by an effective two-baryon potential, despite the involvement of internal structure of two interacting objects during the collision. The second process is the compound state formation in which each baryon has lost its own identity and a completely different q^6 configuration is excited.

This qualitative picture of q^6 dynamics is supported by the resonating group method[19–21] calculations of NN interaction within the nonrelativistic quark model. These calculations indicate that the quark exchange mechanism between two nucleons in an s wave can be effectively represented by a short-range repulsive nucleon-nucleon force as we have conventionally determined from NN phase shifts. We therefore argue that the conventional phenomenology for treating the short range BB interaction can effectively include this "fast" exchange process of quark dynamics. On the other hand, the compound state formation of quark dynamics is beyond the description of the resonating group method and must be added into the theory separately. It is our assumption that this unknown compound state formation mechanism, being mainly due to the confining force, can convert the incoming two baryon into a dibaryon state D with masses predicted by the q^6 bag model calculation.[34] In the same way that we use a $\pi N \leftrightarrow \Delta$ vertex to describe the Δ resonance, we also introduce a $BB \leftrightarrow D$ vertex[37] to describe the excitation of the dibaryon state D. The detailed structure of this dibaryon coupling form factor must be related to the complicated hadronization mechanism of quark degrees of freedom. Clearly, we can only treat this object phenomenologically at the present time. Our approach is clearly different from the P-matrix approach proposed by Jaffe and Low,[23] and explored by Mulders.[24] It is also different from the approach by Henley *et al.*[25] We will see that our model is the most economic way to obtain a theory which is directly related to the existing unitary πNN models. Many existing numerical methods which take into account the essential two- and three-body unitarities can then be readily applied in the study of dibaryon resonances.

In accord with the above arguments, we now assume that the most general model Hamiltonian for the coupled NN+πNN system takes the following form

$$H = H_0 + H_{\text{int}} \, , \tag{2.1}$$

$$H_{\text{int}} = h_{\pi B \leftrightarrow B'} + V_{B_1 B_2, B_1' B_2'} + H_{B_1 B_2 \leftrightarrow D} \, , \tag{2.2}$$

where H_0 is the sum of kinetic energy operators, D denotes the dibaryon states, and B can be N or Δ (the theory can be extended[16] to include higher mass isobars). Because of the vertex interaction $h_{\pi B \leftrightarrow B'}$, one sees that the one-particle states $|\,\mathrm{N}\rangle$ and $|\,\Delta\rangle$ are not stable, and the model can generate multipion states. This nature of the model causes difficult theoretical problems in deriving a mathematically rigorous but also *manageable* πNN scattering theory. In particular, in order to rigorously define the asymptotic πNN and NN wave functions, an appropriate approximation must be introduced to derive from the vertex interaction $h_{\pi B \leftrightarrow B'}$ a consistent description of both the mass of the *physical* nucleon and πN scattering in the P_{11} channel. In addition, the same derivation must also lead to a πNN scattering theory which satisfies the essential two- and three-body unitarity relations. As seen in a series of lengthy publications by Afnan and Blankleider,[1] and also by Avishai and Mizutani,[3] it is not easy to resolve the complexities involved in achieving these theoretical requirements even for the traditional model containing only a $\pi N \leftrightarrow N$ vertex. Needless to say, by considering a general $h_{\pi B' \leftrightarrow B}$ vertex, the problem will be even more complicated.

In this work we take a somewhat less ambitious approach. Following Ref. 4, the first simplification is to keep only the $\pi N \leftrightarrow \Delta$ part of the vertex interaction h. This approximation drastically simplifies the πNN scattering theory, because no mass renormalization problem of the nucleon will ever occur. However, some important πN physics is omitted by this simplification. First, πN scattering can only occur through the process $\pi N \leftrightarrow \Delta \leftrightarrow \pi N$ in the P_{33} channel. Second, pion absorption or production by two nucleons cannot happen except through the formation of Δ resonance, i.e., $NN\pi \leftrightarrow N\Delta \leftrightarrow NN$. To correct this shortcoming without complicating the scattering theory, we add a two-body potential $v_{\pi N}$ to describe πN scattering in channels other than P_{33}, and introduce a transition operator $F_{\pi NN \leftrightarrow NN}$ to describe the nonresonant pion absorption mechanism. Then the interaction H_{int} Eq. (2.2) takes the form

$$H_{\text{int}} \to H_{\text{int}} = h_{\pi N \leftrightarrow \Delta} + v_{\pi N} + V_{B_1 B_2, B_1' B_2'} + F_{\pi NN \leftrightarrow NN} + H_{B_1 B_2 \leftrightarrow D} \, . \tag{2.3}$$

A few words are needed to further justify the model defined by Eq. (2.3). Methods exist in the literature for constructing $v_{\pi N}$ and $F_{\pi NN \leftrightarrow NN}$. For practical calculations, it is simple to construct a separable model of $v_{\pi N}$ to fit the πN phase shifts. One can also apply the conventional reduction method[38] to derive $v_{\pi N}$ from the field theoretical amplitudes. The same method can also be used to derive the nonresonant pion production operator $F_{\pi NN \leftrightarrow NN}$ from the mesonic processes. For example, the earlier work by Koltun and Reitan[39] had constructed an effective operator $F_{\pi NN \leftrightarrow NN}$ from the pion rescattering processes. Their work was later extended[40–42] by several authors. A detailed model based on the chiral Lagrangian can be found in the work by Hachenberg and Pirner.[42] These classical works can be taken as a reasonalbe starting point to define the operator $F_{\pi NN \leftrightarrow NN}$ in our model. Therefore, the structure of each term in Eq. (2.3) is to a large extend known within meson theory. Our model is therefore not completely phenomenological in nature.

The rest of this paper is devoted to deriving, from the model Hamiltonian Eq. (2.3), a set of scattering equations for the study of all processes listed in Eq. (1.1). The essential two-body (NN) and three-body (πNN) unitarity cuts are built into the scattering theory by considering all of the πNN interactions in the model space spanned by NN, NΔ, $\Delta\Delta$, and πNN states. Because of the absence of the nucleon mass renormalization problem, the derivation of πNN scattering equations is straightforward. The main point of our derivation is to cast the scattering theory in a form such that numerical calculations can be efficiently carried out within the capacities of the existing computers. This is achieved by employing the projection technique to decompose the calculation into two parts. The first part is to solve a familiar Faddeev-type three-body equation. The solution is then used to do the πd calculation and to construct an effective baryon-baryon interaction which contains all of the dynamics due to the coupling of the baryon-baryon states to the πNN state. The second part of the calculation then only involves the solution of a set of coupled two-body scattering equations. The advantage of this decomposition is that the important meson-exchange baryon-baryon interactions which, as shown in Refs. 13–17, are essential for a correct description of NN scattering, can be treated exactly within the capacity of most existing computers. In carrying out calculations based on the unified formulation of Refs. 1 and 3, separable NN and πNN interactions have been used, mainly due to the fact that the number of the coupled NN and πNN channels is too large to be handled numerically. The use of the separable representation could be the reason why the NN results of Refs. 1, 2, 5, and 6 are not satisfactory.

III. MESONIC EXCITATION MECHANISM

In this section, we apply the formal scattering theory[43] to develop πNN scattering equations governed by the mesonic excitation mechanisms. Neglecting the coupling to the dibaryon state D, the remaining model Hamiltonian can be written as the following form

$$H = H_0 + H_{\rm int} , \tag{3.1}$$

where H_0 is the sum of kinetic energy operators for π, N, and the Δ isobar. The interactions between these three elementary degrees of freedom of the πNN system are defined by

$$H_{\rm int} = h_{\pi N \leftrightarrow \Delta} + v_{\pi N} + V_{BB} + F_{\pi NN \leftrightarrow NN} , \tag{3.2}$$

where V_{BB} is the simplified notation of $V_{B_1B_2,B_1'B_2'}$ of Eq. (2.3), which defines all possible direct interactions between NN, NΔ, and $\Delta\Delta$ two-baryon (BB) states. Each term of Eq. (3.2) is graphically represented in Fig. 1. To simplify our presentation, we will sometimes use shortened notations, h for $h_{\pi N \leftrightarrow \Delta}$, and F for $F_{\pi NN \leftrightarrow NN}$.

Since single pion production is the dominant inelastic channel in NN collision up to $\sim$2 GeV, it is reasonable to assume that all processes listed in Eq. (1.1) can be described in the subspace $BB \oplus {\rm NN}\pi$, where the baryon-baryon (BB) state can be NN, NΔ, or $\Delta\Delta$ states. The effects of πNΔ, $\pi\Delta\Delta$, and multipion states are neglected (all of the following derivations can be easily extended to include πNΔ and $\pi\Delta\Delta$. For notational simplicity, we keep only πNN in our presentation). Our task is to construct a set of dynamical equations which determine the transitions between the following three channels:

Channel	i
BB	1
NNπ	2
πd	3

Note that only the NN state of the BB channel ($i=1$) can exist asymptotically. The NΔ and $\Delta\Delta$ states only exist in the interaction region.

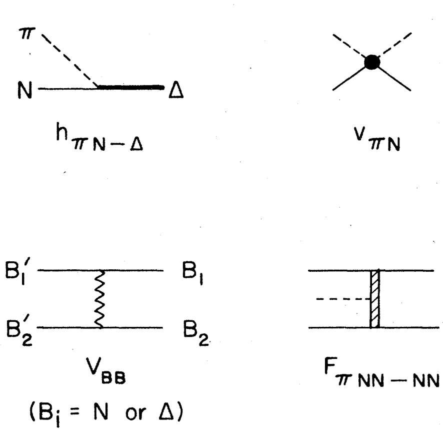

FIG. 1. Graphical representations of the mechanisms contained in the model Hamiltonian Eq. (3.2).

The channel Hamiltonians H_i for the $i=1,2,3$ channel are chosen to be

$$H_1 = (H_0)_1 , \tag{3.3a}$$

$$H_2 = (H_0)_2 , \tag{3.3b}$$

$$H_3 = (H_0)_3 + V_{\rm NN,NN} , \tag{3.3c}$$

where $V_{\rm NN,NN}$ is the NN$\to$NN part of V_{BB}. $(H_0)_i$ denotes the part of H_0 of Eq. (3.1) which is just the sum of kinetic energy operators of particles in the channel i. The channel wave functions (omitting spin-isospin indices) are defined by

$$H_1 \,|\, \mathbf{p}_1,\mathbf{p}_2\rangle = \left(\sqrt{m_1^2+p_1^2} + \sqrt{m_2^2+p_2^2}\,\right) |\, \mathbf{p}_1,\mathbf{p}_2\rangle , \tag{3.4a}$$

$$H_2 \,|\, \mathbf{p}_1,\mathbf{p}_2,\mathbf{k}\rangle = \Big(\sqrt{m_1^2+p_1^2} + \sqrt{m_2^2+p_2^2} + \sqrt{\mu^2+k^2}\Big) |\, \mathbf{p}_1,\mathbf{p}_2,\mathbf{k}\rangle , \tag{3.4b}$$

$$H_3 \,|\, \alpha \mathbf{d},\mathbf{k}\rangle = \left(\sqrt{M_{\rm d}^2+d^2} + \sqrt{\mu^2+k^2}\right) |\, \alpha\mathbf{d},\mathbf{k}\rangle , \tag{3.4c}$$

where $\mathbf{p}_i$ and $\mathbf{k}$ are, respectively, the momentum of baryon and pion, $|\alpha\mathbf{d}\rangle$ is the deuteron state, m_i, μ, and $M_{\rm d}$ denote, respectively, the mass of baryon, pion, and deuteron.

Following formal scattering theory,[43] the transition matrices between the considered three channels are defined as

$$\mathcal{T}_{ij}(E) = V_i' + V_i' \frac{1}{E - H + i\epsilon} V_j' , \quad i,j, = 1,2,3 , \tag{3.5}$$

where $V_i' = H - H_i$ is the interaction in the channel i. From the definition (3.3), we have

$$V_1' = V_2' = H_{\rm int} , \tag{3.6a}$$

$$V_3' = (V_{BB} - V_{\rm NN,NN}) + h_{\pi N\leftrightarrow\Delta} + v_{\pi N} + F_{\pi NN \leftrightarrow NN} . \tag{3.6b}$$

Because of the presence of vertex interactions $h_{\pi N\leftrightarrow \Delta}$ and $F_{\pi NN\leftrightarrow NN}$, the main feature of the transition amplitude is to provide mechanisms connecting two-particle baryon-baryon (BB) and three-particle πNN states. Clearly, Eq. (3.5) leads to a very large number of coupled integral equations. As an example, we list in Table I all channels of $J^\pi = 2^+$, $T=1$ which can couple to each other in NN and πd scattering. Unless one makes drastic simplifications, such as the use of a separable representation of the baryon-baryon interaction V_{BB}, it is very difficult (if not impossible) to solve these coupled integral equations. Since the main objective of our subsequent work will be to explore the extent to which the πNN data can be described by meson theory, it is necessary to develop a scheme in which this kind of simplification of the meson-exchange dynamics can be avoided in our numerical calculation. The main point of the derivation presented in this paper is to cast Eq. (3.5) in a form such that the dynamics in the BB and πNN subspaces can be calculated separately. Then we will see that well-established numerical methods for solving the two-body coupled-channel integral equation[44] and the Faddeev-type three-body equation[45] can be applied within our present computational power. The core of our formalism is a baryon-baryon coupled-channel scattering equation. We derive this equation in subsection A for the study of NN scattering. In

TABLE I. The baryon-baryon and πNN states which must be included in solving the πNN+NN coupled equation in the $J^\pi=2^+$, $T=1$ eigenchannel. l_π is the pion angular momentum relative to the NN pair. Only the $l\leq 3$ NN pair in the πNN state is considered.

T	J^π	NN	NΔ	$\Delta\Delta$	l_π	πNN NN
1	2^+	$^1D_2$	$^5S_2$	$^5S_2$	0	$^3P_2+{}^3F_2$
			$^5D_2$	$^7D_2$	1	$^1S_0, {}^1D_2, {}^3S_1+{}^3D_1, {}^3D_2$
			$^3D_2$	$^5D_2$	2	$^3P_1, {}^3P_2+{}^3F_2, {}^1F_3$
			$^5G_2$	$^3D_2$	3	$^1D_2, {}^3S_1+{}^3D_1, {}^3D_2$
				$^1D_2$	4	$^3P_2+{}^3F_2$
				$^7G_2$		
				$^5G_2$		

subsections B and C, we show that the resulting BB amplitudes are basic inputs to the formalisms for the πd scattering and pion absorption or production.

A. NN scattering

According to Eq. (3.5), the NN scattering amplitude is defined by

$$T_{\rm NN,NN}(E)=\langle {\rm NN}\,|\,\mathcal{T}_{11}(E)\,|\,{\rm NN}\rangle\ . \tag{3.7}$$

We will show that this amplitude can be obtained by solving a two-body integral equation. To proceed, we define the projection operator on the baryon-baryon (BB) space

$$\begin{aligned}P&=|\,BB\,\rangle\langle BB\,|\\ &=P_{\rm NN}+P_{\rm N\Delta}+P_{\Delta\Delta}\\ &=|\,{\rm NN}\rangle\langle{\rm NN}\,|+|\,{\rm N}\Delta\rangle\langle{\rm N}\Delta\,|+|\,\Delta\Delta\rangle\langle\Delta\Delta\,|\ ,\end{aligned} \tag{3.8a}$$

where $|\,{\rm NN}\rangle$, $|\,{\rm N}\Delta\rangle$, and $|\,\Delta\Delta\rangle$ are the eigenstates of the kinetic energy operator H_0 [as defined in Eq. (3.4a)]. Since πNN is the only three-particle state considered, the numerator of the propagator of Eq. (3.5) can be decomposed as

$$1=P+Q\ , \tag{3.8b}$$

with

$$Q=1-P=|\,\pi{\rm NN}\rangle\langle\pi{\rm NN}\,|\ , \tag{3.8c}$$

where $|\,\pi{\rm NN}\rangle$ is also the eigenstate of H_0 [as defined in Eq. (3.4b)]. The next step is to write Eq. (3.5) in such a way that the interaction in the πNN Q space can be separately handled by the standard three-body method.

Let us introduce a baryon-baryon transition operator which is defined by projecting the operator $\mathcal{T}_{11}(E)$ of Eq. (3.5) on the P space

$$T(E)=P\mathcal{T}_{11}(E)P\ , \tag{3.9a}$$

where, according to Eq. (3.6a) for V_1' and Eq. (3.8),

$$\mathcal{T}_{11}(E)=H_{\rm int}+H_{\rm int}\frac{P+Q}{E-H+i\epsilon}H_{\rm int}\ . \tag{3.9b}$$

The on-energy-shell matrix element of T between $|\,{\rm NN}\rangle$ states defines the NN elastic scattering, i.e.,

$$\begin{aligned}T_{\rm NN,NN}(E)&=\langle{\rm NN}\,|\,\mathcal{T}_{11}(E)\,|\,{\rm NN}\rangle\\ &=\langle{\rm NN}\,|\,T(E)\,|\,{\rm NN}\rangle\ .\end{aligned}$$

We will see in subsections B and C that other parts of the matrix elements of T in P space are the basic inputs to the calculations of all NN and πd reactions listed in Eq. (1.1).

By employing the standard Feshbach projection procedure, we get from Eq. (3.9) that (from now on, $+i\epsilon$ in the propagator will be omitted)

$$T(E)=V(E)+V(E)\frac{P}{E-H_0-V(E)}V(E)\ , \tag{3.10a}$$

where the effective BB interaction is

$$V(E)=PH_{\rm int}P+PH_{\rm int}Q\frac{Q}{E-QHQ}QH_{\rm int}P\ . \tag{3.10b}$$

By using Eq. (3.2), we have

$$\begin{aligned}PH_{\rm int}P&=V_{BB}\ ,\\ QH_{\rm int}P&=h_{\pi{\rm N}\leftrightarrow\Delta}+F_{\pi{\rm NN}\leftrightarrow{\rm NN}}\ ,\\ QHQ&=H_0+v_{\pi{\rm N}}+V_{\rm NN,NN}\ .\end{aligned}$$

The above relations allow us to write Eq. (3.10) explicitly as

$$\begin{aligned}V(E)=V_{BB}+(h_{\pi{\rm N}\leftrightarrow\Delta}+F_{\pi{\rm NN}\leftrightarrow{\rm NN}})&\frac{Q}{E-H_0-v_{\pi{\rm N}}-V_{\rm NN,NN}}\\ &\times(h_{\pi{\rm N}\leftrightarrow\Delta}+F_{\pi{\rm NN}\leftrightarrow{\rm NN}})\ .\end{aligned} \tag{3.11}$$

Using the following well-known operator relations for any two operators A and B,

$$\frac{1}{A-B}=\frac{1}{A}+\frac{1}{A}B\frac{1}{A-B}\ , \tag{3.12a}$$

$$=\frac{1}{A}+\frac{1}{A-B}B\frac{1}{A}\ , \tag{3.12b}$$

$$=\frac{1}{A}+\frac{1}{A}\tau\frac{1}{A}\ , \tag{3.12c}$$

with

$$\tau=B+B\frac{1}{A-B}B\ , \tag{3.12d}$$

we can write the propagator of Eq. (3.11) in Q space as

$$\begin{aligned}&\frac{Q}{E-H_0-v_{\pi{\rm N}}-V_{\rm NN,NN}}\\ &\quad=\frac{Q}{E-H_0}+\frac{Q}{E-H_0}t_Q(E)\frac{Q}{E-H_0}\ ,\end{aligned} \tag{3.13a}$$

where

$$\begin{aligned}t_Q(E)=(V_{\rm NN,NN}+v_{\pi{\rm N}})&+(V_{\rm NN,NN}+v_{\pi{\rm N}})\\ &\times\frac{Q}{E-H_0-V_{\rm NN,NN}-v_{\pi{\rm N}}}(V_{\rm NN,NN}+v_{\pi{\rm N}})\ .\end{aligned} \tag{3.13b}$$

$t_Q(E)$ is the $\pi{\rm NN}\rightarrow\pi{\rm NN}$ amplitude which can be dealt with by employing the standard three-body method.[45]

Substituting Eq. (3.13a) into Eq. (3.11), we can write the effective two-baryon interaction as

$$V(E)=V_{BB}+\Sigma_\Delta(E)+V_E(E)+V_3(E)+V_a(E)\ , \tag{3.14}$$

where

$$\Sigma_\Delta(E)=\sum_{i=1}^{2} h_i^\dagger \frac{Q}{E-H_0} h_i\ , \tag{3.15a}$$

$$V_E(E)=\sum_{i\neq j}^{2} h_i^\dagger \frac{Q}{E-H_0} h_j\ , \tag{3.15b}$$

$$V_3(E)=\sum_{i,j}^{2} h_i^\dagger \frac{Q}{E-H_0} t_Q(E) \frac{Q}{E-H_0} h_j\ , \tag{3.15c}$$

$$V_a(E)=F^\dagger\left[\frac{Q}{E-H_0}+\frac{Q}{E-H_0}t_Q(E)\frac{Q}{E-H_0}\right]F + \sum_i \left[F^\dagger\left[\frac{Q}{E-H_0}+\frac{Q}{E-H_0}t_Q\frac{Q}{E-H_0}\right]h_i + h_i^\dagger\left[\frac{Q}{E-H_0}+\frac{Q}{E-H_0}t_Q\frac{Q}{E-H_0}\right]F\right]\ , \tag{3.15d}$$

h_i and F are, respectively, the shortened notation for the transition operators $h_{\pi N\leftrightarrow\Delta}$ and $F_{\pi NN\leftrightarrow NN}$. Each term of the above effective BB interactions is graphically illustrated in Fig. 2. Clearly, Σ_Δ is the Δ self-energy, V_E is the one-pion-exchange interaction between NΔ states, V_3 and V_a contain all of the dynamics in the πNN intermediate state.

To emphasize the Δ resonant effect, we regroup the terms of $V(E)$ into

$$V(E)=V'_{BB}+\Sigma_\Delta(E)+V_c(E)\ , \tag{3.16}$$

where

$$V'_{BB}=V_{BB}-V_{N\Delta,N\Delta}\ , \tag{3.17a}$$

$$V_c(E)=V_{N\Delta,N\Delta}+V_E(E)+V_3(E)+V_a(E)\ . \tag{3.17b}$$

$V_{N\Delta,N\Delta}$ is the N$\Delta\to$NΔ part of the BB interaction V_{BB}. V_c clearly contains all of the "connected" BB interactions due to the coupling to πNN. Our next task is to write the scattering equation (3.9) in terms of Σ_Δ, V'_{BB}, and V_c. This can be done straightforwardly by finding algebraic relations between the following resolvents

$$R(E)=\frac{P}{E-H_0-V(E)} = \frac{P}{E-H_0-V'_{BB}-V_c(E)-\Sigma_\Delta(E)}\ , \tag{3.18a}$$

$$R_c(E)=\frac{P}{E-H_0-\Sigma_\Delta(E)-V_c(E)}\ , \tag{3.18b}$$

$$R_\Delta(E)=\frac{P}{E-H_0-\Sigma_\Delta(E)}\ . \tag{3.18c}$$

Note that the only interaction in $R_\Delta(E)$ is the Δ self-energy $\Sigma_\Delta(E)$. The "connected" BB interaction V_c is isolated in $R_c(E)$. $R(E)$ is the exact BB propagator in Eq. (3.9). By using the operator properties Eq. (3.12), it is easy to see that

$$R(E)=R_c(E)+R_c(E)V'_{BB}R(E)\ , \tag{3.19a}$$

$$=R_c(E)+R(E)V'_{BB}R_c(E)\ , \tag{3.19b}$$

$$=R_c(E)+R_c(E)T_0(E)R_c(E)\ , \tag{3.19c}$$

where

$$T_0(E)=V'_{BB}+V'_{BB}R(E)V'_{BB}\ . \tag{3.20}$$

By comparing the second terms on the right-hand sides of Eq. (3.19) it is evident that

$$V'_{BB}(E)R(E)=T_0(E)R_c(E)\ , \tag{3.21a}$$

$$R(E)V'_{BB}(E)=R_c(E)T_0(E)\ . \tag{3.21b}$$

Substituting Eq. (3.21) into Eq. (3.20), we get the following integral equation form of T_0

$$T_0(E)=V'_{BB}+V'_{BB}R_c(E)T_0(E)\ , \tag{3.22a}$$

$$=V'_{BB}+T_0(E)R_c(E)V'_{BB}\ . \tag{3.22b}$$

Because R_c contains the interaction V_c in the denominator, Eq. (3.22) is not useful for a practical calculation. A more useful form can be derived by using the following relations

$$R_c(E)=R_\Delta(E)+R_\Delta(E)V_c(E)R_c(E)\ , \tag{3.23a}$$

$$=R_\Delta(E)+R_c(E)V_c(E)R_\Delta(E)\ , \tag{3.23b}$$

$$=R_\Delta(E)+R_\Delta(E)T_c(E)R_\Delta(E)\ , \tag{3.23c}$$

where

$$T_c(E)=V_c(E)+V_c(E)R_c(E)V_c(E)\ . \tag{3.24}$$

Equations (3.23) leads to

$$V_c(E)R_c(E)=T_c(E)R_\Delta(E)\ , \tag{3.25a}$$

$$R_c(E)V_c(E)=R_\Delta(E)T_c(E)\ , \tag{3.25b}$$

and hence,

$$T_c(E)=V_c(E)+V_c(E)R_\Delta(E)T_c(E)\ , \tag{3.26a}$$

$$=V_c(E)+T_c(E)R_\Delta(E)V_c(E)\ . \tag{3.26b}$$

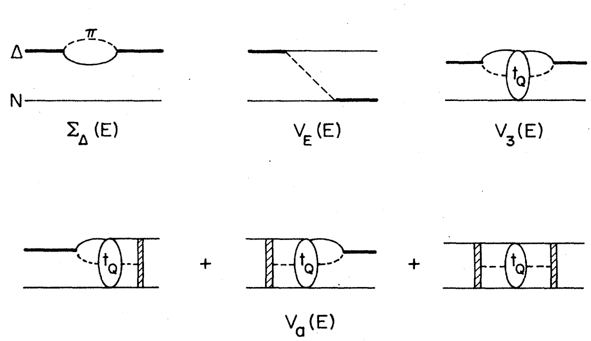

FIG. 2. Graphical representations of the effective baryon-baryon interactions defined by Eq. (3.15).

Substituting Eq. (3.23c) into Eq. (3.22), we obtain the following integral equation [recalling Eq. (3.18c) for $R_\Delta(E)$]

$$T_0(E)=V'_{BB}+V'_{BB}\frac{P}{E-H_0-\Sigma_\Delta(E)}T_0(E)+V'_{BB}\frac{P}{E-H_0-\Sigma_\Delta(E)}T_c(E)\times\frac{P}{E-H_0-\Sigma_\Delta(E)}T_0(E)\,, \qquad (3.27a)$$

or

$$T_0(E)=V'_{BB}+T_0(E)\frac{P}{E-H_0-\Sigma_\Delta(E)}V'_{BB}+T_0(E)\frac{P}{E-H_0-\Sigma_\Delta(E)}T_c(E)\times\frac{P}{E-H_0-\Sigma_\Delta(E)}V'_{BB}\,. \qquad (3.27b)$$

If we set T_c to zero, Eq. (3.27) reduces to the form of the conventional coupled-channel equation.[13–18] It is important to note here that, in our approach, the "width" $\Sigma_\Delta(E)$ is the propagating *off-shell* Δ is defined in terms of the $\pi N\leftrightarrow\Delta$ vertex as described by Eq. (3.15a). This is the consequence of the πNN unitary cut, as discussed in Ref. 16.

We now show that the full BB scattering T matrix, Eq. (3.10), can be expressed in terms of T_0 and T_c. To facilitate this derivtion, we need to relate $R(E)$ to $R_\Delta(E)$ by [using Eq. (3.12) again]

$$R(E)=R_\Delta(E)+R_\Delta(E)[V'_{BB}+V_c(E)]R(E)\,, \qquad (3.28a)$$

$$=R_\Delta(E)+R(E)[V'_{BB}+V_c(E)]R_\Delta(E)\,, \qquad (3.28b)$$

$$=R_\Delta(E)+R_\Delta(E)T_{BB}(E)R_\Delta(E)\,, \qquad (3.28c)$$

where

$$T_{BB}(E)=[V'_{BB}+V_c(E)]+[V'_{BB}+V_c(E)]\times R(E)[V'_{BB}+V_c(E)]\,. \qquad (3.29)$$

It follows that

$$R(E)[V'_{BB}+V_c(E)]=R_\Delta(E)T_{BB}(E)\,, \qquad (3.30a)$$

$$[V'_{BB}+V_c(E)]R(E)=T_{BB}(E)R_\Delta(E)\,. \qquad (3.30b)$$

By using all of the above properties, it is straightforward to express T_{BB} in terms of T_0 and T_c as follows

$$\begin{aligned}T_{BB}&=(V'_{BB}+V_c)+(V'_{BB}+V_c)R(V'_{BB}+V_c)\\&=(V'_{BB}+V'_{BB}RV'_{BB})+(V_c+V_cRV'_{BB}+V'_{BB}RV_c+V_cRV_c)\\&=T_0+V_c+V_cR_cT_0+T_0R_cV_c+V_c(R_c+R_cV'_{BB}R)V_c\\&=T_0+V_c+T_cR_\Delta T_0+T_0R_\Delta T_c+T_cR_\Delta V_c+T_cR_\Delta T_0R_\Delta T_c\\&=(V_c+T_cR_\Delta V_c)+(T_0+T_cR_\Delta T_0+T_0R_\Delta T_c+T_cR_\Delta T_0R_\Delta T_c)\\&=T_c(E)+[1+T_c(E)R_\Delta(E)]T_0(E)[1+R_\Delta(E)T_c(E)]\,. \qquad (3.31)\end{aligned}$$

Defining the scattering operators

$$\Omega_c^{(+)}(E)=1+R_\Delta(E)T_c(E)\,, \qquad (3.32a)$$

$$\Omega_c^{(-)+}(E)=1+T_c(E)R_\Delta(E)\,, \qquad (3.32b)$$

we can write T_{BB} as a familiar distorted-wave form

$$T_{BB}(E)=T_c(E)+\Omega_c^{(-)+}(E)T_0(E)\Omega_c^{(+)}(E)\,. \qquad (3.33)$$

The physics of Eq. (3.33) is clear. The connected operators T_c and $\Omega_c^{(\pm)}$ describe pion multiple scattering between two baryons. The operator T_0 couples this multiple scattering process to other πNN dynamics through the transition interaction $V'_{BB}=V_{BB}-V_{N\Delta,N\Delta}$ [see T_0 in Eq. (3.27)].

The last step of our derivation is to use Eq. (3.16) for the effective BB interaction $V(E)$, and the above operator relations to write the BB scattering operator $T(E)$ Eq. (3.10) as

$$T(E)=V(E)+V(E)\frac{1}{E-H_0-V(E)}V(E)$$
$$=(V'_{BB}+\Sigma_\Delta+V_c)+(V'_{BB}+\Sigma_\Delta+V_c)R(V'_{BB}+\Sigma_\Delta+V_c)$$
$$=[(V'_{BB}+V_c)+(V_{BB}+V_c)R(V'_{BB}+V_c)]$$
$$+[\Sigma_\Delta+\Sigma_\Delta R(V'_{BB}+V_c)]+[(V'_{BB}+V_c)R\Sigma_\Delta+\Sigma_\Delta R\Sigma_\Delta]$$
$$=T_{BB}+(\Sigma_\Delta+\Sigma_\Delta R_\Delta T_{BB})+[T_{BB}R_\Delta\Sigma_\Delta+\Sigma_\Delta(R_\Delta+R_\Delta T_{BB}R_\Delta)\Sigma_\Delta]$$
$$=[\Sigma_\Delta(E)+\Sigma_\Delta(E)R_\Delta(E)\Sigma_\Delta(E)]+[1+\Sigma_\Delta(E)R_\Delta(E)]T_{BB}[1+R_\Delta(E)\Sigma_\Delta(E)]$$
$$=t_\Delta(E)+\frac{E-H_0}{E-H_0-\Sigma_\Delta}T_{BB}\frac{E-H_0}{E-H_0-\Sigma_\Delta(E)}\ , \tag{3.34}$$

where

$$t_\Delta(E)=\Sigma_\Delta(E)+\Sigma_\Delta(E)R_\Delta(E)\Sigma_\Delta(E)$$
$$=\frac{E-H_0}{E-H_0-\Sigma_\Delta(E)}\Sigma_\Delta(E) \tag{3.35}$$

describes the "disconnected" NΔ interaction. We will see later that t_Δ plays an important role in deriving equations for the study of the πd reaction.

It is evident from the definitions of Σ_Δ [Eq. (3.15)] that

$$\langle \mathrm{NN}\,|\,t_\Delta\,|\,\mathrm{NN}\rangle=0\ ,$$
$$\frac{E-H_0}{E-H_0-\Sigma_\Delta}\,|\,\mathrm{NN}\rangle=|\,\mathrm{NN}\rangle\ .$$

Therefore, the NN elastic scattering amplitude is simply

$$T_{\mathrm{NN,NN}}=\langle \mathrm{NN}\,|\,\mathscr{T}_{11}(E)\,|\,\mathrm{NN}\rangle$$
$$=\langle \mathrm{NN}\,|\,T(E)\,|\,\mathrm{NN}\rangle$$
$$=\langle \mathrm{NN}\,|\,T_c+\Omega_c^{(-)+}T_0\Omega_c^{(+)}\,|\,\mathrm{NN}\rangle\ . \tag{3.36}$$

Recalling the definition Eq. (3.32) for $\Omega_c^{(\pm)}$, we see that the NN amplitude can be completely expressed in terms of T_c and T_0, which can be obtained by solving two separate integral equations, Eqs. (3.26) and (3.27). We note that the πNN branch cut is isolated in T_c which only diverges logarithmically. Therefore, the kernals of the integral Eq. (3.27) for T_0 are compact. This equation can be solved by using standard matrix methods on the real momentum axis. Of course, care must be taken to handle the πNN branch cut in evaluating T_c. The standard contour rotation method[45] is most convenient in our approach. Since only two-baryon states are coupled to each other in Eq. (3.27) for T_0, the size of the resulting integral equation can be handled by most existing computers. In this way, calculations with meson-exchange models of V_{BB}, such as the one derived from the Paris potential, can be done exactly. No separable approximation is needed.

If we neglect the nonresonant pion production interaction $F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$, then

$$\Omega_c^{(\pm)}\,|\,\mathrm{NN}\rangle\to|\,\mathrm{NN}\rangle\ , \tag{3.37a}$$
$$\langle \mathrm{NN}\,|\,T_c\,|\,\mathrm{NN}\rangle\to 0\ . \tag{3.37b}$$

The NN scattering amplitude becomes

$$T_{\mathrm{NN,NN}}=\langle \mathrm{NN}\,|\,T_0(E)\,|\,\mathrm{NN}\rangle\ . \tag{3.38}$$

Equation (3.38) is precisely the equation used in the study by Betz and Lee.[4] This approximation should be reasonable to study NN and πd scattering in the region where the nonresonant pion production is not important.

B. πd scattering

The πd scattering has been extensively studied by using the well-studied three-body method.[45] A more useful and practical approach to study πd reactions is, therefore, to separate the three-body multiple scattering process from the rest of the πNN dynamics. In this section, we introduce such an approach to show that all πd amplitudes listed in Eq. (1.1) can be expressed in terms of the solution of a Faddeev-type equation and the BB amplitude T of Eq. (3.34). For notational simplicity, we neglect the less important $\Delta\Delta$ state in the following presentation. Including the $\Delta\Delta$ state is straightforward.

The only tool needed in the following derivations is again the operator relation Eq. (3.12). The first step is to decompose the total Hamiltonian Eq. (3.1) into two parts

$$H=H_F+V_{\mathrm{off}}\ , \tag{3.39}$$

where

$$H_F=H_0+v_{\pi\mathrm{N}}+h_{\pi\mathrm{N}\leftrightarrow\Delta}+V_{\mathrm{NN,NN}}+V_{\mathrm{N\Delta,N\Delta}}\ , \tag{3.40}$$
$$V_{\mathrm{off}}=(V'_{BB}-V_{\mathrm{NN,NN}})+F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$$
$$=(V_{BB}-V_{\mathrm{NN,NN}}-V_{\mathrm{N\Delta,N\Delta}})+F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$$
$$=V_{\mathrm{NN}\leftrightarrow\mathrm{N\Delta}}+F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}\ . \tag{3.41}$$

The main feature of H_F is that it does not couple the πNN channel to the NN state. Therefore, H_F defines the standard pion or Δ multiple scattering process which can be handled by a three-body method (such as the method described in Refs. 45 and 46).

In the considered space $BB\oplus\pi NN$, the πd reaction amplitudes determined solely by H_F can be formally written as

$$T_{ij}^F = V_i^F + V_i^F \frac{1}{E-H_F} V_j^F , \tag{3.42a}$$

$$= V_i^F \omega_j^{(+)} , \tag{3.42b}$$

$$= \omega_i^{(-)+} V_j^F , \tag{3.42c}$$

where $i,j=1,2,3$ denote, respectively, the $N\Delta$, πNN, and πd states. Note that the NN channel can be excluded in solving Eq. (3.42), because there is no pion absorption mechanism in H_F. The channel interactions V_i^F are defined by

$$\begin{aligned} V_1^F &= V_2^F \\ &= H_F - H_0 \\ &= h_{\pi N\leftrightarrow\Delta} + v_{\pi N} + V_{NN,NN} + V_{N\Delta,N\Delta} , \end{aligned} \tag{3.43}$$

$$\begin{aligned} V_3^F &= H_F - K_\pi - H_d \\ &= h_{\pi N\leftrightarrow\Delta} + v_{\pi N} + V_{N\Delta,N\Delta} , \end{aligned} \tag{3.44}$$

where $H_d = H_0 + V_{NN,NN}$ is the Hamiltonian for the deuteron. The scattering operators are defined by

$$\omega_j^{(\pm)} = 1 + \frac{1}{E-H_F \pm i\epsilon} V_j^F . \tag{3.45}$$

The channel wave function can be written as

$$\begin{aligned} |\chi_j\rangle_\pm &= \omega_j^{(\pm)} |j\rangle \\ &= \sum_i |i\rangle \left[\delta_{ij} + \frac{1}{E-H_i} T_{ij}^F \right] , \end{aligned} \tag{3.46}$$

where H_i has been defined in Eq. (3.3).

For our later calculations, we need to solve Eq. (3.42) in order to determine the following Faddeev amplitudes

$$\begin{aligned} T_{\pi d,\pi d}^F &= \langle \pi d | T_{33}^F | \pi d\rangle , \\ T_{\pi d,N\Delta}^F &= \langle \pi d | T_{31}^F | N\Delta\rangle , \text{ etc.} \end{aligned} \tag{3.47}$$

Methods of calculating these amplitudes have been widely discussed in the literature[45] and will be described in our subsequent papers in which we present our numerical results. Here, we focus on the role of $V_{\rm off}$.

According to Eqs. (3.6) and (3.44), we see that the channel interaction V_i' defined previously in Sec. III A is $V_i' = V_i^F + V_{\rm off}$. Then, the scattering amplitude Eq. (3.5) for πd scattering can be written as

$$T_{\pi d,\pi d} = \langle \pi d | \mathscr{T}_{33} | \pi d\rangle , \tag{3.48}$$

where

$$\mathscr{T}_{33} = (V_3^F + V_{\rm off}) + (V_3^F + V_{\rm off}) \frac{1}{E-H} (V_3^F + V_{\rm off}) . \tag{3.49}$$

The next step is to use Eq. (3.39) to write [via Eq. (3.12)]

$$\frac{1}{E-H} = \frac{1}{E-H_F} + \frac{1}{E-H_F} T_{\rm off} \frac{1}{E-H_F} , \tag{3.50}$$

where

$$T_{\rm off} = V_{\rm off} + V_{\rm off} \frac{1}{E-H} V_{\rm off} . \tag{3.51}$$

It follows that

$$V_{\rm off} \frac{1}{E-H} = T_{\rm off} \frac{1}{E-H_F} , \tag{3.52a}$$

$$\frac{1}{E-H} V_{\rm off} = \frac{1}{E-H_F} T_{\rm off} . \tag{3.52b}$$

By using Eq. (3.50), we have

$$\begin{aligned} V_3^F + V_3^F \frac{1}{E-H} V_3^F &= V_3^F + V_3^F \frac{1}{E-H_F} V_3^F \\ &\quad + V_3^F \frac{1}{E-H_F} T_{\rm off} \frac{1}{E-H_F} V_3^F . \end{aligned} \tag{3.53}$$

Equations (3.50)–(3.53) lead us to obtain

$$\begin{aligned} \mathscr{T}_{33} &= \left[V_3^F + V_3^F \frac{1}{E-H_F} V_3^F \right] + \left[V_{\rm off} + V_{\rm off} \frac{1}{E-H} V_{\rm off} \right] \\ &\quad + V_3^F \frac{1}{E-H} V_{\rm off} + V_{\rm off} \frac{1}{E-H} V_3^F + V_3^F \frac{1}{E-H_F} T_{\rm off} \frac{1}{E-H_F} V_3^F \\ &= T_{33}^F + T_{\rm off} + V_3^F \frac{1}{E-H_F} T_{\rm off} + T_{\rm off} \frac{1}{E-H_F} V_3^F + V_3^F \frac{1}{E-H_F} T_{\rm off} \frac{1}{E-H_F} V_3^F \\ &= T_{33}^F + \left[1 + V_3^F \frac{1}{E-H_F} \right] T_{\rm off} \left[1 + \frac{1}{E-H_F} V_3^F \right] \\ &= T_{33}^F + \omega_3^{(-)+} T_{\rm off} \omega_3^{(+)} , \end{aligned} \tag{3.54}$$

where $\omega_3^{(\pm)}$ have been defined in Eq. (3.45). By using Eqs. (3.46) and (3.47) we then can write the πd amplitude Eq. (3.48) in a compact distorted-wave form

$$T_{\pi d,\pi d}=T^F_{\pi d,\pi d}+\langle \chi^{(-)}_{\pi d} \mid T_{\rm off} \mid \chi^{(+)}_{\pi d}\rangle \ . \qquad (3.55)$$

The above derivation can be readily extended to obtain the amplitude for breakup process

$$T_{\pi NN,\pi d}=T^F_{\pi NN,\pi d}+\langle \chi^{(-)}_{\pi NN} \mid T_{\rm off} \mid \chi^{(+)}_{\pi d}\rangle \ . \qquad (3.56)$$

We now need to evaluate the second term of Eqs. (3.55) and (3.56). Since the Faddeev wave function $\mid \chi^{(\pm)}_{\pi d}\rangle$ or $\mid \chi^{(\pm)}_{\pi NN}\rangle$ does not have NN, πNΔ, or $\pi\Delta\Delta$ components, it is obvious from Eq. (3.41) that

$$V_{\rm off} \mid \chi^{(\pm)}_{\pi d}\rangle=(V_{NN\leftrightarrow N\Delta}+F_{NN\leftrightarrow \pi NN}) \mid \chi^{(\pm)}_{\pi d}\rangle$$
$$=P_{NN}(V_{NN\leftrightarrow N\Delta}+F_{NN,\pi NN}) \mid \chi^{(\pm)}_{\pi d}\rangle \ , \qquad (3.57)$$

where P_{NN} is the projection onto the NN state. It then follows that

$$\langle \chi^{(-)}_{\substack{\pi d\\ \pi NN}} \mid V_{\rm off} \mid \chi^{(+)}_{\pi d}\rangle \equiv 0 \ .$$

By using Eq. (3.51) and the above property, we get

$$\langle \chi^{(-)}_{\pi d} \mid T_{\rm off} \mid \chi^{(+)}_{\pi d}\rangle=\left\langle \chi^{(-)}_{\pi d} \left| V_{\rm off}\frac{P_{NN}}{E-H}V_{\rm off} \right| \chi^{(+)}_{\pi d}\right\rangle \ . \qquad (3.58)$$

Making use of Eq. (3.8) for the definition of the *BB* transition operator, we have

$$\frac{P_{NN}}{E-H}=\frac{P_{NN}}{E-H_0}+\frac{P_{NN}}{E-H_0}\left[H_{\rm int}+H_{\rm int}\frac{1}{E-H}H_{\rm int}\right]\frac{P_{NN}}{E-H_0}$$
$$=\frac{P_{NN}}{E-H_0}+\frac{P_{NN}}{E-H_0}T_{NN,NN}\frac{P_{NN}}{E-H_0} \ . \qquad (3.59)$$

Note that $T_{NN,NN}$ is the exact NN amplitude defined by Eq. (3.36).

By Eq. (3.59), we get

$$\langle \chi^{(-)}_{\pi d} \mid T_{\rm off} \mid \chi^{(+)}_{\pi d}\rangle=\langle \chi^{(-)}_{\pi d} \mid (V_{N\Delta,NN}+F_{\pi NN,NN})\tilde{G}_{NN}(E)(V_{NN,N\Delta}+F_{NN,\pi NN}) \mid \chi^{(+)}_{\pi d}\rangle \ , \qquad (3.60)$$

where

$$\tilde{G}_{NN}=\frac{1}{E-H_0}+\frac{1}{E-H_0}T_{NN,NN}\frac{1}{E-H_0} \ .$$

The physical meaning of Eq. (3.60) is clear. It describes the effects of pion absorption on πd scattering because of the appearance of an intermediate NN (no pion) state.

Substituting Eq. (3.60) into Eq. (3.55), we get the final form for πd scattering

$$T_{\pi d,\pi d}=T^F_{\pi d,\pi d}+\langle \chi^{(-)}_{\pi d} \mid (V_{N\Delta,NN}+F_{\pi NN,NN})\left[\frac{1}{E-H_0}+\frac{1}{E-H_0}T_{NN,NN}\frac{1}{E-H_0}\right](V_{NN,N\Delta}+F_{NN,\pi NN}) \mid \chi^{(+)}_{\pi d}\rangle \ . \qquad (3.61)$$

Recalling Eq. (3.46), we see that Eq. (3.61) can be completely expressed in terms of $T^F_{\pi d,\pi d}$, $T^F_{\pi d,N\Delta}$, and $T^F_{\pi d,\pi NN}$. These amplitudes can be calculated by using the numerical method recently developed by one of us.[46] The same calculation will also yield the breakup amplitude $T_{\pi d,\pi NN}$.

C. Pion absorption and production

Following the above derivations, we will show that the πNN$\leftrightarrow$NN transition amplitude can be calculated from the Faddeev amplitudes T^F_{ij} defined by Eq. (3.42) and the *BB* amplitude T defined by Eq. (3.34). First, we consider the πd$\rightarrow$NN process. Since the interaction Eq. (3.6) in the NN channel ($i=3$) can be written as $V_3'=V_3^F+V_{\rm off}$, where $V_{\rm off}$ and V_3^F are given in Eqs. (3.41) and (3.44), we can write, according to the definition Eq. (3.5),

$$T_{\pi d,NN}=\langle \pi d \mid \mathscr{T}_{31} \mid NN\rangle$$
$$=\left\langle \pi d \left| (V_3^F+V_{\rm off})\frac{1}{E-H}H_{\rm int} \right| NN\right\rangle$$
$$+\langle \pi d \mid F_{\pi NN\leftrightarrow NN} \mid NN\rangle \ . \qquad (3.62)$$

By using Eq. (3.50), we get

$$T_{\pi d,NN}=\left\langle \pi d \left| V_{\rm off}\frac{1}{E-H}H_{\rm int} \right| NN\right\rangle$$
$$+\left\langle \pi d \left| V_3^F\frac{1}{E-H_F}\left[1+V_{\rm off}\frac{1}{E-H}\right]H_{\rm int} \right| NN\right\rangle$$
$$+\langle \pi d \mid F_{\pi NN\leftrightarrow NN} \mid NN\rangle \ . \qquad (3.63)$$

Neglecting the contribution from πNΔ and $\pi\pi$NN states, the first term on the right-hand side of Eq. (3.63) can be evaluated as follows

$$\left\langle \pi d \left| V_{\text{off}} \frac{1}{E-H} H_{\text{int}} \right| NN \right\rangle = \left\langle \pi d \left| F_{\pi NN \to NN} \frac{P_{NN}}{E-H} H_{\text{int}} \right| NN \right\rangle$$
$$= \left\langle \pi d \left| F_{\pi NN \to NN} \frac{P_{NN}}{E-H_0} \left[H_{\text{int}} + H_{\text{int}} \frac{1}{E-H} H_{\text{int}} \right] \right| NN \right\rangle$$
$$= \hat{F}_{\pi d, NN} \frac{1}{E-H_0} (T)_{NN,NN} , \tag{3.64}$$

where

$$\hat{F}_{\pi d, NN} = \langle \pi d \mid F_{\pi NN \to NN} \mid NN \rangle . \tag{3.65}$$

By using the definitions of Faddeev amplitudes [Eq. (3.47)], the second term of Eq. (3.63) can be evaluated as follows

$$\left\langle \pi d \left| V_3^F \frac{1}{E-H_F} H_{\text{int}} \right| NN \right\rangle = T^F_{\pi d, N\Delta} \frac{P_{N\Delta}}{E-H_0} V_{N\Delta, NN} + T^F_{\pi d, \pi NN} \frac{P_{\pi NN}}{E-H_0} F_{\pi NN, NN} , \tag{3.66}$$

$$\left\langle \pi d \left| V_3^F \frac{1}{E-H_F} V_{\text{off}} \frac{1}{E-H} H_{\text{int}} \right| NN \right\rangle = T^F_{\pi d, N\Delta} \frac{1}{E-H_0} V_{N\Delta, NN} \frac{1}{E-H_0} T_{NN,NN}$$
$$+ T^F_{\pi d, \pi NN} \frac{1}{E-H_0} F_{\pi NN, NN} \frac{1}{E-H_0} T_{NN,NN} . \tag{3.67}$$

Substituting Eqs. (3.64)—(3.67) into Eq. (3.63), we obtain

$$T_{\pi d, NN} = T^F_{\pi d, N\Delta} \frac{1}{E-H_0} V_{N\Delta, NN} \left[1 + \frac{1}{E-H_0} T_{NN,NN} \right]$$
$$+ \left[1 + T^F_{\pi d, \pi NN} \frac{1}{E-H_0} \right] F_{\pi NN, NN} \left[1 + \frac{1}{E-H_0} T_{NN,NN} \right] . \tag{3.68}$$

Equation (3.68) can be obviously extended to describe pion production from NN scattering

$$T_{\pi NN, NN} = T^F_{\pi NN, N\Delta} \frac{1}{E-H_0} (T)_{N\Delta, NN}$$
$$+ T^F_{\pi NN, \pi NN} \frac{1}{E-H_0} F_{\pi NN, NN}$$
$$\times \left[1 + \frac{1}{E-H_0} (T)_{NN,NN} \right] + F_{\pi NN, NN}$$
$$\times \left[1 + \frac{1}{E-H_0} (T)_{NN,NN} \right] . \tag{3.69}$$

This completes our deviations of πNN scattering equations for the study of all NN and πd processes listed in Eq. (1.1), assuming that the coupling to dibaryon state D can be neglected.

IV. COUPLING TO DIBARYON STATE D

In this section we introduce the coupling of a dibaryon state D to NN and πd reactions. Following the notations of Sec. III, the considered total Hamiltonian Eq. (2.3) is of the form

$$H = H_0 + H_{\text{int}} + H_{D \leftrightarrow BB} , \tag{4.1}$$

where H_{int} has been defined in Eq. (3.2). The transition T matrices Eq. (3.5) between the considered three reaction channels NN, πd, and πNN then take the form

$$\hat{\mathscr{T}}_{ij}(E) = \hat{V}'_i + \hat{V}'_i \frac{1}{E-H_0-H_{\text{int}}-H_{D\leftrightarrow BB}+i\epsilon} \hat{V}'_j , \tag{4.2a}$$

$$\hat{V}'_i = V'_i + H_{D \leftrightarrow BB} , \tag{4.2b}$$

where V'_i has been defined in Eq. (3.6). Note that with the choice of channel Hamiltonian Eq. (3.3), all channel wave functions do not have a dibaryon component.[47] Therefore, we have

$$\langle i \mid H_{D \leftrightarrow BB} \mid j \rangle = 0, \quad \text{for } i,j = 1,2,3 . \tag{4.3}$$

By using Eq. (4.3) and the properties of Eq. (3.12), it is straightforward to write Eq. (4.2) as

$$\hat{\mathscr{T}}_{ij}(E) = \mathscr{T}_{ij}(E) + \mathscr{T}^{(D)}_{ij}(E) , \tag{4.4a}$$

$$\mathscr{T}^{(D)}_{ij}(E) = \omega_i^{(-)+}(E) H_{D \leftrightarrow BB} \Omega_j^{(+)}(E) , \tag{4.4b}$$

where the scattering operators are defined by

$$\omega_i^{(\pm)}(E) = 1 + \frac{1}{E-H_0-H_{\text{int}} \pm i\epsilon} V'_i , \tag{4.5}$$

$$\Omega_j^{(\pm)}(E) = 1 + \frac{1}{E-H_0-H_{\text{int}}-H_{D\leftrightarrow BB} \pm i\epsilon} (V'_j + H_{D\leftrightarrow BB}) . \tag{4.6}$$

$\mathscr{T}_{ij}$ and $\omega_i^{(+)}(E)$ are only determined by mesonic processes and can be computed by using the formalism developed in Sec. III. The task now is to develop a practical method for calculating the second term of Eq. (4.4a).

By using Eq. (4.5) and the operator relation Eq. (3.12), we can write Eq. (4.6) as

$$\Omega_j^{(\pm)}(E)=1+\frac{1}{E-H_0-H_{\rm int}-H_{D\leftrightarrow BB}}H_{D\leftrightarrow BB}+\frac{1}{E-H_0-H_{\rm int}}V_j'+\frac{1}{E-H_0-H_{\rm int}-H_{D\leftrightarrow BB}}\times H_{D\leftrightarrow BB}\frac{1}{E-H_0-H_{\rm int}}V_j'$$
$$=\left[1+\frac{1}{E-H_0-H_{\rm int}-H_{D\leftrightarrow BB}}H_{D\leftrightarrow BB}\right]\omega_j^{(\pm)} . \quad (4.7)$$

Since $\omega_i^{(-)+}H_{D\leftrightarrow BB}\omega_j^{(+)}=0$ [because of Eq. (4.3)], we can use Eq. (4.7) to write Eq. (4.4b) as

$$\mathscr{T}_{ij}^{(D)}(E)=\omega_i^{(-)+}(E)H_{D\leftrightarrow BB}\Omega_j^{(+)}(E)$$
$$\equiv\omega_i^{(-)+}(E)H_{BB\to D}g_{DD}(E)H_{D\to BB}\omega_j^{(+)}(E) , \quad (4.8)$$

where

$$g_{DD}=\left\langle D\left|\frac{1}{E-H_0-H_{\rm int}-H_{D\leftrightarrow BB}}\right|D\right\rangle . \quad (4.9)$$

Since the dibaryon state D does not couple *directly* to the πNN state in our model (see Ref. 38), a straightforward operator algebra yields

$$g_{DD}=\frac{1}{E-K_D-B_D(E)} , \quad (4.10)$$

where K_D is the kinetic energy of the dibaryon D. The effects of all mesonic interactions on the propagation of D are contained in

$$B_D(E)=H_{D\to BB}\frac{P}{E-H_0-H_{\rm int}}H_{BB\to D}$$
$$=H_{D\to BB}\frac{P}{E-H_0}H_{BB\to D}+H_{D\to BB}\frac{P}{E-H_0}T(E)\frac{P}{E-H_0}H_{BB\to D} , \quad (4.11)$$

where $T(E)$ is the BB scattering matrix defined in Eq. (3.9) or (3.34).

By using Eqs. (4.8)—(4.10), we finally obtain

$$\hat{\mathscr{T}}_{ij}(E)=\mathscr{T}_{ij}(E)+\omega_i^{(-)+}(E)H_{BB\leftrightarrow D}\frac{1}{E-K_D-B_D(E)}\times H_{D\leftrightarrow BB}\omega_j^{(+)}(E) . \quad (4.12)$$

Equation (4.12) has a familiar form employed in many studies[31–33] of the dibaryon resonance. However, our Hamiltonian formulation of the problem has clearly separated the "trivial" mesonic processes from the coupling to the six-quark state D. Our approach rigorously satisfies the essential two-body (NN) and three-body (πNN) unitarity relations and is, therefore, distinctly different from the existing prescriptions.[31–33]

To illustrate the structure of the above equations, let us evaluate B_D for the special case that only the NΔ state is coupled to the dibaryon D state $H_{D\leftrightarrow BB}\equiv H_{D\leftrightarrow {\rm N}\Delta}$. This is the case which will be explored first in our subsequent calculation. By using Eqs. (3.34) and (3.33), we can explicitly evaluate the N$\Delta\to$NΔ T matrix needed in calculating the second term of Eq. (4.11).

$$B_D=H_{D\leftrightarrow {\rm N}\Delta}\frac{1}{E-H_0}H_{{\rm N}\Delta\to D}+H_{D\leftrightarrow {\rm N}\Delta}\frac{1}{E-H_0}t_\Delta\frac{1}{E-H_0}H_{{\rm N}\Delta\to D}$$
$$+H_{D\to {\rm N}\Delta}\frac{1}{E-H_0}\frac{E-H_0}{E-H_0-\Sigma_\Delta}T_{{\rm N}\Delta,{\rm N},\Delta}\frac{E-H_0}{E-H_0-\Sigma_\Delta}\frac{1}{E-H_0}H_{{\rm N}\Delta\leftrightarrow D} . \quad (4.13)$$

Recalling Eq. (3.35) for t_Δ, it is simple [use Eq. (3.12)] to see that

$$\frac{1}{E-H_0}+\frac{1}{E-H_0}t_\Delta\frac{1}{E-H_0}=\frac{1}{E-H_0}+\left[\frac{1}{E-H_0}\left[\Sigma_\Delta+\Sigma_\Delta\frac{1}{E-H_0-\Sigma_\Delta}\Sigma_\Delta\right]\frac{1}{E-H_0}\right]$$
$$=\frac{1}{E-H_0}+\left[\frac{1}{E-H_0}\Sigma_\Delta\frac{1}{E-H_0-\Sigma_\Delta}\right]$$
$$=\frac{1}{E-H_0-\Sigma_\Delta} . \quad (4.14)$$

From Eq. (3.33), we have

$$T_{{\rm N}\Delta,{\rm N}\Delta}=\left[T_c+\left[1+T_c\frac{1}{E-H_0-\Sigma_\Delta}\right]T_0\left[1+\frac{1}{E-H_0-\Sigma_\Delta}T_c\right]\right]_{{\rm N}\Delta,{\rm N}\Delta} . \quad (4.15)$$

Substituting Eqs. (4.14) and (4.15) into Eq. (4.13), we then have

$$B_D(E)=\widetilde{X}_D(E)+\widetilde{Y}_D(E)+\widetilde{Z}_D(E)+\widetilde{W}_D(E)\ , \tag{4.16}$$

where

$$\begin{aligned}
\widetilde{X}_D(E)&=H_{D\to N\Delta}\frac{1}{E-H_0-\Sigma_\Delta}H_{N\Delta\to D}\ ,\\
\widetilde{Y}_D(E)&=H_{D\to N\Delta}\frac{1}{E-H_0-\Sigma_\Delta}T_c\frac{1}{E-H_0-\Sigma_\Delta}H_{N\Delta\to D}\ ,\\
\widetilde{Z}_D(E)&=H_{D\to N\Delta}\frac{1}{E-H_0-\Sigma_\Delta}T_0\frac{1}{E-H_0-\Sigma_\Delta}H_{N\Delta\to D}\ ,\\
\widetilde{W}_D(E)&=H_{D\to N\Delta}\frac{1}{E-H_0-\Sigma_\Delta}T_c\frac{1}{E-H_0-\Sigma_\Delta}T_0\frac{1}{E-H_0-\Sigma_\Delta}H_{N\Delta\to D}\\
&\quad+H_{D\to N\Delta}\frac{1}{E-H_0-\Sigma_\Delta}T_0\frac{1}{E-H_0-\Sigma_\Delta}T_c\frac{1}{E-H_0-\Sigma_\Delta}H_{N\Delta\to D}\\
&\quad+H_{D\to N\Delta}\frac{1}{E-H_0-\Sigma_\Delta}T_c\frac{1}{E-H_0-\Sigma_\Delta}T_0\frac{1}{E-H_0-\Sigma_\Delta}T_c\frac{1}{E-H_0-\Sigma_\Delta}H_{N\Delta\to D}\ .
\end{aligned} \tag{4.17}$$

Finally, it is instructive to see the structure of Eq. (4.8) in NN scattering:

$$\begin{aligned}
T^{(D)}_{NN,NN}&=\langle NN\,|\,\mathcal{T}^{(D)}_{11}\,|\,NN\rangle\\
&=\langle\phi^{(-)}_{NN}\,|\,N\Delta\rangle H_{N\Delta,D}\frac{1}{E-K_D-B_D}H_{D,N\Delta}\langle N\Delta\,|\,\phi^{(+)}_{NN}\rangle\\
&=(T)_{NN,N\Delta}\frac{1}{E-H_0-\Sigma_\Delta+i\epsilon}\left[1+(T_c)_{N\Delta,N\Delta}\frac{1}{E-H_0-\Sigma_\Delta+i\epsilon}\right]\left[H_{N\Delta\leftrightarrow D}\frac{1}{E-K_D-B_D}H_{D\leftrightarrow N\Delta}\right]\\
&\quad\times\left[1+\frac{1}{E-H_0-\Sigma_\Delta+i\epsilon}(T_c)_{N\Delta,N\Delta}\right]\frac{1}{E-H_0-\Sigma_\Delta+i\epsilon}(T)_{N\Delta,NN}\ .
\end{aligned} \tag{4.18}$$

Similar forms can also be obtained for describing the dibaryonic excitations in other πNN processes. As seen in this example, all of the matrix elements of $\mathcal{T}^{(D)}_{ij}$ can be calculated entirely from the matrix elements of T_0, T_c, and the Faddeev amplitude T^F. This completes our derivations.

V. SUMMARY

We have presented a theory of mesonic and dibaryonic excitations in the πNN system. The theory not only extends our conventional meson theory of nuclear force to include the Δ excitation and the production of on-mass-shell pions, but also makes contact with six-quark dynamics. Taking into account the complexities involved in formulating a rigorous and tractable scattering theory for a *practical* calculation, we postulate that the πNN dynamics can be described by the model Hamiltonian Eq. (2.3). Assuming that the coupled πNN and NN dynamics can be described in the subspace $NN\oplus N\Delta\oplus\Delta\Delta\oplus\pi NN\oplus D$, we have derived the πNN scattering equations for a unified study of all NN and πd processes listed in Eq. (1.1). By employing the standard projection techniques, we show that all NN and πd amplitudes, given explicitly in Eqs. (3.36), (3.61), (3.68), and (3.69), can be expressed in terms of three basic matrix elements of T_0, T_c, and T^F. Equation (3.26) for T_c and Eq. (3.42) for T^F contain the three-body πNN branch cut. Both can be handled by using the three-body method.[45,46] The two-body Eq. (3.27) for T_0 can be solved by using the standard matrix method.[44] With this decomposition of the three-body and two-body dynamics, we plan to demonstrate, in future work, that all NN and πd scattering can be studied starting from the most sophisticated nucleon-nucleon potential, such as that done in Ref. 16, based on the Paris potential.

Our approach to the bibaryon resonance is different from existing studies.[31–33] In a Hamiltonian formulation presented in Sec. IV, our theory separates the effects due to the "trivial" mesonic processes from the six-quark dynamics. In addition, our formulation respects the essential two-body (NN) and three-body (πNN) unitarity relations. Our study of the dibaryon resonance will be discussed in future work.

To close this paper, we want to mention that the baryon-baryon Eq. (3.27) can also be used with a suitable modification[48] to calculate the matrix elements of NΔ interaction in nuclear matter, which can be used to study both the phenomenological Δ-nucleus spreading potential[49] extracted from pion-nucleus scattering, and the so-called Δ-hole Landau parameter,[50] which plays an important role in the application of the Landau-Migdal theory of nuclear excitations. In the near future, we will make our first attempt to use the πNN matrix elements generated from our theory to carry out a microscopic study of pion-nucleus reactions. In particular, we will reexamine the work by Ohta, Thies, and Lee,[51] in order to resolve the most fundamental problem of pion physics: How the pion gets absorbed by nuclei.

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