Reexamination of fission fragment angular distributions and the fission process: Formalism

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The theory of fission fragment angular distributions is examined and the universally used expression is found to be valid only under restrictive assumptions. A more general angular distribution formula is derived and applied to recent data of high spin systems. At the same time it is shown that the strong anisotropies observed from such systems can be understood without changing the essential basis of standard fission theory. The effects of reaction mechanisms other than complete fusion on fission fragment angular distributions are discussed and possible angular distribution signatures of noncompound nucleus formation are mentioned.

I. INTRODUCTION

Much of the understanding about the fission process has come through the successful application of theory based upon the assumptions that:[1] (1) fission arises from decay of a deformed compound nucleus and (2) the essential fission parameters and compound nuclear shapes at the saddle point are given by the rotating liquid drop model (RLDM).[2] Recently, however, several experiments[3–7] found large anisotropies in fission fragment angular distributions from nuclei of high angular momentum, which were interpreted as conflicting with those assumptions. These findings prompted the suggestion of a new process called quasifission[3] (noncompound nucleus formation[5,7]) and a reexamination of the basis of the RLDM.[4,6,8] In a subsequent publication[9] it was pointed out that the angular distribution formula which was used in those analyses and has been universally used for many years in analyses of all fission fragment angular distributions, is not generally valid and fails dramatically for decay of high spin, near-spherical saddle point systems. It was concluded that the suggested failure of the standard hypotheses given above, if based on angular distribution information alone, was premature. In this work, a more in-depth discussion than Ref. 9, a reexamination of fission fragment angular distributions and the fission process is made. This investigation is divided into two successive works, the present one in which the formalism is developed and the succeeding one in which analysis of several experimental results is made.

The pioneering work on fission fragment angular distributions[10–12] from low spin nuclei in the late 1950's was based on the transition state theory of fission. The derivation of the angular distribution formula, which has been used for 25 years, relies upon the assumptions that: fission proceeds along the symmetry axis of a deformed compound nucleus, a statistical density of states is reached at the saddle point, and the value of K (the projection of J along the symmetry axis) is frozen from the saddle point to scission. For a deformed nucleus the density of states is given by

$$\rho(R,K) \propto e^{-[(R^2/2\mathscr{I}_\perp)+(K^2/2\mathscr{I}_{\|})]\hbar^2/T} , \tag{1}$$

where R is the component of the angular momentum J perpendicular to the symmetry axis and K the component along the symmetry axis, $\mathscr{I}_\perp$ and $\mathscr{I}_{\|}$ are the corresponding moments of inertia, T is the nuclear temperature, and $J^2=R^2+K^2$. Substitution for R in Eq. (1) leads to the normalized density of states expressed in terms of K at the saddle point[1]

$$\rho(K)=\frac{e^{-K^2/2K_0^2}}{\sum_{K=-J}^{J} e^{-K^2/2K_0^2}} , \tag{2}$$

where

$$\frac{1}{K_0^2}=\left[\frac{1}{\mathscr{I}_{\|}}-\frac{1}{\mathscr{I}_\perp}\right]\hbar^2/T\equiv\frac{\hbar^2}{\mathscr{I}_{\text{eff}}T} .$$

The values of $\mathscr{I}_{\text{eff}}$ have historically been taken for saddle point shapes from the RLDM.[2]

For compound nuclei formed with spin projection $M=0$ along the beam direction this choice of $\rho(K)$ for the weighting of K leads to the angular distribution formula, which has been universally used and found to work well for many cases of fission,[1]

$$W(\theta) \propto \sum_{J=J_{\min}}^{J_{\max}} (2J+1)T_J \sum_{K=-J}^{J} \left[\frac{2J+1}{2}\right] |D_{0K}^J(\theta)|^2 \frac{e^{-K^2/2K_0^2}}{\sum_K e^{-K^2/2K_0^2}} . \tag{3}$$

The term $(2J+1)T_J$ reflects the formation cross section for a compound nucleus of spin J (T_J is the transmission coefficient), the factor $[(2J+1)/2]\,|D^J_{0\nu}(\theta)|^2$ is the properly normalized angular distribution function[13] for the state of spin J to decay at an angle θ to the beam direction if the projection of J along the direction of emission is ν. The assumptions that a deformed nucleus fissions along its symmetry axis and K is frozen leads to $K=\nu$. The exponential term generally gives an unequal weighting to the different values of K.

At high angular momentum, J, or at high fissility, the RLDM (Ref. 2) predicts that the saddle point shapes become near-spherical, ($\mathscr{I}_{\|}\simeq\mathscr{I}_{\perp}$) which leads to $K_0^2\to\infty$. This produces a nearly uniform weighting of K, and hence, from Eq. (3) a nearly isotropic angular distribution of fission fragments, since $\sum_K |D^J_{0K}|^2=1$. This predicted tendency toward isotropy for fission fragments from nuclei at high spin is not seen in experiments[3-7] and led to the suggested failures of the standard fission theory mentioned above. Under these circumstances (high J and near-spherical saddle points shapes) there are several problems with the standard angular distribution prescription. The condition $K_0^2\to\infty$ leads to the conclusion that fission is *equally* probable in the direction parallel to the spin J ($K=J$) of the compound nucleus as perpendicular to it ($K=0$). This result is in sharp contrast both to the classical expectation and to a basic condition of the RLDM that $K=0$. Since $\mathscr{I}_{\text{eff}}$ has generally been taken from the RLDM, there is a serious inconsistency in its use with the $\rho(K)$ of Eq. (2) for high J cases. This problem, together with the expectation that angular distributions from high spin compound nuclei should approach $1/\sin\theta$, prompted the present and previous investigations[9] of the standard fission fragment angular distribution formula. It has only been in recent experiments that the conditions necessary to observe a breakdown of Eq. (3) have been acheived.

A potential problem with the transition state model is that the saddle point is part way through the fission process so that one must be generally very clever in choosing the correct distribution of K to reproduce the realistic physical situation (which includes dynamics). *Implicit* in the use of the transition state model [Eqs. (2) and (3)] is that whatever fission fragment spin distributions are necessary to conserve angular momentum will be realistic. It will be shown below that the assumed $\rho(K)$ of Eq. (2) leads to an unphysical division between the angular momentum in relative motion and the spins in the fragments for fission from nuclei formed at high angular momentum. Specifically, the prediction of $K_0^2\to\infty$ implies it is equally probable to have *all* of the compound nuclear spin in the fragments with none in relative motion as it is to have a large amount in relative motion with a lesser amount in the fragments.

The explicit treatment of the exit channel[14-16] in terms of transmission coefficients and the density of states available in the fragments avoids this potential problem and is the *general* way to calculate angular distributions for evaporation of light particles from nuclei. When applied to fission from low spin systems, however, it has been believed that this method does not reproduce data.[1] It will be suggested below that this failure has arisen at least partially from an inappropriate choice for the density of available final states. While Ref. 9 emphasized fission from high spin nuclei, the discussion here will be more general.

This work is organized as follows. In Sec. II a derivation of the general angular distribution formula for decay of a compound nucleus into two arbitrary, spherical fragments is given [Eq. (6)], and the approximations made in obtaining a practical formula [Eq. (9)] when a statistical density of final states is available, are discussed. In Sec. III the unique features of fission are examined and approximate formulae for fission fragment angular distributions are developed [Eqs. (10) and (11)]. In Sec. IV the angular distribution formulae for fission which result from analysis at the saddle point and at scission are discussed and it is suggested that with a more appropriate choice of $\rho(K)$ in the transition state model they give nearly identical results. In Sec. V the sensitivity of fission fragment angular distributions to processes other than complete fusion are investigated. It is concluded that at this point there is no need to invoke other than standard processes to account for fission fragment angular distributions from low and high spin systems, although it is suggested that the standard analysis of fission fragment angular distributions, which has been made for 25 years with Eqs. (2) and (3), must be modified. Possible signatures of fission without compound nucleus formation are mentioned.

II. ANGULAR DISTRIBUTIONS FROM COMPOUND NUCLEI

The general angular distribution formula for a compound nucleus to decay into two arbitrary spherical fragments has been discussed by many authors[17-19] and is based upon explicitly treating the exit channel. It is rederived here primarily because the result is written in a different form from that which is generally given. This form allows a direct comparison of Eq. (3) and also makes the dependences on various parameters more obvious. An analytic formula for high spin systems is also presented for the case of a statistical density of available final states.

We consider the case of a compound nucleus formed via complete fusion and decaying into two arbitrary fragments in specific states whether they be fission fragments, neutron plus residual nucleus, etc. Following Ref. 19 we write

$$\frac{d\sigma}{d\Omega}=\frac{1}{k_i^2}\frac{1}{(2I_A+1)(2I_a+1)}\sum_{M_aM_AM_bM_B}\left|\sum_{J=0}^{\infty}\sum_{\substack{ll'c\\ ss'c'\\ m_sm_s'\\ \mu'}} i^{l-l'}\sqrt{\pi(2l+1)}\,\langle I_aM_aI_AM_A\,|\,sm_s\rangle\langle sm_sl0\,|\,JM\rangle \times S^{J\pi}_{cc'}\langle I_bM_bI_BM_B\,|\,s'm_s'\rangle\langle s'm_s'l'\mu'\,|\,JM\rangle Y^{\mu'}_{l'}(\theta,\phi)\right|^2, \qquad (4)$$

where I_A and I_a are the spins of the projectile and target and s is their channel spin. The final fragments have spins I_b, I_B and couple to channel spin s', the relative orbital angular momentum in the initial and final channels are denoted as l and l', and the total spin of the compound nucleus is J with projection M along the quantization axis (chosen here as the beam axis). The scattering matrices between channel c and c' are denoted as $S_{cc'}^{J\pi}$. The quantum numbers carried by c include l and s. The index c' will also eventually denote the specific nuclei and their excitation energy. Note that the initial and final M states are summed over incoherently.

In carrying out the square in Eq. (4) it is common to replace the combination of Clebsch-Gordan coefficients with Racah coefficients and then Z coefficients. However, the dependence of the angular distributions on essential parameters is more obvious if that is not done. For reasons which will become obvious below, we rewrite the last two factors in Eq. (4)

$$\langle s'm_s' l'\mu' \mid JM\rangle Y_{l'}^{\mu'}(\theta,\phi)=\sum_{\nu}(-1)^{s'-m_s'}\langle J\nu s'-\nu \mid l'0\rangle \times\left[\frac{2J+1}{4\pi}\right]^{1/2} D^{s'\,*}_{-m_s'\,-\nu}(\phi,\theta,0)D^{J*}_{M\nu}(\phi,\theta,0)$$

and then square. The incoherent sum in Eq. (4) over initial and final M states means that the sum over m_s and m_s' is also incoherent as is the sum over M (since $m_s = M$). We also have

$$\sum_{M_a}\langle I_a M_a I_A M_A \mid s m_s\rangle\langle I_a M_a I_A M_A \mid s'' m_s\rangle=\delta_{ss''}\,,$$

i.e., the sum over channel spin s is incoherent. Similarly, the sum over M_b makes the sum over the exit channel spin s' incoherent.

The assumption of a compound nucleus, i.e., that there are many states of a given J,π which are populated with random phases, leads to[20]

$$\overline{\langle S_{cc'}^{J\pi}S_{c''c'''}^{J'\pi'*}\rangle}=\delta_{JJ'}\delta_{\pi\pi'}\delta_{cc''}\delta_{c'c'''}\frac{T_c^J T_{c'}^J}{\sum_d T_d^J}\quad\text{for } c\neq c'\,, \tag{5}$$

where the left-hand side denotes an average over a finite excitation energy ($\Delta E \gg$ level spacing) in the compound nucleus and we have dropped terms not dependent upon angular momentum. The sum in the denominator extends over all available decay channels d. The compound nucleus assumption [Eq. (5)] then removes all interferences between partial waves in both the entrance and exit channels and produces the following angular distribution formula

$$W(\theta)=\sum_{\substack{ss'\\ m_s'\\ c,c'}}\sum_{\substack{JM\\ ll'\\ \nu,\nu'}}(2l+1)\langle sMl0 \mid JM\rangle^2 \times\frac{T_c^J T_{c'}^J}{\sum_{c''}T_{c''}^J}\frac{(2J+1)}{4}\langle J\nu s'-\nu \mid l'0\rangle\langle J\nu' s'-\nu' \mid l'0\rangle D^{s'\,*}_{-m_s'\,-\nu}(\Omega)D^{s'}_{-m_s'\,-\nu'}(\Omega)D^{J*}_{M\nu}(\Omega)D^{J}_{M\nu'}(\Omega)\,,$$

where $\Omega\equiv(0,\theta,0)$. Note that because of the incoherent sum over m_s' and M, the ϕ dependence has been removed. The sum over $m_{s'}$ can be performed and gives $\sum_{m_s'}D^{s'\,*}_{m_s'\nu}D^{s'}_{m_s'\nu'}=\delta_{\nu\nu'}$, i.e., the sum over ν is incoherent.

When, as in fission, there are many unresolved final states which are populated, a density of states for I_b and I_B is introduced which leads to a density of available final channel spins, $\rho(s')$ (see Appendix A). Some consideration must also be given to the denominator in Eq. (5). This term is the total width for the state J to decay into all channels

$$\Gamma(J)=\sum_{\substack{s'',\nu''\\ l''\\ c''}}\left|\langle s''\nu'' l''0 \mid J\nu''\rangle\right|^2(2l''+1)T_{c''}^J\rho(s'') \times\int\left|D^J_{M\nu''}(\theta)\right|^2 d\Omega\,.$$

Under the assumption that $T_{c''}^J$ depends only upon l'' this reduces to a more familiar form. The angular integral gives $8\pi^2/2J+1$, and after rewriting the Clebsch-Gordan coefficient as

$$\frac{2J+1}{2l''+1} |\langle s''\nu''J-\nu''|l''0\rangle|^2$$

and summing over ν'', we obtain

$$\Gamma(J)=\sum_{c''s''l''} T^J_{l''}\rho(s'') .$$

For reasons which will be clear below we use the full expression for $\Gamma(J)$.

We finally arrive at the general angular distribution formula for a compound nucleus to be formed by complete fusion and to decay into two arbitrary, spherical fragments.

$$W(\theta)=\sum_{\substack{ss'\\Jll'\\\nu M\\cc''}} \langle sMl0|JM\rangle^2(2l+1)T^J_c\frac{(2l'+1)T^J_{c'}}{\Gamma(J)} \times\langle J\nu l'0|s'\nu\rangle^2\frac{\rho(s')}{2s'+1}\frac{2J+1}{2}|D^J_{M\nu}(\theta)|^2 . \tag{6}$$

As stated earlier this is not the usual form in which the angular distribution generally appears but it demonstrates more clearly the influence of the important physical quantities. The cross section splits into parts which reflect the Bohr hypothesis of a formation cross section times a decay probability. The vector coupling coefficients ensure angular momentum conservation; the D function serves to rotate the coordinate system which originally defined the quantization direction as the beam axis to the new coordinate system, which the direction of emission is defined as the Z axis. Thus ν is just the projection of J (and s') along the emission axis. Although it no longer explicitly appears, the value of s' is determined from the spins of the final states. For large values of s' (and hence I_b and I_B) the sum over ν is limited by J, but when s' is less than J, clearly the sum over ν is limited to s' via the Clebsch-Gordan coefficient. Finally, if the transmission coefficients depend only on l we can write T_l, but that will not always be valid.

The derivation of an analytic form for the angular correlation becomes complicated as a result of the factor $\rho(s')$. When a uniform density of states $\rho(s')=(2s'+1)$ is available for all states $|J-l'|\le s'\le J+l'$ the sum over s' can be made in Eq. (6) (provided the transmission coefficients have no s' dependence), which then allows the sum over ν to be made. Since $\sum_\nu |D^J_{M\nu}|^2=1$, this results in an isotropic angular distribution and demonstrates the well-known fact that anisotropy requires a nonuniform density of final states.

It is not surprising that the approximation of a uniform density of states is generally not valid and particularly not for decay of high spin systems. In Appendix A a form for the density of states $\rho(s')$ for two spherical nuclei is derived and it takes the form

$$\frac{\rho(s')}{2s'+1}=e^{-(s'+1/2)^2/2s_0^2} \tag{7}$$

which will be valid for reasonably high values of E^*.

The angular distribution which results from this density of states can be approximated when the angular momentum $\langle J\rangle>s_0$ and the values of l' are not restricted severely by T^J_c. These conditions limit $s'<J,l'$. The expression for $\rho(s')$ given above is substituted into Eq. (6) and the Clebsch-Gordan coefficient is approximated as in Appendix B for $J,l'>s'$ as

$$|\langle J\nu l'0|s'\nu\rangle|^2\sim\frac{(2s'+1)}{2J+1}|D^{s'}_{\nu,J-l'}(\pi/2)|^2 .$$

Since s' is also not very small either, we write[13]

$$|D^{s'}_{\nu,J-l'}(\pi/2)|^2\sim\frac{1}{\pi[(s'+\frac{1}{2})^2-\nu^2-(J-l')^2]^{1/2}} ,$$

and change the sums over s' and s'' to integrals in Eq. (6). We assume that all T^J_c are functions of l only and that a single process dominates the decay of the compound nucleus so that the sums over l' and l'' are the same.

The integrals over s' have the form

$$I(J,l',\nu)=\frac{1}{\pi(2J+1)} \times\int_{s'_{\min}}^{s'_{\max}}\frac{\rho(s')}{[(s'+\frac{1}{2})-\nu^2-(J-l')^2]^{1/2}}ds' , \tag{8}$$

where

$$s'_{\min}+\tfrac{1}{2}=\sqrt{\nu^2+(J-l')^2}$$

and, in principle, $s'_{\max}=J+l'$. The approximation for the Clebsch-Gordan coefficient is only valid for $s'<J,l'$, so, properly, the integral should be limited to $s'_{\max}=J$ and a second integral whose limits are $s_{\min}=J$ and $s_{\max}=J+l'$ and in which the Clebsch-Gordan coefficient is approximated for $J<l',s'$, should be added. Since we have assumed $\rho(s')$ limits $s'<J$, we can instead extend the integral (8) to $J+l'$.

The density of states [Eq. (7)] is then inserted and the integral I [Eq. (8)] becomes, after a change of variable

$$I(J,l',\nu)\simeq\frac{e^{-\nu^2/2s_0^2}e^{-(J-l')^2/2s_0^2}}{\pi(2J+1)} \times\int_0^{[(4Jl'-\nu^2)/2s_0^2]^{1/2}}e^{-x^2}dx .$$

The exponential in front of the integral favors $J\simeq l'$ and we have assumed $J>s_0,\nu$ so the integral (which is the error function) is approximately independent of ν. This assumption will be made throughout this work and is valid as long as $(2J/s_0)\gtrsim 2$. The function I can then be approximately written as

$$I(J,l',\nu)\simeq e^{-\nu^2/2s_0^2}f(J,l') ,$$

where $f(J,l')$ is a function only of J,l'. Substitution of I into the sums over l' and l'' (which are presumed to be equal) in Eq. (6) leads to the generalized angular distribution formula for compound nuclei with $\langle J\rangle>s_0$,

$$W(\theta)=\sum_{\substack{Ml\\ \nu J\\ s}}\langle sMl0\,|\,JM\rangle^2(2l+1)T_l\left[\frac{2J+1}{2}\right]\times|D^J_{M\nu}(\theta)|^2\frac{e^{-\nu^2/2s_0^2}}{\sum_{\nu=-J}^{J}e^{-\nu^2/2s_0^2}}\qquad(9)$$

which holds for spherical final fragments. It should be remembered that ν is the projection of J along the emission axis. Equation (9) shows a remarkable similarity in form to Eq. (3) and yet no special consideration of the fission process has been made (see also Ref. 14). The projection of angular momentum along the emission axis is restricted by the spin distributions in the final fragments, and the anistropy observed in standard compound nucleus decay results.

To summarize this section, the major points in deriving Eqs. (6) and (9) are that conservation of angular momentum has been ensured and the important parameters which determine the angular distributions are identified. We now turn to the unique features of the fission process.

III. FISSION FRAGMENT ANGULAR DISTRIBUTIONS

A. General discussion

There are several aspects of fission which modify the considerations which went into the derivation of Eq. (9). The RLDM, as well as earlier models, predict that the fissioning nucleus becomes deformed on its way to fission. Thus, in addition to J and M, the quantum number K, which is the projection of J along the symmetry axis, is introduced. The connection between the parameter K and ν, which is the projection of J along the direction of emission of the fission fragments in Eq. (6), can be made if the assumption is made that fission proceeds along the symmetry axis. This leads to $K=\nu$ (the frozen K assumption) since the angular momentum in relative motion has projection 0 along this axis. Note that in general K may not be defined because of nonaxial symmetry, but ν always is.

In Eq. (6) the nonuniform distribution of ν (or K) for fission, required to produce anisotropic angular distributions, can arise in two ways. Of course, it can arise through the density of states of the final nuclei, but it can also result from the fact that the fission barrier prevents all values of K from contributing equally to fission, i.e., there is a K dependence (equivalently an s' dependence) of the transmission coefficients. Since the emphasis in this paper and the succeeding one is on fission from fairly high spin systems at high excitation energy, the K dependence from $T^J_{c'}$ is expected to be weak and is neglected here, but this possible dependence is potentially important and is discussed in Appendix C.

The crucial factor is the density of available final states. In Sec. II the density of states at scission was calculated for two spheres; however, at least for extended saddle point shapes this is certainly a questionable assumption. More generally we assume that fission fragments are deformed and are emitted with their symmetry axis aligned along the direction of emission. This assumption seems reasonable, but it should be noted that it contains an implied dynamics.[32] In the derivation of the density of states for spherical nuclei there was an implicit assumption that each projection of s' along the quantization axis had equal weight. For deformed nuclei this is not the case since the density of states depends upon the projection of spin along the symmetry axis of the deformed nucleus (for example see Ref. 14). As is shown in Appendix A we more generally replace $\rho(s')/(2s'+1)$ in Eq. (6) by

$$\rho(s',\nu)=e^{-(s'+1/2)^2/2\sigma_\perp^2}e^{-\nu^2/2\sigma_{\text{eff}}^2},\qquad(7')$$

where

$$\sigma_\perp^2=(\mathscr{I}_{\perp_1}T_1+\mathscr{I}_{\perp_2}T_2)/\hbar^2,$$

$$\frac{1}{\sigma_{\text{eff}}^2}=\frac{1}{\sigma_{\parallel}^2}-\frac{1}{\sigma_\perp^2},$$

and

$$\sigma_{\parallel}^2=(\mathscr{I}_{\parallel_1}T_1+\mathscr{I}_{\parallel_2}T_2)/\hbar^2.$$

This reduces to the spherical case [Eq. (7)] when $\sigma_{\parallel}\simeq\sigma_\perp\simeq s_0$. In Ref. 9 it was assumed incorrectly that the ν (or K) dependence was given by K_0 of Eq. (2) and the s dependence by s_0, but the effect on the cases studied there is not major.

B. Approximate forms of angular distributions

In general the complete expression [Eq. (6) with Eq. (7′)] could be used to calculate the angular distributions, however, there are two limits for which good approximations can be made to produce analytic forms which allow more physical insight to be made. In *all* of the following it is presumed that for partial waves from $l_{\min}\le l'\le l_{\max}$, fission is the dominant decay channel. This means that the sums over l'' in Γ and l' in the numerator in Eq. (6) are the same and allows us to obtain closed form expressions. More generally, a factor $\Gamma_f(J)/\Gamma(J)$ can be introduced. It is also assumed that l' is allowed to take on values up to $l'\sim J$. The general procedure to obtain an angular distribution formula follows that of Sec. II in deriving Eq. (9), but the criterion now depends upon the relative size of J and $\sigma_\perp$. As a practical rule of thumb $\sigma_\perp\simeq 10$—20.

1. $J_{\max}<\sigma_\perp$

Under this condition it is clear from Eq. (6) that it is J rather than s' which will restrict the sum on ν (or K) and the standard transition state model [Eq. (3)] is likely to be valid. We approximate that all values of s' from $|l-J|$ to $l+J$ are available so that the sum over s' can be made. This results in

$$W(\theta)=\sum_{\substack{Ml\\ J\nu}}\langle sMl0\,|\,JM\rangle^2(2l+1)T_l\times\frac{e^{-\nu^2/2\sigma_{\text{eff}}^2}(2J+1/2)\,|D^J_{M\nu}(\theta)|^2}{\sum_\nu e^{-\nu^2/2\sigma_{\text{eff}}^2}}\qquad(10)$$

which is seen to be essentially identical to the transition state model [Eq. (3)], except that here the moments of inertia and temperature are determined from the final fragments, whereas in the transition state model these quantities are determined at the saddle point. In the limit of spherical fragments ($\sigma_{\rm eff}=\infty$) the condition $J_{\max}<\sigma$ results in an isotropic angular distribution since there is a uniform density of states s'. Under this angular momentum condition there is no problem with the fact that isotropic angular distributions may be predicted since the available spin distributions are greater than J and it is the deformed nature of the nuclei which produces an anisotropy. Finally, for this condition of $J<\sigma_\perp$, which might be the case for low energy neutron capture, the excitation energy may be near the fission barrier so that the K or (ν) dependence of the transmission coefficients may also be important, and the assumption of 100% fission probability may not be valid.

2. $\langle J\rangle>\sigma_\perp$

If the fragments are near-spherical the angular distribution formula will be given by Eq. (9). For the more general case of two deformed fission fragments we obtain for the function I [Eq. (8)],

$$I(J,l',\nu)\simeq e^{-\nu^2[(1/\sigma_\perp^2)+(1/\sigma_{\rm eff}^2)]/2}g(J,l')\ ,$$

where we have assumed $\langle J\rangle/\sigma_\perp>1$. Since $(1/\sigma_\perp^2)+(1/\sigma_{\rm eff}^2)=1/\sigma_{\|}^2$, the general angular distribution formula for fission from fairly high J systems is

$$W(\theta)=\sum_{\substack{lJs\\ M\nu}}\frac{|\langle sMl0\,|\,JM\rangle|^2(2l+1)T_l^Je^{-\nu^2/2\sigma_{\|}^2}}{\sum\limits_{\nu=-J}^{J}e^{-\nu^2/2\sigma_{\|}^2}}\times\left[\frac{2J+1}{2}\right]|D^J_{M\nu}(\theta)|^2\ . \tag{11}$$

The form of $W(\theta)$ is nearly identical to that of Eqs. (3), (9), and (10) except that the width parameter of the ν distribution is given by $\sigma_{\|}^2$.

Equation (11) then is the expected angular distribution formula for a compound nucleus to be formed at high spin by complete fusion and decay into two deformed fission fragments. The ν weighting depends only upon the sum of the fragment moments of inertia parallel to their symmetry axes. The connection to parameters of the RLDM would come in the transmission coefficients and perhaps in the estimate of $\sigma_{\|}^2$. If we assume two fragments of equal temperature, then $\sigma_{\|}^2=(\mathscr{I}_{\|_1}+\mathscr{I}_{\|_2})T/\hbar^2$. When a liquid drop has a very extended saddle point shape a reasonable expectation is that $\mathscr{I}_{\|_1}+\mathscr{I}_{\|_2}\sim\mathscr{I}_{\|}^{\rm RLDM}$. For near-spherical saddle point shapes the situation is more complex since the saddle point configuration and the scission point configuration are very different. For near-spherical fragments an estimate of

$$\mathscr{I}_{\|}\simeq\mathscr{I}_0^{\rm RLDM}/2^{2/3}\simeq0.63\mathscr{I}_0^{\rm RLDM}$$

is more appropriate. As a practical limit

$$0.45\mathscr{I}_0\le\mathscr{I}_{\|_1}+\mathscr{I}_{\|_2}\le0.63\mathscr{I}_0\ ,$$

which is a rather narrow range. The perpendicular moments of inertia of the fragments would not be so easily estimated since $\mathscr{I}^{\rm RLDM}$ is approximately the sum of these moments and the moment of inertia associated with relative motion of the two fragments. A more comprehensive discussion of fragment shapes is given in Appendix D.

For deformed fragments, $\sigma_{\|}<s_0$ and stronger anisotropies are expected than for near-spherical fragments if the two systems are formed at the same angular momentum. For very high spin J the value of J/σ is very large and angular distributions quickly become $1/\sin\theta$ rather independent of shape except at the most forward angles. This latter conclusion is in sharp contrast to the prediction of the traditional transition state model.

Note that no approximate expression has been given for the case of $\langle J\rangle\approx\sigma_\perp$ because several of the approximations break down. However, the result should lie between the two expressions [Eqs. (10) and (11)] given above.

IV. DISCUSSION

As was noted above, angular distributions for evaporation of particles from a compound nucleus and for fission have historically been treated quite differently. It has been suggested here that generally they should be treated in the same way with proper account taken of the unique features of fission. Equation (6) together with Eq. (7′) is a general angular distribution expression for a compound nucleus to decay into two deformed fragments with their symmetry axes aligned along the direction of emission.[32]. Under the conditions that the compound nucleus is formed with an average spin larger than the spin cutoff parameter (σ) in the final nuclei, and at high excitation energy so that the exit channel transmission coefficients are approximately 1, the general expression for decay can be approximated by Eq. (11). As in standard compound nucleus decay, the anisotropy of fission fragment angular distributions is governed by the ratio of $J_{\max}/\sigma$, where σ is determined by the bombarding energy, Q value, and nuclear shape. It should be emphasized that the general angular distribution formula does not depend upon axial symmetry of the compound nucleus. Rather it is the projection of J along the emission axis which is the crucial quantum number.

We now return to a discussion of the transition state model. As has been pointed out above, there are conditions where the ν distribution may be better determined by the traditional density of states at the saddle point [Eq. (2)], but there are also cases where it clearly can not be. While the standard transition state model is likely to be valid for the low spin cases for which it was initially proposed,[10,11] there is little reason to believe that generally one knows how to make the correct choice for the weighting of K at the saddle point, which is part way through the fission process. Nevertheless it is worth reconsidering the transition state model for high spin systems.

Much of the recent controversy about fission fragment angular distributions and nonfrozen K distributions has arisen because it has been stated in the recent literature that Eq. (2) expresses a statistical distribution of the K mode at the saddle point. In fact, Eq. (2) was derived from Eq. (1) and so represents the K dependence when there is a statistical distribution of *both K and R*. The procedure of normalizing in Eq. (2) has given the incorrect impression that one is assuming that only the K mode is statistically distributed. As can be seen in Eq. (6), the value of K at the saddle point must appear in the intrinsic spins of the fragments. The orthogonal R mode, however, generally divides between the fragment spins and the angular momentum of relative motion. By using Eqs. (1) and (2) one is forcing R to go entirely into intrinsic spin for near-spherical saddle point shapes and certain emission angles. The cases for which Eq. (2) was originally proposed[10,11] were ones where the fragment spin distributions were larger than the compound nuclear spin, so Eq. (2) was not an unreasonable assumption. For high spin systems it is much more reasonable to assume, as the RLDM does, that dynamics leads to $R \simeq J$ and $K \simeq 0$. At the saddle point one allows the *K mode* to be statistically distributed, i.e., the K dependence given in Eq. (1) ($K_0^2 = \mathscr{I}_{\|} T/\hbar^2$) is the proper $\rho(K)$ to be used in Eq. (3). This assumption makes the use of the RLDM to calculate shapes at the saddle point a reasonable procedure since consistent assumptions are made.

The change of the parameter K_0 in Eq. (3) from $K_0^2 = \mathscr{I}_{\text{eff}} T/\hbar^2$ to $K_0^2 = \mathscr{I}_{\|} T/\hbar^2$ as a function of the compound nuclear spin would be expected to be rather quick but a specific analytic form is not obvious. It is reasonable to expect that

$$\mathscr{I}_{\|} T(\text{saddle}) \cong \mathscr{I}_{\|_1} T_1 + \mathscr{I}_{\|_2} T_2$$

so with the choice of $K_0^2 = \mathscr{I}_{\|} T/\hbar^2$ the transition state model and the general formula from scission [Eq. (11)] become essentially identical and there is no problem with either model in reproducing the anisotropic angular distributions from high spin, near-spherical saddle point nuclei. Note that for extended saddle point shapes $\mathscr{I}_{\|_1} + \mathscr{I}_{\|_2} \simeq \mathscr{I}_{\|}$ and thus $T(\text{saddle}) \simeq T(\text{fragments})$. For compact (near-spherical) saddle point shapes this is not the case since $\mathscr{I}_{\|}$ (saddle) $> \mathscr{I}_{\|_1}\, \mathscr{I}_{\|_2}$ so it is expected that $T(\text{saddle}) < T(\text{fragments})$.

Since the angular distribution expressions from either method are expected to give essentially identical results at high spin, it is a matter of choice as to which one should be used. The general form for the angular distribution, which has been derived above, is less model dependent—it is not necessary to have a stable configuration from the RLDM, the expression is generally valid for all angular momentum conditions and it is valid for processes other than fission. The difficulty is knowing what the exit channel transmission coefficients and the fragment shapes are. On the other hand there is clearly an appealing tie to saddle point parameters through the use of the transition state model, and for low spins and excitation energies near the barrier it may be preferable.

We now see that a consistent picture of fission fragment angular distributions for low and high spin systems, for deformed or near-spherical saddle point shapes can be built which is based upon the standard assumptions of fission: compound nucleus formation, the use of the RLDM for essential fission parameters, fission along the symmetry axis (or in general along the longest axis), a frozen K distribution, and a statistical distribution of angular momentum modes at the saddle point which evolves from both K and R being statistically populated at low spin to only K being statistically populated at high spin.

Of course, if data are found to be in agreement with the calculations, it does not prove these assumptions; rather, that the data are consistent with such assumptions. In light of suggestions of noncompound nucleus formation it is worthwhile to identify experimental angular distribution signatures of such a process. In cases where specific masses are not identified, fission fragment angular distributions are guaranteed to be symmetric around 90° no matter what the formation mechanism is. Thus a minimum requirement for conclusively demonstrating noncompound nucleus formation is an unequal yield for a specific mass at angles symmetric around 90°.[33]

V. EFFECTS OF $M \neq 0$ AND INCOMPLETE FUSION

The effects of these conditions on the angular distributions will be covered in some detail, since improper conclusions have been reached in much of the literature. There are at least three ways that the fissioning system can have $M \neq 0$. The most obvious one is when the target or projectile have spin. In addition, prefission particle emission might change M and, perhaps most importantly, reaction mechanisms which lead to incomplete fusion of the projectile and target can leave the compound nucleus in $M \neq 0$.

The effect of $M \neq 0$ relative to $M = 0$ can be seen from the form of the normalized D function, which is

$$(J+\tfrac{1}{2})\,|D_{MK}^{J}(\theta)|^2 \simeq \frac{J+\frac{1}{2}}{[(J+\frac{1}{2})^2\sin^2\theta - M^2 - K^2 + 2MK\cos\theta]^{1/2}} .$$

At 90° the square of the D function is larger for $M \neq 0$ and hence the cross section is larger than for $M = 0$. At very forward angles the square of the D function can be the same for $M = 0$ and $M \neq 0$ if $M = K$. However, values of $K \neq 0$ are suppressed [see Eq. (11)] so that $W(\theta)$ is smaller at small angles for $M \neq 0$ than for $M = 0$. Thus the effect of $M \neq 0$ is to *reduce* the anisotropy of the correlation. Similarly, if less angular momentum is transferred to the nucleus, i.e., J is smaller, more isotropic angular distributions result. These conclusions will be modified if the value of σ^2 in Eq. (11) reduced due to a lower temperature or a reduced value of $\mathscr{I}$, but these latter two effects will be most important at low excitation energies.

Ground state spins are generally ≤ 4 and the orientation is averaged over so except for very low values of J, the consequences for the angular distribution are negligible (see also Ref. 11). Prefission particle emission may produce a reduction in the angular momentum of the fissioning system and a change in the projection M. For high spin systems the effect of the reduction in J due to prefission *evaporation* particle emission is not likely to be major and will not be dealt with here. The effects of other reaction mechanisms can be quite significant. In particular, we now consider processes other than complete fusion for compound nucleus formation. Clearly the effects of preequilibrium particle emission and these reaction mechanisms are not always easy to separate.

In systems formed by bombarding with high energy heavy ions (≥ 10 MeV/A), processes other than complete fusion can compete in producing compound nuclei which will fission. For fissile targets simple transfer or inelastic scattering may contribute, while for lighter targets more massive transfer will be required to exceed the fission barrier. In experiments which detect fission fragments not in coincidence with outgoing particles the effects of both complete fusion and noncomplete fusion will be present so that it is useful to know what to expect from both processes. Note that compound nucleus formation is still assumed.

The process of incomplete fusion[21] (or massive transfer) is viewed as capture of only a fraction of the projectile with the remaining part continuing on at roughly beam velocity. We define the captured mass as m_{capt} and can then estimate that the average angular momentum transferred to the nucleus is $J = m_{\text{capt}} l_p / m_p$ where l_p is the incident angular momentum and m_p the projectile mass. Thus the angular momentum brought into the nucleus is less than if complete fusion took place. The M distribution is also changed relative to complete fusion because there is an outgoing particle which generally does not travel along the beam axis. This means that values of $M \neq 0$ are populated. Both of these effects serve to *reduce* the anisotropy over what would have been obtained for complete fusion of the same partial waves. This conclusion is opposite to that of Ref. 4.

Since at high angular momenta the effect of target and projectile spin is small let us consider spin 0 projectile and target and spin 0 ejectile for the incomplete fusion process. Calculating the angular distribution for this process is not as straightforward as for complete fusion because the formation amplitude is not known. We define the amplitude for formation of the state J,M as $\alpha_J^M(\theta_x,\phi_x)$ where θ_x,ϕ_x indicate the angle of the continuing projectilelike fragment. The expected form of

$$\alpha_J^M(\theta_x,\phi_x) \simeq \langle l_x M J -M \mid l_p 0 \rangle \times [2l_p+1]^{1/2} Y_{l_x}^M(\theta_x,\phi_x) A_{J l_p l_x} ,$$

where A does not depend upon the M quantum number but influences l_x. We consider only one type of particle x so $A \sim 1$ and have

$$\sigma_J = \int \sum_M |\alpha_J^M|^2 d\Omega_x \simeq (2l_p+1) = \frac{2m_p J}{m_{\text{capt}}} + 1 . \quad (12)$$

We replace the formation amplitude in Eq. (4) by $\alpha_J^M(\theta_x,\phi_x)$ and square. As before we assume interferences between different partial waves vanish due to the compound nucleus assumption and obtain

$$W(\theta) = \sum_{\substack{JM\nu \\ M'}} \alpha_J^M(\theta_x,\phi_x) \alpha_J^{M'*}(\theta_x,\phi_x) \frac{e^{-\nu^2/2\sigma_\parallel^2}}{\sum e^{-\nu^2/2\sigma_\parallel^2}} \frac{2J+1}{2} \times D_{M\nu}^J(\phi_f,\theta_f,0) D_{M'\nu}^{J*}(\phi_f,\theta_f,0) . \quad (13)$$

Note that an azimuthal angle dependence ($e^{i(M-M')(\phi_x-\phi_f)}$) exists because the projectilelike fragment is presumed to be detected in coincidence with the fission fragments. In cases where the outgoing particle is not measured in coincidence an integration over ϕ_x can be made and one obtains $\delta_{MM'}$, and the ϕ_x and ϕ_f dependence vanishes. Then Eq. (13) reverts to a form very similar to Eq. (11) except that the M projection of the angular momentum brought into the nucleus is no longer $M=0$ and J is less than for complete fusion.

In order to proceed, further assumptions must be made about α_J^M. Limits on M are easily obtained. If the outgoing particle x is presumed to go at an average angle θ_x relative to the beam with average angular momentum $l_x = (1 - m_{\text{capt}}/m_p) l_p$, the maximum value of M is restricted by the minimum of $M \leq (1 - m_{\text{capt}}/m_p) l_p \sin\theta_x$ or by $M \leq (m_{\text{capt}}/m_p) l_p$. Thus for light particles emitted at very forward angles the former restriction will limit M and if light particles are captured the second will be the restriction. The dynamics of the reaction will determine the specific relative population of the M population. We make some simplifying "reasonable" approximations.

The kinematical conditions for incomplete fusion generally lead to differences between l_p and l_x being large. This condition results in[22] the transferred angular momentum being oriented approximately perpendicular to the plane determined by the outgoing particle x and beam axis. Thus along the beam axis $|\alpha_J^M|^2 \alpha |D_{JM}^J(\pi/2)|^2$ or using Eq. (12) and the approximation in Appendix B

$$|\alpha_J^M|^2 \alpha \frac{1}{(J+\frac{1}{4}-M^2)^{1/2}}$$

which peaks at $M \simeq \sqrt{J}$ (see Fig. 1). This can only be

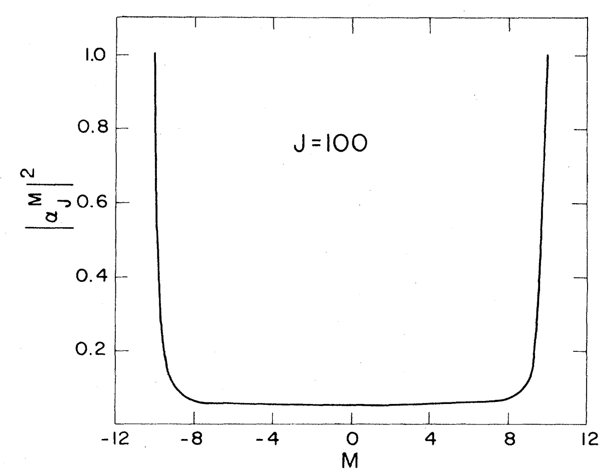

FIG. 1. Relative M population along the beam axis for $J=100$ if J is assumed to be normal to the plane determined by the beam and outgoing particle axis.

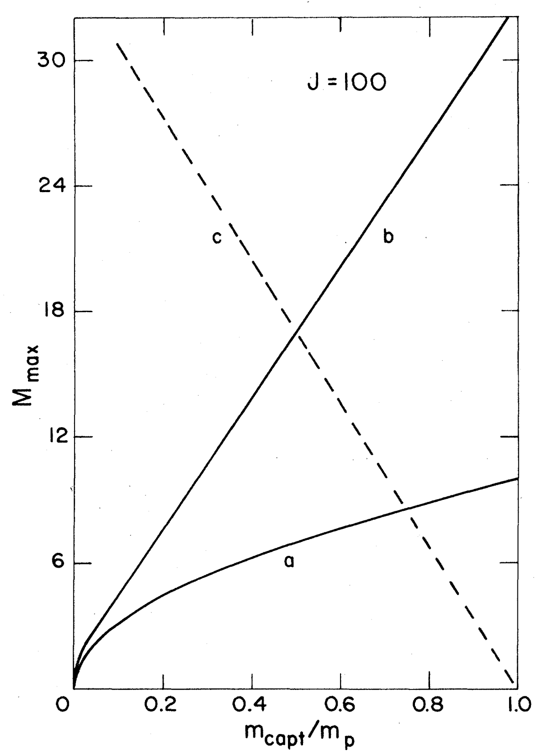

FIG. 2. The maximum values of M for $J=100$ as a function of the captured mass determined for the following assumptions: (a) $M=J$ (J is normal to the beam-outgoing particle plane), (b) $M=0.95J$, and (c) the outgoing particle exits at 20° to the beam axis. Note that for high captured mass curve c determines the maximum value of M, while for low captured mass curves a or b determine M_{max}.

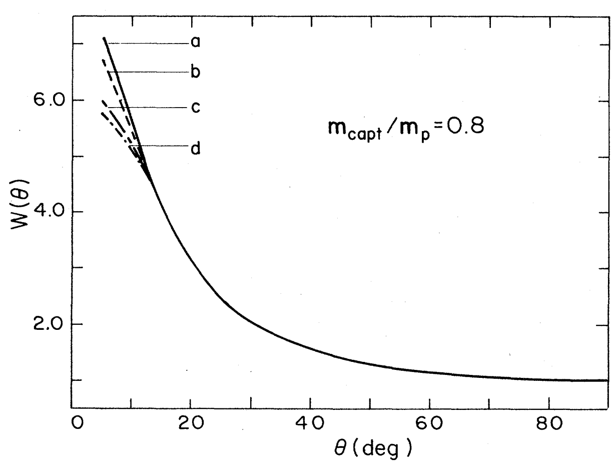

FIG. 3. Calculated angular distributions for $J_{max}=100$ for, a, complete fusion and for several cases of incomplete fusion. Curves b,c,d all presume $\frac{1}{3}$ of the fission cross section (the highest partial waves) is due to incomplete fusion with $m_{capt}/m_p=0.8$; b, the outgoing particle goes at 20°; c, the angular momentum is oriented normal to the plane ($M_{perp}=J$); d, $M_{perp}=0.98J$.

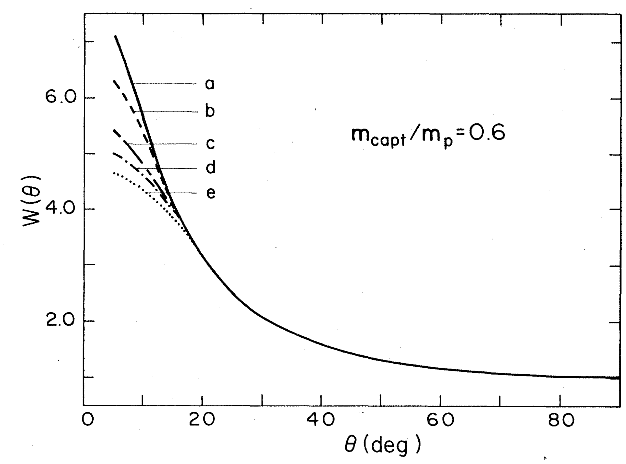

FIG. 4. Same as in Fig. 3 except for $m_{capt}/m_p=0.6$ and, e, one-half of the fission cross section is due to incomplete fusion.

valid when the restriction on M from the outgoing particle does not supersede it. Shown in Fig. 2 are curves which demonstrate limits on M as a function of m_{capt}/m_p.

Shown in Fig. 3 are several calculations including incomplete fusion with $m_{capt}/m_p=0.8$ for a system with $J_{max}\sim 100\hbar$ and $\sigma_{||}\sim 11$ together with a calculation for complete fusion only. With the assumption $M=0$ the curve for incomplete fusion hardly differs from that of complete fusion. Clearly the effect of the reduced J alone *in this case* is a small effect, the major influence is caused by the angular momentum orientation which serves to reduce the anisotropy.

Of course, the assumption that $|M_{perp}|=J$ is not reasonable as there will be a width to the distribution. Very small changes in that restriction, e.g., $|M_{perp}|=0.98J$ produce large changes in the angular distribution. Included in Fig. 3 is the calculated angular distribution for $|M_{perp}|=0.98J$ and a large decrease of the anisotropy is seen.

If the average captured mass corresponds to $m_{capt}/m_p=0.6$ the effects on the fission angular distribution are stronger. Figure 4 shows several calculations of fission fragment angular distributions for this condition.

In summary, the effects of compound nucleus formation by processes other than complete fusion can be significant and generally lead to reduced anisotropies. Only through complete measurements will the individual contributions be unraveled.

VI. CONCLUSIONS

A general angular distribution formula for fission following compound nucleus formation has been derived and it has been shown that the previously used angular distribution prescription is valid only for a restricted range of conditions and fails dramatically for high spin systems. Thus conclusions, based on analysis with that formula, about noncompound nucleus formation or failure of the RLDM, are suspect. At the same time it has been demonstrated that with a more reasonable choice for the K distribution at the saddle point the transition state model and the general expression derived from scission give essentially the same result for high spin systems so that there is no need to change the essential assumptions about the fission process to produce strongly anisotropic angular distributions. For low spin, extended saddle point shapes, the angular distributions are sensitive to RLDM parameters (or fission fragment shapes), but for high spin systems the angular distributions quickly become $1/\sin\theta$, rather independent of shape. It has been shown that incomplete fusion can have a considerable impact on fission fragment angular distributions and so must be considered in analysis of singles data. In the succeeding work several experiments are reanalyzed in order to demonstrate the substantial impact of the results of this work on the recent interpretations of data from high spin compound nuclei.

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APPENDIX A: DENSITY OF STATES FOR CHANNEL SPIN

The density of states for two spherical nuclei of spins j_1 and j_2 coupled vectorially to channel spin s is given by

$$\rho(s)=\sum_{j_1 j_2}\rho(j_1)\rho(j_2)\ , \tag{A1}$$

where $\rho(j_i)=(2j_i+1)e^{-j_i^2/2\sigma_i^2}$ and the sum is restricted so $\mathbf{s}=\mathbf{j}_1+\mathbf{j}_2$. Classically, we can write[14]

$$\rho(\mathbf{s})=\int\rho(\mathbf{j}_1)\rho(\mathbf{j}_2)\delta^3(\mathbf{s}-\mathbf{j}_1-\mathbf{j}_2)\ ,$$

where

$$\rho(\mathbf{j})=e^{-[j+(1/2)]^2/2\sigma^2}j^2dj\,d\Omega\ .$$

After substituting for the δ function, we carry out the integrations and obtain

$$\frac{\rho(s)}{2s+1}\alpha e^{-[s+(1/2)]^2/2s_0^2}\ ,$$

where $s_0^2=\sigma_1^2+\sigma_2^2$ (see also Ref. 14). The values of σ_1 and σ_2 are given by

$$\sigma_i^2=\frac{\mathscr{I}_i T_i}{\hbar^2}\sim\frac{2}{5}\frac{m_i r_i^2 T_i}{\hbar^2}\ ,$$

where m is the fragment mass, r is the radius of that fragment, and T is the nuclear temperature which is related to the excitation energy E_i^* by $T\cong[(8/m)E_i^*]^{1/2}$ and $E_1^*+E_2^*=E_{\text{c.m.}}+Q-E_{\text{CB}}-E_{\text{rot}}$ where E_{CB} is the Coulomb barrier at separation and E_{rot} is the rotational energy in relative motion of the two fragments. For fission the best approximation for the sum of these last two terms comes from the fragment kinetic energies or from Viola systematics.[23]

In general it is much more likely that emitted fission fragments are deformed and that their symmetry axes are aligned, i.e., $K_1+K_2=K$. For a deformed nucleus we have

$$\rho(j_1,K_1)\alpha e^{-[j_1+(1/2)]^2/\sigma_\perp^2}e^{-K_1^2/2\sigma_{\text{eff}}^2}\ ,$$

where $\sigma_\perp^2=\mathscr{I}_\perp T/\hbar^2$ and

$$\frac{1}{\sigma_{\text{eff}}^2}=\frac{1}{\mathscr{I}_{\|}T/\hbar^2}-\frac{1}{\sigma_\perp^2}\ .$$

Since the density of states depends upon the K quantum number the calculation is somewhat more involved than for the spherical case but follows the same line. The spins j_1 and j_2 are allowed to have arbitrary orientation with the constraints $\mathbf{s}=\mathbf{j}_1+\mathbf{j}_2$ and $K=K_1+K_2$ where $K_i=j_i\cos\theta_i$. The integrations are made with the result that the corresponding expression to $\rho(s')/(2s'+1)$ is

$$\rho(s',K)=e^{-[s'+(1/2)]^2/2\sigma_\perp^2}e^{-K^2/2\sigma_{\text{eff}}^2}\ , \tag{A2}$$

where $\sigma_\perp^2=(\mathscr{I}_{\perp_1}T_1+\mathscr{I}_{\perp_2}T_2)/\hbar^2$ and

$$\frac{1}{\sigma_{\text{eff}}^2}=\frac{1}{(\mathscr{I}_{\|_1}T_1+\mathscr{I}_{\|_2}T_2)/\hbar^2}-\frac{1}{\sigma_\perp^2}\ .$$

A more model dependent way of estimating the density of final states can also be obtained from the model of two touching nuclei.[24] It is assumed that these nuclei rotate as a complex so that $\mathscr{I}_{\|}=\mathscr{I}_{\|_1}+\mathscr{I}_{\|_2}$ and $\mathscr{I}_\perp=\mathscr{I}_{\perp_1}+\mathscr{I}_{\perp_2}+\mathscr{I}_{\text{rel}}$. The total angular momentum J is divided up as in a deformed nucleus into R, the component normal to the axis through the nuclei, and K, the component along that axis. Under these assumptions the angular momentum in the relative motion and in the fragments can be derived in terms of J and K. The result for the fragment spins is identical to (A2). This latter model has been used to suggest specific collective modes of excitation of the fission fragments.[25]

In general the parameters σ for the two fragments could be appreciably different and depend in a more complicated way on T. In particular for some systems the fragment masses may change as a function of E^*, for example in fission from uranium where there is a change from asymmetric to symmetric fission as E^* increases. Properly, σ_1 and σ_2 should be averaged over all final channels. It should also be noted that T should be calculated for excitation energies after prefission particle emission but before particle emission from the fission fragments.

For spherical fragments the average value of the magnitude of the spin j (or $j+\frac{1}{2}$) is given by

$$\langle |j| \rangle = \frac{\int j e^{-j^2/2\sigma^2} j^2 dj\, d\Omega}{\int e^{-j^2/2\sigma^2} j^2 dj\, d\Omega}$$

which gives $\langle |j| \rangle = 4\sigma/\sqrt{2\pi}$. An estimate of s_0^2 can be obtained from multiplicity measurements since in such experiments $\langle |j| \rangle$ is determined and we can derive s_0^2 from

$$s_0^2 = \sigma_1^2 + \sigma_2^2 = \frac{2\pi}{16}[\langle |j_1| \rangle^2 + \langle |j_2| \rangle^2] ,$$

which if $\langle |j_1| \rangle = \langle |j_2| \rangle$ gives $s_0^2 = 4\pi/16\langle |j| \rangle^2$. A more realistic calculation to compare to experiment would involve using Eq. (6) to calculate $\langle |s| \rangle$.

APPENDIX B: USEFUL APPROXIMATIONS

Summarized here are various approximations which are used in this paper. For Clebsch-Gordan coefficients whose angular momenta are large compared to 1 but where one value of the angular momentum is less than the other two,[26,27]

$$\langle l_1\mu_1 l_2\mu_2 | LM \rangle \sim \left[\frac{2L+1}{2l_1+1}\right]^{1/2} d^L_{M,l_1-l_2}(\theta) ,$$

where $\theta = \cos^{-1}\mu_2/\sqrt{l_2(l_2+1)}$ and $L < l_1, l_2$. In fission, a large number of partial waves are generally important so that an average value of $|d^L_{MK}(\theta)|^2$ is all that is needed. For angles $\theta \gg |M-K|/(L+\frac{1}{2})$ this is given by[13]

$$|d^L_{MK}(\theta)|^2 \sim \frac{1}{\pi} \frac{1}{[(L+\frac{1}{2})^2\sin^2\theta - M^2 - K^2 + 2MK\cos\theta]^{1/2}} . \tag{B1}$$

APPENDIX C: TRANSMISSION COEFFICIENTS

The transmission coefficients obviously play a crucial role for excitations near and below the fission barrier and less of a role far above the barrier. In order to show the possible problems we consider a parabolic fission barrier,[28] although analytic expressions for more complex shapes have also been given.[29] For a simple parabolic barrier the transmission coefficients have the form[28]

$$T^J_{c'} = \frac{1}{1+\exp\frac{2\pi(B_f - E)}{\hbar\omega}} , \tag{C1}$$

where E is the excitation energy, B_f is the fission barrier, and $\hbar\omega$ is the barrier curvature. Clearly for energies far above the fission barrier $T^J_{c'} = 1$ and the effect of the barrier is small. At energies far below the barrier

$$T^J_{c'} \simeq e^{-2\pi/\hbar\omega(B_f - E)} .$$

The J dependence of the fission barrier, B_f, can be expressed in the RLDM (Ref. 2) as

$$B_f(J) = B_f(J=0) + E_{\rm rot} - E^0_{\rm rot} ,$$

where $E_{\rm rot}$ and $E^0_{\rm rot}$ are the rotational energies of the deformed nucleus at the saddle point and in its ground state (with moment of inertia $\mathscr{I}_0$). Substitution for the last two terms gives

$$B_f(J,K) = B_f(J=0) + \hbar^2 J^2 \left[\frac{1}{2\mathscr{I}_\perp} - \frac{1}{2\mathscr{I}_0}\right] + \hbar^2 K^2 \left[\frac{1}{2\mathscr{I}_\parallel} - \frac{1}{2\mathscr{I}_\perp}\right] . \tag{C2}$$

Since $\mathscr{I}_\perp > \mathscr{I}_0$ the effect of nonzero angular momentum J is to reduce the fission barrier. The RLDM (Ref. 2) assumes rotation perpendicular to the symmetry axis ($K=0$), so that rotation about an arbitrary axis serves to produce an *increase* in the barrier over what the standard RLDM predicts (see also Ref. 30). This implicit K dependence (which can also be translated into an s' dependence) of the fission barrier then gets reflected in the transmission coefficients. The situation is, of course, much more complex and full calculations should be made for rotation around an arbitrary axis. Substituting Eq. (C2) into (C1) we obtain

$$T^J(K) = \frac{1}{1+\exp\frac{2\pi}{\hbar\omega}[B_f(J) - E^*]\exp(+K^2/2K_1^2)} , \tag{C3}$$

where

$$\frac{1}{K_1^2} = \frac{2\pi\hbar^2}{\hbar\omega}\left[\frac{1}{\mathscr{I}_\parallel} - \frac{1}{\mathscr{I}_\perp}\right] .$$

Thus for high values of E^* the transmission coefficients have a weak K dependence which is neglected here but it should be noted that it is potentially a problem.

It should be added that in Ref. 2 it was concluded that a compound nucleus is not formed when $B_f = 0$. While that is certainly plausible, it is suggested that condition ensures $T^J \to 1$ so there will be no survival of the compound nucleus, but it may still be formed in the sense of Eq. (5).

APPENDIX D: ESTIMATE OF FISSION FRAGMENT SHAPES

The estimates of the fragment moments of inertia described here are based on rigid body values for ellipsoidal shapes. From consideration of constant mass and density we have

$$2b^2 a = r_0^3 A , \tag{D1}$$

where b is the minor axis and a the major axis of each ellipsoid and $r_0 A^{1/3}$ is the radius of the spherical compound nucleus.

The sums of the parallel and perpendicular moments of inertia of the two identical fragments, if they are emitted with their major axes aligned, are

$$\sum \mathscr{I}_\parallel = \tfrac{2}{5} A b^2 , \qquad \sum \mathscr{I}_\perp = \tfrac{1}{5} A(a^2 + b^2) , \tag{D2}$$

where A is the mass of the compound nucleus. The value of a can be estimated from the fragment kinetic energies (or Viola systematics) as

$$\frac{\left[\frac{Ze}{2}\right]^2}{2a+\Delta}=E_K ,$$

where E_K is the sum of the fragment kinetic energies, Z is the compound nucleus charge, and Δ is a distance which accounts for the fact that the nuclei have a diffusivity. With the commonly used choice of $\Delta=2$ fm and the definition $a=a_0(A/2)^{1/3}$ we obtain $a_0 \simeq 1.63$. This value of a_0 in conjunction with Eq. (D1) and the RLDM value of $r_0=1.225$ leads to $b_0 \approx 1.062$ and $a/b=1.53$. The value for the ratio of a/b is considerably smaller than that of Ref. 31.

Finally, we arrive at the values for the sums of the fragment moments of inertia

$$\sum \mathscr{I}_{\parallel} \simeq 0.28A^{5/3}\ \mathrm{fm}^2 ,$$
$$\sum \mathscr{I}_{\perp} \simeq 0.48A^{5/3}\ \mathrm{fm}^2 . \tag{D3}$$

Expressed as a fraction of the spherical compound nucleus moment of inertia $\mathscr{I}_{\mathrm{sph}}$ these values correspond to $\sum \mathscr{I}_{\parallel}=0.47\mathscr{I}_{\mathrm{sph}}$ and $\sum \mathscr{I}_{\perp}=0.79\mathscr{I}_{\mathrm{sph}}$. It should be emphasized that there is nothing magical about the value of $r_0=1.225$ and the moments of inertia need not be rigid or the shapes ellipsoidal. Thus the extracted moments of inertia might be expected to differ somewhat from (D3).

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[1]See, for example, R. Vandenbosch and J. R. Huizenga, *Nuclear Fission* (Academic, New York, 1973).
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