Four-nucleon scattering in the K-matrix approach with improved treatment of the $(2+2)$ channels

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The cross sections for elastic and rearrangement four-nucleon reactions have been calculated above the breakup threshold in the K-matrix approximation to the Grassberger-Sandhas integral equations. In the first order K-matrix approximation we find that the ^{2}H(d,p)^{3}H and ^{2}H(d,n)^{3}He cross sections at low energies are sensitive to the properties of the nuclear force and to its tensor component even at higher energies. At higher energies good agreement is obtained with experiment, but there is quantitative and qualitative disagreement with the data at lower energies. We also performed K-matrix calculations including the principal value part of the $(2+2)$ propagators by means of the generalized-unitary-pole-expansion/energy-dependent-pole-expansion for the $(2+2)$ subamplitudes. At lower energies this improved the agreement with data in the ^{2}H(d,n)^{3}He reaction considerably, and in the case of p+^{3}He elastic scattering even resulted in a spectacular improvement in the forward direction. The lack of structure in the differential cross section [e.g., absence of a second maximum in the ^{2}H(d,n)^{3}He cross section] persisted, however. This qualitative disagreement is probably mainly due to the omission of the contribution of the p-wave three-body subsystem amplitude in the calculations.

I. INTRODUCTION

A practical integral equation approach to four-body scattering and breakup has been developed by Grassberger and Sandhas[1] (GS). In this formulation the original operator relations were reduced to effective two-body equations in two steps by employing separable expansions both for the two-body and for the three-body subamplitudes. After partial wave decomposition, the GS equations are then reduced to manageable one-variable integral equations.

The first four-nucleon calculations based on this method have been performed by Alt, Grassberger, and Sandhas (AGS).[2] There, using separable Yamaguchi potentials, the rearrangement reaction $d+d\rightarrow p+t$ $(n+{}^3He)$ and elastic $p+{}^3He$ scattering have been considered at energies above the breakup threshold. The final effective two-body equations were solved in first-order K-matrix approximation, and, as a further simplification, an approximate analytical representation was used for the $(3+1)$ and $(2+2)$ subamplitudes. In view of the rather drastic approximations made, the results obtained were encouraging. As expected, they improved with increasing energy of the incident particle, but failed to reproduce the second maximum in the differential cross section of the $2+2\rightarrow 3+1$ rearrangement scattering, and were quite unable to reproduce the data in a forward direction for the $3+1\rightarrow 3+1$ case.

These calculations have been repeated in a rather careful way by Becker[3] for separable potentials with Gaussian instead of Yamaguchi form factors.[4] Despite this modification, his results were in fair agreement with the ones of Ref. 2. The general trends, in particular the discrepancies between theory and experiment just mentioned, occurred again. A further calculation using this formalism, which goes up to second-order K-matrix approximation but uses the Bateman expansion, is due to Sawicki and Namyslowski.[5] Their first-order results, however, completely disagree even qualitatively with the calculations in Refs. 2 and 3. And, as we will see, similar disagreement is found between their second-order results and the ones obtained in this paper. We will return to this point when comparing different methods in Sec. IV. Finally, we mention the field theoretical model to four-nucleon scattering by Fonseca and Shanley,[6] which has been applied also above the breakup threshold.[7] Being structurally similar to the GS formalism, it adopts simple three-body subamplitudes with adjustable parameters fitted to three-body on-shell data.

A more accurate calculation based on the semirealistic local Malfliet-Tjon potential has been performed by Tjon,[8] however, for energies below the breakup threshold only. For the $(3+1)$ and $(2+2)$ subamplitudes the Hilbert-Schmidt expansion (HSE) was employed at the expense of great numerical effort. An alternative calculation performed by Kröger and Sandhas,[9] which avoids this expansion by employing Padé techniques for the two-variable integral equations, is extremely time consuming as well. In both calculations the importance of the p-wave contribution to the $(3+1)$ subamplitudes has been recognized. Further details and additional references can be found in Ref. 10.

In view of the complexity of the problem and the extreme computational effort needed, more efficient

methods are required. Hence, most recent work on four-nucleon collisions has concentrated on the development and testing of new approximation schemes. As compared to the HSE,[8] considerable improvement in speed and rate of convergence of the calculations is gained by using the energy-dependent pole expansion[11] (EDPE) or the generalized unitary pole expansion[12] (GUPE) to represent the $(3+1)$ and $(2+2)$ subamplitudes. It has been shown that both EDPE and GUPE are about 15—20 times faster than the HSE for a converged calculation.[13] Calculations confirm that the EDPE is very accurate in bound state calculations.[14] With only two separable terms per subamplitude, a four-nucleon binding energy is obtained which agrees to better than 1.5% with the results of Gibson and Lehman,[15] who solved the corresponding two-variable integral equation numerically. The efficiency of these expansions has, moreover, been demonstrated in molecular bound state calculations,[16] as reviewed by Lim at the Karlsruhe Conference.[17] In the scattering region, successful applications of the EDPE and GUPE and their modifications[18] denoted SE1 and SE2 have been performed very recently by Fonseca,[19] for energies below breakup only.

It is the purpose of the present investigation to calculate elastic and rearrangement four-nucleon processes *above* the breakup threshold along the lines developed in Refs. 1 and 2, however, taking into account the new techniques and experiences discussed above. We first solve the GS integral equations for four-nucleon scattering in the first-order K-matrix approximation for a variety of nucleon-nucleon interactions, including the Malfliet-Tjon potential and a separable force with a tensor component, which is taken into account in the t_{00} approximation. For the $(3+1)$ and $(2+2)$ subamplitudes we employ the GUPE and EDPE, instead of the HSE or Bateman expansion. Subsequently, we go beyond the first-order K-matrix expansion: While the $(3+1)$ propagator, as in the case of the first-order K-matrix approximation, is replaced by the discontinuity along its cut,[2] the $(2+2)$ propagator is evaluated using the EDPE-GUPE. This procedure is practically equivalent to the exact inclusion of the $(2+2)$ propagator proposed by Haberzettl and Sandhas,[20] followed by a first-order K-matrix approximation to the single remaining channel.

In Sec. II we present the four-body formalism and the equations to be solved once the EDPE-GUPE separable expansions have been introduced. Section III deals with the first-order K-matrix approximation and its extension by a full evaluation of the $(2+2)$ propagator. The results of the first-order K-matrix calculations are presented in Sec. IV, followed in Sec. V by those obtained by means of the complete evaluation of the $(2+2)$ propagator. We discuss our results and draw some conclusions in Sec. VI. Finally, the numerical treatment of the integral over the $2+2$ propagator is discussed in the Appendix.

II. FORMALISM

In the AGS theory, four-body scattering is described by the equation[1]

$$U^{\sigma\rho}_{\beta\alpha}=\bar{\delta}_{\sigma\rho}\delta_{\beta\alpha}G_0^{-1}T_\alpha^{-1}G_0^{-1}+\sum_\tau\sum_\gamma\bar{\delta}_{\sigma\tau}U^{\tau}_{\beta\gamma}G_0T_\gamma G_0U^{\tau\sigma}_{\gamma\alpha}\ , \tag{2.1}$$

where, as usual, the labels α, β, and γ denote two-body subsystems, while the labels σ, ρ, and τ denote two-cluster partitions of the four particles. There are four partitions of the $(3+1)$-type (ijk,l) and three of the $(2+2)$-type (ij,kl). The presence of the anti-Kronecker symbol $\bar{\delta}_{\sigma\rho}=(1-\delta_{\sigma\rho})$ guarantees the characteristic property of Faddeev-type equations, that they exclude diagonal contributions from the inhomogeneous term and the kernel of the set of integral equations (2.1).

The operators appearing in Eq. (2.1) are defined as follows:

$$G_0(z)=(z-H_0)^{-1} \tag{2.2}$$

is the resolvent of the four-particle kinetic energy operator H_0 and $T_\alpha(z)$ is the two-particle transition operator, which satisfies the two-body Lippman-Schwinger equation

$$T_\alpha(z)=V_\alpha+V_\alpha G_0(z)T_\alpha(z)\ , \tag{2.3}$$

where V_α is the interaction between the constituents of pair α. The transition operators $U^\tau_{\beta\alpha}(z)$ describe the scattering within the three-body subsystem contained in cluster τ and solve the three-body AGS equation,[1]

$$U^\tau_{\beta\alpha}(z)=\bar{\delta}_{\beta\alpha}G_0^{-1}(z)+\sum_\gamma\bar{\delta}_{\beta\gamma}T_\gamma(z)G_0(z)U^\tau_{\gamma\alpha}(z)\ . \tag{2.4}$$

For $(3+1)$ clusterings $\tau=(ijk,l)$, this is the usual three-body equation for the subsystem (ijk) considered in the four-body space, whereas for a $(2+2)$ partition $\tau=(ij,kl)$, it is only an auxiliary equation necessary in the context of Eq. (2.1). In the latter case, Eq. (2.4) describes the collision of two noninteracting pairs (ij) and (kl) in such a way that the corresponding on-shell solution does not have any immediate physical meaning. In contrast to physically observable systems of this type, it only conserves the total energy of the system, but permits an energy flux between the pairs. Therefore, it does not conserve the pair energies individually.

A separable representation of the $T_\gamma(z)$,

$$T_\gamma(z)=\sum_{nm}|\gamma n\rangle t_{\gamma,nm}\langle\gamma m|\ , \tag{2.5}$$

is well known to reduce the three-body AGS equation [Eq. (2.4)] to an effective two-body Lippman-Schwinger equation, i.e., to a one-variable equation after partial wave decomposition. Repeated application of this procedure allows us to reduce the *four-body* AGS equations to effective two-body LS equations in *two* steps.

Inserting Eq. (2.5) into Eq. (2.1) leads, after multiplication with $\langle\alpha n|G_0$ and $G_0|\beta m\rangle$, to the effective three-body equation

$$\tilde{U}^{\sigma\rho}_{\alpha n,\beta m}=\bar{\delta}_{\sigma\rho}\tilde{G}^{-1}_{0_{\alpha n,\beta m}}+\sum_{\tau\neq\sigma}\sum_{\gamma l}\tilde{T}^{\tau}_{\alpha n,\gamma l}\tilde{G}_{0_{\gamma l,\gamma l}}\tilde{U}^{\tau\rho}_{\gamma l,\beta m} \tag{2.6}$$

with

$$\tilde{U}^{\sigma\rho}_{\alpha n,\beta m}=\langle\alpha n|G_0U^{\sigma\rho}_{\alpha\beta}G_0|\beta m\rangle\ , \tag{2.7a}$$

$$\tilde{T}^{\tau}_{\alpha n,\beta m}=\langle\alpha n|G_0U^{\tau}_{\alpha\beta}G_0|\beta_m\rangle\ , \tag{2.7b}$$

$$\tilde{G}_{0_{\alpha n,\beta m}}=\delta_{\alpha\beta}\delta_{nm}t_{\alpha n}. \tag{2.7c}$$

In this way the four-body problem is formally reduced to a three-body problem. To obtain effective two-body equations it is only necessary, therefore, to insert a separable expansion of the effective three-body T-matrices:

$$\tilde{T}^{\tau}_{\alpha n,\beta m}=\sum_{t}\sum_{\mu\nu}\left|\begin{matrix}\tau t\\ \mu\\ \alpha n\end{matrix}\right\rangle G^{\tau t}_{0,\mu\nu}\left\langle\begin{matrix}\tau t\\ \nu\\ \beta m\end{matrix}\right| \tag{2.8}$$

into Eq. (2.6). Multiplication of the resulting equation by

$$\left\langle\begin{matrix}\sigma s\\ \mu\end{matrix}\right|\tilde{G}_0 \text{ and } \tilde{G}_0\left|\begin{matrix}\rho r\\ \nu\end{matrix}\right\rangle$$

leads to the effective two-body LS equation

$$\mathbf{T}=\mathbf{V}+\mathbf{V}\mathbf{G}_0\mathbf{T} \tag{2.9}$$

with matrix elements

$$\mathbf{T}^{\sigma s,\rho r}_{\mu\nu}=\sum_{\alpha n,\beta m}\left\langle\begin{matrix}\sigma s\\ \mu\\ \alpha n\end{matrix}\right|\tilde{G}_{0,\alpha n}\tilde{U}^{\sigma\rho}_{\alpha n,\beta m}\tilde{G}_{0,\beta m}\left|\begin{matrix}\rho r\\ \nu\\ \beta m\end{matrix}\right\rangle, \tag{2.10a}$$

$$\mathbf{V}^{\sigma s,\rho r}_{\mu\nu}=\bar{\delta}_{\sigma\rho}\sum_{\alpha n}\left\langle\begin{matrix}\sigma s\\ \mu\\ \alpha n\end{matrix}\right|\tilde{G}_{0,\alpha n}\left|\begin{matrix}\rho r\\ \nu\\ \alpha n\end{matrix}\right\rangle, \tag{2.10b}$$

$$\mathbf{G}^{\sigma s,\rho r}_{0,\mu\nu}=\delta_{\sigma\rho}\delta_{sr}\tilde{G}^{\tau t}_{0,\mu\nu}\,. \tag{2.10c}$$

Explicitly written, Eq. (2.9) hence reads

$$\mathbf{T}^{\sigma s,\rho r}_{\mu\nu}=\mathbf{V}^{\sigma s,\rho r}_{\mu\nu}+\sum_{\tau t}\sum_{\mu'\nu'}\mathbf{V}^{\sigma s,\tau t}_{\mu\mu'}\mathbf{G}^{\tau t}_{0,\mu'\nu'}\mathbf{T}^{\tau t,\rho r}_{\nu'\nu}\,. \tag{2.11}$$

For a correct description of four-nucleon scattering, we have to include spin and isospin and have to antisymmetrize. In what follows we only consider central nucleon-nucleon potentials (in one case we include tensor forces, but only in the t_{00} approximation). The total angular momentum J, the total spin S and isospin I and their corresponding three-components, as well as the total orbital angular momentum L, are therefore conserved. With the inclusion of spin and isospin, Eq. (2.11) can, after antisymmetrization and partial wave expansion, be written as

$${}^{ISL}\mathbf{T}^{\sigma s,\rho r}_{\mu\nu}={}^{ISL}\mathbf{V}^{\sigma s,\rho r}_{\mu\nu}+\sum_{\tau t}\sum_{\tau' t'}\sum_{\mu'\nu'}{}^{ISL}\mathbf{V}^{\sigma s,\tau t}_{\mu\mu'}\mathbf{G}^{\tau t;\tau' t'}_{0,\mu'\nu'}\,{}^{ISL}\mathbf{T}^{\tau' t',\rho r}_{\nu'\nu} \tag{2.12}$$

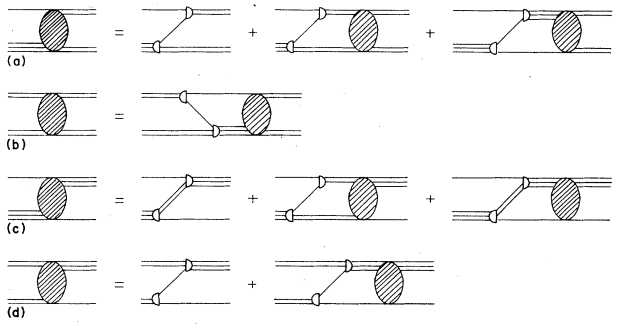

FIG. 1. Graphical representation of the effective two-body Lippman-Schwinger equations of the four-body system [see Eq. (2.13)]. Note the absence of the Born term in the $2+2\rightarrow 2+2$ reaction.

with

$$\mathbf{G}^{\tau t,\tau' t'}_{0,\mu\nu}\equiv\mathbf{G}^{\tau t}_{0,\mu\nu}\delta_{\tau\tau'}\delta_{tt'}\,.$$

The indices σ,ρ now refer to either a $(2+2)$ or a $(3+1)$ clustering only, not to a specific partition of these clusters. We can, therefore, schematically write Eq. (2.12) in the form

$${}^{\sigma}\mathbf{T}^{\rho}={}^{\sigma}\mathbf{V}^{\rho}+\sum_{\tau}{}^{\sigma}\mathbf{V}^{\tau}\mathbf{G}^{\tau}_{0}\,{}^{\tau}\mathbf{T}^{\rho} \tag{2.13}$$

with $\sigma,\rho,\tau=(2+2)$, $(3+1)$. From symmetry considerations it follows that ${}^{2+2}\mathbf{V}^{2+2}=0$. Graphically Eq. (2.13) can be represented as shown in Fig. 1.

It is obvious from the diagrams of Fig. 1 that by eliminating the ${}^{2+2}\mathbf{T}^{2+2}$ and ${}^{2+2}\mathbf{T}^{3+1}$ components we arrive at the two independent relations,

$${}^{3+1}\mathbf{T}^{2+2}={}^{3+1}\mathbf{V}^{2+2}+{}^{3+1}\tilde{\mathbf{V}}^{3+1}\mathbf{G}^{3+1}_{0}\,{}^{3+1}\mathbf{T}^{2+2} \tag{2.14}$$

and

$${}^{3+1}\mathbf{T}^{3+1}={}^{3+1}\tilde{\mathbf{V}}^{3+1}+{}^{3+1}\tilde{\mathbf{V}}^{3+1}\mathbf{G}^{3+1}_{0}\,{}^{3+1}\mathbf{T}^{3+1}\,, \tag{2.15}$$

where the $3+1\rightarrow 3+1$ potential ${}^{3+1}\tilde{\mathbf{V}}^{3+1}$ is now of the more complex form

$${}^{3+1}\tilde{\mathbf{V}}^{3+1}\equiv{}^{3+1}V^{3+1}+{}^{3+1}\mathbf{V}^{2+2}\mathbf{G}^{2+2}_{0}\,{}^{2+2}\mathbf{V}^{3+1}\,. \tag{2.16}$$

More explicitly, for the reaction dd$\rightarrow$pt and pt$\rightarrow$pt (n^3He$\rightarrow n^3$He), Eqs. (2.14) and (2.15) are one variable integral equations of the form

$${}^{ISL}T^{t,dd}_{\mu\nu}(q,q';z)={}^{ISL}B^{t,dd}_{\mu\nu}(q,q';z)+\sum_{\mu'\nu'}\sum_{j=t,qu}4\pi\int_0^{\infty}dq''q''^2\,{}^{ISL}C^{t,j}_{\mu\mu'}(q,q'';z)G^{j}_{0,\mu'\nu'}(z-\tfrac{2}{3}q''^2)\,{}^{ISL}T^{j,dd}_{\nu'\nu}(q'',q';z) \tag{2.17}$$

and

$${}^{ISL}T^{t,t}_{\mu\nu}(q,q';z)={}^{ISL}C^{t,t}_{\mu\nu}(q,q';z)+\sum_{\mu'\nu'}\sum_{j=t,qu}4\pi\int_0^{\infty}dq''\,q''^2\,{}^{ISL}C^{t,j}_{\mu\mu'}(q,q'';z)G^{j}_{0,\mu'\nu'}(z-\tfrac{2}{3}q''^2)\,{}^{ISL}T^{j,t}_{\nu'\nu}(q'',q';z) \tag{2.18}$$

where for simplicity we define

$$A\equiv{}^{3+1}\mathbf{V}^{3+1}\,,$$

$$B\equiv{}^{3+1}\mathbf{V}^{2+2}\,,$$

$$\tilde{B}\equiv{}^{2+2}\mathbf{V}^{3+1}\,,$$

and

$$C\equiv{}^{3+1}\tilde{\mathbf{V}}^{3+1}\,,$$

while

$${}^{ISL}\mathbf{T}^{t,t}\equiv{}^{3+1}\mathbf{T}^{3+1}$$

etc.

Equations (2.17) and (2.18), as well as the effective interaction ${}^{3+1}\tilde{\mathbf{V}}^{3+1}$, are depicted in Fig. 2.

We recall that the indices I, S, and L are the total spin, isospin, and the partial wave number, respectively, while μ and ν indicate the different terms in the separable expansion of the three-body amplitudes. The three-body spin channels corresponding to the triton (spin $\frac{1}{2}$) and the quartet states (spin $\frac{3}{2}$) are denoted by t and qu, respectively.

In the following investigation the effective potentials A, B, and C, as well as the propagators G_0^{3+1} and G_0^{2+2}, are calculated by means of the EDPE (Ref. 11) and GUPE.[12] In terms of the form factors $|\Gamma^{\nu}\rangle$ [or $|\Gamma^{\nu}(E)\rangle$], the potentials A and B are given by

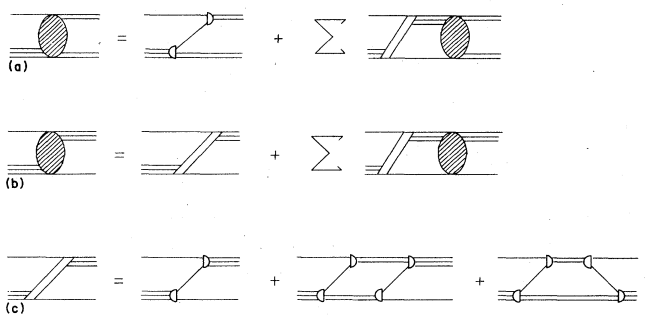

FIG. 2. The reduced equations for (a) the $2+2\rightarrow 3+1$ [Eq. (2.14)]; (b) the $3+1\rightarrow 3+1$ [Eq. (2.15)]; (c) the effective interaction ${}^{3+1}\tilde{\mathbf{V}}^{3+1}$ [Eq. (2.16)].

$$ {}^{ISL}A^{r,s}_{\mu\nu}(q,q';z)=\sum_{i=\phi,\mathrm{d}}\Lambda^{IS}_{rs,i}\sum_{nm}\int_{-1}^{+1}dx\,P_L(x)\Gamma^{r}_{\mu,mi}(Q_1;z)t^{i}_{nm}(Z-W_1)\Gamma^{s}_{\nu,ni}(Q_1';z) \tag{2.19}$$

and

$$ {}^{ISL}B^{r,ii}_{\mu\nu}(q,q';z)=\Lambda^{IS}_{ii,r}\sum_{nm}\int_{-1}^{+1}dx\,P_L(x)\Gamma^{r}_{\mu,mi}(Q_2;z)t^{i}_{mn}(z-W_2)\Gamma^{ii}_{\nu,ni}(Q_2';z) \tag{2.20}$$

and similarly for $\tilde{B}$. The explicit values of arguments are given in Ref. 13. The index i refers to d or ϕ channels [the indices $(2+2)$ or $(3+1)$ are fixed by s, r, and ii and can be omitted]. The n,m refer to the expansion of the two-body transitions given by (2.5). For the GUPE the z dependence of the form factors should be omitted. The explicit form of the propagator $t^{i}_{mn}(z)$, as well as the corresponding form factors, can also be found in Ref. 13.

For potential ${}^{ISL}C^{r,s}_{\mu\nu}$ we find

$$ {}^{ISL}C^{r,s}_{\mu\nu}(q,q';z)={}^{ISL}A^{r,s}_{\mu\nu}(q,q';z)+\sum_{\mu'\nu'}\sum_{i=\mathrm{d},\phi}4\pi\int_0^{\infty}dq''q''^2\,{}^{ISL}\tilde{B}^{ii,r}_{\mu\mu'}(q,q'';z)G^{ii}_{0,\mu'\nu'}(z-\tfrac{1}{2}q''^2)\,{}^{ISL}B^{ii,s}_{\nu'\mu}(q'',q';z)\ . \tag{2.21}$$

The graphical representation of this potential is shown in Fig. 2. The spin-isospin coupling coefficient $\Lambda^{IS}_{rs;i}$ and $\Lambda^{IS}_{ii,r}$, resulting from the antisymmetrization of the $3+1\rightarrow 3+1$ and the $2+2\rightarrow 3+1$ transition amplitudes, respectively, are given in Refs. 3 and 21.

III. THE K-MATRIX APPROXIMATION

In the first part of this section we solve the GS equations after their reduction to one variable integral equations [Eqs. (2.17) and (2.18)] in the first-order K-matrix approximation. In this approximation the p + t and d + d propagators are replaced by the discontinuities along their cuts. In this way all channels corresponding to unphysical particles are eliminated.

The Green's functions $\mathbf{G}_0^{\tau}$ of Eq. (2.13) can be split into two parts

$$\mathbf{G}_0=\mathbf{G}_0^{\delta}+\mathbf{G}_0^{\mathrm{P}}\ , \tag{3.1}$$

a δ function, and a principal value part. The LS equation,

$$\mathbf{T}=\mathbf{V}+\mathbf{V}\mathbf{G}_0\mathbf{T}\ , \tag{3.2}$$

can then be written in the form

$$\begin{aligned}&\mathbf{T}=\mathbf{K}+\mathbf{K}\mathbf{G}_0^{\delta}\mathbf{T}\ ,\\ &\mathbf{K}=\mathbf{V}+\mathbf{V}\mathbf{G}_0^{\mathrm{P}}\mathbf{K}\ .\end{aligned} \tag{3.3}$$

In the first-order K-matrix approximation we set $\mathbf{K}=\mathbf{V}$, and Eq. (3.3) then reduces to

$$\mathbf{T}=\mathbf{V}+\mathbf{V}\mathbf{G}_0^{\delta}\mathbf{T}\ . \tag{3.4}$$

It is obvious that the neglect of the principal value part $\mathbf{G}_0^{\mathrm{P}}$ of $\mathbf{G}_0$ is an approximation which should improve with increasing energy E.

In Eq. (2.13) this approximation means that we set

$$\mathbf{G}_0^{\delta}=\begin{bmatrix}G_0^{\mathrm{dd},\delta} & 0\\ 0 & G_0^{\mathrm{t},\delta}\end{bmatrix} \tag{3.5}$$

with [see Eq. (2.17)]

$$\begin{aligned}G^{\mathrm{t},\delta}_{0,11}(z-\tfrac{2}{3}q^{2\prime\prime})&=-i\pi R_{\mathrm{t}}\delta(E_{\mathrm{t}}+E-\tfrac{2}{3}q''^2)\\ &=-\frac{3}{4}\frac{i\pi}{q}R_{\mathrm{t}}\delta(q-q'')\end{aligned} \tag{3.6}$$

and

$$\begin{aligned}G^{\mathrm{dd},\delta}_{0,11}(z-\tfrac{1}{2}q''^2)&=-i\pi R_{\mathrm{dd}}\delta\left[E-\frac{q''^2}{2}+E_{\mathrm{dd}}\right]\\ &=-i\pi\frac{R_{\mathrm{dd}}}{q'}\delta(q'-q'')\ ,\end{aligned} \tag{3.7}$$

where R_{t} and R_{dd} are the triton and (d + d) residues. The on-shell momenta q and q' are fixed by the energy relation

$$E=-E_t+\tfrac{2}{3}q^2=-E_{dd}+\frac{q'^2}{2}\ . \tag{3.8}$$

Inserting Eqs. (3.6)—(3.8) into Eq. (2.17) reduces it to an algebraic equation with solution

$${}^{ISl}T_{11}^{t,dd}(q,q';E)=\frac{{}^{ISL}B_{11}^{t,dd}(q,q';E)}{1+i3\pi^2qR_t\,{}^{ISL}C_{11}^{t,t}(q,q';E)}\ . \tag{3.9}$$

Similarly,

$${}^{ISL}T_{11}^{t,t}(q,q';E)=\frac{{}^{ISL}C_{11}^{t,t}(q,q';E)}{1+i3\pi^2qR_t\,{}^{ISL}C_{11}^{t,t}(q,q';E)}\ . \tag{3.10}$$

In the first-order K-matrix approximation the potential ${}^{ISL}C_{11}^{t,t}$ is obtained from Eq. (2.21) by replacing G_0^{dd} by its δ-function part $G_0^{dd,\delta}$, with the result that

$${}^{ISL}C_{11}^{t,t}(q,q';E)={}^{ISL}A_{11}^{t,t}(q,q';E)-i4\pi^2R_{dd}q'\,{}^{ISL}\widetilde{B}_{11}^{t,dd}(q,q';E)\,{}^{ISL}B_{11}^{t,dd}(q,q';E)\ . \tag{3.11}$$

It is obvious that the unphysical qu and $\phi\phi$ states have dropped out because they do not possess a bound state and hence no corresponding cut contribution. In the same fashion all the $G_{0,\mu\nu}^{t}$ with $\mu,\nu>1$ are eliminated due to the fact that the bound state is only present in $G_{0,11}^{t}\to G_{0,11}^{t,\delta}$, and similarly $G_{0,11}^{dd}\to G_{0,11}^{dd,\delta}$, if we employ either the GUPE or the EDPE. The two expansions, therefore, reduce to a single term, the energy-dependent pole approximation (EDPA) or generalized unitary pole approximation (GUPA), which, however, are identical (in this case), since the energy dependence of the form factor of the EDPA is fixed at $E=E_t$, or E_{d+d} by the δ function.

The simplest improvement of the first-order K-matrix approximation is obtained by incorporating in the potential ${}^{ISL}C_{11}^{t,t}(q,q';E)$ the principal value parts by means of the EDPE-GUPE. Explicitly,

$$\begin{aligned}{}^{ISL}C_{11}^{t,t}(q,q';z)={}&{}^{ISL}A_{11}^{t,t}(q,q';z)-i4\pi^2R_{dd}q'\,{}^{ISL}\widetilde{B}_{11}^{t,dd}(q,q';z)\,{}^{ISL}B_{11}^{t,dd}(q,q';z)\\&+4\pi\int_0^\infty dq''q''^2\,{}^{ISL}\widetilde{B}_{11}^{t,dd}(q,q'';z)G_{0,11}^{dd,P}(z-\tfrac{1}{2}q''^2)\,{}^{ISL}B^{dd,t}(q'',q';z)\\&+\sum_{\mu\nu>1}4\pi\int_0^\infty dq''q''^2\,{}^{ISL}\widetilde{B}_{1\mu}^{t,dd}(q,q'';z)G_{0,\mu\nu}^{dd}(z-\tfrac{1}{2}q''^2)B_{\nu1}^{dd,t}(q'',q';z)\\&+\sum_{\mu\nu}4\pi\int_0^\infty dq''q''^2\,{}^{ISL}\widetilde{B}_{1\mu}^{t,\phi\phi}(q,q'';z)G_{0,\mu\nu}^{\phi\phi}(z-\tfrac{1}{2}q''^2)B_{\nu1}^{\phi\phi,t}(q'',q';z)\ .\end{aligned} \tag{3.12}$$

The importance of the $\phi\phi$ states for the cross sections can easily be estimated by omitting them in Eq. (3.12). The $(2+2)$ effective potentials contained in Eq. (3.12), unlike the effective $(3+1)$ potentials (see Ref. 10), have no logarithmic singularities. The principal value term and the other ones occurring in Eq. (3.12) can therefore be evaluated numerically using subtraction techniques even above breakup threshold (see the Appendix).

IV. FIRST-ORDER K-MATRIX CALCULATIONS

In the first-order K-matrix approximation we differ from the previous calculations of Alt, Grassberger, and Sandhas,[2] Becker,[3] and Sawicki and Namyslowski[5] in that we employ the EDPE and GUPE, which in this case reduce to their lowest order term, the EDPA or GUPA. We, moreover, employ a variety of two-nucleon potentials, including the local Malfliet-Tjon potential and a separable potential with a tensor component. In the latter case, however, we restrict ourselves to the t_{00} approximation. The potentials employed are given in Table I.

As already emphasized in Ref. 2, the best results should be obtained in first-order K-matrix approximation for the $2+2\to3+1$ reaction $[d+d\to t+p\ \ (n+{}^3He)]$. This approximation, on the other hand, cannot be expected to provide reasonable results for processes where the Born term vanishes, which is the case for elastic deuteron-deuteron scattering. For the elastic $t+p$ $(n+{}^3He)$ scattering one cannot expect too much either, since here the Born term only contributes to the backward scattering. The results of Refs. 2 and 3 confirmed these expectations, and so do our present calculations, which are displayed in Figs. 3—6 (Refs. 22 —25, respectively) for the reaction $d+d\to p+t$ and in Figs. 7 and 8 (Refs. 26 and 27, respectively) for elastic $p+{}^3He$ scattering.

13.8, 25.3, 51.5, and 83 MeV are similar to those obtained in Refs. 2 and 3 (no second maximum in the differential cross section), and too high values compared to experi-

TABLE I. Nucleon-nucleon interactions V_{NN}.

Shape	V_{NN} Symbol	a_d (fm)	a_4 (fm)	r_d (fm)	r_4 (fm)	E_{dd} (MeV)	E_t (MeV)
Yamaguchi	Y_1	5.396	−20.34	1.73	2.84	4.452	10.3
Yamaguchi	Y_2	5.416	−23.68	1.75	2.67	4.450	11.04
Yamaguchi	Y_3	5.48	−23.57	1.92	2.28	4.5774	10.26
Gauss	G	5.43	−23.57	1.87	2.82	4.5428	9.07
Local	MT	5.48	−23.57	1.92	2.82	4.544	8.59
Tensor	P_4	5.397	−17.0	1.727	2.84	3.8322	8.187[a]

[a]In the t_{00} approximation.

ment in the forward direction. However, with increasing energy the agreement with experiment improves considerably. Our results for the Gaussian separable potential G are in agreement with those obtained by Becker[3] at 51.5 and 83 MeV, but are higher for $E<25$ MeV. (This may be due to the fact that Becker used the same approximations to obtain analytical form factors as Alt *et al.*,[2] while the present calculations are based on EDPE-GUPE.) For the Malfliet-Tjon (MT) potential we employed a UPE with three terms at 13.8 MeV and UPA (one term) otherwise. Comparison at 13.8 MeV showed a discrepancy between the three-term UPE and the UPA of less than 2%. The agreement between the results for the MT and Gaussian potentials is remarkably close at all energies, a feature previously noticed. In Refs. 2, 3, and 5, terms containing $x=\cos\theta$ in the effective $(3+1)$ potentials were omitted to simplify the calculations. We tested this approximation and found a maximum error at $\theta_{\text{c.m.}}=0$ of less than 6% at 13.8 MeV increasing with energy to 20% at 83 MeV. In addition to the central separable potentials Y_2, Y_3, and G we also employed a separable Yamaguchi potential P_4, with a tensor component. The four-nucleon calculations in this case are performed in the t_{00} approximation. It is seen from Figs. 3–5 that this potential produces cross sections, which decrease faster with energy than any of the others. While the cross

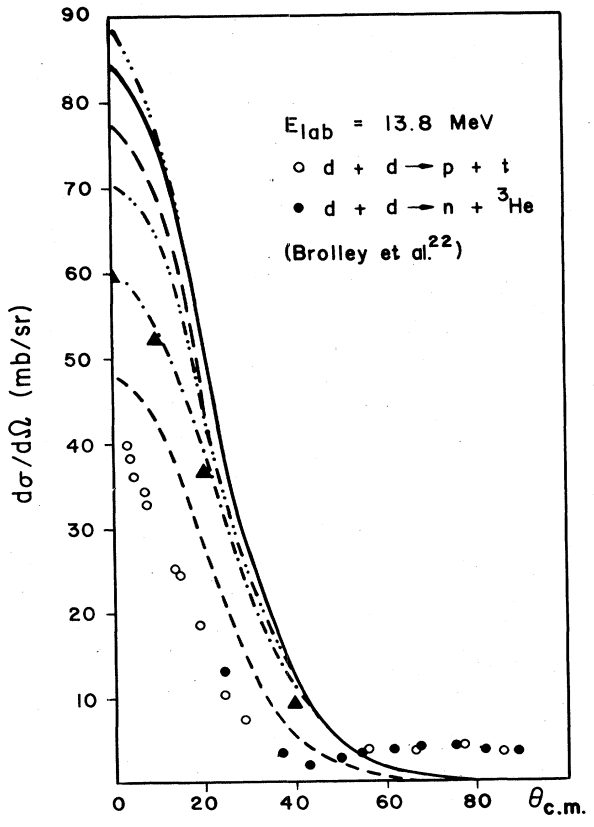

FIG. 3. The differential cross sections for $d+d\rightarrow t+p$ $(n+{}^3He)$, at $E_{lab}=13.8$ MeV in the K-matrix approximation of order one. — — —, (Y_1); —·—·—·—, (Y_2); ·····, (Y_3); ——, (G); ·····, (MT); — — —, (P_4); ▲ (Ref. 3).

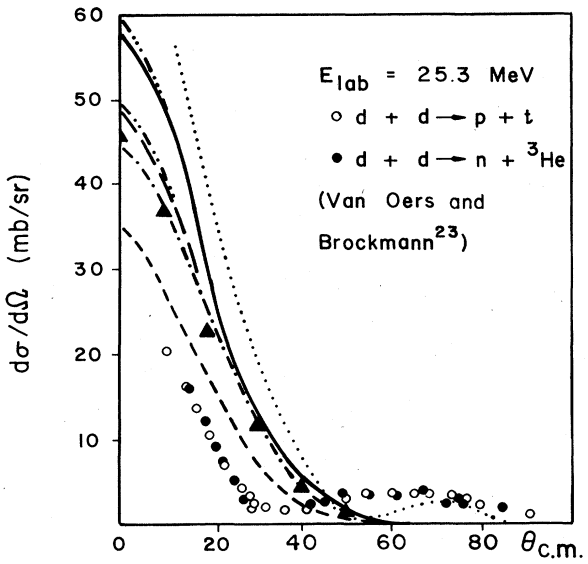

FIG. 4. The same as Fig. 3 but at $E_{lab}=25.3$ MeV. The dotted line is that of Ref. 5.

sections of all the different central potentials closely approximate each other and the data (from above) with increasing energy, the cross section for the tensor force drops even below the data at 83 MeV in the forward direction. However, at least in the t_{00} approximation, there is no indication of a second maximum in the cross section.

We also compare our results with those of Sawicki and Namyslowski,[5] who employed the Bateman expansion (two terms), but with simplified off-diagonal Bateman form factors, constructed from the diagonal ones (this additional approximation reduces the triton binding energy of their potential Y_2 from the correct value of 11.04 to 7.36 MeV). It is seen from Figs. 4 and 5 that there is a qualitative and quantitative disagreement between our cross sections and those of Ref. 5, for potential Y_2. This disagreement increases rapidly with energy. Instead of

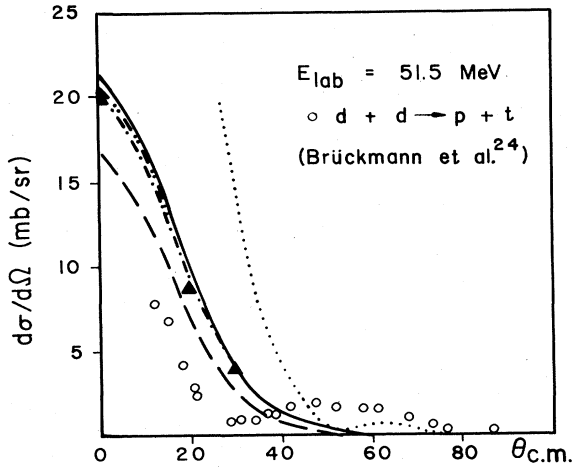

FIG. 5. The same as Figs. 3 and 4 but at $E_{lab}=51.5$ MeV.

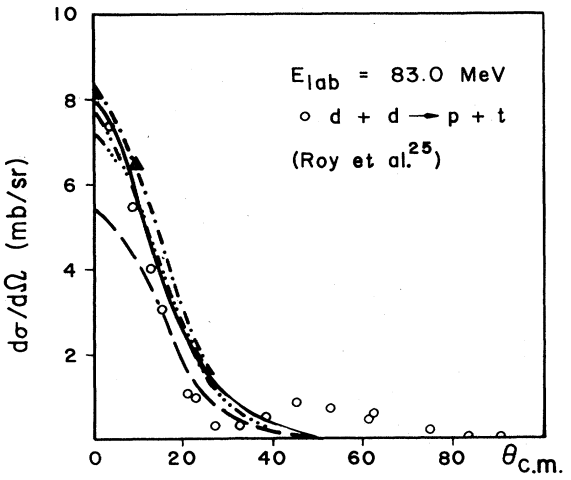

FIG. 6. The same as Fig. 3 but at $E_{lab}=83.0$ MeV.

improving with energy, as in all other cases, the first-order K matrix of Sawicki and Namyslowski becomes worse at higher energies. They also obtained a second maximum in the differential cross section in disagreement with all other calculations.

The results show considerable sensitivity to the properties of the nucleon-nucleon interaction especially at low energies. This is not surprising in view of the widely different values of the triton and the α-particle binding energies these potentials produce, implying considerable differences in the subamplitudes, used in the scattering

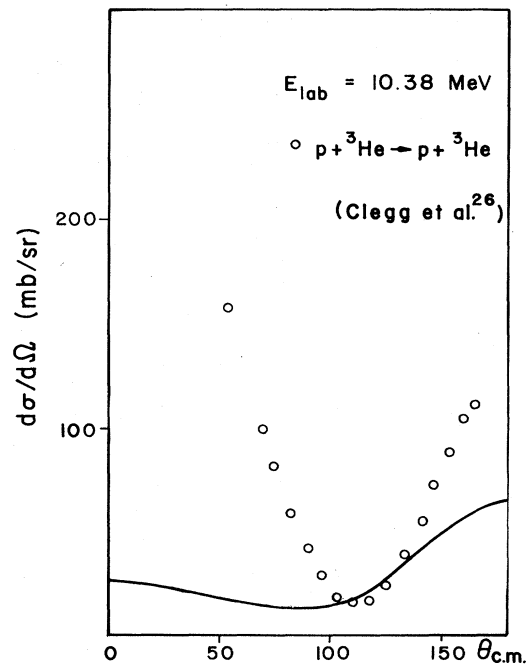

FIG. 7. The differential cross section for $p + {}^3He \rightarrow p + {}^3He$ at $E_{lab}=10.38$ MeV in the K-matrix approximation of order one. ——, (Y_3).

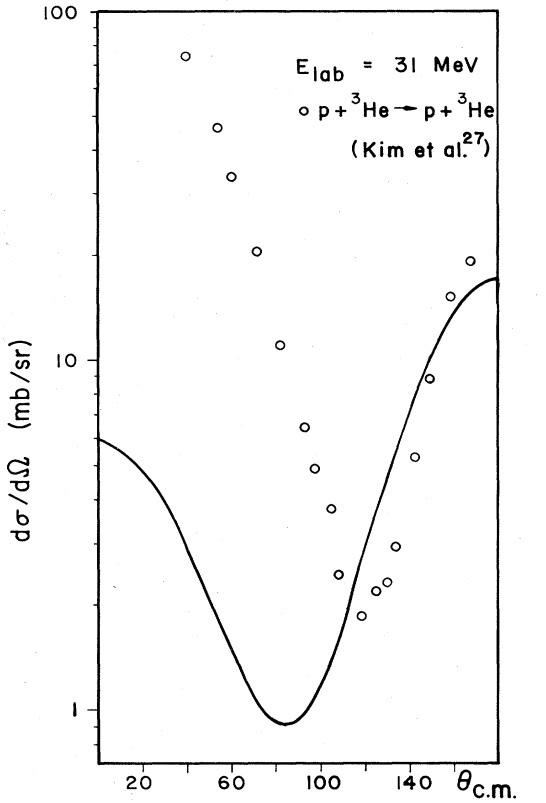

FIG. 8. The same as Fig. 7 but at $E_{lab}=31$ MeV.

calculations, particularly at low energies. In our approach we do not have the possibility of fitting the form factors to the three-nucleon bound state data; consequently low energy four-nucleon scattering is quite sensitive to the nucleon-nucleon potential. This feature is well known in three-nucleon scattering, where a good fit to the triton binding energy (n-d doublet scattering length) is required for a good fit to the scattering data.

As mentioned before, the first-order K-matrix approximation is less well suited for the calculation of elastic $p + {}^3He$ scattering, since in this case the Born term only contributes to the backward scattering. In agreement with the calculations of Refs. 2 and 3 we find for $p + {}^3He$ scattering at $E_{lab}=10.38$ and 31 MeV, using potential Y_3, that our calculations only show a vague similarity with the experimental data at forward angles (see Figs. 7 and 8). Even at backward angles the fit is not good at 10.38 MeV but improves at 31 MeV as expected. In the forward direction, the calculated cross sections lie far below the data. Higher order terms are needed to improve upon this result. A first step towards this direction is the treatment of the principal value part performed in the next section.

V. INCLUSION OF THE PRINCIPAL VALUE PART OF THE (2 + 2) PROPAGATOR

In this section we present the results of our calculations with the inclusion of the principal value parts of the Green's function in the (2 + 2) channels by means of the

EDPE-GUPE. We calculate the potential ${}^{ISL}C_{11}^{t,t}(q,q',E)$ of Eq. (3.12), including all terms, not only those obtained in first-order K-matrix approximation, given by Eq. (3.11).

In view of the computational effort required for inclusion of the principal value part of the $(2+2)$ propagators, these calculations were only performed with the Yamaguchi-type potential Y_3 (see Table I) with $E_t=10.26$ MeV and $E_{dd}=4.5774$ MeV. This is sufficient, however, for the investigation of the qualitative effect of the inclusion of the principal value contributions to the $(2+2)$ channels. We have seen that in the first-order K-matrix approximation the EDPE and GUPE both reduce to a single term, the GUPA. However, when including the principal value part of the $(2+2)$ propagator in our calculations, the two expansions differ. Both the dd and the $\phi\phi$ channels are incorporated. The numerical details are discussed in the Appendix.

The results of our calculations with potential Y_3 are shown in Figs. 9—14. For the $d+d\rightarrow p+t$ ($n+{}^3He$) reaction comparison with Figs. 3—6 shows that the inclusion of the $(2+2)$ channels improves the result of the first-order K-matrix approximation of Sec. IV. At 13.8 MeV (Fig. 9) the improvement is particularly pronounced in the forward direction. The results for the EDPE and the GUPE converge in general to each other after two terms. Since, according to Fig. 10, the first-order K-matrix approximation already leads to a better fit to the data, at 25.3 MeV the correction due to the principal value part is less pronounced but still substantial. The same comments hold for 51.5 MeV and Fig. 11. Note that for the these higher energies the EDPE and GUPE converge to each other already after two terms. At 83 MeV the effect of the inclusion of the $(2+2)$ channels has practically become negligible as is seen from Fig. 12. The reduced importance of the principal value contributions to the $(2+2)$ propagators is, of course, in accordance with physical expectations. The improvements achieved have to be contrasted with the fact that not the slightest indication of the second maximum could be obtained in these calculations, however. We should also mention that the good agreement for $\theta<40$ at 13.8 MeV may be partly due to the special choice of potential Y_3.

In Figs. 13 and 14 the effect of the inclusion of the principal value part of the $(2+2)$ propagators on *elastic* $p+t$ ($p+{}^3He$ or $n+{}^3He$) scattering, is shown. There is a dramatic improvement in forward direction at both energies 10.38 and 31 MeV, although the calculated curves

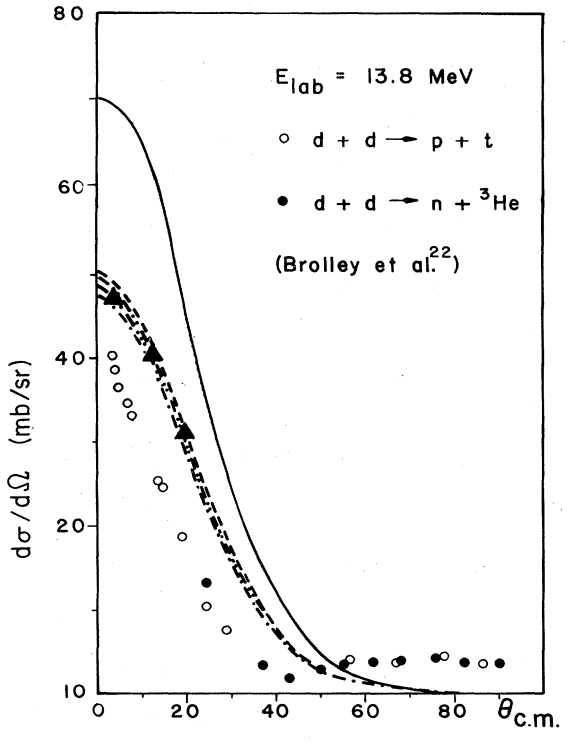

FIG. 9. The differential cross sections for $d+d\rightarrow t+p$ ($n+{}^3He$), at $E_{lab}=13.8$ MeV with the principal value parts of the propagators included in $2+2$ channel for the Y_3 potential. ——, K-matrix of order one; —·—·—·, EDPE $N=1$; —··—··, EDPE $N=2$; —····—····, GUPE $N=1$; — — —, GUPE $N=2$; ▲, GUPE $N=3$.

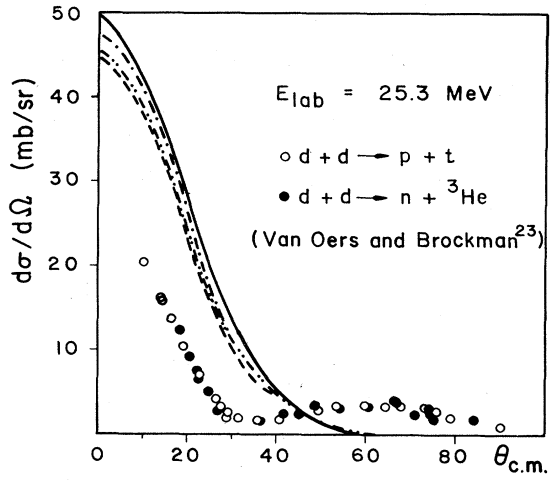

FIG. 10. The same as Fig. 9 but at $E_{lab}=25.3$ MeV.

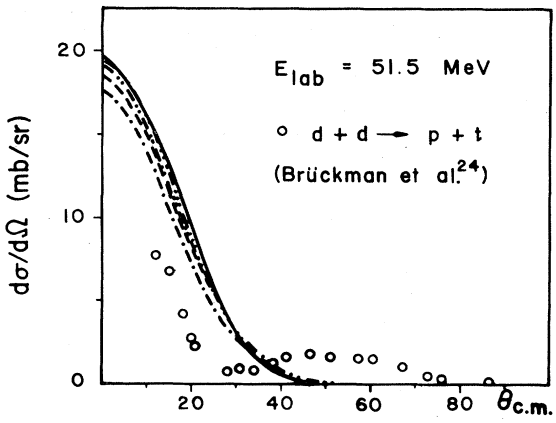

FIG. 11. The same as in Fig. 9 but at $E_{lab}=51.5$ MeV.

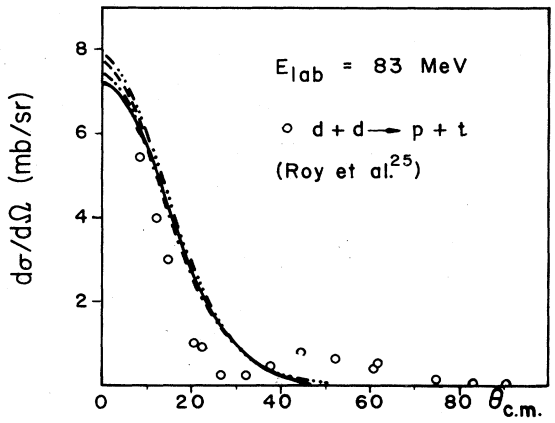

FIG. 12. The same as in Fig. 9 but at $E_{lab}=83.0$ MeV.

still lie far below the data at these angles. In the backward direction the (2 + 2) channels produce no perceptible change and the position of the minimum in the cross section is only slightly shifted to larger angles. Both the EDPE and GUPE appear to have converged and nearly coincide already after two terms.

Finally in Fig. 14 we compare the results of calculations with and without inclusion of the $\phi\phi$ contributions to the potential ${}^{ISL}C_{11}^{t,t}(q,q';z)$ in Eq. (3.12) at 31.8 MeV. Similar results were obtained at other energies. The $\phi\phi$ channels contributions are clearly negligible and can be omitted.

VI. CONCLUSIONS

Our present results for the d + d→p + t (n + ^{3}He) reaction and elastic p + ^{3}He scattering in first-order K-matrix approximation are in good agreement with the original work of Alt, Grassberger, and Sandhas and unpublished calculations of Becker. They are, however, in quantitative and even qualitative disagreement with those of Sawicki and Namyslowski. Unlike previous work, we have employed the separable GUPE-EDPE for the (2 + 2) and (3 + 1) subamplitudes. In the first-order K-matrix approximation these expansions both reduce to a single term, the GUPA. We, moreover, have employed a variety of nuclear forces including the local Malfliet-Tjon potential and a separable nucleon-nucleon potential with a tensor component. At 13.8 MeV the d + d→p + t (n + ^{3}He) differential cross section is quite sensitive in forward direction to the nucleon-nucleon force. This is related to the different triton binding energies and wave functions produced by these potentials. (It should be recalled that a similar phenomenon is found in n-d scattering.) At higher energies this sensitivity is considerably reduced, except for the separable potential with a tensor component P_4. In forward direction the agreement with experiment steadily improves with increasing energy and becomes excellent at 83 MeV.

To determine the origin of the quantitative and partly even qualitative disagreement between our first-order K-matrix calculations and the experimental cross section found already in Refs. 2 and 3, we also performed K-matrix calculations which include the principal value part

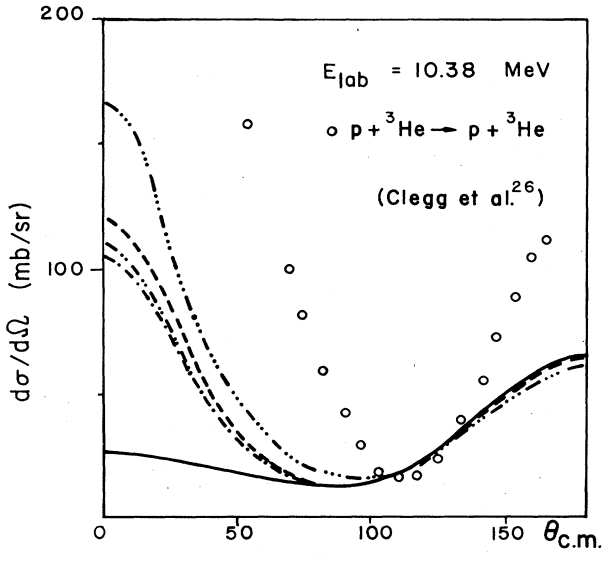

FIG. 13. The differential cross section for p + ^{3}He→p + ^{3}He at $E_{lab}=10.38$ MeV. For explanations see Fig. 9.

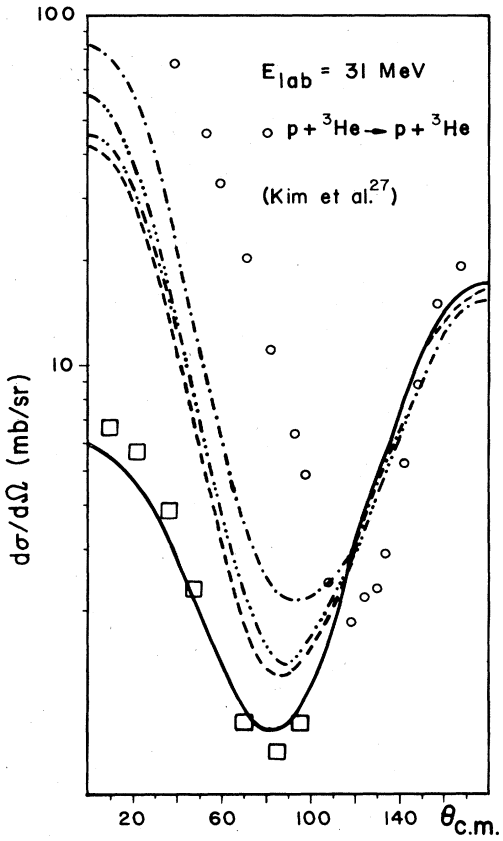

FIG. 14. Same as Fig. 13 but at $E_{lab}=31.10$ MeV; □, $\phi\phi$ channels only (one term).

of the (2 + 2) propagators. For this purpose we employed the EDPE-GUPE separable expansions for the (2 + 2) subamplitudes and found rapid convergence. In the (2 + 2) channels the effective interactions do not contain logarithmic singularities, which simplifies the application of the EDPE-GUPE in four-nucleon scattering *above* breakup. Our calculations also show that the contribution of the $\phi\phi$ channel can be neglected, an additional useful simplification.

The cross section of the d + d→p + t (n + ^{3}He) reaction is considerably diminished in the forward direction, resulting in better agreement with experiment. The corrections become less important with increasing energy and at 83 MeV the effect of the inclusion of the principal value part of the (2 + 2) propagator becomes nearly negligible, as expected. The inclusion of the principal value parts in the (2 + 2) channels also does not change the results for larger angles. There is not the slightest hint of a second maximum in the cross section between 45° and 75°, as required by the data.

For elastic p + ^{3}He scattering the effect of the full inclusion of the (2 + 2) channel turned out to be more spectacular. In the forward direction the differential cross is raised by a factor of the order of 5, but it still lies far below the experimental data, while the minimum is slightly shifted to the right in better agreement with experiment. For backward angles, however, the influence of the full inclusion of the (2 + 2) channels is practically negligible.

We can therefore only conclude that the lack of structure in the calculated differential cross section, in particular in first-order K-matrix approximation for the d + d→p + t (n + ^{3}He) reaction, is not a result of the neglect of the full contribution of the (2 + 2) channel. The additional structure in the cross section (second maximum) must come from information present in the (3 + 1) channel, which is not incorporated in our calculations. This argument is supported by the fact that the first-order K-matrix approximation at higher energies in general appears to be excellent but, nonetheless, does not show this maximum. This lack of structure in the differential cross section is not unexpected in view of the work of Tjon and of Fonseca's recent results below breakup. Tjon showed that one can expect the p-wave three-body subsystem amplitudes to contribute strongly to the d + d→p + ^{3}H (n + ^{3}He) observables, a fact confirmed in a different way by Kröger and Sandhas. It is, therefore, natural to expect that the absence of the p wave (3 + 1) subamplitudes in our K-matrix calculation is mainly responsible for the disagreement with experiment.

The neglect of the principal value part of the (3 + 1) propagators may be responsible for some other quantitative disagreements with the data, at least at lower energies. At higher energies [e.g., 83 MeV for d + d→p + t (n + ^{3}He)] it seems likely that this contribution will be unimportant, as was the case for the (2 + 2) propagator. In this case the p waves in the (3 + 1) subamplitude are most likely to be mainly responsible for the second maximum in the differential cross section. Investigations on this question are in progress.

The importance of the principal value part of the (2 + 2) channel to the forward angles in the differential cross section in p + ^{3}He *elastic scattering* is quite encouraging. In this case the principal value part of the (3 + 1) propagator can also be expected to provide an important contribution to the forward cross section. To investigate this point a second-order K-matrix calculation would be required, however.

APPENDIX

In this appendix we discuss the numerical treatment of the integral over the (2 + 2) propagator. One has to evaluate the integral

$$I^{ii}_{\nu\mu}(q',q;z)=\int_0^\infty dq''q''^{2L}\tilde{B}^{t,ii}_{1\nu}(q',q'';z) \times G^{ii}_{0,\nu\mu}(z-\tfrac{1}{2}q''^2)^L B^{t,ii}_{1\mu}(q'',q;z)\ . \quad \text{(A1)}$$

For $ii=\phi\phi$ the calculation is straightforward as the propagator $G^{\phi\phi}_{0,\nu\mu}$ has no poles. The same is true for the dd channels with $\nu\times\mu>1$. However, in the latter case with $\nu=\mu=1$, $G^{dd}_{0,11}$ has a pole and direct subtraction techniques cannot be applied. We treated this problem by splitting the propagators into their principal value and δ-function parts:

$$G^{dd}_{0,11}=G^{dd,\delta}_{0,11}+G^{dd,P}_{0,11}\ .$$

Then we have

$$I^{dd}_{11}=I^{dd,\delta}_{11}+I^{dd,P}_{11}\ .$$

The integral I^{δ}_{dd} is the part used in the K-matrix calculations of order one. For the principal value part we write

$$I^{dd,P}_{11}=\int_0^\infty F_{dd}(q'q;q'')G^{P}_{0,11}(E_{q''})dq'' =P\int_0^\infty F_{dd}(q',q;q'')G_{0,11}(E_{q''})\frac{E_{q''}+\epsilon_{dd}}{E_{q''}+\epsilon_{dd}} \quad \text{(A2)}$$

with $E_{q''}=E-q''^2/2$. It should be noted here that $q=q'$ and that $\epsilon_{dd}=|E_{dd}|$. Further

$$\lim_{E_{q''}\to-\epsilon_{dd}}(E_{q''}+\epsilon_{dd})G^{dd}_{0,11}(E_{q''})=R_{dd}$$

or

$$\lim_{q''\to q_0}\left[E+\epsilon_{dd}-\frac{q''^2}{2}\right]G^{dd}_{0,11}(E_{q''})=R_{dd}\ , \quad \text{(A3)}$$

where $q_0^2=2(E+\epsilon_{dd})$. Inserting Eq. (A3) into Eq. (A2) we obtain

$$I^{dd,P}_{11}=\int_0^\infty dq''\frac{[F_{dd}(q,q'')G^{dd}_{0,11}(Eq'')(q_0^2-q''^2)-2R_{dd}F_{dd}(q,q_0)]}{q_0^2-q''^2} \quad \text{(A4)}$$

and therefore we may proceed to evaluate (A4) using subtraction techniques.

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