

Determination of two-nucleon spectroscopic amplitudes from  $(d, \alpha)$  analyzing powers

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Tensor and vector analyzing powers and cross sections have been measured for low-lying levels populated by the  $^{89}\text{Y}(d, \alpha)^{87}\text{Sr}$  reaction at 9.0, 12.0, and 16.0 MeV. Angular distributions of  $d\sigma/d\Omega$ ,  $A_y$ ,  $A_{yy}$ ,  $A_{xx}$ , and  $A_{zz}$  have been analyzed using full finite-range distorted-wave Born approximation calculations including  $\alpha$ -particle  $D$ -state effects. It is shown that the sign and amplitude of two-nucleon spectroscopic amplitudes corresponding to different  $L$  and  $J$  transfers can be determined from the angular distributions of the tensor analyzing powers.

The  $(d, \alpha)$  reaction has long been recognized as a valuable spectroscopic tool for the study of two-hole states in nuclei.<sup>1</sup> Most nuclear structure studies have been performed using differential cross section data to identify the orbital angular momentum  $L$  of the transferred  $n$ - $p$  cluster. The vector analyzing power  $iT_{11}$  has been shown<sup>2</sup> to also be sensitive to the total angular momentum transfer  $J$ . In the present paper we show that the tensor analyzing powers (TAP) of  $(d, \alpha)$  reactions have a distinct and strong dependence on the coherence properties of the two-nucleon wave functions. Previously the usefulness of the TAP for spectroscopic applications has been limited by the problem of evaluating the effects of the  $\alpha$  particle  $D$  state.<sup>3-5</sup> At low energies these effects are determined by the value of  $D_2$ , which is approximately proportional to the asymptotic  $D$  to  $S$  state ratio for  $d$ - $d$  relative motion in the  $\alpha$  particle.<sup>3</sup> Independent determinations of  $D_2$  using different  $(d, \alpha)$  reactions<sup>6,7</sup> and also the  $^2\text{H}(d, \gamma)^4\text{He}$  reaction<sup>8</sup> indicate a value close to  $D_2 = -0.3 \text{ fm}^2$ .

The strong dependence of the TAP on  $L$  and  $J$  is essentially a consequence of the spin  $S=1$  of the transferred  $n$ - $p$  cluster. To clarify this dependence we consider the analyzing power  $A_{yy}$  which, by time reversal invariance, is identical to  $P_{yy} = \langle 3S_y^2 - 2 \rangle$ , where  $S_y$  is the  $y$  component of the deuteron spin vector in the inverse  $(\alpha, d)$  reaction. Since the spin of the transferred  $n$ - $p$  cluster is necessarily antiparallel to the outgoing deuteron spin as a result of the coupling in the spinless  $\alpha$  particle (neglecting  $\alpha$ -particle  $D$ -state effects), we can write  $A_{yy} = \langle 3S_y^2 - 2 \rangle$  where now  $S_y$  is the  $y$  component of the  $n$ - $p$  cluster spin in the composite nucleus. Because of strong absorption, the reaction is peripheral and therefore  $\mathbf{L}$  tends to be perpendicular to the reaction plane. Thus  $\mathbf{L}$  is perpendicular to the  $Z$  axis and, using the Madison coordinate system,<sup>9</sup> this means that only  $|LM\rangle$  states where  $M=0$  contribute to the transfer. With this condition, and using standard angular momentum algebra,<sup>10</sup> we obtain

$$A_{yy} = \langle SJL0 | 3S_y^2 - 2 | SJL0 \rangle \\ = (-1)^2 \sqrt{3/2} (L0L0 | 20) W(L1L1; J2) . \quad (1)$$

For  $J = L$ ,  $L \pm 1$  Eq. (1) gives for  $(A_{yy})_{L,J}$

$$(A_{yy})_{L,L} = -\frac{1}{2} , \quad (2a)$$

$$(A_{yy})_{L,L+1} = \frac{L}{2(2L+3)} , \quad (2b)$$

$$(A_{yy})_{L,L-1} = \frac{L+1}{2(2L-1)} . \quad (2c)$$

The  $A_{yy}$  predicted by Eqs. (2) are large and strongly  $L$  and  $J$  dependent.<sup>11</sup> The same model predicts a vanishing  $A_y$  because the expectation value of  $S_y$  between  $M=0$  states is zero. Furthermore, the matrix element in Eq. (1) is non-vanishing only for spin  $\geq 1$ . For instance in  $(d, p)$  reactions, where  $S = \frac{1}{2}$ , the TAP are essentially a manifestation of the deuteron  $D$  state.<sup>12</sup>

In unnatural parity transitions the orbital angular momentum of the transferred  $n$ - $p$  cluster can be  $L = J \pm 1$ . The effect of this mixing of  $L$  values on the TAP can be studied using a peripheral plane wave model,<sup>13</sup> which predicts that

$$(A_{yy})_J = \frac{(J+2)x^2 - 6[J(J+1)]^{1/2}x + J-1}{2(2J+1)(1+x^2)} , \quad (3)$$

where  $x$  is proportional to the ratio  $G_{J+1,J}/G_{J-1,J}$  of the cluster spectroscopic amplitudes  $G_{LJ}$ . It is the presence of the linear term in  $x$ , in the numerator of Eq. (3), that makes the TAP very sensitive to the relative sign of the  $G_{LJ}$ . Notice that for pure  $L$  transfers ( $x=0$  and  $x=\infty$ ) Eq. (3) reproduces the previous Eqs. (2b) and (2c). The effect of the  $\alpha$ -particle  $D$  state is generally to increase  $A_{yy}$  relative to the value given by Eq. (3).<sup>6</sup>

Angular distributions for vector and tensor analyzing powers and cross sections have been measured for three well-separated low-lying levels in  $^{87}\text{Sr}$ ; the ground state with  $J^\pi = \frac{3}{2}^+$ , the 0.388 MeV  $\frac{1}{2}^-$  state and the 0.873 MeV  $\frac{3}{2}^-$  state. The data were taken at the Triangle Universities Nuclear Laboratory using a vector and tensor polarized deuteron beam from a Lamb-shift polarized source. The beam was incident on a 260- $\mu\text{g}/\text{cm}^2$ -thick  $^{89}\text{Y}$  target and viewed by four pairs of 300- $\mu\text{m}$ -thick solid state detectors situated at  $10^\circ$  intervals to the right and left of the beam. A  $^3\text{He}$  polarimeter was used to monitor the beam polarization during

each eight-step measurement that determined  $d\sigma/d\Omega$ ,  $A_y$ ,  $A_{yy}$ , and  $A_{xx}$ . At 12 and 16 MeV,  $A_{xx}$  was also measured.

The  $^{89}\text{Y}(d,\alpha)^{87}\text{Sr}$  reaction to the low-lying states of  $^{87}\text{Sr}$  is particularly suitable to study  $L$ -mixing effects because the nuclear states involved are relatively well described by the single particle shell model.<sup>14</sup> In particular, the  $^{89}\text{Y}$  ground state is found to be 88% a  $\pi(2p_{1/2})$  proton coupled to a  $^{88}\text{Sr}$  closed shell. The low-energy spectrum of  $^{87}\text{Sr}$  has been successfully described<sup>15,16</sup> by coupling the single neutron hole motion in the  $1g_{9/2}$ ,  $2p_{1/2}$ ,  $1f_{5/2}$ , and  $2p_{3/2}$  orbitals to collective vibrations of the even core. This suggests that the  $^{89}\text{Y}(d,\alpha)^{87}\text{Sr}$  reaction to the lowest  $\frac{9}{2}^+$ ,  $\frac{1}{2}^-$ , and  $\frac{3}{2}^-$  states in  $^{87}\text{Sr}$  is populated predominantly by pickup from the  $1g_{9/2}$ ,  $2p_{1/2}$ , and  $2p_{3/2}$  neutron and proton shells. If it is assumed that the proton ( $1f_{5/2}$ ) and ( $2p_{3/2}$ ) shells in the ground state of  $^{87}\text{Sr}$  are fully occupied, the  $^{89}\text{Y}(d,\alpha)^{87}\text{Sr}$  reaction to the  $\frac{9}{2}^+$  ground state can only involve the shell-model configuration  $\pi(2p_{1/2})\nu(1g_{9/2})$  and this circumstance has been previously used<sup>6</sup> to estimate  $\alpha$ -particle  $D$ -state effects. However, mixing of different shell-model configurations is likely to occur in the transitions to the 0.388 MeV ( $\frac{1}{2}^-$ ) and 0.873 MeV ( $\frac{3}{2}^-$ ) states.

The present  $(d,\alpha)$  data were analyzed using the cluster model of two-nucleon transfer reactions<sup>1,17</sup> in which the spectroscopic amplitudes  $G_{LJ}$  for the allowed  $(L,J)$  transfers are given as a sum over shell-model configurations  $\eta = (n_1 l_1 j_1, n_2 l_2 j_2)$

$$G_{LJ} = \sum_{\eta} \mathcal{S}(\eta) G_{LJ}(\eta) . \quad (4)$$

Here  $\mathcal{S}(\eta)$  is the spectroscopic amplitude for the configuration  $\eta$  and  $G_{LJ}(\eta)$  is an amplitude equal to the product of a symmetrized  $LS-jj$  recoupling coefficient by a Talmi-Moshinsky coefficient.<sup>17</sup> The  $^{89}\text{Y}(d,\alpha)^{87}\text{Sr}$  reaction to the 0.388-MeV state can be populated by  $(L,J)$  transfers of  $(0,1)$  and  $(2,1)$ , corresponding to the amplitudes  $G_{01}$  and  $G_{21}$ . The  $A_{yy}$  angular distribution data at  $E_d=12.0$  MeV are positive and oscillate around an average value of  $A_{yy} \cong 0.5$ , as shown in Fig. 1. According to Eq. (2c), this indicates that the transition is predominantly  $L=2$ . The inclusion of the  $\alpha$ -particle  $D$  state increases  $A_{yy}$  but this effect is compensated by a small admixture of the  $L=0$  amplitude. DWBA calculations were performed with the code PTOLEMY<sup>18</sup> using measured optical model parameters, as in Ref. 6. Figure 1 shows the result of calculations with fixed  $D_2 = -0.30 \text{ fm}^2$  and for different values of  $G_{01}/G_{21}$ . We find that the sensitivity to variations in  $G_{01}/G_{21}$  is considerably larger in the TAP than in the cross section or vector analyzing power. Furthermore, only the TAP depend strongly on the relative sign between  $G_{01}$  and  $G_{21}$ . The best fit to the angular distributions at the three energies is obtained for  $-0.15 \leq G_{01}/G_{21} \leq -0.05$ . Assuming that the population of the 0.388-MeV state results primarily in the pickup from the two shell-model configurations  $\eta_1 = \pi(2p_{1/2})\nu(2p_{1/2})$  and  $\eta_2 = \pi(1f_{5/2})\nu(1f_{5/2})$  the solution of Eq. (4) gives for the corresponding spectroscopic amplitudes (normalized to unit probability)  $\mathcal{S}(\eta_1) = 0.69 \pm 0.07$  and  $\mathcal{S}(\eta_2) = -0.72 \pm 0.07$ .

For the  $^{89}\text{Y}(d,\alpha)^{87}\text{Sr}$  reaction to the 0.873-MeV ( $\frac{3}{2}^-$ ) state the allowed  $(L,J)$  values are  $(0,1)$ ,  $(2,1)$ , and  $(2,2)$  and correspond to the mixing of natural and unnatural parity transitions. This implies that the relative phase between  $G_{21}$  and  $G_{22}$  cannot be determined because the reaction ob-

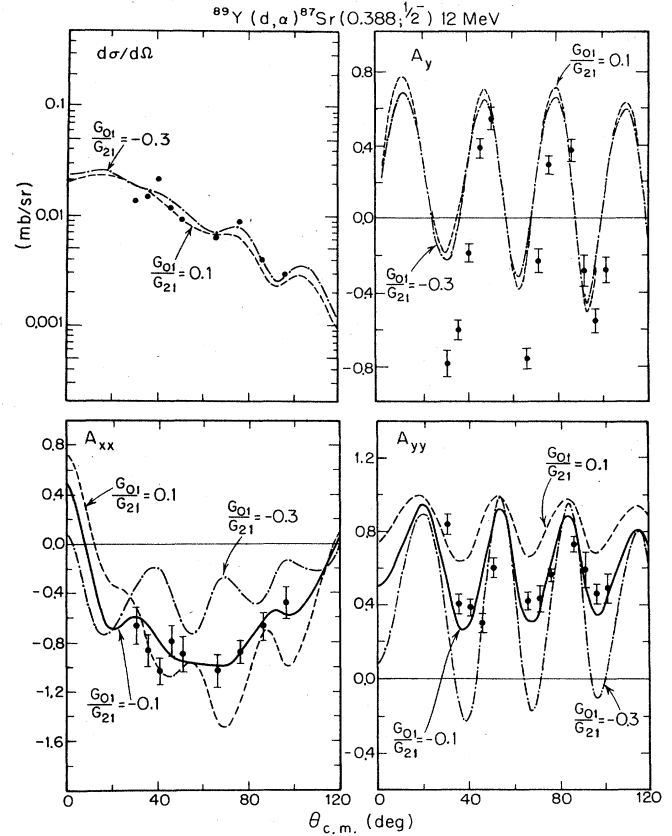


FIG. 1. The angular distributions of cross section, vector, and tensor analyzing power for the  $^{89}\text{Y}(d,\alpha)^{87}\text{Sr}_{0.388 \text{ MeV}}$  reaction initiated by 12-MeV polarized deuterons. The curves correspond to  $D_2 = -0.30 \text{ fm}^2$  and  $G_{01}/G_{21} = 0.1$  (dashed curve),  $G_{01}/G_{21} = -0.1$  (solid curve), and  $G_{01}/G_{21} = -0.3$  (dot-dashed curve). Here, and in Fig. 2, the calculated differential cross section is normalized to experiment.

servables are incoherent summations over  $J$ . The analysis of  $A_y$  data indicates that  $G_{01}$  is the dominant amplitude. On the other hand, the value of  $A_{yy}$  is close to 0.5 which shows that  $G_{21}$  is nonvanishing. By mixing the amplitudes  $G_{01}$ ,  $G_{21}$ , and  $G_{22}$  it is possible to obtain agreement with TAP data both in phase and in magnitude. The best fit is obtained for  $0.45 \leq G_{21}/G_{01} \leq 0.60$  and  $0.55 \leq G_{22}/G_{01} \leq 0.70$ , as shown in Fig. 2. The three highest-energy shell-model configurations that can contribute to the population of the 0.873-MeV ( $\frac{3}{2}^-$ ) state are

$$\begin{aligned} \eta_2 &= \pi(1f_{5/2})\nu(1f_{5/2}) , \\ \eta_3 &= \pi(2p_{1/2})\nu(2p_{3/2}) , \\ \eta_4 &= \pi(2p_{3/2})\nu(2p_{3/2}) \end{aligned}$$

and the corresponding spectroscopic amplitudes (normalized to unit probability) determined from Eq. (4) with  $G_{22}/G_{01} > 0$  are  $\mathcal{S}(\eta_2) \cong \mathcal{S}(\eta_4) = -0.7 \pm 0.17$  and  $\mathcal{S}(\eta_3) = 0.03 \pm 0.004$ . If  $G_{22}/G_{01}$  is taken to be negative, the amplitudes  $\mathcal{S}(\eta_2)$  and  $\mathcal{S}(\eta_4)$  are almost unchanged and  $\mathcal{S}(\eta_3)$  becomes negative. In both cases we find that the configurations  $\eta_2$  and  $\eta_4$  share most of the pickup probability and the corresponding spectroscopic amplitudes have the same sign.

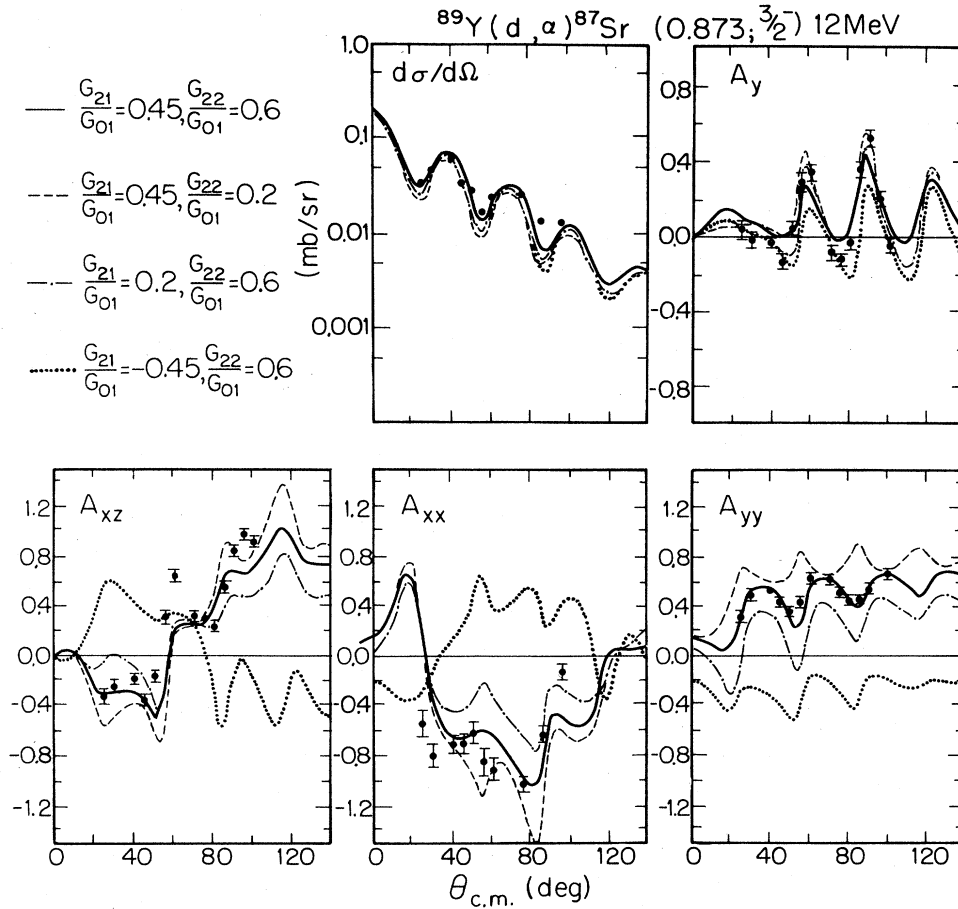


FIG. 2. The angular distributions of cross section, vector, and tensor analyzing powers for the  $^{89}\text{Y}(d, \alpha)^{87}\text{Sr}_{0.873 \text{ MeV}}$  reaction initiated by 12-MeV polarized deuterons. The curves correspond to an  $S$ - and  $D$ -state admixture in the  $\alpha$  particle having  $D_2 = -0.3 \text{ fm}^2$ . The solid, dotted, dashed, and dot-dashed curves pertain to different values of  $G_{21}/G_{01}$  and  $G_{22}/G_{01}$ , as shown in the figure.

For both the 0.388- and 0.873-MeV states, the probability of pickup from the  $\eta_2$  configuration is about half of the total. This probability, which is strongly enhanced by the large occupancy number of the  $\eta_2$  orbitals in the target, indicates the presence of proton particle-hole excitations in the low-lying states of  $^{87}\text{Sr}$ .

Our results show that the analysis of the angular distributions of the tensor analyzing powers in  $(d, \alpha)$  reactions provides new information on the phase and amplitude of the two-nucleon cluster amplitudes that is not obtainable from the cross section and vector analyzing power. The sensi-

tivity of the TAP to the coherence properties of nuclear states is found to be a unique feature of transfer reactions where the spin transferred is  $S \geq 1$ . The same type of phenomena is also expected to be present in other reactions transferring spin-one particles, such as  $(^6\text{Li}, \alpha)$ .

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<sup>1</sup>N. K. Glendenning, Phys. Rev. 137, B102 (1965).

<sup>2</sup>E. J. Ludwig, T. B. Clegg, W. W. Jacobs, and S. A. Tonsfeldt, Phys. Rev. Lett. 40, 441 (1978).

<sup>3</sup>F. D. Santos, S. A. Tonsfeldt, T. B. Clegg, E. J. Ludwig, Y. Tagishi, and J. F. Wilkerson, Phys. Rev. C 25, 3243 (1982).

<sup>4</sup>F. D. Santos, Prog. Theor. Phys. 70, 1679 (1983).

<sup>5</sup>J. A. Tostevin, Phys. Rev. C 28, 961 (1983).

<sup>6</sup>B. C. Karp, E. J. Ludwig, W. J. Thompson, and F. D. Santos, Phys. Rev. Lett. 53, 1619 (1984).

<sup>7</sup>J. A. Tostevin, J. M. Nelson, O. Karban, A. K. Basak, and S. Roman, Phys. Lett. 149B, 9 (1984).

<sup>8</sup>F. D. Santos, A. Arriaga, A. M. Eiró, and J. A. Tostevin, Phys. Rev. C 31, 707 (1985).

<sup>9</sup>Polarization Phenomena in Nuclear Reactions, edited by H. H. Barshall and W. Haeblerli (Univ. of Wisconsin Press, Madison,

- 1971), p. xxv.
- <sup>10</sup>M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1963).
- <sup>11</sup>L. J. B. Goldfarb and R. C. Johnson, Nucl. Phys. **18**, 353 (1960); **21**, 462 (1960); F. D. Santos, in *Proceedings of the Fourth International Symposium on Polarization Phenomena*, edited by W. Gruebler and V. König (Birkhäuser-Verlag, Basel, 1976), p. 205.
- <sup>12</sup>L. D. Knutson and W. Haeberli, Phys. Rev. Lett. **35**, 558 (1975).
- <sup>13</sup>F. D. Santos and A. M. Eiró, Port. Phys. **15**, 65 (1984).
- <sup>14</sup>J. D. Vergados and T. T. S. Kuo, Nucl. Phys. **A168**, 225 (1971).
- <sup>15</sup>S. K. Basu and S. Sen, Nucl. Phys. **A220**, 580 (1974).
- <sup>16</sup>J. E. Kitching, Z. Phys. **258**, 22 (1973).
- <sup>17</sup>R. D. Lawson, *Theory of the Nuclear Shell Model* (Oxford Univ. Press, Cambridge, 1980).
- <sup>18</sup>M. H. Macfarlane and S. C. Pieper, Argonne National Laboratory Report No. ANL-76-11 Rev. 1 (unpublished); revised by R. P. Goddard (unpublished).