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Electron scattering transverse sum rule: Tentative comparison with experimental data

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A first systematic attempt to extract the transverse sum rule from experimental data on ^{40}Ca, ^{48}Ca, and ^{56}Fe is made. The comparison with the theoretical predictions in a correlated model for the ground states, considering tensor correlations explicitly, turns out to be satisfactory. The necessity of invoking modifications of the nucleon properties when immersed in the nuclear medium is discussed.

The recent electron scattering experiments performed in Saclay[1] and at MIT[2,3] giving the separated longitudinal and transverse response functions of some nuclei in the deep inelastic region, have stimulated many discussions and investigations.[4-15]

The main point is to understand which nuclear structure and dynamical effects are important in one channel and which are important in the other; the problem lies in the difficulty in finding a consistent treatment which explains the data of both channels.

In the course of these investigations a useful tool has been represented by sum rules (SR). In particular, for the longitudinal case, it has been well known for a long time that the Coulomb SR is a measure of the proton-proton spatial correlations of nuclei,[16] and it has been the concern of experimentalists[1-3] and theoreticians[8-17] to provide data and predictions of it.

Indeed, violations of the total sum are much more serious than any disagreement with the detailed shape of the calculated excitation spectra, because the Coulomb SR is nearly model independent.

In previous works[13,14] we have discussed the comparison between theoretical and experimental Coulomb SR's, and have also presented evidence of how they are modified by short range and tensor correlations in the ground state, through the modification of the p-p correlation function.

The main point of that paper[14] was that the discrepancy between theoretical evaluations and experimental observations could mostly be ascribed to the contributions coming from the high energy tail of the experimental spectra, an interpretation which gives a reasonable explanation of the observed missing longitudinal strength at high momentum transfer, without invoking a much more drastic and deep hypothesis on the nucleonic structure in the nuclear medium.[5,9,18]

Up to now there has not been much interest regarding the transverse SR's because of the complications they present both in the theoretical calculations and in the experimental estimates.

On the theory side, the main problems are the consistent treatments of two body current operators and 2p-2h correlations in the ground state of finite nuclei, besides the difficulty in including Δ's and free pions in the calculations.

On the experimental side, the main difficulty lies in the possibility of performing sums which can be compared with the theoretical SR values. In contrast with the longitudinal case, the data on the transverse response do not show a tendency to converge, owing to the excitation of the Δ isobar. Besides all that, an intrinsic difficulty underlies such a comparison; transverse SR will exist only if the transverse response function converges.

About this point there has been little theoretical investigation, except in the case of deuteron.[19] For such a nucleus it has been shown that the transverse response function converges. In particular, in deep inelastic scattering, convection current and meson exchange contributions are of the order of a few percent.

In light of these results and considerations, we felt motivated to investigate a transverse SR for the spin current operator in three nuclei where the transverse response function has been measured; these are ^{40}Ca, ^{48}Ca, and ^{56}Fe,[1-3] (a previous investigation for ^{12}C can be found in Ref. 13), using a method of correlating nuclear ground states described in Ref. 20.

This study is particularly interesting in the region of momentum $q > 200$ MeV/c, because the convection current contribution to the total transverse sum is quite negligible.[17]

Moreover, there is another fundamental reason why the study of such an SR can be interesting. When only the spin current operator is considered the sum exhibits a nice characteristic: It is connected to the spin-spin correlation function of the nuclear ground state[16,17] and also to a model independent term largely dominating[13] for $q > 300$ MeV/c. In fact, one has

$$S_0^T(q) = \int R^T(q,\omega)\,d\omega = \sum_n |\langle n|\mathbf{J}_{\rm spin}(q)|0\rangle|^2 = \langle 0|\mathbf{J}^+_{\rm spin}(q)\mathbf{J}_{\rm spin}(q)|0\rangle \;, \tag{1}$$

with

$$\mathbf{J}_{\rm spin}(q) = \sum_k \frac{\mu_k(q)}{2M}\mathbf{q}\times\boldsymbol{\sigma}_k e^{i\mathbf{q}\cdot\mathbf{r}_k} \;. \tag{2}$$

When $\mathbf{J}_{\rm spin}(q)$ is inserted in (1),

$$S_0^T(q) = e^2(q)q^2/(2M^2)\left[(Z\mu_p^2 + N\mu_n^2) + \tfrac{1}{2}\left\langle 0\left|\sum_{i\neq k}\mu_i\mu_k(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k - \sigma_i^3\sigma_k^3)e^{i\mathbf{q}\cdot\mathbf{r}_{ik}}\right|0\right\rangle\right] \;. \tag{3}$$

Equation (3) shows that, analogously to the Coulomb case, $S(q)$ has a model independent behavior driven by the proton form factor $[e^2(q)]$ by the full gyromagnetic moment values μ_p and μ_n ($\mu_p = 2.79$, $\mu_n = -1.91$ in the free case), and by the nucleonic mass M.

In Figs. 1, 2, and 3 the transverse SR is shown as a function of q for ^{40}Ca, ^{48}Ca, and ^{56}Fe. The dashed line represents the model independent term [first term in Eq.

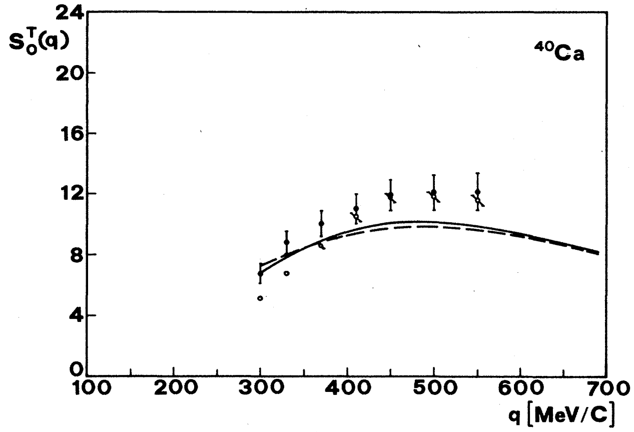

FIG. 1. Transverse sum rule for ^{40}Ca. Dashed line: model independent term [see Eq. (3)]; continuous line: result of the correlated model of Ref. 20. ●: "quasi experimental" data, i.e., when the contribution of the high energy tail is properly taken into account (see the text); ×: sum of the Saclay data (Ref. 23).

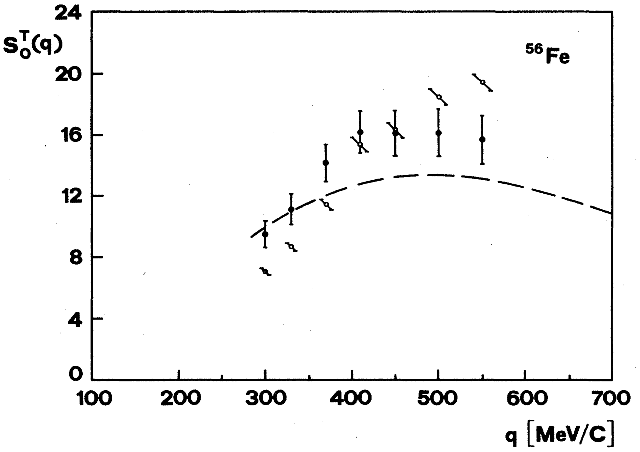

FIG. 3. Transverse sum rule for ^{56}Fe. Notations as in Fig. 1.

(3)]. The continuous line is $S_0^T(q)$ when the spin-spin term is also added. The latter has been evaluated in the frame of a model,[20] which allows the accommodation of short range and tensor correlations in the ground state of a finite nucleus. It turns out[21,22] that tensor correlations affect that term, which has a negative sign, very much (it almost doubles in the q region between 200 and 400 MeV/c), which tends to lower the SR further. But the net effect is practically negligible because of the dominance of the first term at such values of momentum transfer. We remark that in $N=Z$ nuclei one can write the spin term as a sum of an isoscalar and an isovector part:

$$\sum_{i\neq k}\langle 0|\mu_i\mu_k(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k-\sigma_i^3\sigma_k^3)e^{i\mathbf{q}\cdot\mathbf{r}_{ik}}|0\rangle$$
$$=\tfrac{1}{4}[(\mu_p+\mu_n)^2\delta(\tau=0)+(\mu_p-\mu_n)^2\delta(\tau=1)]\ , \quad (4)$$

where

$$\delta(\tau=1)=\left\langle 0\left|\sum_{i\neq k}\tau_i^3\tau_k^3(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k-\sigma_i^3\sigma_k^3)e^{i\mathbf{q}\cdot\mathbf{r}_{ik}}\right|0\right\rangle\ , \quad (5)$$

and

$$\delta(\tau=0)=\left\langle 0\left|\sum_{i\neq k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k-\sigma_i^3\sigma_k^3)e^{i\mathbf{q}\cdot\mathbf{r}_{ik}}\right|0\right\rangle\ . \quad (6)$$

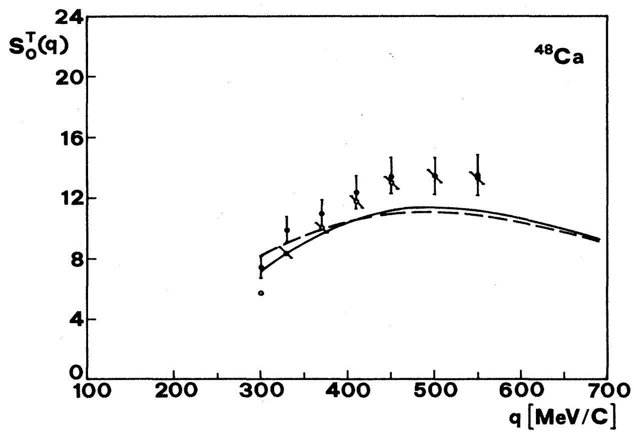

FIG. 2. Transverse sum rule for ^{48}Ca. Notations as in Fig. 1.

It turns out that tensor correlations affect the isovector and isoscalar terms in two different directions, quenching the former and enhancing the latter. Anyway, because of the factors in front, the former is dominant.

In Figs. 1, 2, and 3 the comparison of our calculations with "quasi experimental" points is shown. From the calculations of the transverse response functions in deuteron[19] and in middle weight nuclei[7,8,10] (showing how the different parts, shell model or Fermi gas, p-h correlations, pion production, and Δ contributions sum up into the total response), we argue that, in the present approach, we are testing little more than the integral under the quasielastic peak and that, therefore, we have to compare our calculations with that experimental area. So, analogously to what was done in Ref. 14, we have performed a best fit with a smooth cutoff of the Saclay data[1] after the quasielastic peak. The form of the cutoff varies around ω^{-3}, a hint borrowed from the deuteron case. Adding this tail contribution to the sum of the experimental values up to shortly after the quasielastic peak, we obtain the full circles in the figures. The error bars embody both systematic and statistical experimental errors, and uncertainty in the cutoff.

It is curious to note that the solid points lie very near those (open circles) obtained by simply summing, for every q, all the available experimental values, exactly in the same way the data on the Coulomb SR in Ref. 1 were obtained. This happens because the area which is sharply cut out by the cutoff is compensated by the tail contribution. A series of comments can be done on the previous comparison.

(i) The pictures for ^{40}Ca, ^{48}Ca, and ^{56}Fe are very similar, and they look like Fig. 4 of Ref. 13, where the same comparison is shown for ^{12}C. In all three cases the "quasi experimental" points slightly overestimate the theoretical predictions. Dynamical correlations give a very small contribution in better achieving that agreement (see Figs. 1 and 2). The major source of the remaining discrepancy should be ascribed to meson exchange current effects, which have not been included in the present calculations, and which should enhance the sum by a further 5% (cf. Refs. 7 and 8).

(ii) Similar to the longitudinal case,[14] the agreement between theoretical and experimental areas achieved when the high energy tail of the excitation spectrum is properly taken into account seems to make questionable the hypothesis of a substantial change of nucleon properties (bag radius, charge, and magnetic form factors) when it is immersed in the nuclear medium.

Moreover, in contrast with the longitudinal case, ^{40}Ca does not show a striking disagreement between theoretical and experimental areas. In the Coulomb sum it happens that the model independent dominant term, i.e, the charge number, being the same in both Ca isotopes, fits the data well on ^{48}Ca but does not fit the data on ^{40}Ca, which are somewhat smaller. In the transverse case, on the contrary, the equivalent model independent contribution is bigger in ^{48}Ca than in ^{40}Ca [see Eq. (3)], following the trend of the data and giving the same kind of agreement in the two cases.

(iii) If one would like to study the role of correlations, one should compare the theoretical and experimental inverse weighted integral of the response function

$$S^{T}_{-1}(q)=\int \frac{1}{\omega} R^{T}(q,\omega)\,d\omega \quad .$$

In fact, the uncertainty of the cutoff would be reduced, and the lower energy sector, where correlations are believed to affect the response function, would be stressed. However, it is well known that S_{-1} is not properly a sum rule, since it is impossible to write it as a mean value of an operator on the nuclear ground state. So, it is not clear if the approximations one must do to evaluate it affect the result more or less than the uncertainty on the tail in S^{T}_{0}. Investigations are in progress in this direction.

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