Meson theoretical basis for Dirac impulse approximation

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(Received 27 February 1985)

Basic features of proton-nucleus optical potentials for use in the Dirac equation are discussed. The original form of the Dirac impulse approximation follows from using Fermi covariants (scalar, vector, tensor, pseudoscalar, and axial vector) to extend physical NN amplitudes into operators in the full Dirac space. Overly large scalar and vector optical potentials are shown to follow at low energy in this case due to forcing pion exchange contributions to be pseudoscalar. The pair contributions to proton-nucleus scattering are much too large at low energy. A variant of the impulse approximation is developed by replacing pseudoscalar covariants by pseudovector ones. Much reduced scalar and vector strengths are obtained at low energy in the pseudovector case. The pair contributions are similarly reduced to reasonable values. However, the large differences between optical potentials based on pseudoscalar and pseudovector covariants are not controlled by physical NN scattering data. These differences represent a basic ambiguity in NN amplitudes when the only constraint is positive energy scattering data. Using a complete and unambiguous set of NN Lorentz invariant amplitudes obtained from a relativistic one-meson-exchange model, the scalar and vector optical potential strengths are found to be reasonably constant over the range of 50 to 1000 MeV of proton energy. The meson theory results for nuclear matter are found to be comparable to those obtained when the pseudoscalar covariant is replaced by the pseudovector covariant in the original impulse approximation.

I. INTRODUCTION

Recent work has shown that proton-nucleus scattering observables can be successfully predicted based on the impulse approximation Dirac optical potential.[1–3] Spin observables for elastic scattering are described particularly well in the Dirac approach. The most noted result is a very close reproduction of experimental data for analyzing power, A_y, and spin rotation, Q, at 500 MeV proton energy.[2,3] Nonrelativistic impulse approximation results do not give an adequate description of A_y or Q at this energy.[4] However, the impulse approximation is not as successful at low energy where it predicts overly large scalar and vector terms in the potential.[1,5]

The Dirac optical potential is derived from a five-term representation of the NN amplitude based upon the Fermi covariants: scalar, vector, tensor, pseudoscalar, and axial vector.[6] Each of the five invariant amplitudes is determined from positive energy NN scattering data (e.g., phase shifts). Therefore no parameters in the Dirac potential can be varied.

An essential feature of the relativistic treatment based on the Dirac equation is the prediction of significant $N\bar{N}$ virtual pair contributions.[7] Calculations by Hynes *et al.*[8] support the conclusion that the Dirac equation description of proton-nucleus scattering differs from the nonrelativistic analysis essentially because of pair contributions. This point is particularly clear in a momentum space treatment[8] which focuses on the projection of the Dirac scattered wave onto positive energy states of the free Dirac equation, ψ^+. Negative energy projections of the full wave function can occur only as virtual intermediate states. These can be formally eliminated to obtain an equation for ψ^+ as follows:

$$(E-E_P-U^{++}-U_{\text{pair}})\psi^+=0\ , \tag{1.1}$$

where U_{pair} is a virtual pair contribution,

$$U_{\text{pair}}=U^{+-}(E+E_P-U^{--})^{-1}U^{-+}\ . \tag{1.2}$$

Matrix elements of the full Dirac potential, U, are defined by

$$U_{s's}^{\rho'\rho}\equiv\bar{u}_{s'}^{(\rho')}(\mathbf{p}')U(\mathbf{p}',\mathbf{p})u_s^{(\rho)}(\mathbf{p})\ , \tag{1.3}$$

where u_s^+ and u_s^- are free Dirac spinors for positive and negative energy, respectively, with spin labels s and s'. Spin indices are omitted in (1.1) for simplicity. There are two main points. First, U^{++} is the same whether a relativistic or nonrelativistic impulse approximation is used provided that the nuclear density has no negative energy components. Second, neglect of U_{pair} in (1.1) produces good agreement with nonrelativistic analyses, while retention of U_{pair} produces the Dirac results.[8] Thus U_{pair} must be considered the essential new ingredient of relativistic approaches.

In the impulse approximation,[1] the Dirac potential $U\simeq S+\gamma^0V$ consists of a scalar and time-component-of-vector term, neglecting a tensor term,[9,10] which plays a minor role. The pair contribution is therefore predicted from (1.2) once S and V are known, and the latter are fixed by the Lorentz invariant scalar and vector components of the NN amplitude and the nuclear density. In a crude but useful contact approximation, the pair term is given by

$$U_{\rm pair}=\bar{u}^{+}(\mathbf{p}')\left[\frac{1-\gamma^0}{2}\right]\frac{(S-V)^2}{E+m}u^{+}(\mathbf{p})\;, \tag{1.4}$$

$U_{\rm pair}$ is analogous to the $\mathbf{A}^2/(2m)$ electromagnetic term in the Schrödinger equation. The difference of scalar and vector potentials controls the pair contribution in the Dirac impulse approximation. At low energy, the impulse approximation predicts very large S and V which have opposite signs. Consequently, the pair contribution becomes too large.

In Sec. II of this paper, a simple formulation of the Lorentz invariant NN amplitude is presented which builds upon work by Goldberger, Grisaru, MacDowell, and Wong (GGMW).[11] In particular, the large S and V at low energy are shown to arise from pseudoscalar covariants whose associated invariant amplitudes include pion exchange contributions. The overly large $U_{\rm pair}$ is identified as being due to the pionic contributions.

Pair suppression in NN scattering is usually accomplished by use of a pseudovector πN coupling.[12] In Sec. III, the one-pion contribution based on pseudovector coupling is examined and is found to require a more complicated representation for the NN amplitude in the sense that a new term involving negative energy projection operators is needed. A new version of the impulse approximation is suggested in which pseudoscalar covariants are replaced by pseudovector ones. Since the physical positive energy matrix elements are the same as in the pseudoscalar case in view of the equivalence theorem,[13] the same formulas as in Sec. II determine the invariant amplitudes of the new representation from NN phase shifts. We show that the choice of pseudovector covariants leads, however, to cancellation of the one-pion contributions to the Dirac optical potential, and thus to much smaller S, V, and $U_{\rm pair}$ at low energy.

The large differences due to the choice of pseudoscalar or pseudovector covariants provide an example of a fundamental ambiguity in the determination of the full NN amplitude, $\hat{F}$, from knowledge of its positive energy matrix elements. The issue of ambiguities was first pointed out by Adams and Bleszynski.[14] Suppose $\hat{F}^1$ is a representation of the NN amplitude restricted to five terms which are then fixed by equating positive energy matrix elements to the five independent helicity amplitudes. As mentioned above, there can be additional contributions, $\hat{F}^2$, involving negative energy projectors such that the sum of $\hat{F}^1$ and $\hat{F}^2$ is the full amplitude,

$$\hat{F}=\hat{F}^1+\hat{F}^2\;.$$

By definition the positive energy matrix elements of $\hat{F}^2$ vanish; however, this does not mean that $\hat{F}^2$ or its contribution to the Dirac optical potential vanishes. The impulse approximations using pseudoscalar and pseudovector covariants amount to two different assumptions about the negative energy part, $\hat{F}^2$, which is inaccessible to experiments. Large differences in $U_{\rm pair}$ between these choices are found to come entirely from the $\hat{F}^2$ portion.

In a relativistic theory, the full $\hat{F}$ is, in principle, defined by the same dynamics which explains the physical NN matrix elements.[15] The Lorentz covariant dynamics specifies $\hat{F}^2$ and eliminates ambiguities in the choice of covariants. Such a dynamical treatment is necessary to provide a theoretical basis for $U_{\rm pair}$ since it explicitly depends on negative energy matrix elements, i.e., $\hat{F}^2$. A central theoretical issue is therefore to define $\hat{F}^2$ within existing theory, namely, conventional meson exchange dynamics for NN scattering.

In Ref. 16 we develop a complete and general representation for $\hat{F}$ and describe how to determine its invariant amplitudes from solutions of relativistic NN integral equations[17] based on meson theory. Section IV develops expressions for the proton optical potential in nuclear matter based on the general Lorentz invariant representation of $\hat{F}$. Using a dynamical model with meson exchange, these amplitudes can be determined. Numerical calculations are presented using the formalism of Refs. 16 and 17. Comparable results are obtained for the nuclear matter optical potential using the full meson theory, and using the simpler impulse approximation based upon pseudovector covariants discussed in Sec. III. Some conclusions are presented in Sec. V.

II. FIVE-TERM LORENTZ INVARIANT REPRESENTATION: PSEUDOSCALAR CASE

The representation of NN amplitudes based on the five Fermi covariants has been discussed in detail in the work of Goldberger, Grisaru, MacDowell, and Wong.[11] In particular, a simple connection of five invariant amplitudes f_n to helicity amplitudes has been derived in GGMW (see Appendix A). In an alternative approach, Ref. 6 derived a 5×5 matrix to relate a different but closely related set of five invariant amplitudes, F_n, to five Wolfenstein amplitudes in the c.m. frame. Calculations of invariant amplitudes F_n were done by numerically inverting the matrix and applying it to the known Wolfenstein amplitudes. The GGMW analysis avoids matrix inversion and thus gives a very simple and elegant, but equivalent, definition of the invariant amplitudes in terms of known helicity amplitudes. With some minor changes of notation, we review the GGMW approach and show the relation of the GGMW amplitudes, f_n, to the amplitudes, F_n, of Ref. 6.

Following GGMW, the invariant amplitude is written in a symmetrized form

$$\hat{F}=(2ip)^{-1}[f_1(S-\tilde{S})+\tfrac{1}{2}f_2(T+\tilde{T})-f_3(A-\tilde{A}) +f_4(V+\tilde{V})+f_5(P-\tilde{P})]\;, \tag{2.1}$$

where the Fermi covariants used in this paper are[18]

$$S=1,\quad T=\sigma_1^{\mu\nu}\sigma_{2\mu\nu},\quad A=\gamma_1^5\gamma_1^\mu\gamma_2^5\gamma_{2\mu}\;,\qquad V=\gamma_1\cdot\gamma_2,\quad P=\gamma_1^5\gamma_2^5\;. \tag{2.2}$$

Covariant, $\tilde{S}$,[11,19] interchanges Dirac spinor indices of particles 1 and 2,

$$\tilde{S}u_\sigma(1)u_\tau(2)=u_\tau(1)u_\sigma(2)\ . \tag{2.3}$$

The other interchange covariants are defined by $\tilde{T}=\tilde{S}T$, $\tilde{A}=\tilde{S}A$, $\tilde{V}=\tilde{S}V$, and $\tilde{P}=\tilde{S}P$. Each interchange covariant can be alternatively expressed in terms of the Fermi covariants by the Fierz matrix as follows:[11]

$$\begin{bmatrix}\tilde{S}\\ \tilde{V}\\ \tilde{T}\\ \tilde{A}\\ \tilde{P}\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1 & \frac{1}{2} & -1 & 1\\ 4 & -2 & 0 & -2 & -4\\ 12 & 0 & -2 & 0 & 12\\ -4 & -2 & 0 & -2 & 4\\ 1 & -1 & \frac{1}{2} & 1 & 1\end{bmatrix}\begin{bmatrix}S\\ V\\ T\\ A\\ P\end{bmatrix}\ . \tag{2.4}$$

For isospin I, the Pauli principle requires the GGMW amplitudes to have definite symmetry when $\theta\to\pi-\theta$;

$$f_n^I(\theta)=(-)^{n+I}f_n^I(\pi-\theta)\ . \tag{2.5}$$

Clearly it is possible to eliminate the interchange covariants altogether to arrive at a simpler representation based on the five Fermi covariants as follows:

$$\hat{F}=F_1S+F_2V+F_3T+F_4P+F_5A\ . \tag{2.6}$$

Linear independence of the Fermi covariants coupled with Eqs. (2.1), (2.4), and (2.6) requires the following relation between the amplitudes:

$$\begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\\ F_5\end{bmatrix}=(8ip)^{-1}\begin{bmatrix}3 & 6 & -4 & 4 & -1\\ -1 & 0 & -2 & 2 & 1\\ -\frac{1}{2} & 1 & 0 & 0 & -\frac{1}{2}\\ -1 & 6 & 4 & -4 & 3\\ 1 & 0 & -6 & -2 & -1\end{bmatrix}\begin{bmatrix}f_1\\ f_2\\ f_3\\ f_4\\ f_5\end{bmatrix}\ . \tag{2.7}$$

Equation (2.6) is the representation assumed in Ref. 6 and subsequently used in the Dirac impulse approximation for p-nucleus scattering.

As shown in GGMW, the f_i can be expressed rather simply and elegantly in terms of helicity amplitudes ϕ_i. Alternatively, we can express the f_i in terms of helicity amplitudes h_i, $i=1$ to 5, which have been defined by Arndt *et al.*[20] These relations are given in Appendix A. Phase shift analyses of NN scattering data have been performed by Arndt and collaborators[20] using the h_i amplitudes. Using the relations given in Appendix A, the available phase shifts can be readily used to calculate the f_i and by use of (2.7), the F_i, $i=1$ to 5. In this way, the invariant amplitudes are completely determined from NN data.

One-boson exchange contributions to the invariant amplitudes illustrate the normalization convention and they also provide some useful insight. The amplitude for pseudoscalar (π), scalar (ϵ), and vector (ω) meson exchange takes the form:

$$\hat{F}=\frac{1}{2ip}\left[\frac{m^2}{2\pi\sqrt{s}}\right]\hat{M}\ , \tag{2.8}$$

where $\hat{M}$ is the Feynman amplitude

$$\begin{aligned}\hat{M}=&-g_\pi^2\left[\frac{(-3+4I)P}{m_\pi^2-t}-\frac{(3-2I)\tilde{P}}{m_\pi^2-u}\right]\\ &+g_\epsilon^2\left[\frac{S}{m_\epsilon^2-t}-\frac{(2I-1)\tilde{S}}{m_\epsilon^2-u}\right]\\ &-g_\omega^2\left[\frac{V}{m_\omega^2-t}-\frac{(2I-1)\tilde{V}}{m_\omega^2-u}\right]\ .\end{aligned} \tag{2.9}$$

In Eq. (2.9) the π-N coupling is pure γ^5 and ω-N coupling is pure γ^μ for illustration. Moreover, I is the isospin quantum number and $s=4(p^2+m^2)$, $t=-2p^2(1-\cos\theta)$, and $u=-2p^2(1+\cos\theta)$ are the Mandelstam invariants. The isospin factor $(-3+4I)$ is the usual $\tau_1\cdot\tau_2$ coupling and the factor $(3-2I)$ results from applying the isospin exchange operator to $\tau_1\cdot\tau_2$. Terms involving $\tilde{S}$, $\tilde{P}$, and $\tilde{V}$ are associated with nucleon interchange processes.

The proton-nucleus optical potential U in infinite nuclear matter involves the Dirac trace with respect to particle 2 of the product of Feynman amplitude $\hat{M}$ and a nuclear density matrix $\hat{\rho}$ as follows:

$$U=\tfrac{1}{4}\mathrm{Tr}_2(-\hat{M}\hat{\rho})\ , \tag{2.10}$$

where the nuclear matter density has scalar and time component of vector terms

$$\hat{\rho}=\rho_S+\gamma_2^0\rho_V\ . \tag{2.11}$$

Various estimates suggest ρ_V and ρ_S are equal to within 2–3 % in nuclear matter. In (2.10), a factorization of the NN interaction from the nuclear wave functions has been assumed even though nucleon interchange processes require a more careful treatment.

For the case where equal numbers of neutrons and protons are present in the nuclear medium, the appropriate average of pp and pn interactions is $\frac{3}{4}(I=1)+\frac{1}{4}(I=0)$. The average is easily obtained by using $I=\frac{3}{4}$ in (2.9). Furthermore, $\hat{M}$ is to be evaluated at zero momentum transfer ($\theta=0$). Using Eq. (2.8) and the relation $p=P_{\rm lab}m/\sqrt{s}$ for c.m. momentum, p in terms of laboratory momentum $P_{\rm lab}$ produces

$$U=\tfrac{1}{4}K\mathrm{Tr}_2(\hat{F}\hat{\rho})\ , \tag{2.12a}$$

where

$$K=-4\pi iP_{\rm lab}/m\ , \tag{2.12b}$$

which is in accord with the normalization condition employed in Refs. 1 and 2. Noting that the only terms in $\hat{F}$ which can survive the Dirac trace have unit or γ_2^0 matrices, with respect to particle 2 it follows that only scalar (S_0) and time component of vector (V_0) terms of Eq. (2.11) are needed to characterize the optical potential in nuclear matter;

$$U=S_0+\gamma^0V_0\ , \tag{2.13a}$$

where

$$S_0=KF_1(0)\rho_S\ , \qquad (2.13b)$$

$$V_0=KF_2(0)\rho_V\ . \qquad (2.13c)$$

The one-boson exchange approximation yields (using $4p^2=2mT_{\rm lab}$ where $T_{\rm lab}$ is the proton's kinetic energy in the laboratory frame)

$$F_1(0)=K^{-1}\left[-\tfrac{3}{8}g_\pi^2\frac{1}{m_\pi^2+2mT_{\rm lab}}-g_\epsilon^2\left[\frac{1}{m_\epsilon^2}-\frac{1}{8}\frac{1}{m_\epsilon^2+2mT_{\rm lab}}\right]-\tfrac{1}{2}g_\omega^2\frac{1}{m_\omega^2+2mT_{\rm lab}}\right], \qquad (2.14)$$

$$F_2(0)=K^{-1}\left[\tfrac{3}{8}g_\pi^2\frac{1}{m_\pi^2+2mT_{\rm lab}}+\tfrac{1}{8}g_\epsilon^2\frac{1}{m_\epsilon^2+2mT_{\rm lab}}+g_\omega^2\left[\frac{1}{m_\omega^2}+\frac{1}{4}\frac{1}{m_\omega^2+2mT_{\rm lab}}\right]\right]. \qquad (2.15)$$

Scalar and vector parts of nucleon interchange covariants, such as $\widetilde{P}$, survive in accordance with the Fierz matrix (2.4) and they have characteristic dependence on the proton energy. In particular, the pion exchange contribution can easily be checked to give quite large scalar attraction and vector repulsion, particularly at low energy. For $T_{\rm lab}=10$ MeV, the one-pion exchange contribution is approximately -2200 MeV to S_0 and $+2200$ MeV to V_0. The scalar (ϵ) meson contribution to S_0 is about -400 MeV and the vector (ω) contribution to V_0 is about $+300$ MeV. Unitarity effects in $\hat{F}$ at low energy reduce the one-pion exchange contributions by more than 50%; however, the pionic contributions to S_0 and V_0 remain large when U is calculated from Eq. (2.12) using the impulse approximation. The term $f_5\widetilde{P}$ in (2.1) has a large part of the one-pion contribution and Eq. (2.7) shows that this term contributes to scalar (F_1) and vector (F_2) amplitudes with coefficients -1 and 1, respectively. Figure 1 shows S_0 and V_0 as a function of $T_{\rm lab}$ based on using the free NN amplitudes in Eqs. (2.13). The very rapid increase in real parts of S_0 and V_0 at low energy is essentially due to the pseudoscalar one-pion exchange contribution as reduced by unitarity effects. It causes the impulse approximation to severely overestimate the contributions to $U_{\rm pair}$ at low energy.

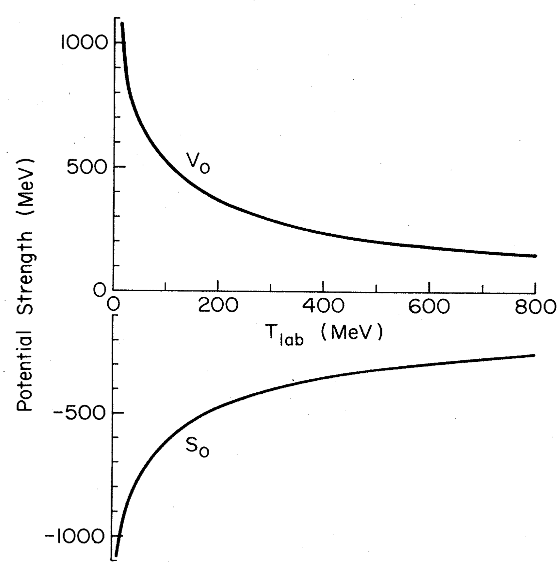

FIG. 1. Scalar and vector potential strengths for nuclear matter based on impulse approximation using pseudoscalar covariants.

III. FIVE-TERM LORENTZ INVARIANT REPRESENTATION: PSEUDOVECTOR CASE

The need for pair suppression in the NN interaction when a pseudoscalar πN coupling is present is well known in meson theories of the nuclear force. Theoretical solutions for NN scattering equations are satisfactory only when the πN coupling is taken to be pseudovector.[12] Analyses of the proton optical potential in nuclear matter have been performed in Ref. 21 using pseudovector coupling. Several other effects were also included in Ref. 21 so that the relation to the impulse approximation is not completely clear. However, the results give no evidence for the large S_0 and V_0 seen in Fig. 1.

If the one-pion exchange contribution is pseudovector, the NN amplitude representation of Eq. (2.1) or (2.6) is necessarily inadequate as can be seen by the following argument. Pseudovector coupling yields, for one-pion exchange,

$$\hat{M}_\pi=-g_\pi^2\frac{(-3+4I)}{m_\pi^2-t}PV+g_\pi^2\frac{(3-2I)}{m_\pi^2-u}\widetilde{PV}\ , \qquad (3.1a)$$

where

$$PV=\gamma_1^5\frac{(\not p_1-\not p_1')_1}{2m}\gamma_2^5\frac{(\not p_2-\not p_2')_2}{2m}\ , \qquad (3.1b)$$

$$\widetilde{PV}=\widetilde{S}\gamma_1^5\frac{(\not p_1-\not p_2')_1}{2m}\gamma_2^5\frac{(\not p_2-\not p_1')_2}{2m}\ , \qquad (3.1c)$$

and p_1,p_2 are initial momenta, p_1',p_2' are final momenta, and subscripts indicate which Dirac matrices enter, for example, $(\not p_1-\not p_2')_1=\gamma_1^\mu(p_{1\mu}-p_{2\mu}')$. Here PV is the covariant for direct exchange while $\widetilde{PV}$ is the covariant for π exchange followed by interchange of nucleons 1 and 2. For positive energy, on-shell states, $\hat{M}_\pi$ given by (3.1) is no different from the pion contribution of Eq. (3.13) as is well known.[13] To illustrate that the difference occurs only in negative energy basis states, expand as follows:

$$\gamma_1^5\frac{(p_1-p_1')_1}{2m}=\gamma_1^5-\gamma_1^5\Lambda_1^{(-)}-\Lambda_{1'}^{(-)}\gamma_1^5\ , \qquad (3.2)$$

where $\Lambda^{(-)}$ is a negative energy projection operator,

$$\Lambda_1^{(-)}=\frac{-E_{p_1}\gamma_1^0+\gamma_1\cdot\mathbf{p}_1+m}{2m} \tag{3.3}$$

and $\Lambda_{1'}^{(-)}$ is similarly defined with $\mathbf{p}_1'$ and $E_{p_1'}$ in place of $\mathbf{p}_1$ and E_{p_1}. The momenta, p_1, p_2, etc., in Eq. (3.1) are assumed to be on mass shell, otherwise additional terms are needed in (3.2). Substituting Eq. (3.2) into (3.1), using the definition $\widetilde{P}=\widetilde{S}P$, the following expansion for $\widehat{M}_\pi$ is obtained:

$$\widehat{M}_\pi=-g_\pi^2\frac{(-3+4I)}{m_\pi^2-t}[P-P(\Lambda_1^{(-)}+\Lambda_2^{(-)})-(\Lambda_{1'}^{(-)}+\Lambda_{2'}^{(-)})P+P\Lambda_1^{(-)}\Lambda_2^{(-)}+\Lambda_{1'}^{(-)}\Lambda_{2'}^{(-)}P+\Lambda_{2'}^{(-)}P\Lambda_1^{(-)}+\Lambda_{1'}^{(-)}P\Lambda_2^{(-)}]$$
$$+g_\pi^2\frac{(3-2I)}{m_\pi^2-u}[\widetilde{P}-\widetilde{P}(\Lambda_1^{(-)}+\Lambda_2^{(-)})-(\Lambda_{1'}^{(-)}+\Lambda_{2'}^{(-)})\widetilde{P}+\widetilde{P}\Lambda_1^{(-)}\Lambda_2^{(-)}+\Lambda_{1'}^{(-)}\Lambda_{2'}^{(-)}\widetilde{P}+\Lambda_{1'}^{(-)}\widetilde{P}\Lambda_1^{(-)}+\Lambda_{2'}^{(-)}\widetilde{P}\Lambda_2^{(-)}]\ . \tag{3.4}$$

Note that the interchange covariant $\widetilde{PV}$ is expanded using (3.2) except with momenta $\mathbf{p}_1'$ and $\mathbf{p}_2'$ interchanged. The $\widetilde{S}$ operator acts, for example, as

$$\widetilde{S}\Lambda_1^{(-)}(\mathbf{p}_2')=\Lambda_2^{(-)}(\mathbf{p}_2')\widetilde{S}$$

since it interchanges Dirac indices. Thus PV and $\widetilde{PV}$ take similar forms; however, note that the last two terms of $\widetilde{PV}$ are not obtained by just replacing P by $\widetilde{P}$ in the PV covariant.

The P and $\widetilde{P}$ terms with no projectors in (3.4) gives precisely the pseudoscalar result of (2.9). However, equally large contributions are present for amplitudes not contained in the simple five-term representation. It is due to the absence of terms involving negative energy projectors that Eq. (2.1) is not adequate to describe pseudovector one-pion contributions.

If we assume the πN coupling is pseudovector, a more appropriate representation of the NN amplitudes might be to replace P and $\widetilde{P}$ in Eq. (2.1) by PV and $\widetilde{PV}$. This new pseudovector representation is

$$\widehat{F}=(2ip)^{-1}[f_1(S-\widetilde{S})+\tfrac{1}{2}f_2(T+\widetilde{T})-f_3(A-\widetilde{A})$$
$$+f_4(V+\widetilde{V})+f_5(PV-\widetilde{PV})]\ . \tag{3.5}$$

Precisely the same amplitudes f_i are required in Eqs. (2.1) and (3.5) since the positive energy matrix elements of P and $\widetilde{P}$ are the same as those of PV and $\widetilde{PV}$. Thus f_1 to f_5 are determined just as before from positive energy helicity amplitudes.

It is convenient to proceed as before to eliminate some of the interchange covariants. Using the quantities inside brackets in Eq. (3.4) to define PV and $\widetilde{PV}$, and employing the Fierz transformation (2.4), we may rewrite $\widehat{F}$ as

$$\widehat{F}=F_1S+F_2V+F_3T+F_4P+F_5A$$
$$+F_{PV}(PV-P-\widetilde{PV}+\widetilde{P})\ , \tag{3.6}$$

where F_1 to F_5 are exactly the same as in Eq. (2.6). The new contribution due to the pseudovector covariants contains only terms with Λ^- projection operators. The amplitude of this term originates in the f_5 term, i.e.,

$$F_{PV}=\frac{1}{2ip}f_5\ . \tag{3.7}$$

Implications for the proton-nucleus optical potential are obtained from Eq. (2.10). Once again only scalar and time component of vector contributions (with respect to particle 2) survive the trace and we find

$$\tfrac{1}{4}\mathrm{Tr}_2\widetilde{P}\widehat{\rho}=\tfrac{1}{4}(\rho_S-\gamma_1^0\rho_V)$$

and

$$\tfrac{1}{4}\mathrm{Tr}_2\Lambda_2^{(-)}\widetilde{P}\widehat{\rho}=-\tfrac{1}{8}(1+\gamma_1^0)(\rho_V-\rho_S)\ .$$

Consequently the optical potential takes the form

$$U=K[F_1(0)\rho_S+\gamma_1^0F_2(0)\rho_V]+\tfrac{1}{4}KF_{PV}(0)\{(\rho_S-\gamma_1^0\rho_V)\Lambda_1^{(-)}+\Lambda_1^-(\rho_S-\gamma_1^0\rho_V)-\Lambda_1^{(-)}(\rho_S-\gamma_1^0\rho_V)\Lambda_1^{(-)}$$
$$-\tfrac{1}{2}[1+\gamma_1^0-\Lambda_1^{(-)}(1+\gamma_1^0)-\Lambda_1^{(-)}(1+\gamma_1^0)](\rho_V-\rho_S)\}\ . \tag{3.8}$$

After substituting (3.3) for Λ^- and some rearranging, there are three contributions as follows:

$$\widehat{U}=S_0+\gamma^0V_0+C_0\frac{(-\not p_1+m)}{m}\ , \tag{3.9}$$

where

$$S_0=K\left[F_1(0)\rho_S+\frac{E_p}{4m}F_{PV}(0)\rho_V-\tfrac{1}{8}\left[1+\frac{E_p}{m}\right]F_{PV}(0)(\rho_V-\rho_S)\right]\ , \tag{3.10a}$$

$$V_0=K[F_2(0)\rho_V-\tfrac{1}{4}F_{PV}(0)\rho_V]\ , \tag{3.10b}$$

$$C_0=K\left[\tfrac{1}{8}F_{PV}(0)\left[\rho_S-\frac{E_p}{m}\rho_V\right]+\tfrac{1}{8}F_{PV}(0)(\rho_V-\rho_S)\right]. \tag{3.10c}$$

The main feature to be noted in these equations is the role of amplitude F_{PV} which carries the new part of the pion exchange contribution. From Eqs. (2.7) and (3.7), the usual scalar and vector amplitudes already contain contributions $F_1=-\frac{1}{4}F_{PV}+\cdots$, and $F_2=+\frac{1}{4}F_{PV}+\cdots$, which are large at low energy. Since $\rho_S\simeq\rho_V$, these contributions are seen to be cancelled at low energy in Eqs. (3.10) by the new term from Eq. (3.6).

A second point worth noting is that contributions to $\hat{F}$ which contain negative energy projection for the target nucleon, i.e., $\Lambda_2^{(-)}$, enter the optical potential proportional to the lower component density $\rho_l=\frac{1}{2}(\rho_V-\rho_S)$. These contributions are suppressed since $\rho_l/\rho_V\simeq 2\%$ to 3%.

In the Dirac equation,

$$(E\gamma_1^0-\gamma_1\cdot\mathbf{p}-m-\hat{U})\psi=0\ , \tag{3.11}$$

it is convenient to define effective scalar and vector potentials as follows:

$$(E\gamma_1^0-\gamma_1\cdot\mathbf{p}-m-S_0^{\rm eff}-\gamma_1^0V_0^{\rm eff})\psi'=0\ , \tag{3.12a}$$

where

$$\psi'=(1+C_0/m)\psi\ , \tag{3.12b}$$

$$S_0^{\rm eff}=\frac{S_0}{1+C_0/m}\ , \tag{3.12c}$$

$$V_0^{\rm eff}=\frac{V_0}{1+C_0/m}\ . \tag{3.12d}$$

Since the $(1+C_0/m)$ factor can be absorbed into the wave function, the problem is equivalent to Dirac propagation with the effective scalar and vector potentials. A small scalar potential term, $[(E-E_{p_1})C_0/(m+C_0)]$, has been neglected going from (3.11) to (3.12). Note that there is no need to absorb the $1+C_0/m$ factor into the wave function and we do so only to facilitate comparison with the original impulse approximation which, like (3.12a), has only scalar and vector terms. Figure 2 shows the one-pion exchange contributions to effective scalar and vector potentials for the cases of pseudoscalar and pseudovector πN coupling. The large pionic contribution of the pseudoscalar case is greatly suppressed in the pseudovector case. Similar behavior is found for the impulse approximation potentials using pseudovector covariants in place of pseudoscalar ones. Two choices for F_{PV} in Eq. (3.6) are illustrated in Fig. 3. The first, defined by Eq. (3.7) and shown by dashed lines, represents the straightforward replacements of $P\rightarrow PV$ and $\tilde{P}\rightarrow\widetilde{PV}$ as discussed above. The second, defined by $F_{PV}=(f_5+f_4-f_2)/(2ip)$ and shown by dotted lines, corresponds to changing the full one-pion contribution associated with nucleon interchange from pseudoscalar to pseudovector. As shown in Appendix B, the full one-pion contribution relevant to the optical potential is not contained in invariant amplitude f_5, but rather involves the combination $f_5+f_4-f_2$. In either case, one sees a substantial reduction of the optical potentials at low energy and less energy dependence.

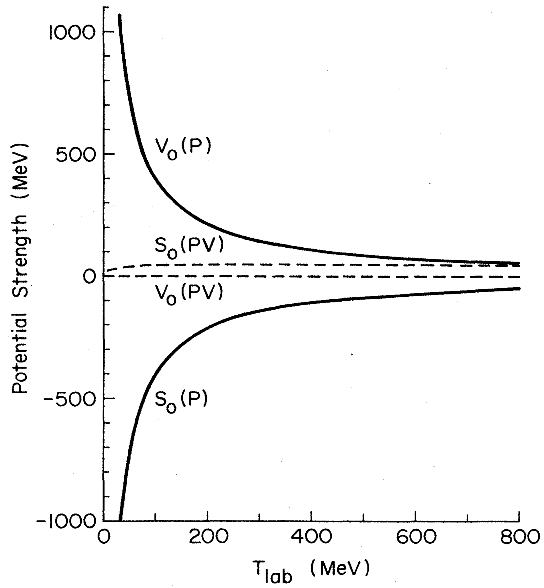

FIG. 2. Scalar and vector potential strengths for nuclear matter based on one-pion exchange. Solid lines show results for pseudoscalar πN coupling. Dash lines show results for pseudovector πN coupling.

Figure 3 suggests that an improvement of the Dirac impulse approximation of Ref. 1 might be made by adopting the pseudovector covariants. Such a change is motivated by meson theory. Pair contributions to p-nucleus scatter-

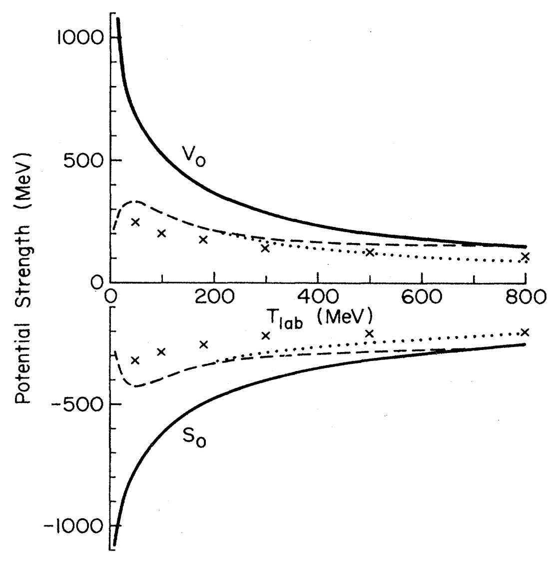

FIG. 3. Scalar and vector potential strengths for nuclear matter based on impulse approximation. Solid lines are obtained using pseudoscalar covariants. Dash and dotted lines are obtained using pseudovector covariants where $F_{PV}=f_5/(2ip)$ (dashed line) and $F_{PV}=(f_5+f_4-f_2)/(2ip)$ (dotted line). Results based on full meson theory NN amplitude are shown by $\times$.

ing are reduced at low energy and much closer agreement with the optical potential found in Ref. 21 is obtained. However, the large differences between the two versions of the impulse approximation represented by Figs. 1 and 3 can also be interpreted as an example of a fundamental ambiguity in the representation of NN amplitudes when the only input is positive energy scattering data. Indeed, we could equally well replace F_{PV} in (3.6) by λF_{PV}, where λ is an arbitrary constant. This corresponds to replacement of P and $\widetilde{P}$ in the original impulse approximation by covariants $(1-\lambda)P+\lambda PV$ and $(1-\lambda)\widetilde{P}+\lambda\widetilde{PV}$, respectively. Since positive energy matrix elements of P and PV are identical, λ cannot be determined by physical matrix elements. However, the scalar and vector optical potential strengths depend on λ as also does U_{pair}, but, of course, U^{++} is independent of λ. The various curves in Fig. 3 correspond to different choices of λ.

It clearly is not a satisfactory procedure simply to hypothesize which is the correct set of covariants to be used. Rather there must be some theoretical guidance which can eliminate the ambiguity. The central question to be answered is whether the Dirac successes are in accord with an underlying meson theory which predicts the pair couplings from an interaction Lagrangian conventionally used in nuclear physics.

IV. RECONSTRUCTION OF S AND V FROM THE GENERAL LORENTZ INVARIANT REPRESENTATION

In Ref. 16 a general Lorentz invariant representation of the NN scattering amplitude is developed in the following form:

$$\hat{F}=\hat{F}^{11}+\Lambda_{2'}^{(-)}\hat{F}^{12}+\hat{F}^{13}\Lambda_{2}^{(-)}+\Lambda_{2'}^{(-)}\hat{F}^{14}\Lambda_{2}^{(-)}+\Lambda_{1'}^{(-)}(\hat{F}^{21}+\Lambda_{2'}^{(-)}\hat{F}^{22}+\hat{F}^{23}\Lambda_{2}^{(-)}+\Lambda_{2'}^{(-)}\hat{F}^{24}\Lambda_{2}^{(-)})$$
$$+(\hat{F}^{31}+\Lambda_{2'}^{(-)}\hat{F}^{32}+\hat{F}^{33}\Lambda_{2}^{(-)}+\Lambda_{2'}^{(-)}\hat{F}^{34}\Lambda_{2}^{(-)})\Lambda_{1}^{(-)}+\Lambda_{1'}^{(-)}(\hat{F}^{41}+\Lambda_{2'}^{(-)}\hat{F}^{42}+\hat{F}^{43}\Lambda_{2}^{(-)}+\Lambda_{2'}^{(-)}\hat{F}^{44}\Lambda_{2}^{(-)})\Lambda_{1}^{(-)} . \quad (4.1)$$

There are 16 $\hat{F}^{ij}$. The first superscript of $\hat{F}^{ij}$ is the class index, $i=1$ to 4, which refers to the associated negative energy projectors for particle 1 (projectile), and the second superscript of $\hat{F}^{ij}$ is the subclass index, $j=1$ to 4, which refers to the associated negative energy projectors for particle 2 (target). The form of (4.1) is chosen so that the representation of Sec. II is naturally embedded as $\hat{F}^{11}$. The use of negative energy projection operators guarantees that positive energy matrix elements, $(++\,|\,\hat{F}\,|\,++)$, involve only the $\hat{F}^{11}$ term. Furthermore, the representation contains all possible negative energy components in contrast with the limited set found to be needed in (3.4) to expand the pseudovector covariants.

Each $\hat{F}^{ij}$ in (4.1) is expanded in an overcomplete set of nine covariants:

$$\hat{F}^{ij}=\sum_{n=1}^{9}F_n^{ij}\mathscr{K}_n \quad (4.2)$$

where

$$\mathscr{K}_n=\{S,V,T,P,A,\gamma_2\cdot Q_1,\gamma_1\cdot Q_2,P\gamma_2\cdot Q_1,P\gamma_1\cdot Q_2\} \ . \quad (4.3)$$

Here $Q_1^\mu=(p_1+p'_1)^\mu/(2m)$ and $Q_2^\mu=(p_2+p'_2)/(2m)$, where p_1 and p_2 are initial momenta, p'_1 and p'_2 are final momenta. There exists one linear relation among the helicity basis matrix elements of the nine $\mathscr{K}_n$ which reduces them to eight linearly independent covariants. Equation (4.1) is capable of representing a general parity conserving NN interaction since only half of the $4^4=256$ matrix elements of $\hat{F}$ are independent when parity is conserved.

In Ref. 16, the invariant amplitudes F_n^{ij} are related to c.m. frame helicity amplitude ϕ_m^{ij}, $m=1$ to 8. A quasipotential equation for NN scattering is solved to determine the ϕ_m^{ij}. Although somewhat complicated in practice, this is, in principle, a straightforward procedure. The result is a determination of all F_n^{ij} from the meson exchange dynamics for which we use the coupled (NN,NΔ,ΔΔ) channel formalism of Ref. 17. Because we need negative energy matrix elements, the calculations of Ref. 17 are extended to include also negative energy NN intermediate states in the dynamical equations. With a modest adjustment of the scalar meson coupling constant, a satisfactory description of NN phase shifts has been obtained. Details of these calculations are presented in Ref. 16.

From the general Lorentz invariant representation of $\hat{F}$ we may readily construct the proton-nucleus optical potential in nuclear matter. Employing (2.12) and our general form for $\hat{F}$ given by (4.1), the proton optical potential is found to have scalar and vector contributions from each class $i=1$ to 4, as follows:

$$\hat{U}=S_0^1+\gamma_1^0V_0^1+\Lambda_1^{(-)}(S_0^2+\gamma_1^0V_0^2)+(S_0^3+\gamma_1^0V_0^3)\Lambda_1^{(-)}$$
$$+\Lambda_1^{(-)}(S_0^4+\gamma_1^0V_0^4)\Lambda_1^{(-)} . \quad (4.4)$$

In terms of the invariant amplitudes, the components of the potential are

$$S_0^i=K\left[F_1^{i1}(0)\rho_S+\frac{E_p}{m}F_6^{i1}(0)\rho_V\right]$$
$$+K\sum_{j=2}^{4}\left[-F_1^{ij}(0)+\frac{E_p}{m}F_6^{ij}(0)\right]\rho_I , \quad (4.5)$$

$$V_0^i=K[F_2^{i1}(0)\rho_V+F_7^{i1}(0)\rho_S]$$
$$+K\sum_{j=2}^{4}[F_2^{ij}(0)-F_7^{ij}(0)]\rho_I , \quad (4.6)$$

where

$$\rho_l = \tfrac{1}{2}(\rho_V - \rho_S) . \tag{4.7}$$

As in Sec. III, subclass 1 amplitudes enter proportional to the full density ρ_S or ρ_V, and subclass 2, 3, and 4 amplitudes which involve projectors $\Lambda_2^{(-)}$ enter proportional to the lower component density ρ_l, which is much smaller.

Proceeding to multiply out Λ^- projection operators as in Sec. III, the optical potential simplifies to four terms:

$$U = S_0 + \gamma_1^0 V_0 + C_0 \left[\frac{-p_1 + m}{m} \right] + F_0 \frac{\boldsymbol{\alpha} \cdot \mathbf{p}}{m} , \tag{4.8}$$

where

$$S_0 = S_0^1 - \frac{1}{2} \frac{E_p}{m} (V_0^2 + V_0^3) , \tag{4.9}$$

$$V_0 = V_0^1 + \tfrac{1}{2}(V_0^2 + V_0^3) , \tag{4.10}$$

$$C_0 = \tfrac{1}{2} \left[S_0^2 + S_0^3 + S_0^4 - \frac{E_p}{m} V_0^4 \right] , \tag{4.11}$$

$$F_0 = \tfrac{1}{2}(V_0^3 - V_0^2) . \tag{4.12}$$

Due to time-reversal invariance, $F_0 \equiv 0$, and thus there are only three nonzero terms as in Eq. (3.9). Effective potentials consequently take the same form as in Eqs. (3.12). Figure 3 shows the effective scalar and vector strengths for nuclear matter based on the full meson theory as x's. Reasonable agreement is observed between the five-term pseudovector representation of Sec. III and the results of using the full meson theory.

V. CONCLUSIONS

The important pair contributions to proton-nucleus elastic scattering which are implicit in a Dirac equation analysis are overly large at low energy in the original form of the impulse approximation. This flaw is clear associated with a representation of the NN amplitudes which admits only pseudoscalar πN coupling. The pseudoscalar pion exchange contribution associated with nucleon interchange (as required by the Pauli principle) causes the overly large scalar and vector potentials at low energy.

When equivalent (on positive energy states) pseudovector covariants are used in place of the pseudoscalar covariants, the scalar and vector optical potentials are reduced in magnitude and they vary less with respect to energy. The pair contribution is consequently reduced at low energy. Reasonable accord is found with the potential strengths used in phenomenological fits to proton scattering data.[22] However, simply replacing covariants is an ambiguous procedure since negative energy components cannot be determined from positive energy NN data. It is possible to add an arbitrary amount of an amplitude whose positive energy matrix elements vanish. Since the Dirac potentials are changed by such an addition they are ambiguous without some further input beyond the NN data.

The only existing model for the fully relativistic NN scattering which explains the scattering data is relativistic meson theory. Due to Lorentz covariance, once the coupling constant and form factors of the meson theory are fixed (by fitting NN data), one can predict a complete set of Lorentz invariant NN amplitudes. This procedure eliminates ambiguities associated with the choice of NN covariants. There is, however, model dependence in the meson theory. For example, some admixture of pseudoscalar and pseudovector πN coupling could presumably be used and still obtain good phase shifts. The first meson theory analysis has been carried out in Ref. 16, and in the present paper the results are used to calculate the Dirac optical potential in nuclear matter within the context of a $t\rho$ impulse approximation.

Reasonable agreement is found between the full meson theory analysis for nuclear matter and the simpler analysis in which the original impulse approximation is altered by replacement of pseudoscalar covariants with pseudovector covariants. The use of pseudovector πN coupling in the meson theory accounts for the qualitative difference from the original impulse approximation. The new impulse approximation based on the meson theoretical NN amplitudes provides a more fundamental basis for calculating the virtual pair contributions to p-nucleus scattering. The main features of the Dirac potential in this approach are attractive scalar potentials of 215 to 315 MeV strength and repulsive vector potentials of 150 to 240 MeV strength in the range of proton energies from 50 to 800 MeV. Calculations for p-nucleus scattering show that the meson theoretical approach provides significant improvement for 181 and 200 MeV proton scattering.[23]

Research support of the U.S. Department of Energy and the University of Maryland Computer Science Center is gratefully acknowledged.

APPENDIX A: RELATION OF INVARIANT AMPLITUDES TO c.m. FRAME HELICITY AMPLITUDES

A summary is given of relations which determine the invariant amplitudes f_i from knowledge of c.m. frame helicity amplitudes. Following Arndt *et al.*,[20] five helicity amplitudes h_i are considered for which phase shifts are readily available. The Arndt *et al.* amplitudes can be used to compute a set of helicity amplitudes ϕ_i defined by (Ref. 11) as follows:

$$\begin{aligned} \phi_1 &= (h_5 + h_1)/p , \\ \phi_2 &= (h_5 - h_1)/p , \\ \phi_3 &= (h_2 + h_3)/p , \\ \phi_4 &= (h_2 - h_3)/p , \\ \phi_5 &= -h_4/p , \end{aligned} \tag{A1}$$

where p is the c.m. momentum. Helicity amplitudes ϕ_i possess kinematic singularities in the form of zeros at $\theta = 0$ or π which are branch points with respect to $\cos\theta$. A set of amplitudes $\hat{\phi}_i$ which are free of kinematic singularities (KSF) is defined by

$$\hat{\phi}_1=\phi_1\ ,$$
$$\hat{\phi}_2=\phi_2\ ,$$
$$\hat{\phi}_3=\frac{m^2}{p^2}\frac{\phi_3}{1+\cos\theta}\ , \tag{A2}$$
$$\hat{\phi}_4=\frac{m^2}{p^2}\frac{\phi_4}{1-\cos\theta}\ ,$$
$$\hat{\phi}_5=\frac{-2m\phi_5}{E\sin\theta}\ .$$

Here $E=\sqrt{p^2+m^2}$, where m is the nucleon's mass. Kinematic zeros at $\theta=0$ and $\theta=\pi$ are divided out and thus all five $\hat{\phi}_i$ remain finite at these points. GGMW consider an intermediate set of invariant amplitudes G_i which can be determined directly from the KSF helicity amplitudes as follows:[11]

$$G_1=\frac{m^2}{E^2}(\hat{\phi}_1-\hat{\phi}_2+\hat{\phi}_3-\hat{\phi}_4)-\frac{p^2}{E^2}\cos\theta(\hat{\phi}_3+\hat{\phi}_4)+\cos\theta\hat{\phi}_5\ ,$$
$$G_2=-\hat{\phi}_3-\hat{\phi}_4+\frac{E^2}{p^2}\hat{\phi}_5\ ,$$
$$G_3=-\hat{\phi}_3+\hat{\phi}_4\ , \tag{A3}$$
$$G_4=\hat{\phi}_3+\hat{\phi}_4-\frac{m^2}{p^2}\hat{\phi}_5\ ,$$
$$G_5=-\frac{m^2}{p^2}(\phi_1+\phi_2)-\cos\theta(\hat{\phi}_3+\hat{\phi}_4)+\frac{E^2+m^2}{p^2}\cos\theta\hat{\phi}_5\ .$$

The desired GGMW invariant amplitudes of Eq. (2.1) are simply expressed in terms of the G_i as follows:

$$f_1=\tfrac{1}{8}(G_1+4G_3+3G_5)\ ,$$
$$f_2=\tfrac{1}{2}G_2\ ,$$
$$f_3=\tfrac{1}{8}(G_5-G_1)\ , \tag{A4}$$
$$f_4=\tfrac{1}{2}G_4\ ,$$
$$f_5=\tfrac{1}{8}(3G_1-4G_3+G_5)\ .$$

Equations (A1)–(A4) provide a simple and direct link from the known helicity amplitudes h_i to the invariant amplitudes f_i of Eq. (2.1), or by use of (2.7), to the invariant amplitudes F_i of Eq. (2.6). Numerical calculations have been performed based on these equations and agreement with the matrix inversion method of Ref. 6 has been established as a check.

APPENDIX B: ONE-PION EXCHANGE CONTRIBUTIONS

The one-pion exchange contribution, assuming pure γ^5 coupling, takes the form given as part of Eqs. (2.12) and (2.13).

$$\hat{F}_\pi=(2ip)^{-1}(DP+X\tilde{P})\ , \tag{B1}$$

where direct and exchange terms are

$$D=\frac{m^2}{2\pi\sqrt{s}}\frac{g_\pi^2}{m_\pi^2-t}(3-4I)\ , \tag{B2}$$

$$X=\frac{m^2}{2\pi\sqrt{s}}\frac{g_\pi^2}{m_\pi^2-u}(3-2I)\ , \tag{B3}$$

and I is the isospin.

One-pion contributions to the invariant amplitudes of Eq. (2.1) can be determined by using the identities

$$\tfrac{1}{2}(T+\tilde{T})=S+\tilde{S}+P+\tilde{P}\ , \tag{B4}$$

$$V+\tilde{V}=S+\tilde{S}-P-\tilde{P}\ , \tag{B5}$$

to rewrite (2.1) as follows:

$$\hat{F}=(2ip)^{-1}[(f_1+f_2+f_4)S-(f_1-f_2-f_4)\tilde{S}-f_3(A-\tilde{A})$$
$$+(f_5+f_2-f_4)P-(f_5+f_4-f_2)\tilde{P}]\ . \tag{B6}$$

Equations (B4) and (B5) follow from Eq. (2.4).

Covariants S, $\tilde{S}$, P, $\tilde{P}$, and $A-\tilde{A}$ form a linearly independent set. Therefore when (B1) and (B6) are equated, coefficients of S, $\tilde{S}$, and $A-\tilde{A}$ must be zero, whereas coefficients of P and $\tilde{P}$ must be the direct and exchange one-pion contributions. In particular, the combination $f_5+f_4-f_2=-X$ determines the one-pion contribution which effects the optical potential. The one-pion contributions to individual invariant amplitudes are

$$f_1=0;\quad f_2=\tfrac{1}{4}(D+X);\quad f_3=0;$$
$$f_4=-\tfrac{1}{4}(D+X);\quad f_5=\tfrac{1}{2}(D-X)\ .$$

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