Relativistic effects in the neutron-deuteron scattering lengths

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We have estimated the contribution of relativistic effects to the neutron-deuteron scattering lengths by solving the nonrelativistic and relativistic Faddeev equations for several local and nonlocal potentials that act only in S waves. We found that the relativistic effects increase the quartet scattering length by about 0.2 fm and decrease the doublet scattering length by roughly the same amount. These effects are not small in the case of the doublet scattering length, where they represent a correction of more than 20% of the experimental value.

Relativistic corrections in the three-nucleon problem have been restricted mainly to the calculation of these effects in the binding energy of the triton.[1-3] The results of these calculations indicate that relativistic effects are small, amounting to approximately 3%, since they increase the binding energy by about 0.25 MeV. These relativistic corrections arise partly from the kinetic energy terms of order $(v/c)^2$ and partly from the transformation of the potentials between the two-body and three-body c.m. frames, since in the three-body system the interactions are known in the c.m. frame of each pair but are required in the three-body c.m. frame. As it turns out, the corrections arising from the potential are repulsive while those arising from the kinetic energy are attractive, so that they tend to cancel each other to a large extent.[4]

In order to calculate the relativistic corrections to the neutron-deuteron scattering lengths, we will solve simultaneously the Faddeev equations and their relativistic generalization, which has been proposed by Aaron, Amado, and Young,[5] for several local potentials that act only in S waves. As far as we know, this is the first attempt to calculate the relativistic corrections to the three-nucleon continuum problem.

The Faddeev equations for the neutron-deuteron scattering lengths in the case of potentials that act only in S waves are written in momentum space as

$$\langle p_iq_i \mid T_i^J \mid \phi_0\rangle = 2b_{i1}^J \frac{1}{q_i^2}\delta(q_i)G_0^{-1}(p_iq_i)\phi_0(p_i) + \sum_{j=1}^{2} b_{ij}^J \int_0^\infty q_j^2 dq_j \int_{-1}^{1} d\cos\theta\, t_i(p_i,p_i';E-3q_i^2/4M)G_0(p_jq_j)\langle p_jq_j \mid T_j^J \mid \phi_0\rangle, \tag{1}$$

where M is the mass of the nucleon, t_1 and t_2 are, respectively, the $^3S_1$ and $^1S_0$ nucleon-nucleon T matrices, ϕ_0 is the deuteron wave function,

$$G_0(p_jq_j) = \frac{1}{E-p_j^2/M-3q_j^2/4M+i\epsilon}, \tag{2}$$

$$p_i'^{\,2} = q_j^2+q_i^2/4+q_iq_j\cos\theta, \tag{3}$$

$$p_j^2 = q_i^2+q_j^2/4+q_iq_j\cos\theta. \tag{4}$$

The spin-isospin coefficients b_{ij}^J are

$$b_{11}^{1/2} = b_{22}^{1/2} = \tfrac{1}{4}, \tag{5}$$

$$b_{12}^{1/2} = b_{21}^{1/2} = -\tfrac{3}{4}, \tag{6}$$

for the doublet ($J=\frac{1}{2}$) channel, and

$$b_{11}^{3/2} = -\tfrac{1}{2}, \tag{7}$$

$$b_{12}^{3/2} = b_{21}^{3/2} = b_{22}^{3/2} = 0 \tag{8}$$

for the quartet ($J=\frac{3}{2}$) channel. After solving the integral equations (1), the neutron-deuteron scattering lengths are obtained as

$$a^J = 2M\pi/3 \int_0^\infty q_i^2 dq_i \phi_0(q_i)\langle q_i/2, q_i \mid T_1^J \mid \phi_0\rangle. \tag{9}$$

The relativistic generalization of the Faddeev equations (1) is[5]

$$\langle p_iq_i \mid T_i^J \mid \phi_0\rangle = 2b_{i1}^J \frac{1}{q_i^2}\delta(q_i)G_0^{-1}(p_iq_i)(M^2+p_i^2)^{1/4}\phi_0(p_i) + \sum_{j=1}^{2} b_{ij}^J \int_0^\infty \frac{q_j^2 dq_j}{2\omega_j} \int_{-1}^{1} \frac{d\cos\theta}{2\omega_k} t_i(p_i,p_i';S+M^2-2\sqrt{S}\,\omega_i)G_0(p_jq_j)\langle p_jq_j \mid T_j^J \mid \phi_0\rangle, \tag{10}$$

where S is the invariant mass of the system squared,

$$G_0(p_jq_j)=\frac{2(\omega_i+\omega_j+\omega_k)}{S-(\omega_i+\omega_j+\omega_k)^2+i\epsilon}\ , \tag{11}$$

$$p_i'^{\,2}=(\omega_j+\omega_k)^2/4-q_i^2/4-M^2\ , \tag{12}$$

$$p_j^2=(\omega_i+\omega_k)^2/4-q_j^2/4-M^2\ , \tag{13}$$

$$\omega_i=(M^2+q_i^2)^{1/2}\ , \tag{14}$$

$$\omega_j=(M^2+q_j^2)^{1/2}\ , \tag{15}$$

$$\omega_k=(M^2+q_i^2+q_j^2+2q_iq_j\cos\theta)^{1/2}\ . \tag{16}$$

The nucleon-nucleon T matrices t_i in Eq. (10) must be obtained by solving the Blankenbecler-Sugar equation, but they can be related to the usual Lippmann-Schwinger T matrices by means of the minimal relativity transformation,[6]

$$t_i^{\rm BS}(p_i,p_i';s_i)=4M(M^2+p_i^2)^{1/4}(M^2+p_i'^{\,2})^{1/4}\times t_i^{\rm LS}(p_i,p_i';E_i)\ , \tag{17}$$

where the invariant mass squared of the pair s_i is related to the nonrelativistic energy E_i as

$$s_i=4M(M+E_i)\ . \tag{18}$$

Finally, after solving the integral equations (10), the scattering lengths are obtained as

$$a^J=2M\pi/3\int_0^\infty\frac{q_i^2dq_i}{\omega_i}(M^2+q_i^2)^{1/4}\phi_0(q_i)\times\langle[M(\omega_i-M)/2]^{1/2},q_i\,|\,T_1^J\,|\,\phi_0\rangle\ . \tag{19}$$

The relativistic Faddeev propagator (11) was obtained by Aaron, Amado, and Young[5] (AAY) by performing a dispersion integration in the invariant mass squared S of the three-body system as suggested by Blankenbecler and Sugar.[7] The Blankenbecler-Sugar prescription, however, is not unique, and many other reductions are possible which lead also to three-body integral equations that are relativistically invariant. Thus, for example, Ahmadzadeh and Tjon[8] (AT) applied the Blankenbecler-Sugar prescription by performing the dispersion integral in the invariant mass squared s_i of the two-body subsystem jk which is the interacting pair, so as to get instead of Eq. (11) the new relativistic Faddeev propagator

$$G_0(p_jq_j)=\frac{2[(\omega_j+\omega_k)^2-q_i^2]^{1/2}}{(S-\omega_i)^2-(\omega_j+\omega_k)^2+i\epsilon}\ . \tag{20}$$

We perform the relativistic calculation using both Blankenbecler-Sugar reductions.

In order to solve the two-dimensional integral equations (1) and (10), we used the method of the Padé approximants with a 40-point Gauss mesh for each variable, which gives an accuracy of better than 0.01 fm for the scattering lengths. We took $\hbar^2/M=41.47$ MeV fm^2 and used four different models to represent the nucleon-nucleon interaction as follows: (a) the so-called uncoupled Reid soft-core model,[9] which is equal to the Reid potential for the $^1S_0$ channel; the $^3S_1$ channel is assumed to be uncoupled and equal to the $^1S_0$ potential multiplied by 1.4507, which gives a deuteron binding energy of 2.23 MeV; (b) the Malfliet-Tjon I—III model[10] which also gives a binding energy of 2.23 MeV; (c) the Malfliet-Tjon II—IV model[10] which is not very realistic since it does not contain short-range repulsion, and it reproduces well only the low-energy parameters a and r but not the phase shifts at low energies; (d) the nonlocal separable potentials of Yamaguchi[11] which reproduce well the parameters a and r and the low-energy phase shifts.[12]

We give in Table I our results for the quartet scattering length, where we see that for the four models considered the relativistic effects increase the scattering length by about 0.2 fm, and the results of the two Blankenbecler-Sugar reductions differ from each other by only 0.01 fm. The relativistic effects represent a correction of approximately 3%, which is similar to that found in the bound-state problem.

It is known[14] that the quartet scattering length is correlated strongly to the deuteron binding energy or, in other words, to the nucleon exchange mechanism in which the interaction range is determined by the radius of the deuteron. Relativistic kinematics weakens the binding of the deuteron,[1,15] thus increasing the range of the repulsive effective interaction. This, in turn, leads to an increased quartet scattering length which has been demonstrated also by Payne *et al.*[10] in their calculation with the Malfliet-Tjon V potential. This effect can also be seen by the comparison of the n-d to the p-d scattering lengths: The addition of the repulsive Coulomb force leads to an increased quartet scattering length.

In Table II we give the corresponding results for the doublet scattering length, where we see the opposite effect of the relativistic corrections, since in this case they decrease the scattering length. Here it is known[16] that the doublet scattering length is correlated strongly with the triton binding energy. Since the relativistic corrections lead to a stronger binding in the triton,[1–3] a reduction of the doublet scattering length follows according to Phillips.[16]

The changes in the scattering length due to the relativistic effects are, as in the quartet case, about 0.2 fm for the two relativistic models and for the Yamaguchi model, with the results of the two Blankenbecler-Sugar reductions differing among themselves by about 0.02 fm. The Malfliet-Tjon II—IV model gives both a larger effect for the relativistic corrections and a larger difference between the two Blankenbecler-Sugar reductions. This model,

TABLE I. Quartet scattering length (in fm) calculated using four different models of the nucleon-nucleon interaction in a nonrelativistic formalism (NR) and in the relativistic formalisms of Aaron, Amado, and Young (AAY) and Ahmadzadeh and Tjon (AT). The experimental value is $^4a=6.35\pm0.02$ fm (Ref. 13).

	Uncoupled Reid	Malfliet-Tjon I—III	Malfliet-Tjon II—IV	Yamaguchi
NR	6.37	6.44	6.53	6.29
AAY	6.55	6.61	6.71	6.46
AT	6.56	6.62	6.72	6.46

TABLE II. Doublet scattering length (in fm) calculated using four different models of the nucleon-nucleon interaction in a nonrelativistic formalism (NR) and in the relativistic formalisms of Aaron, Amado, and Young (AAY) and Ahmadzadeh and Tjon (AT). The experimental value is $^2a = 0.65 \pm 0.04$ fm (Ref. 13).

	Uncoupled Reid	Malfliet-Tjon I—III	Malfliet-Tjon II—IV	Yamaguchi
NR	0.63	0.70	−5.62	−0.92
AAY	0.43	0.55	−5.92	−1.14
AT	0.41	0.54	−6.01	−1.16

however, as we have already pointed out, is completely unrealistic, so that if we disregard it we see that the relativistic effects are of the same size for the quartet and doublet channels. The changes in the doublet scattering length coming from the relativistic effects represent a correction of more than 20%.

The question of to what extent the relativistic effects considered in this work will be in accord with the solution of a fully relativistic three-body equation cannot be answered at present. There are, however, first reports on triton binding energy calculations employing three-body Bethe-Salpeter equations[17] confirming the sign of the corrections based on relativistic kinematics, i.e., implying a stronger binding in the triton.

As far as a comparison with experimental values of the scattering lengths (see Table I and II) is concerned, it is too premature to decide whether the corrections will support a better agreement between theory and experiment. Taking, e.g., the scattering lengths obtained with the five-channel Reid potential calculation (the values $^4a = 6.30$ fm and $^2a = 1.76$ fm are given in Ref. 18), the relativistic effect would move ^2a closer to the measured value, whereas the opposite would be true with ^4a. For a more stringent comparison with experiment, however, contributions of three-body forces should also be included. In this context it is interesting that Torre *et al.*[19] as well as Delfino and Glöckle[20] have found that a three-body force can reduce the doublet scattering length by about half a fermi which would move, e.g., the Reid potential result closer to experiment.

To conclude, we have found that the relativistic effects increase the quartet scattering length by about 0.2 fm and decrease the doublet scattering length by a similar amount. This represents a correction of approximately 3% for the quartet case and of more than 20% for the doublet case. The relativistic effect seems to be nearly independent of the underlying nucleon-nucleon interaction and of our choice of relativistic reductions.

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