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Simple parametrization of the π-N amplitude

R. J. McLeod and I. R. Afnan
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042, Australia
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We present a simple parametrization of the *S*-, *P*-, and *D*-wave π-N amplitudes using separable potentials for $T_\pi < 1$ GeV. The effect of the inelasticity is included in the Green's function while maintaining consistency with unitarity. The P_{11} amplitude is written as a pole plus nonpole in order to describe pion absorption in $A \geq 2$.

I. INTRODUCTION

The interest in dibaryon resonances has led in recent years to a large number of polarization measurements in N-N,[1] π-d,[2] and pp$\rightarrow\pi$d (Ref. 3) experiments. Many of these experiments have been carried in the energy region above the Δ(1232) resonance and below the two-pion threshold. Part of the interest in the dibaryon resonance is the possible evidence for quark degrees of freedom, such as six-quark states. One way of establishing the existence of such exotic states is to find a glaring discrepancy between the experimental results for the above reactions, and more conventional nuclear theories based on N, Δ, π,. . . .

For the above reactions, we have such a theory based on the coupling of the NN to the πNN channels, and satisfying two- and three-body unitarity.[4–7] The input to these calculations is the basic π-N amplitude off shell. To simplify the solution of the NN-πNN equations, all calculations to date have assumed separable potentials for the π-N interaction. This has been justified by the fact that separable potentials are a good approximation to a system with resonances,[8] and the π-N system has many such resonances.

One feature of the π-N system above the Δ(1232), that is often ignored in the construction of separable potentials for NN-πNN calculations, is the fact that just above the two-pion thresholds the inelasticity grows suddenly, and there are several π-N resonances. This rapid variation in the phase shifts and inelasticity is illustrated in Fig. 1 for the D_{13} channel. Such rapid variation in phase shifts and inelasticity could affect the parametrization of the π-N amplitude, even below the pion production threshold. This in turn could affect the result of the NN-πNN calculation, or the concentration of a pion-nucleus optical potential.

To include such large and rapidly changing inelasticity into the π-N amplitude, we should include the three-body unitarity cut into the amplitude. However, the resultant *t* matrix would be impractical for use in the NN-πNN calculations, and the determination of the pion-nucleus optical potential. Furthermore, most of the NN-πNN and $\pi-A$ calculations are below the two-pion threshold, and the π-N amplitude must only satisfy two-body unitarity at these energies. We show in Sec. II how we can build the effects of the three-body threshold, and yet maintain the simplicity of two-body separable amplitudes.

Since the lowest of these resonances above the two-pion threshold is the Roper (1440) resonance, we need to include the inelasticity in the P_{11} amplitude, while maintaining the basic structure of this amplitude as a *t* (pole) + *t* (nonpole). In this way, we can maintain the coupling of the πNN to the NN channel. While for π-*A* scattering, it will allow the coupling of the (π,π) and the (π,p) reaction channels. This construction is also presented in Sec. II.

In Sec. III we present our parametrization for the *S*-, *P*-, and *D*-wave π-N amplitudes including inelasticity. We show how the experimental inelasticity is built into the potential following procedures similar to those used by Ernst and Johnson.[9] However, we use the latest π-N

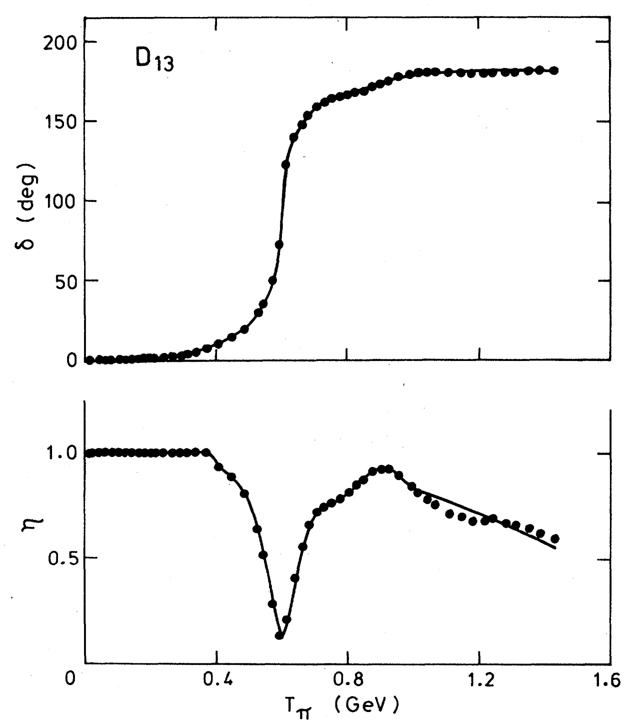


FIG. 1. The phase shifts δ and inelasticity η for the D_{13} channel. The solid curve is the theory, the experimental points (●) are from Refs. 10 and 24.

phase shifts based on the Karlsruhe[10] compilation. We also illustrate how the inclusion of inelasticity can affect the range of the π-N interaction in momentum space.

II. BASIC FORMULATION

The main aim of this analysis is to include the contribution of the two-pion threshold into the π-N amplitude without resorting to three-body theory. This is motivated by the fact that the resultant t matrix will be mainly used below the two-pion production threshold. To achieve this, let us consider the unitarity equation for the partial wave amplitude above the two-pion threshold, i.e.,

$$\begin{aligned}\Delta t_{00} &= \frac{1}{2i}[t_{00}(E+i\epsilon)-t_{00}(E-i\epsilon)] \\ &= -\left[\rho_0\,|\,t_{00}(E)\,|^2+\sum_n \rho_n\,|\,t_{0n}\,|^2\right] ,\end{aligned} \tag{1}$$

where the sum n runs over all the inelastic channels, and ρ_n is the phase space factor for channel n. Since the elastic and total cross section are given by

$$\begin{aligned}&\sigma_{\text{el}} \sim \rho_0\,|\,t_{00}\,|^2 , \\ &\sigma_T \sim \rho_0\,|\,t_{00}\,|^2+\sum_n \rho_n\,|\,t_{0n}\,|^2 ,\end{aligned} \tag{2}$$

then the unitarity equation can be written as

$$\begin{aligned}\Delta t_{00} &= -\rho_0\,|\,t_{00}\,|^2 \frac{\sigma_T}{\sigma_{\text{el}}} \\ &= -\frac{\rho_0}{\hat{\eta}(k)}\,|\,t_{00}(E)\,|^2 ,\end{aligned} \tag{3}$$

where $\rho_0(k)=k^2/(dW/dk)$ with W the total energy of the system, and

$$\hat{\eta}(k)=\frac{\sigma_{\text{el}}}{\sigma_T} . \tag{4}$$

Since $\sigma_T \geq \sigma_{\text{el}}$, we have that $0 \leq \hat{\eta} \leq 1$. In particular, below the two-pion threshold $\hat{\eta}=1$. If we now assume that t_{00} satisfies a two-body equation, then there are several ways we can include the effect of inelasticity:

(i) We can introduce a complex energy independent potential. In fact, Landau and Tabakin[11] constructed such a potential by assuming that it is a rank-one separable potential. The form factors were then determined using inverse scattering theory. These potentials are complex even below the inelastic threshold, and can lead to an unphysical off-shell t matrix. If used in the NN-πNN equations it leads to a violation of two- and three-body unitarity.[12]

(ii) To build the inelastic threshold you can introduce an energy dependent potential. This has been achieved, using separable potentials, by coupling to the inelastic channels.[13] The resultant t matrix has physical off-shell behavior.[14] It is also derivable from a coupled channel problem.[15]

(iii) In the present analysis we propose to build the inelastic threshold into the Green's function. Like the energy dependent separable potentials, this would correspond to the situation where one has a coupled channels problem that is reduced to an effective one-channel problem. The result we get is the same as that of Ernst and Johnson,[9] who use N/D to get their amplitude with inelasticity included. This result is ideally suited for use in three-body—type calculations.

To see how this threshold can be introduced into the Green's function, we consider the unitarity of the t matrix that satisfies the two-body integral equation (e.g., the Lippmann-Schwinger equation) with a real potential. In operator form this unitarity is given by

$$\Delta t = t(E^+)\Delta G(E) t(E^-) . \tag{5}$$

We now want to choose ΔG to include the effect of inelasticity. One such choice for the Green's function is

$$G(E^+)=\frac{\xi}{E+i\epsilon-H_0}=\int_0^\infty dk\,k^2 \frac{|\,k\rangle\xi(k)\langle k\,|}{E+i\epsilon-W(k)} , \tag{6}$$

where ξ is an analytic function to be determined by unitarity, and includes the effects of the two-pion threshold. Here, $W(k)$ is the π-N energy and includes the rest mass of both particles. From Eq. (6) we have that

$$\Delta G(E) = -\pi\xi\delta(E-H) \tag{7}$$

and the corresponding on-shell unitarity is given by,

$$\Delta t(E) = -\rho_0(k)\xi(k)\,|\,t(E)\,|^2 \tag{8}$$

with $E=W(k)$. Comparing Eqs. (8) and (3) we see that

$$\xi(k)=\frac{1}{\hat{\eta}(k)} \tag{9}$$

and the Green's function thus includes the effects of the two-pion threshold, and is given by

$$G(E^+)=\int_0^\infty dk\,k^2 \frac{|\,k\rangle\langle k\,|}{\hat{\eta}(k)[E^+-W(k)]} . \tag{10}$$

In this way we can fold the experimental inelasticity into our parametrization of the π-N amplitude, such that the resultant amplitude satisfies two-body unitarity at all energies of interest. This in turn allows us to improve the fit to the π-N phase shift, maintain two- and three-body unitarity in the NN-πNN system, and introduce no additional complexity to the problem.

In this analysis, we will assume the π-N interaction to be a rank-one separable potential for all partial waves other than the P_{11}. The corresponding π-N amplitudes are of the form

$$t_\alpha(k,k',E)=g_\alpha(k)\tau_\alpha(k)g_\alpha(k') , \tag{11}$$

where α refers to all π-N channels other than the P_{11}, and $g_\alpha(k)$ is the separable potential form factor. In this case,

$$\tau_\alpha(E)=\left\{\lambda_\alpha^{-1}-\int_0^\infty dk\,k^2 \frac{[g_\alpha(k)]^2}{\hat{\eta}_\alpha(k)[E-W(k)]}\right\}^{-1} \tag{12}$$

with λ_α the strength of the potential. This result is identical (except for P-wave pions) to that of Ernst and Johnson[9] who derive it by examining unitarity in an N/D method. Because our inelasticity was included in the Green's function, we can easily extend this result to rank-two potentials. In Eq. (12), $\hat{\eta}(k)$ can be taken directly from experiment, and because of the way it appears in the integral, it can have some influence on the off-shell behavior of the amplitude through $g_\alpha(k)$. This in turn could affect the range of the π-N interaction.

For the P-wave channels, Ernst and Johnson[9] used the Chew-Low model to construct their D function in the N/D method. In particular, they included the u-channel pole into the D function. This, in the static limit, leads to a pole in the off-shell amplitude in the energy variable. This in turn generates unphysical threshold[16] if used in a three-body—type calculation. To avoid this problem, we have used Eqs. (11) and (12) for the π-N amplitude in the P_{13}, P_{31}, and P_{33} channels.

For the P_{11} channel, the phase shifts change sign at $T_\pi \simeq 150$ MeV, and we need to resort to at least a rank-two separable potential. To include the coupling between the NN and πNN channel, the P_{11} amplitude needs to have a pole at the nucleon mass. This can be achieved[12,17–19] by taking an energy dependent rank-two potential of the form

$$v(k,k',E)=f_0(k)\frac{1}{E-m_0}f_0(k')+g(k)\lambda g(k') \ , \qquad (13)$$

where m_0 is a parameter of the potential, and we refer to it as the bare nucleon mass. The motivation for the energy dependence in the first term on the right-hand side (rhs) of Eq. (13) is the inclusion of the diagram in Fig. 2(a), which has a pole in the energy plane at the bare nucleon mass. In that case $f_0(k)$ is the bare πNN vertex. The second term on the rhs of Eq. (13) is a parametrization of all other lowest order contributions to the π-N amplitude in the P_{11} channel. In particular, it includes the crossed diagram in Fig. 2(b).

The amplitude corresponding to the potential in Eq. (13) can be obtained[12] by solving the two-body equation with the Green's function in Eq. (10). Alternatively, we can write, using two-potential theory, the amplitude for the potential in Eq. (13) as

$$t(k,k',E)=f(k,E)d(E)f(k',E)+g(k)\tau(E)g(k') \ , \qquad (14)$$

where $\tau(E)$ is given by Eq. (12), and the second term on the rhs of Eq. (14) is the t matrix for the potential $g(k)\lambda g(k')$. Here, $f(k,E)$ is the dressed πNN form factor, and is given by

$$f(k,E)=f_0(k)+g(k)\tau(E)\langle g \mid G(E) \mid f_0 \rangle \qquad (15)$$

with

$$\langle g \mid G(E) \mid f_0 \rangle = \int_0^\infty dk\, k^2 \frac{g(k)f_0(k)}{\hat{\eta}(k)[E-W(k)]} \ . \qquad (16)$$

In Eq. (14), $d(E)$ is the dressed nucleon propagator, and is given by

$$d(E)=[E-m_0-\Gamma(E)]^{-1} \ , \qquad (17)$$

with

$$\Gamma(E)=\langle f_0 \mid G(E) \mid f(E) \rangle = \int_0^\infty dk\, k^2 \frac{f_0(k)f(k,E)}{\hat{\eta}(k)[E-W(k)]} \ . \qquad (18)$$

The form of the P_{11} amplitude given in Eq. (14) was originally suggested by Mizutani and Koltun[20] for use in the NN-πNN system.

To adjust the parameters of the potential to give the πNN coupling constant, we need to rewrite the first term on the rhs of Eq. (14) as

$$f(k,E)d(E)f(k',E)=f^R(k,E)d^R(E)f^R(k',E) \ , \qquad (19)$$

where $d^R(E)$ is the renormalized dressed propagator which has a simple pole at the nucleon mass, and a residue of one. To get an explicit expression for $d^R(E)$, we need to expand $\Gamma(E)$ about the nucleon mass, i.e.,

$$\Gamma(E)=\Gamma(m_N)+(E-m_N)\Gamma_1(m_N)+(E-m_N)^2\Gamma_2(E) \ , \qquad (20)$$

where

$$\Gamma_1(m_N)=\langle f(m_N) \mid [G(m_N)]^2 \mid f(m_N) \rangle \qquad (21)$$

and

$$\Gamma_2(E)=\langle f(m_N) \mid [G(m_N)]^2 G(E) \mid f(E) \rangle + \langle f(m_N) \mid [G(m_N)]^2 \mid g \rangle \times \tau(m_N) \langle g \mid G(m_N)G(E) \mid fE) \rangle \ . \qquad (22)$$

We now can write the renormalized propagator and πNN form factor as,

$$d^R(E)\{(E-m_N)[1-(E-m_N)\Gamma_2^R(E)]\}^{-1} \qquad (23)$$

and

$$f^R(k,E)=Z_2^{1/2}f(k,E) \ , \qquad (24)$$

where the wave function renormalization constant Z_2 is given by

$$Z_2=[1+\Gamma_1(m_N)]^{-1} = 1-\Gamma_1^R(m_N) \qquad (25)$$

with

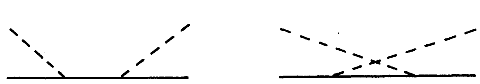
$$\Gamma_i^R=Z_2\Gamma_i \quad (i=1,2) \ . \qquad (26)$$

In this way we can write our P_{11} amplitude in terms of a renormalized πNN form factor and nucleon propagator.

To determine the πNN coupling constant, we follow the procedure of Mizutani *et al.*[17] and compare $f^R(k,E)$ with the πNN vertex using the pseudoscalar coupling, i.e.,

$$-ig_0(k)\bar{u}(\tau\cdot\phi)(i\gamma_5)u \ , \qquad (27)$$

where τ is the Pauli matrix, ϕ is the pion field, and u is the usual Dirac spinor. This comparison at $E=m_N$ and



(a) (b)

FIG. 2. Contribution to the pole (s-channel pole) (a) and nonpole (u-channel pole) (b) to the π-N potential.

$k=k_0$ where

$$k_0^2=m_\pi^2\left[1-\frac{m_\pi^2}{4m_N^2}\right], \tag{28}$$

gives the πNN coupling constant,

$$\begin{aligned} f_{\pi NN}^2(k) &= \frac{g_0^2(k)}{4\pi}\left[\frac{m_\pi}{2m_N}\right]^2 \\ &= \frac{\pi}{3}\frac{m_\pi^2}{4m_N^2}\omega(k)\epsilon(k)[\epsilon(k)+m_N] \\ &\quad\times\left[\frac{m_N+E(k)}{E(k)}\right]\left[\frac{f^R(k,m_N)}{k}\right]^2, \end{aligned} \tag{29}$$

with

$$\begin{aligned} E(k) &= \omega(k)+\epsilon(k)\ , \\ \omega(k) &= (k^2+m_\pi^2)^{1/2}\ , \\ \epsilon(k) &= (k^2+m_N^2)^{1/2}\ . \end{aligned} \tag{30}$$

The requirement that $f_{\pi NN}^2(k_0)=0.079$ will allow us to fix the strength of the form factor $f_0(k)$, while the parameter m_0 is determined by the requirement that $d(E)$ has a pole at $E=m_N$. This gives

$$m_0=m_N-\Gamma(m_N)\ . \tag{31}$$

In this way, the division of the P_{11} amplitude into a pole and nonpole part had the added advantage of determining two of the parameters of the potential in terms of the position of the nucleon pole and the πNN coupling constant. To adjust the rest of the parameters of the potential to fit the phase shifts, we find it simpler to treat the potential in Eq. (13) as a rank-two separable potential.

The above inclusion of inelasticity in the P_{11} channel is similar to that of Ernst and Johnson[9] only after they set $\overline{\eta}_{11}=\overline{\eta}_{12}=\overline{\eta}_{22}=\hat{\eta}$. In their formalism, they have the added freedom of including distinct couplings between the different terms in the rank two potential and the inelastic channels. The effect of not setting all the η's equal has been examined recently.[21] It was found that the results had a limited dependence on the precise value of the η's used. Due to lack of experimental information about the relative magnitude of the η's, we have used a single η in the Green's function, as was the case with the other partial waves.

In addition, Ernst and Johnson[9] have utilized the Chew-Low model to fix the first term in their rank-two potential in the P_{11} channel. In particular, this first term includes both the s-channel and u-channel pole.[22] On the other hand, we have included only the s-channel pole in the first term, while the crossed diagram in Fig. 2(b) is part of the attraction that is parametrized by the separable potential $g(k)\lambda g(k')$ in Eq. (13). In this way, the u-channel pole which leads to a cut after partial wave projection does not appear in either $d(E)$ or $\tau(E)$. This avoids the problem of getting unphysical threshold in a three-body calculation.[16] Thus, the residue of the off-shell P_{11} amplitude at the nucleon pole is different for the two models because of the way the u-channel pole is included.[22]

Finally, comparing our parametrization with that of Coronis and Landau,[23] we note that they get their separable potential form factor numerically by employing inverse scattering theory. The use of such potentials in the three-body calculation is limited because one can no more employ the rotation of contour method to solve the integral equations. With our parametrization, where $f_0(k)$ and $g_\alpha(k)$ are of the Yamaguchi-type, no such limitation is encountered.

III. NUMERICAL RESULTS

Before we can proceed to the detail parametrization of the potentials, we should relate our inelasticity $\hat{\eta}_\alpha$ to the phase shifts δ_α and inelasticity η_α obtained in a phase shift analysis of the π-N data. From Eq. (3), we have for a given partial wave that

$$t_\alpha(k)=-\frac{\hat{\eta}_\alpha(k)}{\rho(k)}e^{i\hat{\delta}_\alpha(k)}\sin\hat{\delta}_\alpha(k)\ , \tag{32a}$$

$$=-\frac{1}{\rho(k)}\frac{(\eta_\alpha e^{2i\delta_\alpha}-1)}{2i}\ . \tag{32b}$$

In this way, we can get $\hat{\eta}_\alpha$ and $\hat{\delta}_\alpha$ from either the Karlsruhe amplitude[10] analysis, or from the more recent phase shift analysis.[24–26] We have found that near the threshold for pion production the $\hat{\eta}_\alpha$ obtained from the Karlsruhe analysis[10] have large fluctuations. This is particularly the case when the phase shifts in that channel are quite small, as is the case in higher partial waves. To avoid this problem, we have fit the inelasticity $\hat{\eta}_\alpha(k)$ using a smooth function to the phase shift analysis of Koch and Pietarinen[24] near the pion threshold, and the Karlsruhe analysis at higher energies. The functional form chosen for $\hat{\eta}_\alpha(k)$ is[9]

$$\hat{\eta}_\alpha(k)=0.5+\sum_{i=1}^{N}\frac{B_\alpha(i-1)-B_\alpha(i)}{1+\exp\{[k-k_\alpha(i)]/C_\alpha(i)\}}\ , \tag{33}$$

where $B_\alpha(0)=1.0$. In addition, we require that

$$\lim_{k\to 0}\hat{\eta}_\alpha(k)=1 \text{ and } \lim_{k\to\infty}\hat{\eta}_\alpha(k)=0.5\ .$$

This functional form allows us to fit all the structures in $\hat{\eta}_\alpha(k)$ for pion energies less than 1.5 GeV. For S- and D-wave pions, we take $N=4$ and the corresponding parameters are given in Table I, while for P-wave pions, we take $N=3$ and the parameters are given in Table II. The fit to the experimental values are given in Figs. 3–11. In general, the fit is very good, and in all cases, $\hat{\eta}_\alpha\to 1$ below the pion production threshold. In some channels, we have not fit all the detailed structure in the data, but then it is not clear if this detail is only due to errors in the data. With this parametrization of $\hat{\eta}_\alpha(k)$, we can now proceed to adjust the parameters of the potential to fit the phase shifts. We note here, that adjusting the parameters of the potential is now the same as was the case in the absence of inelasticity. Furthermore, since $\hat{\eta}_\alpha(k)=1$ for all practical purposes below the threshold for pion production, the two-pion threshold is built into the amplitude and one can fit the rapid changes in the phase shifts above the pion production threshold, without introducing any rapid vari-

TABLE I. Parameters for the inelasticity $\hat{\eta}$ for S and D waves. The parameters are defined in Eq. (33).

	S_{11}	S_{13}	D_{13}	D_{15}	D_{33}	D_{35}
$B(1)$	0.728	−1.09	0.532	0.143	0.026	0.0848
$B(2)$	0.165	0.93	0.327	0.358	0.590	0.207
$B(3)$	1.030	1.12	0.000	0.000	0.269	0.195
$B(4)$	0.50	0.50	0.50	0.500	0.500	0.50
$C(1)$	0.010	0.078	0.010	0.0964	0.0136	0.0139
$C(2)$	0.0713	0.108	0.0477	0.033	0.250	0.119
$C(3)$	0.117	0.063	0.0248	0.186	0.065	0.0318
$C(4)$	0.276	0.614	0.526	0.496	0.512	0.299
$k(1)$	1.91	2.44	1.73	2.07	1.77	2.54
$k(2)$	2.33	2.51	2.55	2.43	3.03	3.45
$k(3)$	2.57	3.24	2.92	3.58	3.61	4.13
$k(4)$	3.02	4.18	3.90	4.70	5.18	5.21

TABLE II. Parameters of the inelasticity $\hat{\eta}$ for the P waves. The parameters are defined in Eq. (33).

	P_{11}	P_{13}	P_{31}	P_{33}
$B(1)$	0.605	0.010	0.458	0.291
$B(2)$	0.418	0.155	1.188	−0.0395
$B(3)$	0.500	0.500	0.500	0.500
$C(1)$	0.0978	0.0229	0.181	0.0461
$C(2)$	0.214	0.0916	0.268	0.170
$C(3)$	0.0136	0.426	0.704	0.761
$k(1)$	1.66	2.63	2.87	2.28
$k(2)$	2.87	2.97	3.79	2.64
$k(3)$	3.175	3.59	4.36	3.90

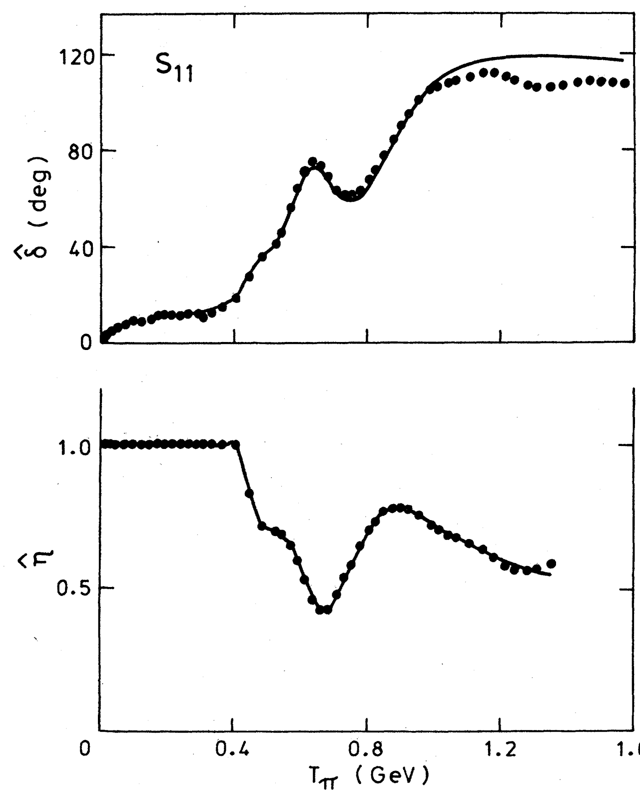


FIG. 3. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the S_{11} channel. The experimental data (●) are from Refs. 10 and 24.

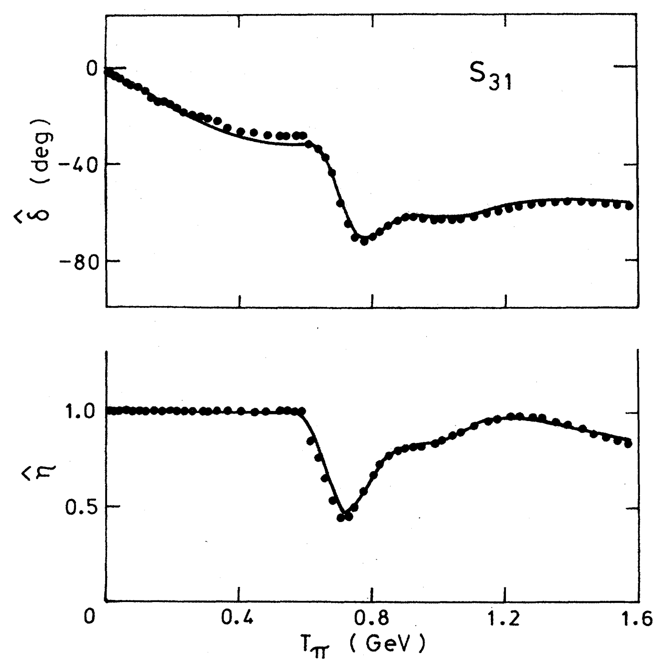


FIG. 4. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the S_{31} channel. The experimental data (●) are from Refs. 10 and 24.

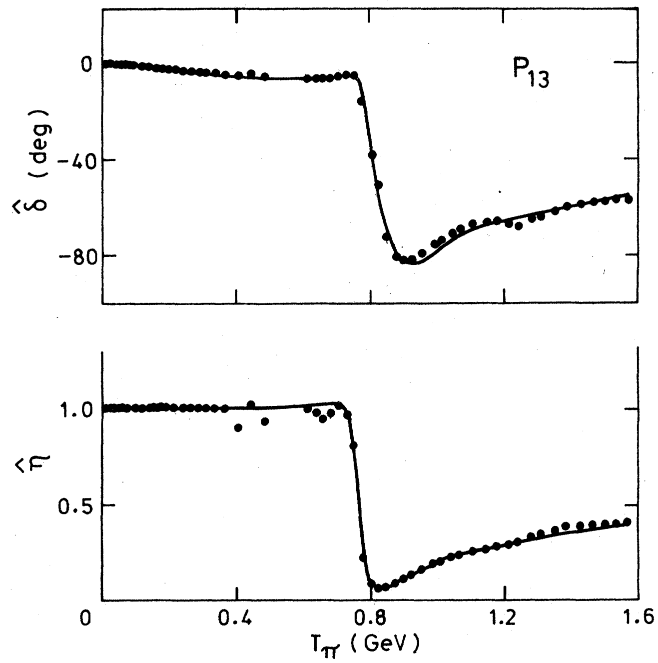


FIG. 5. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the P_{13} channel. The experimental data (●) are from Refs. 10 and 24.

ation in the form factor, or coupling to inelastic channels.

For all partial waves other than the P_{11} channel, we have chosen a rank-one separable potential with a form factor that is the sum of two Yamaguchi types, i.e.,

$$g_\alpha(k)=\frac{k^l}{[\omega(k)]^{1/2}}\left[\frac{C_1}{(k^2+\beta_1^2)^{n_1}}+\frac{C_2 k^{n_2}}{(k^2+\beta_2^2)^{n_3}}\right]. \tag{34}$$

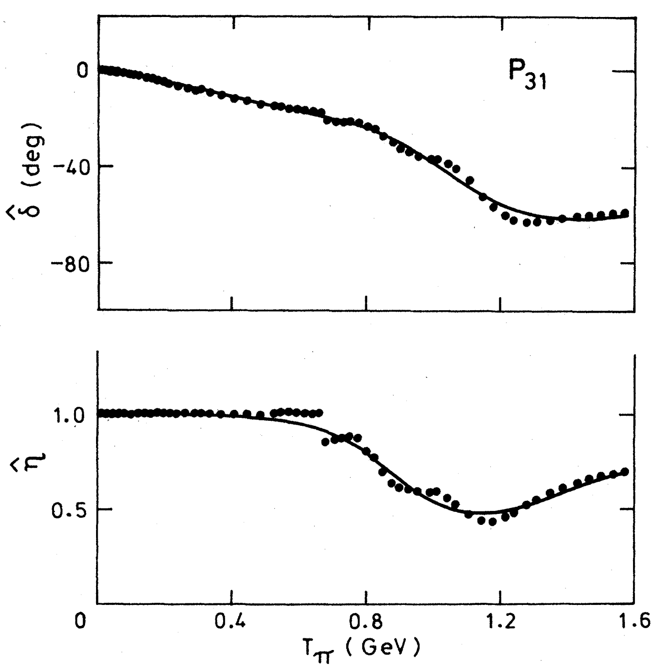


FIG. 6. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the P_{31} channel. The experimental data (●) are from Refs. 10 and 24.

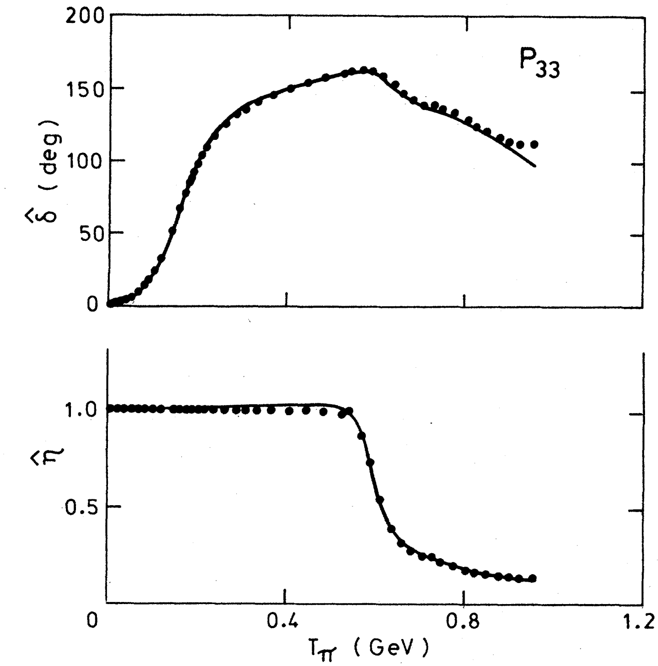


FIG. 7. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the P_{33} channel. The experimental data (●) are from Refs. 10 and 24.

On the other hand for the P_{11} channel, we take

$$f_0(k)=\frac{C_0}{[\omega(k)]^{1/2}}\frac{k}{(k^2+\alpha^2)^{n_0}}\,, \tag{35}$$

while $g_\alpha(k)$ is given by Eq. (34). The advantage of this form factor over a Gaussian[9] or Bessel function[27] is that

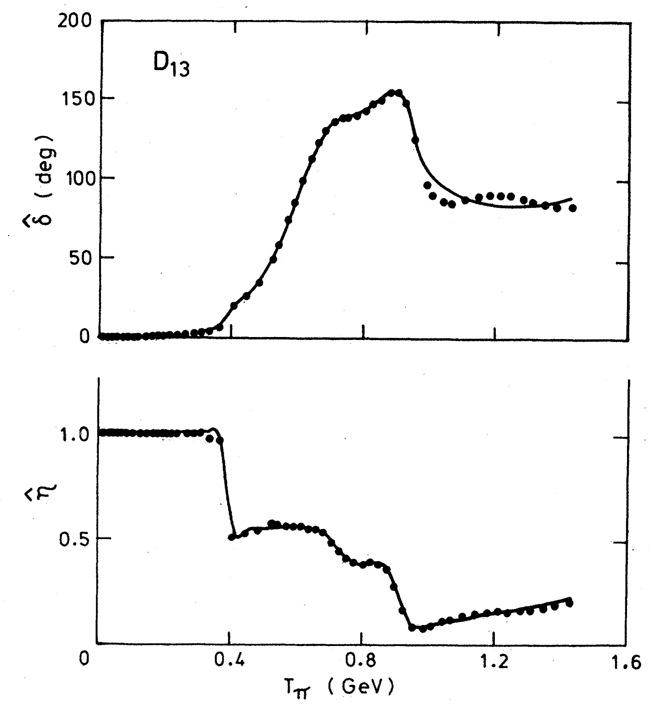


FIG. 8. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the D_{13} channel. The experimental data (●) are from Refs. 10 and 24.

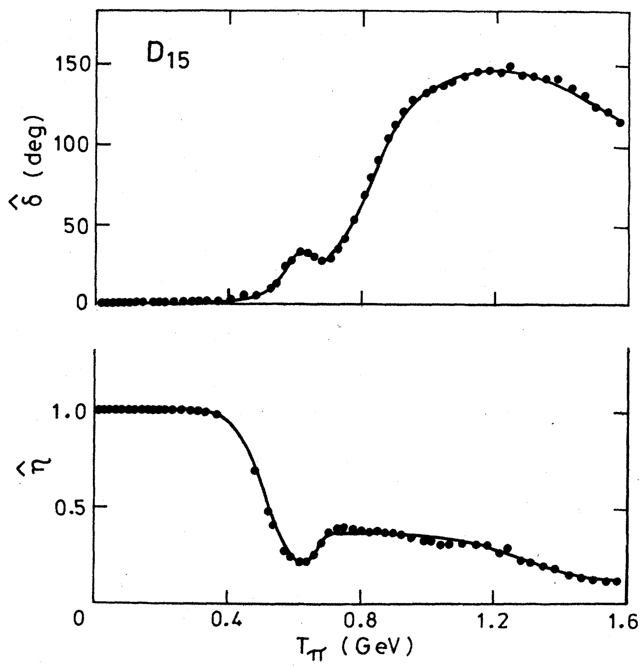


FIG. 9. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the D_{15} channel. The experimental data (●) are from Refs. 10 and 24.

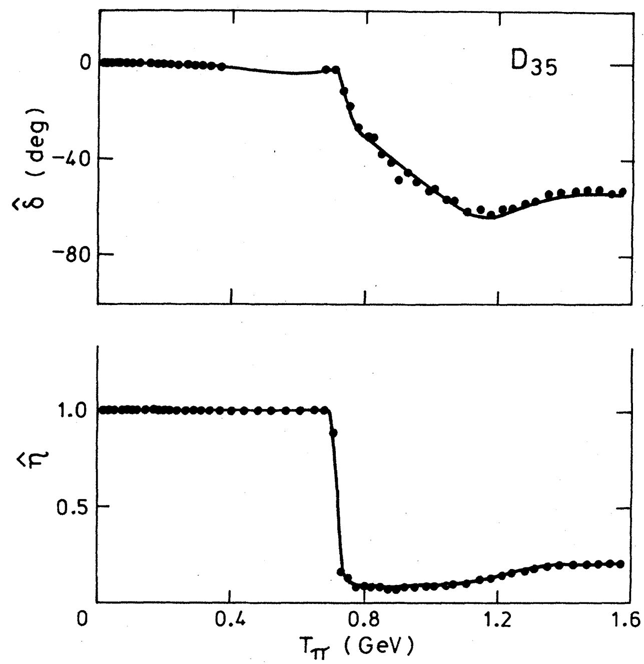


FIG. 11. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the D_{35} channel. The experimental data (●) are from Refs. 10 and 24.

in three-body calculations, one can use the rotation of contour method to solve the integral equations. On the other hand, the Gaussian or Bessel functions vary slowly at low momentum, and in particular between the nucleon pole and threshold. For the P_{11} channel this might have some significance, since the NN-πNN results seem to be sensitive to this behavior.[28]

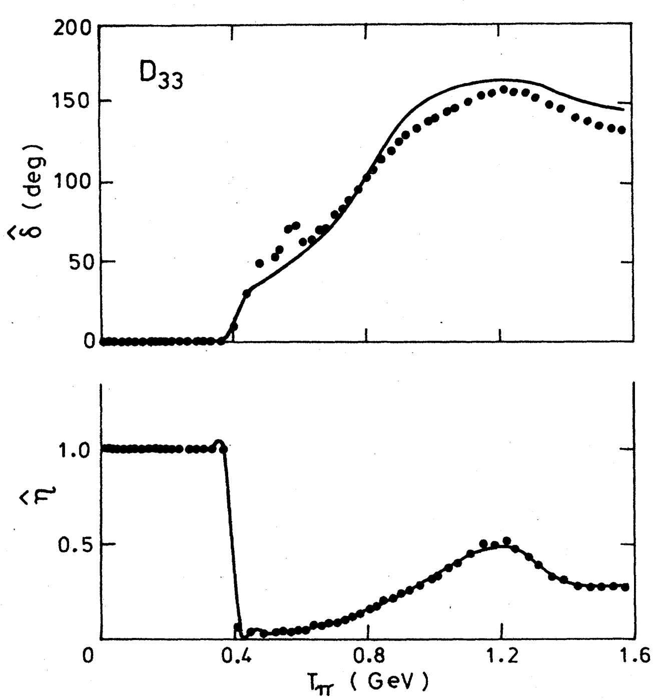


FIG. 10. The phase shifts $\hat{\delta}$ and inelasticity $\hat{\eta}$ for the D_{33} channel. The experimental data (●) are from Refs. 10 and 24.

In Table III, we present the parameters of the potentials in all channels other than the P_{11}. The corresponding phase shifts $\hat{\delta}$ and inelasticities $\hat{\eta}$ are given in Figs. 3—11. Here, we plot both the $\hat{\delta}$ and $\hat{\eta}$ to illustrate the correlation between the opening of the inelastic channel (the sudden decrease in $\hat{\eta}$), and the rapid increase in the phase shifts. We have chosen to include the effect of this rapid variation in $\hat{\eta}(k)$. In this way, our form factors are relatively smooth, which in turn leads to a smooth off-shell behavior. Thus, below the threshold for pion production, our off-shell amplitude is well behaved, even though we do not have a coupled channel problem to solve. In some channels (e.g., the P_{31}), the data, and particularly $\hat{\eta}$, have some oscillation, which we have chosen not to fit. This

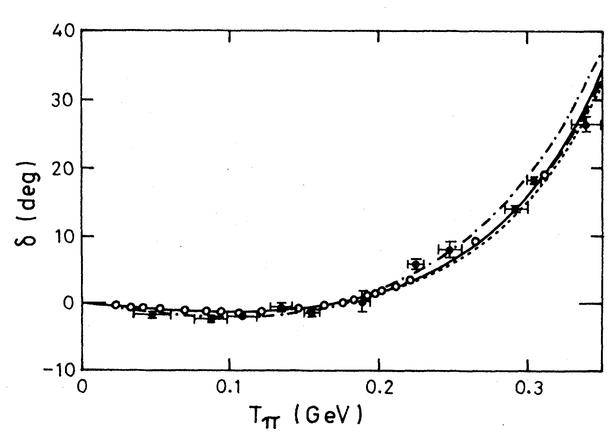


FIG. 12. The phase shifts δ in the P_{11} channel for the potentials PJ (——), $M1$ (— — —), $E2$ (· · ·), and $E1$ (—·—·—·). The experimental data is that of Koch *et al.* (Ref. 24) and Zidell *et al.* (Ref. 25).

TABLE III. Parameters of π-N potentials in all channels other than p_{11}.

	n_1	n_2	n_3	λ	C_1	C_2	β_1 (fm^{-1})	β_2 (fm^{-1})
S_{11}	1	2	2	-1	0.3706	83.45	1.716	9.345
S_{31}	1	2	2	$+1$	61.8	8.4	11.5	1.99
P_{13}	2	2	2	$+1$	0.581	1.66	1.23	1.925
P_{31}	2	2	2	$+1$	70.1	91.4	1.65	2.24
P_{33}	2	2	2	-1	1.08	1.375	1.54	2.20
D_{13}	2	2	4	-1	1.448 97	217 766.0	2.797	14.9335
D_{15}	2	2	4	-1	0.397 61	22 693.0	1.6625	8.561
D_{33}	2	2	4	-1	0.043 868	26 601.0	1.0707	8.6601
D_{35}	2	2	4	$+1$	0.490 07	667.01	1.4044	4.0123

TABLE IV. Parameters of the P_{11} π-N interaction used in the present investigation. All these potentials give a πNN coupling constant $f^{(0)}_{\pi NN(k_0)}=0.079$. Potentials $E1$ and $E2$ have the inelasticity included (i.e., $\hat{\eta}\neq 1$). In this channel $n_1=1$.

Potential	n_0	n_2	n_3	α (fm^{-1})	β_1 (fm^{-1})	β_2 (fm^{-1})	C_1 (fm^{-1})	C_2	C_3	m_0 (fm^{-1})
$E1$	2	1	4	5.7661	2.3220	5.9671	119.5173	1.1348	9237.17	5.8649
$E2$	2	1	4	3.4983	1.2278	4.5545	30.5768	0.3233	21326.9	5.2062
PJ	2	1	3	3.8206	1.2689	5.181	43.5646	0.2907	1420.59	5.1574
$M1$	1	2	3	2.7703	1.4422	2.1982	1.0726	0.3433	7.4026	5.4314

was motivated by the fact that $\hat{\eta}$ is always integrated over in calculating the amplitude [see Eq. (10)], and thus should not affect our final off-shell amplitude. This is particularly the case since the variations are not as distinct in the phase shifts $\hat{\delta}$ as they were in the inelasticity $\hat{\eta}$.

In general, our fit to the Karlsruhe data[10] is very good. The exception is the D_{33} where we could not fit the rapid variation of the phase shifts just above the threshold. We observe that Ernst and Johnson[9] seem to have had this problem also.

Part of the reason for including the inelasticity in the parametrization of the amplitude is that it could have some influence on the off-shell behavior of the amplitude, even below the pion production threshold. Since the P_{11} channel has the lowest energy resonance above the pion production threshold, we have chosen to examine the role of the inelasticity on the off-shell behavior of the amplitude in this channel. For this purpose we have chosen two potentials with inelasticity included in their parametrization, $E1$ and $E2$, and two potentials in which the production threshold was not included, i.e., $\hat{\eta}=1$. These are referred to as PJ and $M1$. In Table IV we present the parameters for these potentials, while the form of the potential is given by Eqs. (34) and (35). In Fig. 12, we compare the low energy phase shifts for these four potentials, while in Table V, we give the scattering volume a_{11}, the wave function renormalization constant Z_2, and $f^2_{\pi NN}(0)$. The phase shifts for the four potentials are similar for $T_\pi < 350$ MeV. In particular, the potentials $E2$ and PJ have similar phase shifts, scattering volume a_{11} and Z_2. To compare their off-shell behavior, we have chosen to compare the πNN form factor $f(k,m_N)$ [see Eq. (15)] since that exhibits the behavior of both $f_0(k)$ and $g(k)$. In Fig. 13, we compare this form factor for the four potentials. Here, we observe that the potentials PJ and $E2$

TABLE V. The scattering volume a_{11}, wave function renormalization Z_2, and the πNN coupling constant at $k=0$ for the different P_{11} potentials. All potentials have a πNN coupling constant of 0.079 [see Eq. (29)].

Potential	a_{11} (m_π^{-3})	Z_2	$f^2_{\pi NN}(0)$
$E1$	-0.1037	0.5355	0.0655
$E2$	-0.0710	0.7939	0.0510
PJ	-0.0706	0.8059	0.0620
$M1$	-0.0721	0.7273	0.0488

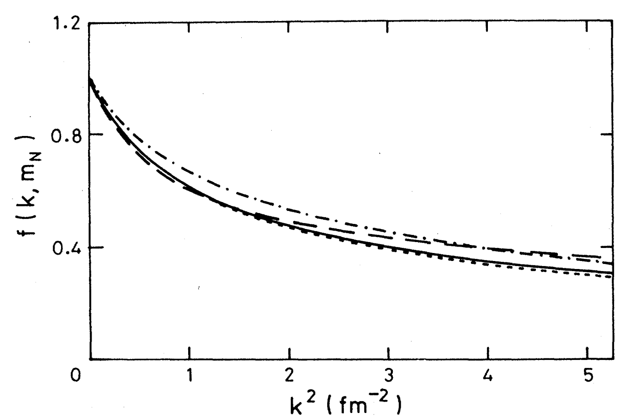


FIG. 13. The dressed πNN form factor $f(k,m_N)$ normalized to one at $k=0$, for the potentials in Fig. 12. The curves are labeled as in Fig. 12.

give almost identical form factors. This is a clear indication that the inclusion of inelasticity does not necessarily influence the off-shell behavior of the amplitude. In fact, the biggest difference is between the two potentials $E1$ and $E2$. These also give a different scattering volume and phase shift, but within the experimental uncertainty.

IV. CONCLUSION

In the above, we have presented separable potentials that fit the π-N phase shifts and include inelasticity in the S-, P-, and D-waves. In all channels other than the P_{11} we have used a rank-one separable potential without introducing any rapid variation in the off-shell behavior of the amplitude. This was achieved by including the inelasticity in the Green's function. In this way the threshold for pion production was included, maintaining consistency with unitarity. The mean feature of these potentials is the choice of form factor $g_\alpha(k)$ which make them suitable for use in three-body calculations using rotation of contour method. In the P_{11} channel the amplitude is written as a part that has the nucleon pole plus an attractive nonpole part. This form is designed to couple the pion elastic channel to the absorption channel.

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