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Comments

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Absorption cross sections and the use of complex potentials in coupled-channels models

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We give a simple and transparent derivation of a result obtained recently by Kim, Udagawa, and Tamura for the loss of flux occurring in the coupled-channels scattering problem when a complex potential is used. The loss of flux, or absorption cross section, consists of three components: absorption from the elastic channel, absorption from the nonelastic channels, and absorption occurring during the elastic-nonelastic transitions. Some discussion is given of these results and their application to fusion reactions.

It is well known[1] that the absorption (or reaction) cross section σ_A implied by the use of a complex optical potential $U=V+iW$ is given by

$$\sigma_A(E_\alpha) = -(2/\hbar v_\alpha)\langle \chi_\alpha^{(+)}|W|\chi_\alpha^{(+)}\rangle \quad . \tag{1}$$

[Note that the expectation value of W has to be negative to satisfy unitarity, so that $\sigma_A \geq 0$. However, in general, it is not necessary even for a local $W(r)$ to be everywhere negative. Counterexamples are provided by potentials U that are the local equivalents to a nonlocal interaction. These may have imaginary parts that are positive in some regions of r although, by construction, they will satisfy $\sigma_A \geq 0$. Examples have been shown in Ref. 2.] Here $\chi_\alpha^{(+)}$ is the scattering wave function at energy E_α, a solution of

$$[E_\alpha + (\hbar^2/2\mu_\alpha)\nabla_\alpha^2 - U_\alpha]\chi_\alpha^{(+)}(\mathbf{r}_\alpha) = 0 \quad , \tag{2}$$

with outgoing scattered waves. Also, v_α is the relative velocity of the colliding systems, μ_α is their reduced mass, and $\mathbf{r}_\alpha$ is the separation of their centers of mass.

Recently, Udagawa, Kim, and Tamura (UKT)[3] used formal manipulations to extend the relation (1) to the use of a complex interaction in a coupled-channels model. We comment that this result can be derived in a simple and more transparent way by explicitly considering the loss of flux from the set of coupled channels. We then append some remarks on the use of these relations.

Consider a set of nonelastic channels, plus the elastic channel (the set $\{\beta\}$, say), which are to be treated explicitly in a coupled-channels (CC) calculation. The various interactions are represented by a complex optical potential matrix $U=V+iW$, whose imaginary part accounts for any loss of flux into those open nonelastic channels (the set $\{\gamma\}$, say) which are not considered explicitly. (The existence of this imaginary part W also implies that the real part V, which is related to it by a dispersion relation,[4] depends upon the energy E_α. This dependence may be particularly marked for energies close to the top of the Coulomb barrier;[5] hence of importance for the interpretation of fusion cross sections at these energies.)

Let the relative motion in each channel β be described by a wave function $\chi_\beta^{(+)}(\mathbf{r}_\beta)$. These are outgoing-wave solutions of the coupled equations

$$[E_\beta + (\hbar^2/2\mu_\beta)\nabla_\beta^2 - U_{\beta\beta}]\chi_\beta^{(+)}(\mathbf{r}_\beta) = \sum_{\beta'\neq\beta} U_{\beta\beta'}\chi_{\beta'}^{(+)}(\mathbf{r}_\beta) \quad . \tag{3}$$

Using Eq. (3), the divergence of the flux $\mathbf{j}_\beta(\mathbf{r}_\beta)$ in channel β is now found in the usual way[1] to be

$$\text{div}\mathbf{j}_\beta(\mathbf{r}_\beta) = (2/\hbar)\sum_{\beta'} \text{Im}[\chi_\beta^{(+)*}(\mathbf{r}_\beta)U_{\beta\beta'}\chi_{\beta'}^{(+)}(\mathbf{r}_\beta)] \quad . \tag{4}$$

The contributions to $\text{div}\mathbf{j}_\beta$ from $\text{Re}U_{\beta\beta'}$ represent the interchange of flux between the β and β' channels, while those from $\text{Im}U_{\beta\beta'}$ correspond to current flowing out of the $\{\beta\}$ set. From Eq. (4) we immediately get the absorption cross section for the total loss from the $\{\beta\}$ set of coupled channels,

$$\sigma_A(\text{CC}) = -(2/\hbar v_\alpha)\sum_{\beta\beta'} \text{Im}\langle \chi_\beta^{(+)}|U_{\beta\beta'}|\chi_{\beta'}^{(+)}\rangle \quad . \tag{5}$$

The sum over channels is symmetric in β and β', while the interaction matrix is also symmetric,[6]

$$U_{\beta\beta'}(\mathbf{r}_\beta,\mathbf{r}_{\beta'}) = U_{\beta'\beta}(\mathbf{r}_{\beta'},\mathbf{r}_\beta) \quad . \tag{6}$$

Here we have allowed the possibility that the potential U is nonlocal. One example where the effective coupling potential $U_{\beta\beta'}$ is nonlocal occurs when the β and β' channels differ by a rearrangement; e.g., $\beta \rightarrow \beta'$ represents a transfer reaction. In general, the coupling term $U_{\beta\beta'}$ then contains both interaction and nonorthogonality terms; it corresponds to the coupling kernel called $K_{\beta\beta'}(\mathbf{r}_\beta,\mathbf{r}_{\beta'})$ in Ref. 7. This kernel also obeys[6] the symmetry relation (6).

With these symmetries, the expression (5) can be reduced to the matrix generalization of Eq. (1),

$$\sigma_A(\text{CC}) = -(2/\hbar v_\alpha)\sum_{\beta\beta'} \langle \chi_\beta^{(+)}|\text{Im}U_{\beta\beta'}|\chi_{\beta'}^{(+)}\rangle \quad , \tag{7}$$

so that the absorption cross section depends only upon

$\mathrm{Im}U$, as expected. This corresponds to Eq. (17) of UKT, although their equation was obtained after assuming $\mathrm{Im}U$ to be diagonal. This is an unnecessary restriction; complex off-diagonal couplings will also contribute to $\sigma_A(\mathrm{CC})$. Indeed, the sum in Eq. (7) may be broken into three pieces,

$$\sigma_A(\mathrm{CC}) = \sigma_{\mathrm{A,el}} + \sigma_{\mathrm{A,nel}} + \sigma_{\mathrm{A,tr}} \ , \tag{8}$$

in which $\sigma_{\mathrm{A,el}}$ represents absorption from the elastic channel ($\beta=\beta'=\alpha$), $\sigma_{\mathrm{A,nel}}$ is due to absorption while propagating in the nonelastic channels ($\beta=\beta'$, but $\beta,\beta'\neq\alpha$), and $\sigma_{\mathrm{A,tr}}$ describes absorption occurring during the transitions ($\beta\neq\beta'$). This last term vanishes if the off-diagonal coupling $U_{\beta\beta'}$ is real, while the first two terms are positive definite if, as is usually done, the diagonal potentials $\mathrm{Im}U_{\beta\beta}$ are chosen to be negative everywhere. In general, however, the only requirement of unitarity is that the sum $\sigma_{\mathrm{A}}(\mathrm{CC})\geq 0$.

UTK postulate that $\mathrm{Im}U = W$, say, can be separated into two terms, $W = W_{\mathrm{F}} + W_{\mathrm{DR}}$, the first of which describes fusion, or compound-nucleus formation, while the other accounts for other direct or semidirect reaction channels (in the $\{\gamma\}$ set) which are not included explicitly in the CC calculations. (From the remarks just made, it does not follow that $|W_{\mathrm{F}}|\leq|W|$ everywhere, but only that the expectation values of W_{F} and W_{DR} be negative or zero.) From Eq. (7), it is clear that $\sigma_{\mathrm{A}}(\mathrm{CC})$ may be separated into two parts in the same way, so that the fusion cross section $\sigma_{\mathrm{F}}(\mathrm{CC})$ is obtained from that equation by replacing $\mathrm{Im}U$ by W_{F}. [However, as stressed by UTK, the CC waves $\chi_{\beta}^{(+)}$ in Eq. (7) are still to be generated by the full potential $U=V+iW_{\mathrm{F}}+iW_{\mathrm{DR}}$.] Then this $\sigma_{\mathrm{F}}(\mathrm{CC})$ can be expressed as a sum of three components corresponding to Eq. (8) for $\sigma_{\mathrm{A}}(\mathrm{CC})$, the first term $\sigma_{\mathrm{F,el}}$ of which is what UTK call σ_{EF}, or elastic fusion, and the second term $\sigma_{\mathrm{F,nel}}$ they call σ_{DRF}, or direct-reaction fusion.

The third term $\sigma_{\mathrm{A,tr}}$ of $\sigma_{\mathrm{A}}(\mathrm{CC})$, or $\sigma_{\mathrm{F,tr}}$ of $\sigma_{\mathrm{F}}(\mathrm{CC})$, arises if the imaginary parts of the off-diagonal couplings are nonzero. It is customary to include these imaginary parts in the usual collective model description of inelastic scattering, and, in principle, they appear also for transfer reactions.[7] They arise from $\beta\leftrightarrow\beta'$ transitions that occur via intermediate channels in the excluded $\{\gamma\}$ set, consequently their importance depends upon the completeness of the $\{\beta\}$ set that is chosen. In practice, this choice usually represents a severe truncation of the full space, so that the off-diagonal W and the corresponding $\sigma_{\mathrm{A,tr}}$ need not be negligible. It is less clear whether the part W_{F} of W should have non-negligible off-diagonal parts, although the physical picture[7] underlying the use of a deformed optical potential for collective inelastic excitations implies that W_{F} should be treated in the same way. Further, we note that in the application of their ideas, UTK find that the W_{F} needed to describe measured heavy-ion fusion cross sections extend to large radii beyond the top of the Coulomb barrier. This suggests that W_{F} could also be important for the nonelastic transitions that are included in a CC treatment.

Finally, we note that an equivalent one-channel optical potential $U_\alpha = U_{\alpha\alpha} + \Delta U_\alpha$, which gives the same elastic scattering as the set (3) of coupled equations, may be obtained by projecting[4,7] the β set of CC onto the elastic channel, $\beta=\alpha$. By construction, U_α then reproduces exactly the elastic wave $\chi_\alpha^{(+)}$ that was obtained by solving the $\{\beta\}$ set of coupled equations. The complex term ΔU_α is the polarization potential which represents the effects of the couplings to the other members of the β set. If its nonlocality is ignored, we may identify U_α as the empirical optical potential which reproduces the observed elastic scattering of the system. The expectation value of $W_{\alpha\alpha}$ taken with respect to $\chi_\alpha^{(+)}$ gives exactly the term $\sigma_{\mathrm{A,el}}$ of Eq. (8). Similarly, the expectation value of $(W_{\mathrm{F}})_{\alpha\alpha}$ gives the corresponding $\sigma_{\mathrm{F,el}}$ term. Note that neither of these is the absorption cross section, $\sigma_{\mathrm{A},\alpha}$ say, associated with the optical potential U_α; that is given by the expectation value of $\mathrm{Im}U_\alpha = W_{\alpha\alpha} + \mathrm{Im}\Delta U$. It equals σ_{A} (CC) plus the sum of the nonelastic cross sections to the explicitly coupled β channels, and corresponds to the *total* loss of flux from the elastic channel.

Note added. A derivation of Eq. (4) and a discussion of the flux lost from a system of coupled channels have also been given in the doctoral dissertation of R. Wolf, University of Giessen, 1983 (unpublished).

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