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Effect of the nuclear surface on a propagating density pulse

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We show that the density variation in the region of the diffuse surface of the target nucleus has a substantial effect on the motion of a soliton formed in a nuclear reaction.

In a series of recent papers,[1–3] a novel feature of nuclear hydrodynamics at intermediate energies, namely, that of a solitary density wave formation, has been investigated. In the idealized treatment of Ref. 1, we showed that the dispersive and nonlinear nature of the nuclear fluid might lead to a density oscillation describable by the Korteweg–de Vries (KdV) equation, which admits stable stationary wave solutions usually referred to as solitons. For reasons discussed in Refs. 1 and 2, these solitons are expected to be formed at intermediate-energy nuclear collisions involving small nuclear projectiles on large nuclear targets.

The idealized treatment of Refs. 1 and 2 was corrected to a certain extent in Ref. 3. There we showed that even when the soliton propagates in an infinite medium, the three-dimensional motion in the fluid, which arises from the inherent nonlinearities of the hydrodynamic equations, results in a damping of the propagating soliton-like pulse. The purpose of this note is to draw attention to the point that, independent of the three-dimensional motion, the finite size of the target nucleus also plays a crucial role in the physics of the nuclear soliton. In particular, we shall discuss the effect of the density variations in the surface region of the target nucleus on the motion of the propagating soliton. For the understanding of this effect, it suffices to consider the one-dimensional case allowing for analytical considerations.

As in Ref. 4, we approximate the density distribution by a constant density for the central region followed by a linear falloff to zero near the nuclear surface. The region characterized by the varying density then has a length S of about $4.4a$, where a is the diffuseness parameter. For large nuclei the quantity a ranges between 0.6 and 0.7 fm. If the "equilibrium" density of the nucleus is denoted by ρ_0 (and the local density by ρ), we have

$$\rho_0(r)=\begin{cases}\text{const}, & 0<r<R-S/2\ ,\\ \text{const}-\alpha r, & R-S/2<r<R+S/2\ ,\\ 0, & R+S/2<r\ ,\end{cases}$$

where R is the nuclear radius.

The implied application of the hydrodynamical approximation to the nuclear surface region is in line with the general practice in nuclear physics; see, e.g., Ref. 5 and references therein. In terms of classical dynamics the same holds, since our considerations would then correspond to increasingly shallower water, i.e., a typical region of applicability of the Korteweg–de Vries equation.

We take the motion to be in the Z direction and assume ρ_0 to be independent of time t. Expanding the density ρ and the velocity v around their equilibrium values ρ_0 and 0, respectively,

$$\rho=\rho_0+\epsilon\rho_1+\epsilon^2\rho_2\ ,\quad v=\epsilon v_1+\epsilon^2 v_2\ ,$$

and repeating the calculation of Ref. 1 with consideration of the $r=Z$ dependence of ρ_0, we get the "*forced*" KdV equation[6]

$$\frac{\partial\rho_1}{\partial\zeta}+\frac{3}{2}\frac{\sqrt{c}_1}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi}+\frac{c_2}{2\sqrt{c}_1}\frac{\partial^3\rho_1}{\partial\xi^3}-\frac{\alpha'\sqrt{c}_1}{2\rho_0}\rho_1=0\ , \tag{1}$$

for $R-S/2<|Z|<R-S/2$, and the usual KdV equation

$$\frac{\partial\rho_1}{\partial\zeta}+\frac{3}{2}\frac{\sqrt{c_1}}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi}+\frac{c_2}{2\sqrt{c_1}}\frac{\partial^3\rho_1}{\partial\xi^3}=0 \tag{2}$$

for $0<|Z|<R-S/2$. Here $\xi=\epsilon^{1/2}(Z-\sqrt{c}_1 t)$, $\zeta=\epsilon^{3/2}t$.

Using the Bogoliubov-Mitropolsky procedure[7] for nonlinear equations, Ott and Sudan[8] investigated equations of type (1). Following their work, it can be shown that (1) has an approximate analytical solution of the form

$$\rho_1=N(\tau)\text{sech}^2\eta\ , \tag{3}$$

where

$$\eta=\left[\frac{1}{4}\frac{c_1}{c_2\rho_0}N(\tau)\right]^{1/2}\left[\xi-\frac{1}{2}\frac{\sqrt{c}_1}{\rho_0}\int_0^\tau N(\tau)\,d\tau\right]$$

and

$$N(\tau)=N(0)\exp\left[\frac{2}{3}\frac{\alpha'\sqrt{c}_1}{\rho_0}\tau\right],\quad N(0)>0\ ,$$

with $\alpha=\epsilon^{3/2}\alpha'$.

Equation (3) describes a solitary density pulse with height equal to $N(\tau)$ and width proportional to $[N(\tau)]^{-1/2}$. Since $N(\tau)$ is varying with time τ, the amplitude increases and the width decreases with progressing time. On the other hand, the solitary solution to Eq. (2) is characterized by a constant amplitude and a constant width. So, the physical picture emerging from these considerations is that the soliton (created at $\rho=\rho_0$) propagates unchanged through the region of constant density up to $|Z|<R-S/2$. Then it grows slowly (and simultaneously becomes narrower), the rate of the changes being determined by α'.

At this point, let us remark that the Bogoliubov-Mitropolsky procedure is a perturbative one, appropriate for very small perturbations. But we have also investigated Eq. (1) numerically and have obtained the same behavior as predicted by Eq. (3).

To draw quantitative conclusions, we transform back to the $z-t$ space, and get

$$N(t)=N(0)\exp\left[\frac{2}{3}\frac{\alpha\sqrt{c}_1}{\rho_0}t\right] . \qquad (4)$$

From the considerations of Ref. 4, we approximate α as

$$\alpha\sim\frac{0.8\rho_0^{\rm central}}{S} .$$

With[3] $\sqrt{c}_1=0.16$, $\rho_0=0.17\ {\rm fm}^{-3}$ (in units of the velocity of light), one obtains

$$N(R)=\begin{cases}1.41N(0), & a=0.6\ {\rm fm}\ ,\\ 1.42N(0), & a=0.7\ {\rm fm}\ .\end{cases}$$

The initial amplitude of the density soliton is increased by about 40% during its passage through the nuclear skin. The width suffers a corresponding shrinkage of about 30%. A schematic illustration of the situation is given at the top of Fig. 1.

In the extreme limit of a very strong nonlinearity the full equation could be approximated by

$$\frac{\partial\rho_1}{\partial\tau}+\frac{3}{2}\frac{\sqrt{c}_1}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi}=0 . \qquad (5)$$

Equation (5) yields for an initial KdV soliton at $t=0$ the time evolution sketched in the lower part of Fig. 1. Due to the nonlinearity, the crest of the wave is moving faster than its base leading to multivalued solutions of (5) and eventually to the destruction of the initially well localized distributions. In the case discussed above this "destructive" behavior is apparently counteracted by the change in the background density. Nevertheless, it is obvious that even in an idealized [one-dimensional (1D) nondissipative] treatment the evolution of the solution, as created at $t=0$, is strongly affected by the variations in the (equilibrium) density in the surface region of the target nucleus.

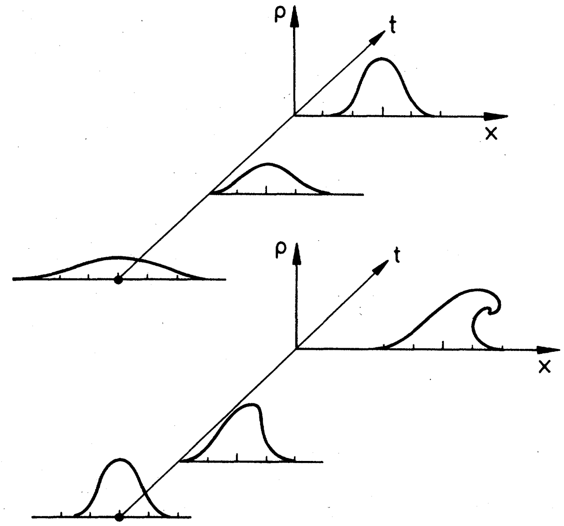

FIG. 1. The evolution of a initial soliton is sketched as a function of x and t (decreasing from front to back). In the upper part this is done for a changing background density $\rho(x)$ and in the lower half for the dispersionless KdV Eq. (2).

However, as shown in Ref. 3, 3D motion of the density disturbance implies that it is coupled to the other coordinates having a similar effect on the soliton as damping. Thus, in reality, its characteristics are already changed before it reaches the surface region. As a consequence, the findings of the present study and of Ref. 3 have to be combined to arrive at more realistic predictions for evolution and experimental signatures of such density disturbances which are prototypes of hot spots.

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