PHYSICAL REVIEW C VOLUME 32, NUMBER 6 DECEMBER 1985

Electric hexadecupole transition strength in ^{32}S and shell-model predictions for $E4$ systematics in the sd shell

B. H. Wildenthal
Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104

B. A. Brown
Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

I. Sick
Physics Department, University of Basel, Basel, Switzerland
(Received 3 June 1985)

The $E4$ form factor for inelastic electron scattering to the first 4^+ state of ^{32}S predicted from a systematic shell-model treatment of $0^+ \rightarrow 4^+$ transitions in the $A = 18$–38 region is found to be in excellent agreement with experiment. This shell-model prediction is quite different from recent deformed Hartree-Fock predictions for the same transition. Corresponding predictions for other doubly even nuclei in the $0d$, $1s$ shell are also presented and discussed.

The electric hexadecupole moments and transition strengths of nuclei provide a vital complement and supplement to electric quadrupole values in experimental and theoretical studies of shape-collective phenomena in nuclear structure. Hexadecupole features are independent of the quadrupole structure and hence a theory which yields agreement with experiment for both $E2$ and $E4$ data has been validated much more firmly than if $E2$ features only are confirmed. Moreover, the $E4$ predictions appear to depend more sensitively upon the details of the theoretical formulations than do the quadrupole values and the higher multipolarity of the $E4$ operator allows more discrimination in identifying critical individual components to total matrix elements.[1]

Simply put, many theoretical formulations for the structure of sd-shell nuclei yield similar results for electric quadrupole phenomena. Every doubly-even system in the $A = 20$–36 range is characterized experimentally by a dominant $E2$ transition to the lowest 2^+ state, the strength of which is not particularly sensitive to the individual nucleus. Any theory worthy of consideration must qualitatively reproduce this basic fact. The situation is quite different for the $E4$ case, however. The experimental picture is less complete, and those data which do exist show considerably more variation from nucleus to nucleus than is found for the $E2$ case. Theoretical predictions also exhibit more variety from one nucleus to the next, but not necessarily in accord with experiment.

A recent study by Jaqaman and Zamick[2] highlights some of these issues by calling attention to the drastic divergences as a function of mass number between the predictions for $E4$ features which are obtained from shell-model calculations on the one hand and deformed Hartree-Fock calculations on the other. While below $A = 28$ the different results are at least qualitatively similar, above $A = 28$ there are order-of-magnitude differences. For example, for ^{32}S the deformed Hartree-Fock calculation with the SII interaction[2] gave $B(E4, 0^+ \rightarrow 4^+) = 0.78 \times 10^3\ e^2\text{fm}^8$ compared with the shell-model result[1] of $50.09 \times 10^3\ e^2\text{fm}^8$. In this work we review the latest refinements of shell-model predictions for the doubly-even sd-shell nuclei and, in particular, compare the results for ^{32}S with recently available medium-energy electron scattering data. The ^{32}S system is one for which definitive experimental information had been lacking and for which wide differences between different theoretical predictions exist.

The formulation of $E4$ matrix elements in the shell model is a straightforward extension of the often treated $E2$ case. Aside from the obvious differences in the higher order of the spherical harmonic and the higher power of the radius in the operator, the essential difference between the two cases results from the fact that the higher multipolarity of the $E4$ excludes a significant fraction of the single-particle transition densities which combine to create the characteristic collectivity of the $E2$ matrix elements. In an sd-shell model space, for example, the only one-body terms which can contribute to the $E4$ calculations are $0d_{5/2} \rightarrow 0d_{5/2}$, $0d_{5/2} \rightarrow 0d_{3/2}$, and $0d_{3/2} \rightarrow 0d_{5/2}$.

Some collectivity can be generated for $E4$ matrix elements even within this small space, but the potentialities are much reduced from what is available with the $E2$ operator. The model space cannot encompass a direct $E6$ transition at all, because of the same sort of restrictions on participating orbits which the assumption of one-body processes (the impulse approximation) imposes. Hence $E4$ phenomena in the sd shell can be viewed as a limiting case in which collectivity is generated out of a small set of possible contributions.

We have discussed shell-model predictions for $E4$ transitions in the sd shell previously.[1,3] The first work[3] employed shell-model amplitudes derived from the "particle" and "hole" empirical Hamiltonians of Chung and Wildenthal.[4] Recent advances in the determination of empirical Hamiltonians for the sd-shell model space have yielded wave function amplitudes for all sd-shell systems which are generated from a single Hamiltonian formulation.[5] We have used these new wave functions to calculate new $\Delta J = 4$ and $\Delta J = 2$ one-body-transition-density matrix elements and used these as the foundation for a study of inelastic electron scattering to the lowest few 2^+ and 4^+ states in doubly even $A = 20$–36 nuclei.[1]

In the present study we focus on the net multiproton and multineutron matrix elements of the $E4$ operator obtained from these newest sd-shell wave functions and, in particular, on the electron scattering form factor for the first 4^+ state in ^{32}S. The definitions, notation, and discussion of auxiliary assumptions which underpin the present calculations of net matrix elements from the one-body-transition-density matrix elements are given in Ref. 3. Specifically, we use harmonic-oscillator single-particle radial wave functions. The length parameter for each nucleus is set so that the calculated rms charge radius calculated in the conventional oscillator-model prescription matches the corresponding experimental value.

The essential results of our calculations for $0^+ \rightarrow 4^+$ transitions, the "model-space" proton and neutron matrix elements A_p and A_n as defined in Ref. 3, are presented in Tables I and II for the $N=Z$ and $N=Z+2$ nuclei, respectively. In these tables we also note the calculated excitation energies of the states considered and the values of the "total" proton and neutron matrix elements M_p and M_n, which result from combining[3] core-polarization corrections ("effective charges" in the simplest limit) with the A_p and A_n. For nuclei with approximately equal numbers of neutrons and protons, such as we consider here, the M matrix elements are related to the A matrix elements by

$$M_p = A_p e_p + A_n e_n ,$$

$$M_n = A_n e_p + A_p e_n .$$

Based on our previous analyses[1] of $E4$ electron scattering, we have used the values $e_p = 1.5e$, $e_n = 0.5e$ in determining the values of M_p and M_n from A_p and A_n. The convention-

TABLE I. $E4$ matrix elements in units of $e\,\mathrm{fm}^4$ for the even-even sd-shell nuclei with $N=Z$. The matrix elements are purely isoscalar with $A_n = A_p$ and $M_n = M_p$.

A	Z	Ex (MeV)	No.	A_p	M_p
20	10	4.213	1	−102.55	−205.11
		9.974	2	−20.98	−41.96
		10.676	3	2.33	4.67
		11.753	4	1.16	2.33
		13.887	5	−1.77	−3.54
		14.279	6	−15.43	−30.85
		14.988	7	−8.29	−16.58
		15.239	8	−20.48	−40.97
24	12	4.379	1	7.77	15.55
		5.935	2	99.38	198.75
		8.374	3	−13.04	−26.08
		9.640	4	8.64	17.29
		11.039	5	14.73	29.46
		11.378	6	−0.28	−0.56
		11.746	7	14.30	28.60
		12.128	8	19.25	38.50
28	14	4.659	1	−83.27	−166.53
		7.037	2	83.73	167.47
		9.493	3	−0.03	−0.07
		9.978	4	−20.59	−41.19
		10.661	5	0.65	1.31
		11.224	6	14.49	28.99
		11.552	7	−1.84	−3.69
		11.881	8	21.72	43.44
32	16	4.698	1	−111.90	−223.81
		6.265	2	−74.47	−148.95
		6.866	3	19.22	38.44
		8.131	4	−2.67	−5.34
		8.990	5	−30.88	−61.76
		9.427	6	−1.48	−2.97
		10.303	7	−9.64	−19.29
		10.743	8	−15.97	−31.94
36	18	4.564	1	97.87	195.73
		6.357	2	−38.36	−76.72
		8.543	3	50.43	100.85
		11.129	4	15.88	31.75
		12.416	5	−19.87	−39.74
		13.033	6	−24.48	−48.97
		13.917	7	3.57	7.14
		14.750	8	−6.19	−12.38

TABLE II. $E4$ matrix elements in units of $e\,\mathrm{fm}^4$ for the even-even sd-shell nuclei with $N=Z+2$.

A	Z	Ex (MeV)		A_p	A_n	M_p	M_n
18	8	3.782	1	0.00	−105.37	−52.68	−158.05
		8.750	2	0.00	−83.18	−41.59	−124.77
22	10	3.378	1	−80.36	6.46	−117.31	−30.49
		5.480	2	27.46	61.65	72.01	106.20
		6.430	3	−49.28	−59.22	−103.52	−113.46
		6.992	4	21.89	−56.55	4.55	−73.89
		8.189	5	−5.15	−53.27	−34.37	−82.48
		8.725	6	29.40	20.90	54.55	46.05
		9.422	7	−10.44	46.89	7.79	65.12
		10.089	8	−4.79	18.34	1.98	25.11
26	12	4.532	1	−55.85	−35.72	−101.63	−81.50
		4.932	2	56.82	89.33	129.89	162.40
		5.472	3	22.05	−56.33	4.91	−73.48
		6.008	4	−57.92	−20.59	−97.18	−59.85
		6.777	5	−4.48	21.24	3.91	29.62
		7.410	6	39.76	−34.51	42.38	−31.89
		7.940	7	38.26	6.12	60.46	28.31
		8.413	8	36.96	1.03	55.95	20.02
30	14	5.506	1	−77.36	−122.84	−177.45	−222.94
		5.913	2	−96.08	−10.09	−149.17	−63.18
		6.997	3	−47.75	49.56	−46.85	50.46
		8.051	4	−40.65	−29.36	−75.65	−64.37
		8.606	5	−14.58	16.92	−13.42	18.09
		8.696	6	−6.92	−10.12	−15.44	−18.64
		8.933	7	24.18	0.44	36.49	12.74
		9.321	8	31.29	27.26	60.57	56.54
34	16	4.896	1	−106.94	−78.78	−199.80	−171.64
		6.819	2	36.89	60.31	85.49	108.91
		6.987	3	−45.55	−1.34	−68.99	−24.78
		7.623	4	−83.93	54.45	−98.66	39.71
		8.554	5	−78.85	−25.51	−131.04	−77.70
		9.323	6	−27.27	19.35	−31.23	15.39
		9.807	7	−3.24	0.29	−4.72	−1.18
		10.044	8	−7.74	11.17	−6.02	12.89
38	18	8.520	1	162.97	0.00	244.46	81.49
		17.208	2	2.30	0.00	3.45	1.15

al reduced transition strength $B(E4)$ is related to M_p by

$$B(E4,J_i \rightarrow J_f) = (2J_i+1)^{-1}(M_p)^2 \ .$$

The values of A_p and A_n (and M_p and M_n) can be calibrated against the value of a unit-strength $0d_{5/2} \rightarrow 0d_{3/2}$ matrix element. For ^{32}S this value is 250 e fm^4 for M_p and the variations in this value due to differences in the radii of other sd-shell nuclei amount to less than 10%. We note that the strongest calculated 4^+ transitions are slightly smaller than this "single-particle" value, but that the total amount of low-lying strength in each nucleus is roughly one and a half times greater.

The present results are, overall, quite similar to the earlier[3] Chung-Wildenthal results. The most significant differences occur for the lowest states of ^{30}Si and ^{36}Ar. In ^{30}Si the new results predict more strength for the first 4^+ and a general reshuffling of the total and relative neutron and proton strengths in the first several states. In ^{36}Ar, the new matrix element for the first 4^+ is 20% smaller than the old value, the missing strength appearing in the second 4^+. It is also interesting to note that the predicted strength for the first 4^+ in ^{24}Mg, which is observed to have a weak, anomalously shaped form factor,[6] is reduced considerably further in the new predictions from the small value obtained in the old calculation.

In the $N=Z(T=0)$ nuclei, the neutron and proton matrix elements are, of course, the same. Probes will excite these systems with strength profiles independent of different probe-neutron and probe-proton interaction strengths. In the $N=Z+2(T=1)$ nuclei, the neutron and proton matrix elements differ from each other as functions of the detailed structures of the individual states. Therefore, probes with differential sensitivities to neutrons and protons will produce different profiles of excitation strength for a given nucleus.[7] Discussions of how the predicted values of M_p and M_n combine to yield predictions for nonelectromagnetic probes have been presented previously,[3,7] and they apply equally well to the new predictions of M_p and M_n. Experimental tests of the predicted ratios of M_p to M_n for the first two 4^+ states in the $A=22$, 26, 30, and 34 systems would provide a fundamental and discriminating critique of the present calculations.

Existing experimental inelastic electron scattering results have made it possible to test the $E4$ predictions for nuclei in the $A=20$–28 range.[1] It has been found that within the $0d, 1s$ shell-model approximation, best agreement is obtained between experimental form factors and theory by introducing the core-polarization corrections in the form of a Tassie-model radial shape, rather than in shapes which are identical to the model-space transition densities. The magnitude of the core-polarization correction is normalized to produce a matrix element at $q=0$ which is equal to that calculated with the $e_p=1.5, e_n=0.5$ effective charge assumption. With this formulation, reasonably complete $E4$ form factors in ^{20}Ne, ^{24}Mg, ^{28}Si, ^{19}F, and ^{27}Al are well reproduced, along with more fragmentary results for ^{22}Ne and ^{26}Mg.[1] These large $E4$ effective charges have their origin in the coupling of the sd-shell valence particles to the giant $E4$ resonances.[8]

Between $A=30$ and 40, no $E4$ electron scattering data have been available for analysis until now, leaving the significant differences which exist between various theories a matter of speculation. The most logical candidate for improving this situation would seem to be ^{32}S, for which extensive $E2$ results are available. The difficulty which has inhibited progress with ^{32}S concerns the closeness of the first 4^+ and the second 2^+ states, which have an experimental energy separation so small as to have precluded past experiments of resolving them. Our success in interpreting the 2^+ and 4^+ form factors of the lighter sd-shell nuclei suggested circumventing this problem by analyzing a composite experimental form factor of this doublet with our ^{32}S wave functions.

Inelastic electron scattering data at 250 and 500 MeV were obtained at Stanford[9] in an experiment on ^{24}Mg, ^{28}Si, and ^{32}S. The results for the ground states and first excited 2^+ states and the ^{24}Mg 4^+ states have been published previously.[9] The newly reanalyzed form factor for the ^{32}S doublet is shown in Fig. 1. The solid curve in Fig. 1 is the sum of our predictions for the second 2^+ and first 4^+ form factors of ^{32}S, as presented in Ref. 1. The individual quadrupole and hexadecupole contributions are indicated by the $\times$'s and the dashed line, respectively. As can be seen, beyond 1.5 fm^{-1}, the form factor is predicted to result almost totally from $E4$ excitation.

Overall, the agreement between the data and the combined 2^+ and 4^+ theoretical form factors is excellent. We conclude, therefore, that the $E4$ matrix element calculated in the shell model for the first 4^+ state of ^{32}S is, like those for the first 4^+ states of ^{20}Ne and ^{28}Si and the second 4^+ state of ^{24}Mg, nicely consistent with experiment. The details of the form factor out to 2.5 fm^{-1} are consistent with the calculated curve which incorporates the Tassie-model shape for the core-polarization term with a constant effective charge of $0.5e$.

Shell-model calculations for $0^+ \rightarrow 4^+$ transition strengths in doubly even sd-shell nuclei predict undiminished intensi-

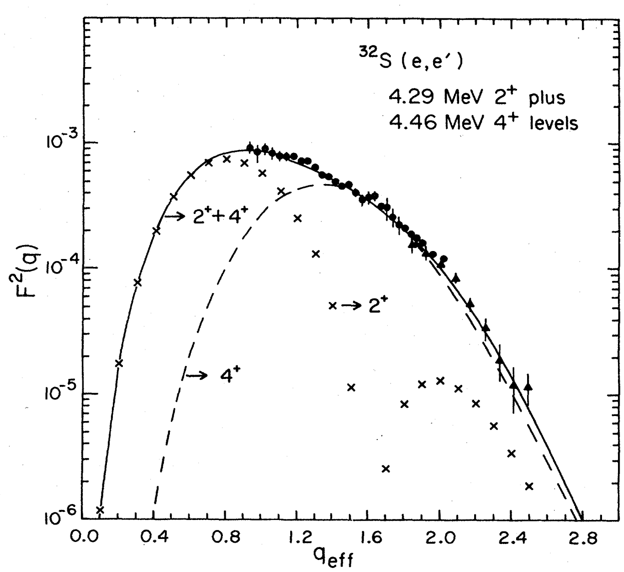

FIG. 1. Experimental and theoretical form factors for the first 4^+-second 2^+ doublet at 4.4 MeV in ^{32}S. The $E2$ form factor is indicated by $\times$ and the $E4$ form factor by the dashed line. The solid line represents the sum of the $E2$ and $E4$ contributions. The 250 MeV experimental data are shown by the circles with error bars and the 500 MeV data are shown by the triangles with error bars.

ties from $A=18$ through $A=38$. For the $T=0$ nuclei, the results for ^{20}Ne and ^{36}Ar are similar in the dominance of the lowest 4^+, at essentially the same strength. The dominant strength in ^{28}Si is equally split between the lowest two states, the second 4^+ is dominant in ^{24}Mg and, while the lowest 4^+ dominates in ^{32}S, the second 4^+ has significant strength. The predicted features of ^{20}Ne and ^{24}Mg are confirmed experimentally, as are the properties of the lowest 4^+ states in ^{28}Si and, with the present work, ^{32}S. This variety is to be contrasted with the uniform behavior of the $E2$ strength function in these nuclei.

The results for $T=1$ nuclei are more varied still than those for the $T=0$ cases. The lowest 4^+ tends to dominate (^{26}Mg is the exception), but not by much, and the remaining strength is distributed over several higher states, not just the second. Moreover, of course, the $T=1$ predictions encompass a variety of features pertaining to the relative importance of the neutron and proton components of the transitions. Very few aspects of these $T=1$ predictions have been experimentally tested.

The basic structure of the shell-model predictions is fairly transparent. The single-particle transition $0d_{5/2}\rightarrow 0d_{3/2}$ dominates every strong transition except that of ^{20}Ne, for which the $0d_{5/2}\rightarrow 0d_{5/2}$ component is competitive. The centers of gravity of the 4^+ excitation energies characterized by this transition are about 6 MeV through the whole shell, until ^{38}Ar. This can be taken as reflecting the high degree of configuration mixing in the shell-model wave functions, which has the consequence of blurring the zero-order distinctions which otherwise might be expected to appear with the increase of nucleon number from 18 to 36. The effects of core polarization are treated independently of the mass and excitation energy, reflecting the assumption that the origins of these effects lie in the giant $E4$ resonances, whose excitation energies are much higher than those considered here.[8]

The large differences in the $A=30$–40 region between these shell-model predictions and those of recent Hartree-Fock calculations[2] need to be analyzed further. More work will have to be done along the lines of identifying the individual constituents of the Hartree-Fock predictions and their sensitivities to the details of the assumed interaction and other parameters before a meaningful comparison with the shell-model results can be made. Further experimental work above $A=32$ will be valuable to confirm the conclusions of the present study, but the high degree of stability and cohesion of the shell-model wave functions suggests that, in the interim before these tests are made, the present results offer a reasonable guide to potentially observable phenomena.

This research was supported in part by the National Science Foundation under Grants No. PHY-83-12245 and No. PHY-85-09736.

[1]B. A. Brown, R. Radhi, and B. H. Wildenthal, Phys. Rep. **101**, 314 (1983).

[2]H. R. Jaqaman and L. Zamick, Phys. Rev. C **30**, 1719 (1984).

[3]B. A. Brown, W. Chung, and B. H. Wildenthal, Phys. Rev. C **21**, 2600 (1980).

[4]W. Chung, Ph. D. thesis, Michigan State University, 1976 (unpublished); B. H. Wildenthal, in *Elementary Modes of Excitation in Nuclei, Proceedings of the International School of Physics "Enrico Fermi," Course LXIX,* edited by R. Broglia and A. Bohr (North-Holland, Amsterdam, 1977).

[5]B. H. Wildenthal, *Progress in Particle and Nuclear Physics* (Pergamon, New York, 1983), Vol. 11, p. 5.

[6]H. Zarek *et al.*, Phys. Lett. **80B**, 26 (1978).

[7]B. A. Brown and B. H. Wildenthal, Phys. Rev. C **21**, 2107 (1980).

[8]H. Sagawa and B. A. Brown, Phys. Lett. **150B**, 247 (1985).

[9]G. C. Li, M. R. Yearian, and I. Sick, Phys. Rev. C **9**, 1861 (1974).