PHYSICAL REVIEW C VOLUME 32, NUMBER 6 DECEMBER 1985

Neutrino reactions in ^{13}C and the behavior of the axial current form factor

S. L. Mintz
Physics Department, Florida International University, Miami, Florida 33199
(Received 26 December 1984)

The total and differential cross sections $\sigma(\nu_\mu + {}^{13}\text{C} \rightarrow {}^{13}\text{N}_{\text{g.s.}} + \mu^-)$ and $d\sigma/d\Omega(\nu_\mu + {}^{13}\text{C} \rightarrow {}^{13}\text{N}_{\text{g.s.}} + \mu^-)$ are calculated from threshold to $E_\nu = 260$ MeV. The dependence of the cross section on the axial current form factor $F_A(q^2)$ is obtained for a range of assumptions. The dependence of the cross section on the pseudoscalar form factor is found to be negligible.

The elementary particle model (EPM)[1,2] can, in principle, be used to make model independent calculations for weak processes in nuclei if the form factors describing the matrix elements of the axial vector current and vector current are known. In practice, the form factors describing the weak vector current are reasonably well known from electromagnetic scattering data via the conserved vector current hypothesis (CVC). Unfortunately, there exists at present no model independent way for obtaining the form factors describing the axial vector current matrix element.

The axial vector form factor $F_A(q^2)$ [see Eq. (3b)] is usually obtained by making use of a scaling relationship[1] derived from the impulse approximation, namely, that $F_A(q^2)/F_A(0) = F_M(q^2)/F_M(0)$, where F_M is the weak magnetism form factor. This relationship comes from the fact that both $F_A(q^2)$ and $F_M(q^2)$ are proportional to $\langle f|\sum_{i=1}\tau_{(i)}^{\pm}\sigma_i e^{i\mathbf{q}\cdot\mathbf{r}}|i\rangle$ when only the leading term in the impulse approximation is kept. Thus, departures from this scaling are a measure of the size of the neglected components, and non-nucleon terms in the impulse approximation such as pion exchange current terms.

The situation for the pseudoscalar form factor is worse. A relationship[3] based on work by Nambu yields

$$F_P = -(1+\epsilon)m_\pi(M_f + M_i)F_A(q^2)/(q^2 - m_\pi^2) \ ,$$

where estimates[3] for ϵ are in the range of -0.015 to -0.29. It would therefore be very useful to be able to experimentally test these assumptions. There is currently only enough data to make a test of the scaling relationship between $F_A(q^2)$ and $F_M(q^2)$ in the $^{12}\text{C} \leftrightarrow {}^{12}\text{B}$, ^{12}N transitions at $q^2 = -0.74m_\mu^2$. This has been done by Nozawa, Kohyama, and Kubodera[4] using data for the total muon-capture rate Γ_μ, the average polarization P_R, and the ratio of the average polarization to the longitudinal polarization. Writing

$$F_A(q^2)/F_A(0) = [F_M(q^2)/F_M(0)](1+\eta) \ , \qquad (1)$$

they obtained $|\eta| \leq 0.08$, consistent with scaling when order of αZ symmetry breaking effects are considered. It would obviously be desirable to have an estimate of η over a wider range of q^2.

We present here a calculation for the $\frac{1}{2}^- \leftrightarrow \frac{1}{2}^-$, neutrino reaction $\nu_\mu + {}^{13}\text{C} \rightarrow \mu^- + {}^{13}\text{N}_{\text{g.s.}}$. This reaction depends, in principle, on only two axial current[5] form factors $F_A(q^2)$ and $F_P(q^2)$ plus $F_V(q^2)$ and $F_M(q^2)$, which are determined from existing electron scattering data via CVC. Furthermore, as will be noted, the reaction is extremely insensitive to $F_P(q^2)$ in the range of q^2 used here. Thus, a measurement of this reaction effectively allows the determination of the form factor $F_A(q^2)$. This is useful, of course, for its own sake and would allow a determination of η in Eq. (1), thereby providing a test of the scaling assumption. If the scaling assumption is violated above the 10% level ($|\eta| \geq 0.1$), the phenomenological form of $\eta(q^2)$ would help in testing model calculations for the additional impulse approximation terms which would then have to be examined.

The reaction $\nu_\mu + {}^{13}\text{C} \rightarrow \mu^- + {}^{13}\text{N}_{\text{g.s.}}$ is also interesting for other reasons. Most of the $\frac{1}{2} \leftrightarrow \frac{1}{2}$ weak tansitions studied in detail have been light ones ($\text{n} \leftrightarrow \text{p}$, and $^3\text{H} \leftrightarrow {}^3\text{He}$) which may not have sufficient complexity for examining the question of whether F_A and F_M scale in the same manner. Finally, an accurate determination of $F_A(q^2)$ from the neutrino reaction should enable a reasonably accurate calculation near threshold for the cross section of the pion photoproduction reaction[6,7] $\gamma + {}^{13}\text{C} \rightarrow \pi^- + {}^{13}\text{N}_{\text{g.s.}}$, about which there is at present some controversy and which depends strongly on $F_A(q^2)$ but only marginally on $F_P(q^2)$.

In this Brief Report we shall obtain the matrix elements for the reaction $\nu_\mu + {}^{13}\text{C} \rightarrow {}^{13}\text{N}_{\text{g.s.}} + \mu^-$ and use them to calculate $d\sigma/d\Omega$ and σ for the case of scaling between $F_A(q^2)$, i.e., $\eta = 0$ in Eq. (1) and for various assumptions for $\eta(q^2)$. Thus, if experimental results become available the assumption of scaling between $F_A(q^2)$ and $F_M(q^2)$ can be checked.

The transition matrix element for the process $\nu_\mu + {}^{13}\text{C} \rightarrow \mu^- + {}^{13}\text{N}_{\text{g.s.}}$ may be written as

$$M = \frac{G}{\sqrt{2}}\cos\theta_c \langle {}^{13}\text{N}_{\text{g.s.}}|J_\alpha^+(0)|{}^{13}\text{C}\rangle \bar{u}_\mu \gamma^\alpha (1-\gamma_5)u_\nu \ , \qquad (2)$$

where $G(=1.02\times 10^{-5}/m_p^2)$ is the weak coupling constant, θ_C is the Cabbibo angle, and $J_\alpha^+ = V_\alpha^+ - A_\alpha^+$ is the weak nuclear charge raising current. The form of the matrix elements $\langle {}^{13}\text{N}_{\text{g.s.}}|V_\alpha^+|{}^{13}\text{C}\rangle$ and $\langle {}^{13}\text{N}_{\text{g.s.}}|A_\alpha^+|{}^{13}\text{C}\rangle$ is well known and they may be written

$$\langle {}^{13}\text{N}_{\text{g.s.}}|V_\mu^+(0)|{}^{13}\text{C}\rangle = \bar{u}_f[\gamma_\mu F_V(q^2) + iF_M(q^2)\sigma_{\mu\nu}q^\nu]u_i \ , \qquad (3\text{a})$$

$$\langle {}^{13}\text{N}_{\text{g.s.}}|A_\mu^+(0)|{}^{13}\text{C}\rangle = \bar{u}_f\left[\gamma_\mu\gamma_5 F_A(q^2) + \frac{q_\mu\gamma_5}{m_\pi}F_P(q^2)\right]u_i \ . \qquad (3\text{b})$$

Thus the problem in determining any weak process is that of determining $F_V(q^2)$, $F_M(q^2)$, $F_A(q^2)$, and $F_P(q^2)$. Electron scattering data exists[8] which enables a determination[9] of $F_V(q^2)$ and $F_M(q^2)$ via the CVC, using the SU(2) current commutation relations. The results are

$$F_V(q^2)=F_f^{(1)}(q^2)-F_i^{(1)}(q^2) \ , \tag{4a}$$

$$F_M(q^2)=F_f^{(2)}(q^2)-F_i^{(2)}(q^2) \ , \tag{4b}$$

where i refers to ^{13}C and f refers to $^{13}N_{g.s.}$. The form factors $F_i^{(1)}(q^2)$ and $F_i^{(2)}(q^2)$ may be obtained from electron scattering data,[8] $e+{}^{13}C \rightarrow e'+{}^{13}C$, and are well fit by the standard dipole form

$$F_i(q^2)=F_i(0)/(1-q^2/M^2)^2 \ . \tag{5}$$

The values $F_f^{(1)}(0)$ and $F_f^{(2)}(0)$ are known and a result[10] based on charge symmetry implies that the F_f have the same form and therefore the same dipole mass as the F_i. We are thus able to obtain

$$F_V(q^2)=1/(1-q^2/m_V^2)^2, \quad M_V^2=3.95m_\pi^2 \tag{6a}$$

and

$$F_M(q^2)=F_M(0)/(1-q^2/M_M^2)^2, \quad M_M^2=1.1m_\pi^2 \tag{6b}$$

with $F_M(0)=-1.101/(2M_P)$.

We next examine the axial current form factors. The quantity $F_A(0)$ may be obtained from the beta decay $^{13}N_{g.s.} \rightarrow {}^{13}C+e^+ +\nu_e$. We find[11] from $\log ft=3.667 \pm 0.001$,

$$F_A(0)=0.31 \pm 0.01 \ , \tag{7}$$

where the positive sign is assigned on the basis of an impulse approximation calculation. As discussed above, since we would like to test the assumption of $F_A(q^2)$, $F_M(q^2)$ scaling we write from Eq. (1)

$$F_A(q^2)=F_A(0)\frac{F_M(q^2)}{F_M(0)}(1+\eta) \tag{8a}$$

or

$$F_A(q^2)=[0.31/(1-q^2/M^2)^2](1+\eta), \quad M^2=1.1m_\pi^2 \ , \tag{8b}$$

and where η is in general a function of q^2.

Finally, it is necessary to obtain $F_P(q^2)$. To do this we make use of a relation due to Nambu[3,12] and write

$$F_P(q^2)=\frac{-m_\pi(M_f+M_i)F_A(q^2)}{(q^2-m_\pi^2)}(1+\epsilon) \ , \tag{9}$$

where M_f is the $^{13}N_{g.s.}$ mass and M_i is the ^{13}C mass. As was previously mentioned, the parameter ϵ is estimated[3,12,13] to be of the order $-0.15 \sim -0.30$. In any case, the neutrino reaction cross sections are very insensitive to the values of F_P over a substantially larger variation of ϵ than that given above as was determined by direct computation. Thus, Eqs. (3a) and (3b) combined with Eqs. (6), (6b), (8), and (9) completely determine the transition matrix element Eq. (2).

The differential cross section for the process $\nu_\mu+{}^{13}C \rightarrow \mu^- + {}^{13}N_{g.s.}$ may be written as

$$\frac{d\sigma}{d\Omega}=\frac{m_\nu m_\mu M_f}{16\pi^2\nu}\frac{|M|^2}{|M_i+\nu-(\mu_0/|\boldsymbol{\mu}|)\cos\theta|} \ , \tag{10}$$

where ν is the magnitude of the neutrino momentum or energy, μ_0 and $\boldsymbol{\mu}$ are the muon energy and momentum, respectively, and θ is the muon angle with respect to the neutrino beam. The quantity $|M|^2$ is given by

$$\begin{aligned}|M|^2 = {} & 8F_V^2M_i\nu[\mu_0(M_i-\delta)+M_f|\boldsymbol{\mu}|\cos\theta+(\nu-\mu_0)A_\mu-m_\mu^2]+8F_VF_MM_i\nu[2A_\mu\delta(\nu-\mu_0)+4\nu A_\mu^2-3m_\mu^2A_\mu-\delta m_\mu^2] \\ & +4M_i\nu F_M^2\{A_\mu[4\nu M_fA_\mu-3m_\mu^2M_f-4\mu_0(M_i\mu_0+\nu A_\mu-m_\mu^2) \\ & -4\nu(M_i\nu-\nu A_\mu)+m_\mu^2(M_i+\nu-\mu_0)]+4m_\mu^2(M_i\nu-\nu A_\mu)\}+16M_i(M_i+M_f)\nu F_AF_M \\ & \times[A_\mu(\mu_0+\nu)-m_\mu^2]+8F_A^2M_i^2\nu\left[(2+M_f/M_i)\mu_0-\frac{M_f}{M_i}|\boldsymbol{\mu}|\cos\theta+\frac{(\nu-\mu_0)}{M_i}A_\mu-m_\mu^2/M_i\right]-\frac{8F_AF_p}{m_\pi^2} \\ & \times M_im_\mu^2\nu(\delta+A_\mu)+\frac{4F_P^2}{m_\pi^2}m_\mu^2M_i\nu A_\mu(\nu-\mu_0-\delta)+16F_VF_AM_i\nu[\nu\mu_0+|\boldsymbol{\mu}|^2-A_\mu(\mu_0+\nu)] \ ,\end{aligned} \tag{11}$$

where

$$A_\mu=\mu_0-|\boldsymbol{\mu}|\cos\theta \ , \tag{12}$$

and

$$\delta=M_f-M_i \ . \tag{13}$$

In Fig. 1 we evaluate $d\sigma/d\Omega$ for a variety of incoming neutrino energies from near thereshold to 260 MeV.

We then integrate Eq. (10). The results are given from threshold to $E_\nu=260$ MeV in Fig. 2. We also obtain curves for σ for various assumptions concerning $\eta(q^2)$ which are shown in Fig. 2.

An examination of Eq. (11) shows that the contributions of $|F_A|^2$ to it is roughly three times that of $|F_V|^2$. However, the small size of $F_A(0)$ combined with the small size of M_A, which forces a rapid falloff in the size of the dipole form factor F_A, leads to a relatively small contribution from the axial part of the weak nuclear current. Thus this process $\nu_\mu+{}^{13}C \rightarrow \mu^- + {}^{13}N_{g.s.}$ is dominated by the vector current. The cross sections are however, sensitive to departures from the expected shape of the axial vector current form factor (i.e., to η).

We consider several possibilities for η. If η is constant over the range of q^2 considered here, an $\eta=0.3$ would increase the cross section about 10% over its lower range and about 5% at the upper range. For the few cases which have some evidence of scaling, namely, the muon-capture results, for ^{12}C, ^{6}Li, and ^{3}H, $M_V \simeq M_M$ so that on a purely phenomenological basis one could have $F_A(q^2)/F_A(0) \simeq F_V(q^2)/F_V(0)$; this would correspond to an $\eta(q^2)$ in Eq. (8b) given by

$$\eta(q^2)=\frac{8^2}{m_\pi^2}\left[0.762\frac{q^2}{m_\pi^2}-0.625\right]\Big/[1-q^2/(3.95m_\pi^2)]^2 \ . \tag{14}$$

This is not a whimsical choice because the vector current form factor mass is closely related to the charge radius

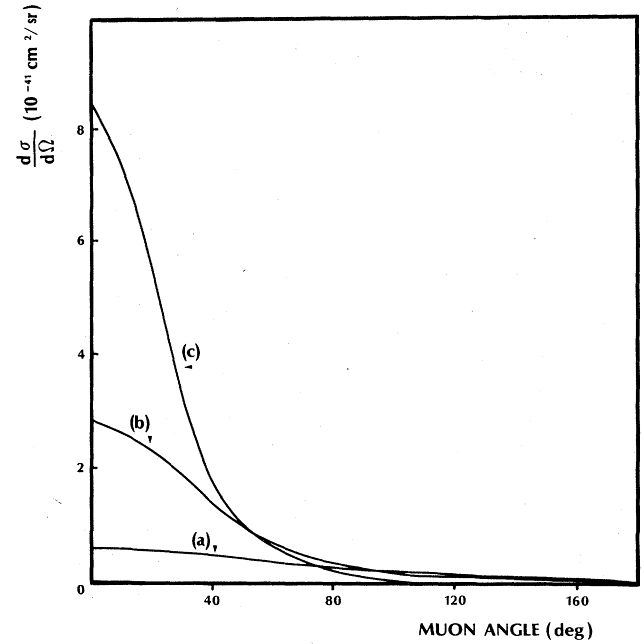

FIG. 1. Plot of the differential cross section $d\sigma/d\Omega(\nu_\mu + {}^{13}C \rightarrow \mu^- + {}^{13}N)$ as a function of muon laboratory angle. Curves (a), (b), and (c) are for incident neutrino energies of 119, 170, and 260 MeV, respectively.

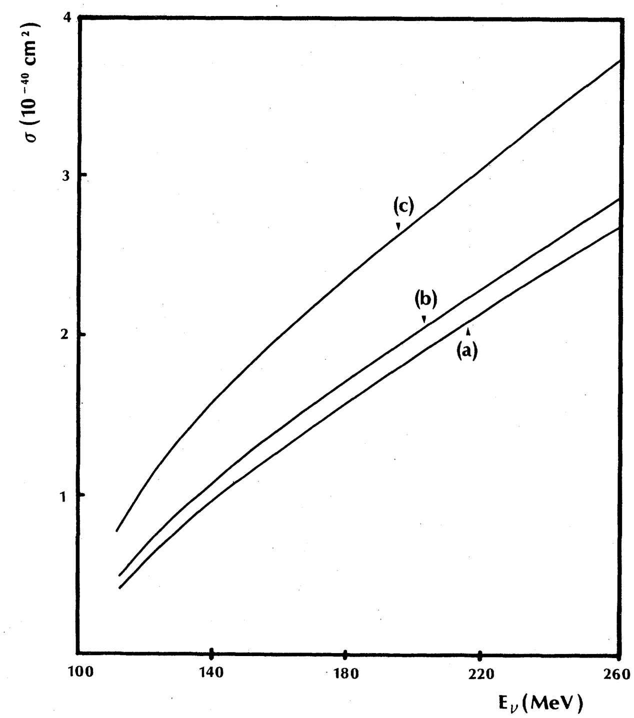

FIG. 2. Plot of the cross section $\sigma(\nu_\mu + {}^{13}C \rightarrow \mu^- + {}^{13}N)$ as a function of the parameters $F_A(0)$, the axial form factor at $q^2=0$, and M_A the mass used in the dipole form for $F_A(q^2)$. Curve (a) is for $F_A(0)=0.31$ and $M_A^2=1.1m_\pi^2$. Curve (b) is for $F_A(0)=0.31$. and $M_A^2=3.95m_\pi^2$. Curve (c) is for $F_A(0)=0.62$ and $F_A(0)=3.95m_\pi^2$.

which is a characteristic size measurement of the nucleus and which might be relevant for other form factors. Behavior for $\eta(q^2)$ given by Eq. (14) would lead to an increase in σ of 20% in the lower part of the energy range given here and an increase in σ of about 15% in the upper range. Finally, if a value for η double that of Eq. (14) were used, σ would double near threshold and increase by 40% in the upper energy range obtained here.

Thus, although accurate neutrino cross section measurements for nuclei are not yet available it is hoped and expected that this situation shall change. Measurements of the reaction discussed here would be very useful in testing the scaling assumption used in obtaining $F_A(q^2)$ from the q^2 dependence of $F_M(q^2)$, and the phenomenological form of significant departures from scaling would be of use in testing model calculations for other terms discussed previously which should contribute to $F_A(q^2)$.

[1]C. W. Kim and H. Primakoff, Phys. Rev. B **140**, 566 (1965).
[2]C. W. Kim and H. Primakoff, Phys. Rev. B **139**, 1447 (1965).
[3]C. W. Kim and H. Primakoff, in *Mesons in Nuclei*, edited by M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979).
[4]S. Nozawa, Y. Kohyama, and K. Kubodera, Prog. Theor. Phys. **70**, 892 (1983).
[5]In ^{12}C case there is an F_T term. We have assumed no second class currents.
[6]John J. LeRose *et al.*, Phys. Rev. C **25**, 1702 (1982).
[7]B. Bosco, C. W. Kim, and S. L. Mintz, Phys. Rev. C **25**, 1986 (1982).
[8]See L. Lapikás, G. Box, and H. de Vries, Nucl. Phys. **A253**, 324 (1975); J. Heisenberg, J. S. McCarthy, and I. Sick, *ibid.* **A157**, 435 (1970); C. S. Yang, E. L. Tomusiak, R. K. Gupta, and H. S. Kaplan, *ibid.* **A162**, 71 (1971).
[9]See Ref. 6 or J. Frazier and C. W. Kim, Phys. Rev. **177**, 2568 (1969).
[10]Using the operator for isospin rotation $R=e^{i\pi I_2}$ and $e^{i\pi I_2}J_\mu^3 e^{-i\pi I_2}=-J_\mu^3$, $e^{i\pi I_2}|^{13}C\rangle=-|^{13}N\rangle$, $e^{-\pi I_2}|^{13}N\rangle=|^{13}C\rangle$, one obtains $\langle ^{13}C|J_\mu^3(0)|^{13}C\rangle=-\langle ^{13}N|J_\mu^3(0)|^{13}N\rangle$ so that the form factors are essentially of the same form in so far as charge symmetry holds.
[11]F. Ajzenberg-Selove, Nucl. Phys. **A360**, 1 (1981).
[12]Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960).
[13]C. W. Kim and J. S. Townsend, Phys. Rev. D **11**, 656 (1975).