Distribution of charge and matter in nuclei: Charge density difference of ^{206}Pb and ^{205}Tl

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We contrast two calculations of the charge-density difference of ^{206}Pb and ^{205}Tl. In the simplest model this difference in charge density is due to the occupation of an additional $3s_{1/2}$ orbital in ^{206}Pb. A standard mean-field calculation of the charge difference does not yield a satisfactory result. One may modify this result by assigning the $3s_{1/2}$ orbital an occupation probability of 70%, with a corresponding increase to 30% of the occupation probability of a $2d_{3/2}$ orbital. However, this modification of the mean-field analysis, while solving one problem, is seen to create a new problem in the fit to the data. In this work we present an alternative analysis: We maintain unit occupation probability for the $3s_{1/2}$ orbital, but use the medium-modified proton electromagnetic form factor we have calculated previously. Our model is able to give a better fit to the data without the introduction of free parameters into the analysis. Medium-modified form factors have recently been shown to be effective in explaining the charge distribution of ^{208}Pb, and their application to the interpretation of the ^{206}Pb-^{205}Tl charge-density difference yields a result which is consistent with the experimental data and superior to that obtained in the adjusted mean-field analysis described above.

The problem of explaining the charge distribution of ^{208}Pb has received much attention and has even led to questions concerning the applicability of mean-field theory (and the nuclear shell model) in the description of the physics at the center of a large nucleus.[1] In general, the use of the mean-field theory and free-space nucleon electromagnetic form factors leads to theoretical charge distributions that have oscillations not seen in the experimentally determined charge distribution of ^{208}Pb. This situation was part of the motivation for experiments which determined the charge difference between ^{206}Pb and ^{205}Tl. In the simplest model this charge difference is due to the occupation of an additional $3s_{1/2}$ orbital in ^{206}Pb. Since this orbital has a characteristic shape it was found that the experimental data could readily be interpreted as a measurement of the charge distribution of a single shell-model orbital, and one could conclude that the mean-field picture gave a generally satisfactory description of the physical situation at the center of a large nucleus.[2]

In this work we are concerned with some of the details of the mean-field calculations. First we note that if one compares the charge density difference calculated in the mean-field analysis[2] to the charge-density difference of ^{206}Pb and ^{205}Tl determined experimentally, there is a significant disagreement—see Fig. 1. In part, this disagreement may be removed by modifying the mean-field analysis by assuming an occupation probability of 0.7 for the particles in the $3s_{1/2}$ shell and 0.3 for a particle in the $2d_{3/2}$ shell. In this manner a good fit is achieved for the ratio of cross sections for scattering from ^{205}Tl and ^{206}Pb.[2] There is a residual problem seen, however, when the theoretical charge-density difference is compared to the data—see Fig. 2. In particular, the inclusion of the $2d_{3/2}$ orbital, which has a peak in its contribution to the charge distribution near 3 fm,[2] leads to some disagreement with the data in the region between 2 to 4 fm, where the adjusted mean-field theory yields a result that is about a factor of 2 higher than the data. (See Fig. 2.)

We now present an alternative analysis of the data which yields a good result for the charge-density difference without the introduction of a free parameter into the analysis and without the use of what might be considered an excessively small occupation factor for the $3s_{1/2}$ orbital.

Let $\rho_{\rm Pb}^{\rm mat}(r)$ represent the theoretical (mean-field) proton matter distribution of ^{206}Pb and let $\rho_{\rm Tl}^{\rm mat}(r)$ stand for the corresponding quantity for ^{205}Tl. Further, let $\rho_{\rm Pb}^{\rm mat}(q^2)$ and $\rho_{\rm Tl}^{\rm mat}(q^2)$ be the corresponding proton *matter* form factors.

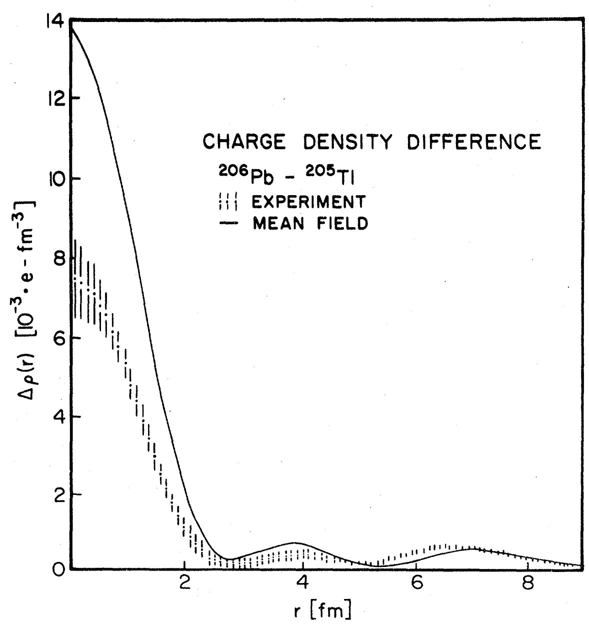

FIG. 1. Comparison of the experimental and theoretical charge-density difference of ^{206}Pb and ^{205}Tl in an unadjusted mean-field theory. (See Ref. 2.) The theoretical result includes the polarization contribution described in the text and depicted in Ref. 2.

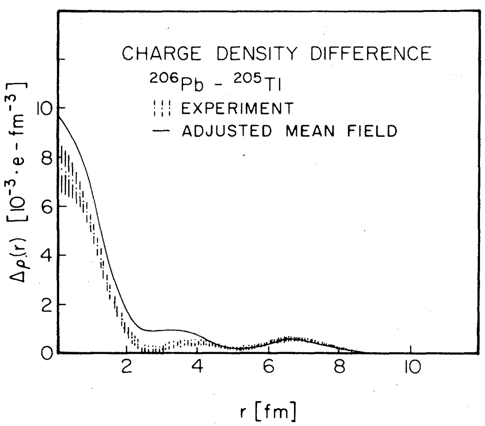

FIG. 2. Comparison of the experimental and the adjusted (mean-field) theoretical charge-density difference of ^{206}Pb and ^{205}Tl. (See Ref. 2.) (The theoretical result includes the polarization contribution described in the text and depicted in Ref. 2.) Note that the value of $\Delta\rho(r)$ at the origin is 70% of the value shown in Fig. 1, in accordance with the 70% occupation probability assigned to the $3s_{1/2}$ orbital.

In the standard analysis the charge form factors of ^{206}Pb and ^{205}Tl may be obtained as

$$F_{\rm Pb}(q^2)=G_E^{\rm p}(q^2)\rho_{\rm Pb}^{\rm mat}(q^2) \ , \tag{1}$$

$$F_{\rm Tl}(q^2)=G_E^{\rm p}(q^2)\rho_{\rm Tl}^{\rm mat}(q^2) \ , \tag{2}$$

where $G_E^{\rm p}(q^2)$ is the proton electromagnetic form factor.

It is useful to separate off the form factor of a *single* $3s_{1/2}$ orbital from the theoretical quantity, $\rho_{\rm Pb}^{\rm mat}(q^2)$, and write

$$F_{\rm Pb}(q^2)=G_E^{\rm p}(q^2)[\tilde{\rho}\,{}_{\rm Pb}^{\rm mat}(q^2)+\rho_{3s_{1/2}}^{\rm mat}(q^2)] \ . \tag{3}$$

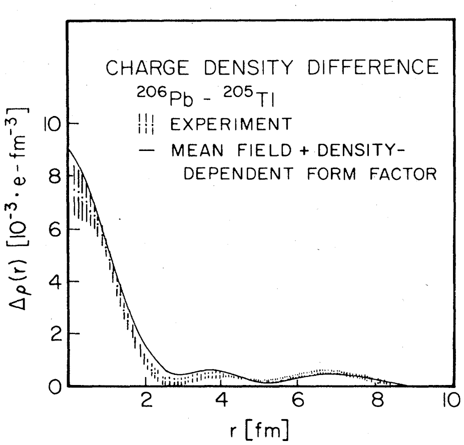

FIG. 3. Comparison of the experimental and theoretical charge-density difference of ^{206}Pb and ^{205}Tl. The theoretical curve is obtained with medium-modified electromagnetic form factors of the proton (Ref. 5) and unit occupation probability of the $3s_{1/2}$ orbital. The polarization contribution [see Eqs. (9) and (10)] is included in the theoretical result. The $3s_{1/2}$ wave function used was provided to us by C. Horowitz (Ref. 13).

We may form the difference,

$$F_{\rm Pb}(q^2)-F_{\rm Tl}(q^2)=G_E^{\rm p}(q^2)[\tilde{\rho}\,{}_{\rm Pb}^{\rm mat}(q^2)-\rho_{\rm Tl}^{\rm mat}(q^2)] + G_E^{\rm p}(q^2)\rho_{3s_{1/2}}^{\rm mat}(q^2) \ . \tag{4}$$

The first of the two terms on the right-hand side of Eq. (4) serves to define the *polarization contribution* to the difference of the charge form factors, and this quantity is readily obtained using a mean-field theory.[2-4] One may write Eq. (4) in coordinate space as

$$\Delta\rho^{\rm ch}(r)=\rho_{\rm pol}^{\rm ch}(r)+\rho_{3s_{1/2}}^{\rm ch}(r) \ . \tag{5}$$

[The quantity $\rho_{\rm pol}^{\rm ch}(r)$ is shown in Fig. 9 of Ref. 2.] We remark at this point that the above equations represent the *unadjusted* mean-field analysis, as described in Ref. 2.

We now wish to consider the possibility that nucleon electromagnetic form factors are modified in nuclei.[5] [There is a significant body of evidence which supports that assumption. For example, one can understand the quenching of the longitudinal response in (e,e′) reactions near the (nucleon) quasielastic peak,[6,7] and also the charge distribution of ^{208}Pb,[8,9] if nucleons are larger in nuclei than in free space. The increase in nucleon size in nuclei may also be used to explain the so-called European Muon Collaboration (EMC) effect.[10]] Now let us rewrite Eqs. (1)–(5) assuming that $G_E^{\rm p}(q^2)$ should be replaced by $G_E^{\rm p}(q^2,\rho_M(r))$. Here $G_E^{\rm p}(q^2,\rho_M(r))$ is a medium-modified form factor[5] and $\rho_M(r)$ is the total matter density of both protons and neutrons. Thus we have[9]

$$\hat{F}_{\rm Pb}(q^2)=\frac{1}{Z}\int e^{i\mathbf{q}\cdot\mathbf{r}}G_E^{\rm p}(q^2,\rho_M(r))\rho_{\rm Pb}^{\rm mat}(r)\,d\mathbf{r} \tag{6}$$

$$=\frac{1}{Z}\int e^{i\mathbf{q}\cdot\mathbf{r}}G_E^{\rm p}(q^2,\rho_M(r))[\tilde{\rho}\,{}_{\rm Pb}^{\rm mat}(r)+\rho_{3s_{1/2}}^{\rm mat}(r)]\,d\mathbf{r} \ , \tag{7}$$

$$\hat{F}_{\rm Tl}(q^2)=\frac{1}{Z}\int e^{i\mathbf{q}\cdot\mathbf{r}}G_E^{\rm p}(q^2,\rho_M(r))\rho_{\rm Tl}^{\rm mat}(r)\,d\mathbf{r} \tag{8}$$

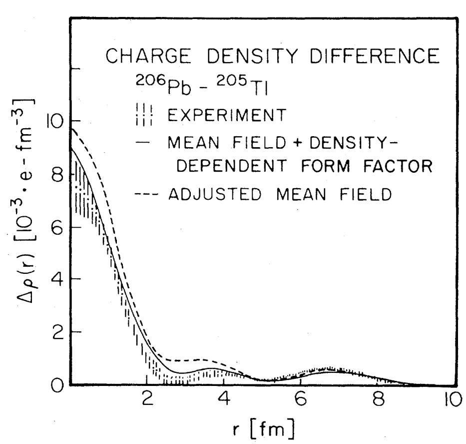

FIG. 4. The adjusted mean-field result shown in Fig. 2 (dashed line) is compared with the result obtained using medium-modified form factors (solid line)—see Fig. 3.

TABLE I. Charge-density difference at the origin of ^{206}Pb and ^{205}Tl.

	Mean-field analysis (Ref. 2)	This work
$\rho^{ch}_{3s_{1/2}}(0)$: calculated with free-space form factors	0.0169 (e fm^{-3})	0.0150 (e fm^{-3})
$\hat{\rho}^{ch}_{3s_{1/2}}(0)$: medium-modified form factors	...	0.0117
Polarization correction:		
$\rho^{ch}_{pol}(0)$	−0.0033	...
$\hat{\rho}^{ch}_{pol}(0)$	...	−0.0028
$\Delta\rho^{ch}(0)$:		
$\rho^{ch}_{3s_{1/2}}(0)+\rho^{ch}_{pol}(0)$	0.0136	...
$\hat{\rho}^{ch}_{3s_{1/2}}(0)+\hat{\rho}^{ch}_{pol}(0)$	...	0.008 87
Adjustment (occupation factor 0.7)	0.009 52	...

and

$$\hat{F}_{Pb}(q^2)-\hat{F}_{Tl}(q^2)=\frac{1}{Z}\int e^{i\mathbf{q}\cdot\mathbf{r}}G_E^p(q^2,\rho_M(r))[\tilde{\rho}^{mat}_{Pb}(r)-\rho^{mat}_{Tl}(r)]\,d\mathbf{r}+\frac{1}{Z}\int e^{i\mathbf{q}\cdot\mathbf{r}}G_E^p(q^2,\rho_M(r))\rho^{mat}_{3s_{1/2}}(r)\,d\mathbf{r}\ . \tag{9}$$

Finally, the coordinate-space version of Eq. (2.9) is

$$\Delta\rho^{ch}(r)=\hat{\rho}^{ch}_{pol}(r)+\hat{\rho}^{ch}_{3s_{1/2}}(r)\ . \tag{10}$$

[We remark that in Eqs. (6)–(9) we have neglected the small difference between $\rho_M(r)$ for ^{206}Pb and ^{205}Tl since the analysis would be totally insensitive to this difference.]

We may now compare the (theoretical) quantity on the right-hand side of Eq. (10) with the experimental charge-density difference. This comparison is made in Fig. 3. The result given in Fig. 3 may also be compared to the results of the *adjusted* mean-field theory given in Fig. 2. (See Fig. 4.) As we have noted earlier, the problem introduced by the addition of contributions of the $2d_{3/2}$ orbital in the adjusted mean-field calculation is not present in the analysis based upon Eq. (10).

It is of interest to see how our model is able to provide a fit to the data with an occupation factor for the $3s_{1/2}$ orbital near unity, while the adjusted mean-field theory, with an occupation factor of 0.7 for the $3s_{1/2}$ orbital, yields a result that is still too large at the origin—see Fig. 2. There are two factors responsible: First we have used a solution of the Dirac equation[13] for the $3s_{1/2}$ orbital rather than the nonrelativistic wave function used in the calculations reported in Ref. 2. This choice is quite consistent with our use of *relativistic* mean-field matter densities[13] to explain the charge distribution of ^{208}Pb.[8] We find that the value of $\rho^{mat}_{3s_{1/2}}(r)$ at the origin is somewhat smaller for the relativistic wave function than for the nonrelativistic $3s_{1/2}$ wave function. Second, we have used medium-modified from factors to obtain the charge density from the matter density. These effects are shown in Table I, where we present values of the charge density at the origin for the (adjusted) mean-field theory and for our analysis.

The fundamental ambiguity in this analysis lies in the fact that we do not have a direct experimental measure of the occupation probability of the $3s_{1/2}$ orbit. The reduction of this probability to 70% in the adjusted mean-field calculation may be excessive. For example, random-phase-approximation (RPA) calculations of Decharge and Gogny[11,12] lead to occupation probabilities of the $3s_{1/2}$ orbital in ^{208}Pb of about 90%. Occupation factors of that size ultimately lead to a good account of the charge distribution in ^{208}Pb if one uses our medium-modified form factors to obtain the charge distribution from theoretical matter distributions.[8]

We may conclude that the use of medium-modified form factors and occupation factors of about 90 to 100% for the $3s_{1/2}$ orbital can give good account of *both* the charge distribution in ^{208}Pb and the charge-density difference between ^{208}Pb and ^{205}Tl. Of course, it will be useful to provide additional tests of our prediction[5] of medium-modified electromagnetic form factors of the nucleon, and such work is in progress.

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