

Ground-state properties of nuclear matter using the Λ approximations of the Green's function theory

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The nuclear matter parameters and single-particle properties are calculated in the so-called $\Lambda^{(00)}$, $\Lambda^{(10)}$, and $\Lambda^{(11)}$ approximation of the Green's function theory. For the nucleon-nucleon interaction we used the Hamada-Johnston, the Reid-soft-core, and the Paris potential. The results show no significant differences with those obtained from the Brueckner theory. For comparison we used also the Galitskii approximation.

I. INTRODUCTION

For the treatment of the nuclear matter system several methods are available. Most papers utilize either Brueckner's theory¹⁻³ or the coupled-cluster method.⁴ Extensive reviews of Brueckner's approach to nuclear matter, in which the different variants of the theory are extensively discussed, are given, for instance, in Refs. 5 and 6. Martin and Schwinger⁷ have developed a method for the thermodynamic Green's functions, which makes possible the consideration of two, three, and higher correlations in a systematic way. Because, however, the latter is rather difficult to implement, only a few investigations—mostly with separable potentials—applied this method to the nuclear matter problem.⁸⁻²¹ In this scheme one decouples the infinite hierarchy of Green's function equations by a "nonperturbative superposition" approximation—analogueous to the Born-Green-Kirkwood-Bogolyubov decomposition in statistical mechanics—which factorizes the sixpoint function into fourpoint functions and single-particle propagators.²² By keeping only dynamical correlations connected with the two-body potential, one obtains approximate theories, the so-called $\Lambda^{(00)}$, $\Lambda^{(10)}$, and $\Lambda^{(11)}$ approximations.

Contrary to the Brueckner theory, where one has the option between the conventional gap choice or the so-called continuous choice,^{5,6} the Λ theories have the advantage that they give a definite description of the propagator dominators. However, due to the complicated structure

of the Λ theory the Brueckner theory is preferred in most of the nuclear matter calculations. Especially from the viewpoint of realistic potentials, only the Hamada-Johnston potential has been used in the $\Lambda^{(00)}$ and $\Lambda^{(10)}$ approximation.^{15,16} Therefore it seems worthwhile to calculate the nuclear matter properties in the different Λ approximations for more recent potentials and to compare the results between the different Λ approximations and the Brueckner treatment. For this purpose we have selected the Hamada-Johnston (HJ),²³ the Reid soft core (RSC),²⁴ and the Paris potentials.²⁵ Since the formalism is known from the literature,⁸ we present the basic theory only briefly in the following section. A closer inspection of the basic system of coupled equations reveals that it is not practical to perform a fully self-consistent calculation. The main cause is the complicated nature of the intermediate propagator in matter. Therefore we have to introduce some additional simplifications, which will be outlined in Sec. III. Section IV is devoted to the numerical results and the comparison between the different approaches.

II. FORMALISM

The Λ approximations in energy-momentum representation are defined by the following set of three equations⁸ which connect the effective single-particle potential V (mass operator), the one-particle Green's function g , and the (antisymmetrized) effective scattering amplitude T (reducible vertex):

$$\langle \mathbf{p} | \tilde{T}(\mathbf{Q}; z) | \mathbf{p}' \rangle - 2 \langle \mathbf{p} | v | \mathbf{p}' \rangle = \int d^3 q \left[\langle \mathbf{p} | v | \mathbf{q} \rangle \tilde{\Lambda} \left[\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}; z \right] \langle \mathbf{q} | \tilde{T}(\mathbf{Q}; z) | \mathbf{p}' \rangle \right], \quad (2.1)$$

$$\begin{aligned} \tilde{V}(\mathbf{p}; z) = & \int_{-\infty}^0 d\omega \int d^3 q \left\{ \left\langle \frac{\mathbf{p}-\mathbf{q}}{2} \right| \tilde{T}(\mathbf{p}+\mathbf{q}; z+\omega) \left| \frac{\mathbf{p}-\mathbf{q}}{2} \right\rangle A(\mathbf{q}; \omega) \right. \\ & \left. + \frac{i}{2\pi} \left\langle \frac{\mathbf{p}-\mathbf{q}}{2} \right| [\tilde{T}(\mathbf{p}+\mathbf{q}; \omega+i\eta) - \tilde{T}(\mathbf{p}+\mathbf{q}; \omega-i\eta)] \left| \frac{\mathbf{p}-\mathbf{q}}{2} \right\rangle \tilde{g}(\mathbf{q}; \omega-z) \right\}, \end{aligned} \quad (2.2)$$

$$\begin{aligned} \tilde{g}(\mathbf{p}; z) = & \left[z - \frac{\mathbf{p}^2}{2m} - \tilde{V}(\mathbf{p}; z) + \mu \right]^{-1} \\ = & \int_{-\infty}^{+\infty} d\omega \frac{A(\mathbf{p}; \omega)}{z - \omega}. \end{aligned} \quad (2.3)$$

The spectral function $A(\mathbf{p};\omega)$ is given from relation (2.3) as

$$A(\mathbf{p};\omega) = \frac{i}{2\pi} [\tilde{g}(\mathbf{p};\omega+i\eta) - \tilde{g}(\mathbf{p};\omega-i\eta)] \\ = \frac{1}{2\pi} \frac{\gamma(\mathbf{p};\omega)}{\left[\omega - \frac{\mathbf{p}^2}{2m} + \mu - \text{Re}\tilde{V}(\mathbf{p};\omega) \right]^2 + \frac{1}{4}\gamma^2(\mathbf{p};\omega)} . \quad (2.4)$$

The intermediate propagator Λ is defined by

$$\tilde{\Lambda}(\mathbf{p}, \mathbf{p}'; z) = \int_{-\infty}^{+\infty} d\omega d\omega' \frac{A(\mathbf{p};\omega) A(\mathbf{p}';\omega')}{z - \omega - \omega'} \\ \times [\theta(\omega)\theta(\omega') - \theta(-\omega)\theta(-\omega')] \quad (2.5)$$

with

$$\theta(x) := \frac{1}{2} \left[1 + \frac{x}{|x|} \right] . \quad (2.6)$$

We have neglected spin and isospin variables, which will be included later. Functions with a tilde denote analytical functions for $\text{Im}z \neq 0$. The physical functions (without tilde) are given by the following boundary values:

$$\tilde{\Lambda}^{(ij)}(\mathbf{p}, \mathbf{p}'; z) := \int \int d\omega d\omega' \left[\frac{\theta(\omega)\theta(\omega') - \theta(-\omega)\theta(-\omega')}{z - \omega - \omega'} \right] \begin{cases} A^0(\mathbf{p};\omega) A^0(\mathbf{p}';\omega') & i=j=0, \\ \frac{1}{2} [A^0(\mathbf{p};\omega) A(\mathbf{p}';\omega') + A(\mathbf{p};\omega) A^0(\mathbf{p}';\omega')] & i=1, j=0, \\ A(\mathbf{p};\omega) A(\mathbf{p}';\omega') & i=j=1. \end{cases} \quad (2.13)$$

A^0 denotes the free particle spectral function given by [see (2.4) for $\gamma \rightarrow 0$]

$$A^0(\mathbf{p};\omega) = \delta \left[\omega - \frac{\mathbf{p}^2}{2m} + \mu \right] . \quad (2.14)$$

For a full self-consistent solution, one has to solve the coupled systems of Eqs. (2.1)–(2.3), where the intermediate propagator is defined by (2.13). With exception of the $\Lambda^{(00)}$ case, this task is a very complicated problem. The main difficulty is the Λ propagator, for which one needs—in principle—the complete off-shell energy behavior of the mass operator [see definitions (2.4) and (2.5)]. Therefore, the complete treatment would demand the solution of the coupled system (2.1)–(2.3) for the whole momentum-energy range. For realistic potentials we see no possibility of handling this problem without further simplifications, which will be sketched in the next section.

III. APPROXIMATE TREATMENT

The main difficulties in the calculation emerge from the intermediate Λ propagator. Only for the simple $\Lambda^{(00)}$ approximation,^{9,10,12,15,16,18–21} where one is not confronted with a self-consistent determination of the spectral

$$g(\mathbf{p};\omega) = \theta(\omega) \tilde{g}(\mathbf{p};\omega+i\eta) + \theta(-\omega) \tilde{g}(\mathbf{p};\omega-i\eta) , \quad (2.7)$$

$$V(\mathbf{p};\omega) = \theta(\omega) \tilde{V}(\mathbf{p};\omega+i\eta) \\ + \theta(-\omega) \tilde{V}(\mathbf{p};\omega-i\eta), \text{ etc. ,} \quad (2.8)$$

$$\gamma(\mathbf{p},\omega) = i [\tilde{V}(\mathbf{p};\omega+i\eta) - \tilde{V}(\mathbf{p};\omega-i\eta)] \geq 0 . \quad (2.9)$$

For convenience we use the Hamiltonian (μ and N denote the chemical potential and the particle number, respectively):

$$\mathcal{H} = H - \mu N . \quad (2.10)$$

The energy per nucleon and momentum distribution are given by

$$\frac{\langle 0 | H | 0 \rangle}{N} = \frac{1}{2\rho} \int_{-\infty}^0 d\omega \\ \times \int \frac{d^3p}{(2\pi)^3} \left[\omega + \frac{p^2}{2m} + \mu \right] A(\mathbf{p};\omega) , \quad (2.11)$$

$$n(\mathbf{p}) = \int_{-\infty}^0 d\omega A(\mathbf{p};\omega) . \quad (2.12)$$

The different Λ approximations are defined as follows:

function A in the Λ propagator, can one solve directly (after the standard angle averaging procedure³ of Λ) Eq. (2.1) for the T matrix, which is not coupled to Eqs. (2.2) and (2.3). Furthermore, the T matrix has no cut for negative frequencies⁸ and Eqs. (2.2)–(2.5) give immediately⁸

$$A^{(00)}(\mathbf{p};\omega) = \delta \left[\omega - \frac{p^2}{2m} - V^{(00)}(\mathbf{p};\omega) + \mu \right] \quad \omega < 0 , \quad (3.1)$$

where the single-particle energies are determined from the eigenvalue equation

$$\epsilon(\mathbf{p}) = \frac{p^2}{2m} + V^{(00)}(\mathbf{p};\epsilon(\mathbf{p}) - \mu) , \quad (3.2)$$

and the momentum distribution is given by

$$n(\mathbf{p}) = \left[1 - \frac{\partial}{\partial \omega} V^{(00)}(\mathbf{p};\omega) \right]_{\omega=\epsilon(\mathbf{p})-\mu}^{-1} . \quad (3.3)$$

For the $\Lambda^{(10)}$ and $\Lambda^{(11)}$ approximations one has—in principle—to iterate the set (2.1)–(2.3) with the definition (2.13) until self-consistency is achieved. This procedure is numerically not tractable. For this reason one simplifies the Λ propagator by using the single-particle description (3.1) and (3.2) for the spectral functions in (2.13). One obtains^{8,11–13,20,21}

$$\tilde{\Lambda}^{(10)}(p_1, p_2; \omega + i\eta) = \frac{1}{2} \left\{ \left[\omega + i\eta - \frac{p_1^2}{2m} - \frac{p_2^2}{2m} + 2\mu - V^{(10)} \left[p_2; \omega + i\eta - \frac{p_1^2}{2m} + \mu \right] \right]^{-1} \right. \\ \left. - n(p_2) \theta(\mu - \epsilon(p_2)) \left[\omega + i\eta - \frac{p_1^2}{2m} - \epsilon(p_2) + 2\mu \right]^{-1} + (p_1 \leftrightarrow p_2) \right\} \quad (3.4)$$

and^{8,14}

$$\tilde{\Lambda}^{(11)}(p_1, p_2; \epsilon + i\eta) = \frac{N(\mathbf{Q}, \mathbf{q})}{\epsilon + 2\mu - \epsilon(p_1) - \epsilon(p_2) + \frac{i}{2}[\gamma(p_1) + \gamma(p_2)]} \quad (3.5)$$

$\gamma(p)$ denotes the "on shell" imaginary part of the mass operator, defined by Eq. (2.9) as

$$\gamma(p) := i[\tilde{V}(p; \epsilon(p) - \mu + i\eta) - \tilde{V}(p; \epsilon(p) - \mu - i\eta)] \\ = \gamma(p; \omega = \epsilon(p) - \mu). \quad (3.6)$$

The Pauli principle is taken into account via

$$N(\mathbf{Q}, \mathbf{q}) := \theta(p_1 - p_F) \theta(p_2 - p_F) - \theta(p_F - p_1) \theta(p_F - p_2) \quad (3.7)$$

with

$$\mathbf{Q} = \mathbf{p}_1 + \mathbf{p}_2, \quad \mathbf{q} = \frac{\mathbf{p}_1 - \mathbf{p}_2}{2}. \quad (3.8)$$

For the $\Lambda^{(10)}$ approximation these assumptions plus the neglect of $\text{Im} V^{(10)}$ in Eq. (3.4) are sufficient for the numerical solution of the basis set (2.1)–(2.3) for local potentials. The $\Lambda^{(11)}$ version has an even more complicated analytical structure⁸ and turns out to be too complicated for a numerical treatment. In order to overcome this problem we utilize instead of Eq. (2.1) the equivalent equation

$$\tilde{T}^{(11)}(z) = \tilde{T}^{(00)}(z) + \frac{1}{2} \tilde{T}^{(00)}(z) \\ \times [\tilde{\Lambda}^{(11)}(z) - \tilde{\Lambda}^{(00)}(z)] \tilde{T}^{(11)}(z) \quad (3.9)$$

and use only the "distorted Born-type" approximation

$$\tilde{T}^{(11)}(z) \simeq \tilde{T}^{(00)}(z) + \frac{1}{2} \tilde{T}^{(00)}(z) \\ \times [\tilde{\Lambda}^{(11)}(z) - \tilde{\Lambda}^{(00)}(z)] \tilde{T}^{(00)}(z). \quad (3.10)$$

Equation (3.10) combined with Eqs. (2.2) and (2.3) are then taken as the basis set for the approximate treatment of the $\Lambda^{(11)}$ approximation. In this version we achieved self-consistency after five iterations. As in the $\Lambda^{(10)}$ case we use only real single-particle energies in the $\Lambda^{(11)}$ propagator by starting from the values obtained from the simpler Λ versions. We treated additionally, for the purpose of comparison, the so-called Galitskii-version $\Lambda^{(G)}$ of the $\Lambda^{(11)}$ approximation,¹⁸ in which the single-particle energies are replaced by the kinetic energies.

In the numerical treatment we solved first the $\Lambda^{(00)}$ and $\Lambda^{(10)}$ problems. Here we have—contrary to the $\Lambda^{(11)}$ approximation—the advantage of a δ -function-type spectral function for negative frequencies. Nevertheless the $\Lambda^{(10)}$ approximation is not simple, since one has to calculate the off-shell self-energy. In the $\Lambda^{(11)}$ approximation one has to deal with a spectral function, which is still peaked near the single-particle energy.⁸ For this reason we made the simplifying parametrization:

$$A(\mathbf{p}; \omega) = \begin{cases} z(p) \delta(\omega - \epsilon(p) + \mu) & \text{for } \epsilon(p) - \mu - \alpha(p) \leq \omega \leq \epsilon(p) - \mu + \alpha(p) \\ \frac{z(p)}{\pi} F(p) \frac{\alpha(p)}{(\omega - \epsilon(p) + \mu)^2 + \alpha^2(p)} & \text{elsewhere} \end{cases} \quad (3.11)$$

with the definitions

$$z(p) := \left[1 - \text{Re} \frac{\partial V(p; \omega)}{\partial \omega} \right]_{\omega = \epsilon(p) - \mu}^{-1}, \quad (3.12)$$

$$F(p) := 2 \frac{1 - z(p)}{z(p)}, \quad (3.13)$$

$$\alpha(p) := \frac{1}{2} z(p) \gamma(p). \quad (3.14)$$

This ansatz reproduces—it seems—the essential features of the spectral function and fulfills the sum rule for $A(\mathbf{p}; \epsilon)$. (For hard core potentials, see Refs. 26 and 27.) The resulting momentum distribution is given by

TABLE I. Comparison of nuclear matter parameters for different potentials and approximations.

N-N potential	Approximations ^a	$\frac{E_0(N)}{N}$ (MeV)	p_F (fm ⁻¹)	κ^b (MeV)
HJ	$\Lambda^{(00)}$	-6.27	1.245	87.63
RSC	$\Lambda^{(00)}$	-11.45	1.43	148.72
PAR	$\Lambda^{(00)}$	-13.17	1.548	162.03
HJ	$\Lambda^{(10)}$	-5.28	1.145	144.72
RSC	$\Lambda^{(10)}$	-8.55	1.334	80.18
HJ	$\Lambda^{(11)}$	-5.88	1.240	65.98
RSC	$\Lambda^{(11)}$	-10.96	1.461	92.28
PAR	$\Lambda^{(11)}$	-12.19	1.620	86.42
HJ	$\Lambda^{(G)}$	-5.53	1.192	109.95
RSC	$\Lambda^{(G)}$	-9.58	1.378	140.52
PAR	$\Lambda^{(G)}$	-10.99	1.492	97.64
HJ	BR	-7.23	1.27	82.5
RSC	BR (Refs. 32 and 33)	-11.30	1.43	138.0
PAR	BR (Ref. 34)	-16.2	1.62	
PAR	BR (Ref. 35)	-21	1.6	
PAR	BR (Ref. 25)	-11.22	1.51	

^a For more details see Ref. 31.^b $\kappa = r_0^2 (\partial^2 / \partial r_0^2) [E_0(N)/N]$ with $r_0 = (3/4\pi\rho)^{1/3}$.

$$\begin{aligned}
n(p) = & \theta(\epsilon(p) - \mu - \alpha(p)) [1 - z(p)] \left\{ 1 + \frac{2}{\pi} \tan^{-1} \left[\frac{\mu - \epsilon(p)}{\alpha(p)} \right] \right\} \\
& + \theta(\epsilon(p) - \mu + \alpha(p)) \theta(\alpha(p) - \epsilon(p) + \mu) \left\{ z(p) \theta(\mu - \epsilon(p)) + \frac{1}{2} [1 - z(p)] \right\} \\
& + \theta(-\epsilon(p) + \mu - \alpha(p)) \left\{ z(p) + \frac{2}{\pi} [1 - z(p)] \tan^{-1} \left[\frac{\mu - \epsilon(p)}{\alpha(p)} \right] \right\}.
\end{aligned}$$

The numerical outcome for the different Λ theories is given in the next section.

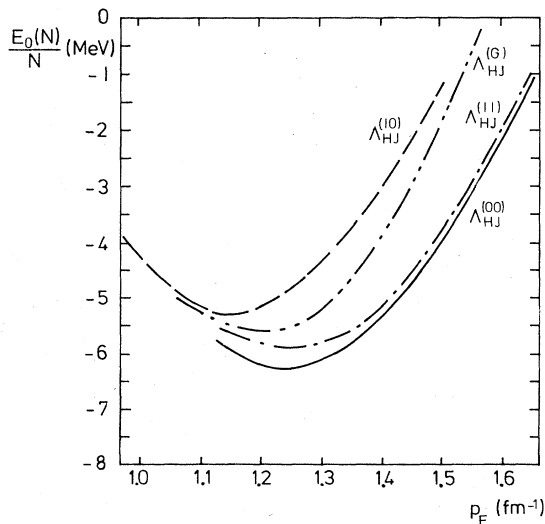


FIG. 1. Energy per nucleon versus Fermi momentum for the HF potential using different Λ approximations.

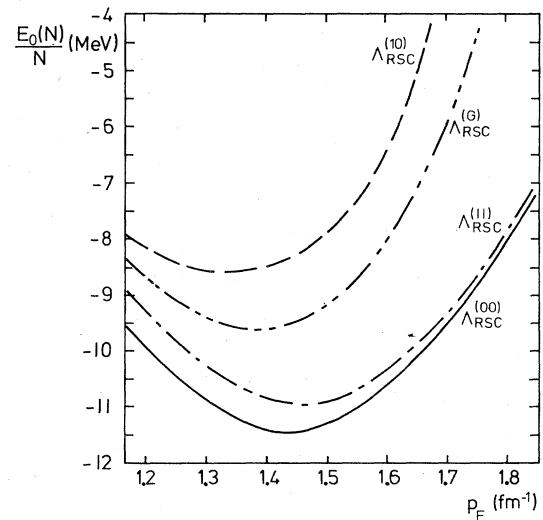


FIG. 2. Energy per nucleon versus Fermi momentum for the RSC potential using different Λ approximations.

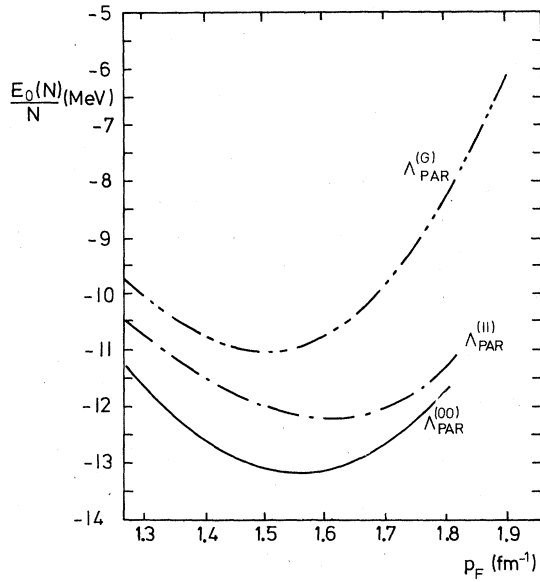


FIG. 3. Energy per nucleon versus Fermi momentum for the Paris potential using different Λ approximations.

IV. NUMERICAL RESULTS AND DISCUSSION

A. General comparison

It is rather difficult to judge which Λ theory is “best” for the treatment of nuclear matter. One ought to select the $\Lambda^{(10)}$ or $\Lambda^{(11)}$ approximation, respectively, since several theoretical reasons favor these approaches. The main arguments in favor of the $\Lambda^{(10)}$ version are¹⁶ the following: (1) It is closer to the outcome of third-order perturbation theory than any other approach.¹² (2) The three-body corrections are small.²⁸ (3) The Bardeen-Cooper-Schrieffer results for superconductivity are reproduced exactly. In favor of the $\Lambda^{(11)}$ approximation are—besides the symmetric structure in the intermediate propagator—the thermodynamic consistency²⁹ and the inclusion of correlation contributions.^{8,30} The advantage of the $\Lambda^{(00)}$ and $\Lambda^{(G)}$ approximations lies in the much simpler numerical treatment. As we shall see later, the

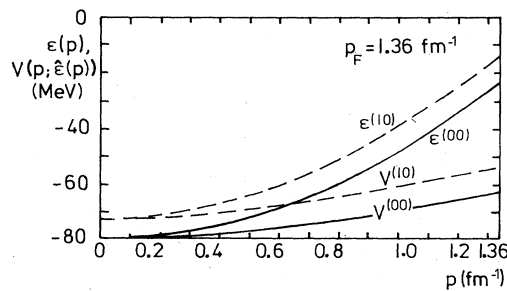


FIG. 4. Comparison of single-particle energies $\epsilon(p)$ and (on-shell) single-particle potentials $V(p; \hat{\epsilon}(p) \equiv \epsilon(p) - \mu$) for the RSC potential using $\Lambda^{(00)}$ and $\Lambda^{(10)}$ approximations.

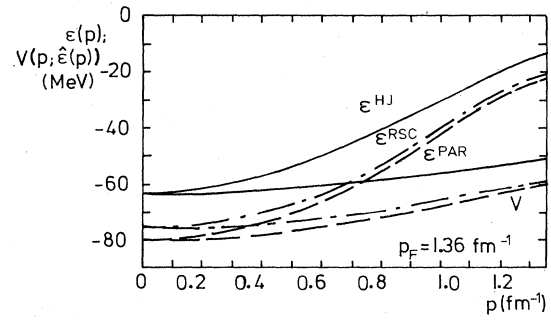


FIG. 5. Single-particle energies $\epsilon(p)$ and (on-shell) single-particle potentials $V(p; \hat{\epsilon}(p))$ for different nucleon-nucleon interactions in the $\Lambda^{(G)}$ approximation.

differences of nuclear matter properties between the Λ theories are not significant.

B. Results and comparison with Brueckner theory

For all approximations we calculated the nuclear matter parameters and the single-particle properties, i.e., real and imaginary parts of the self-energy, k and E mass, and the momentum distribution. The basic nuclear matter properties compared with a sample of Brueckner treatments are given in Table I. In Figs. 1–3 we exhibit the density dependence of the energy per particle.

The momentum dependence of the single-particle energies and the mass operator are shown in Figs. 4–7. Examples of the off-energy shell behavior of the mass operator are given in Fig. 8. Figures 9 and 10 show the k and E mass for the RSC and Paris potentials. The E mass is almost symmetric around the maximum located at the Fermi momentum indicating that both the polarization and correlation contributions⁵ are contained in the $\Lambda^{(11)}$ approximation,³⁰ since neglect of one contribution exhibits

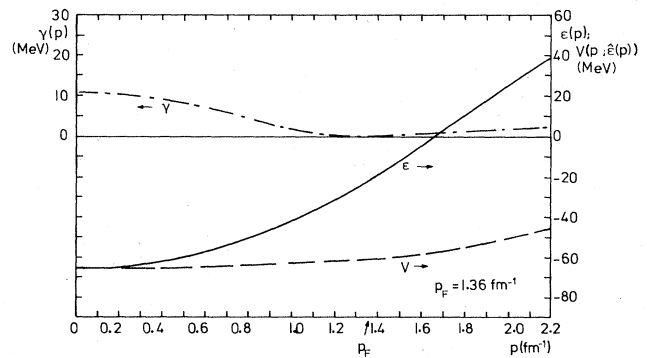


FIG. 6. Single-particle energies $\epsilon(p)$, single-particle potential $V(p; \hat{\epsilon}(p))$, and width $\gamma(p)$ for the HJ potential in the $\Lambda^{(11)}$ approximation.

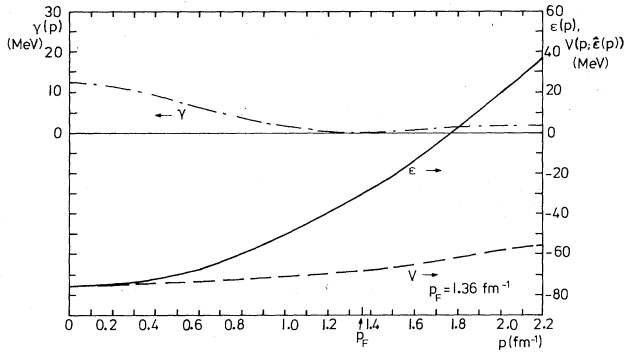
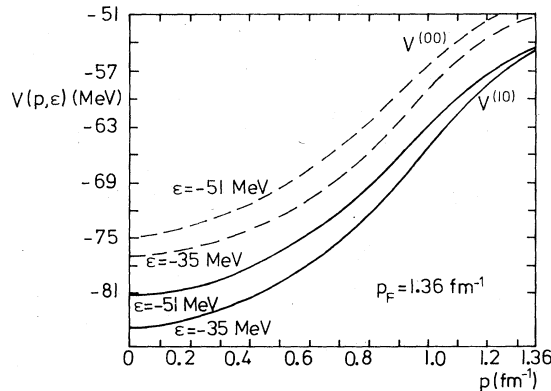
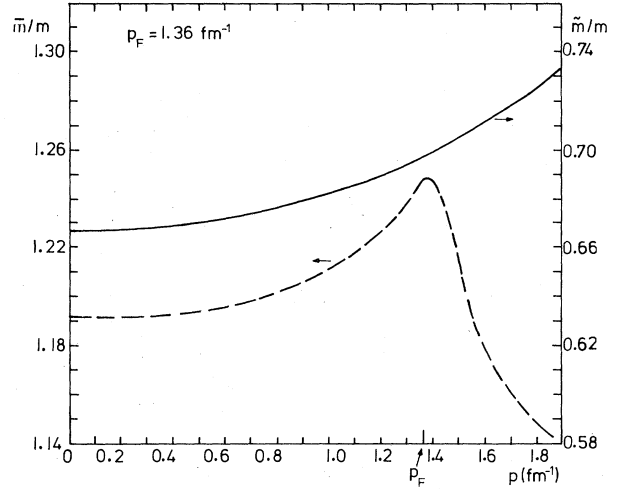


FIG. 7. Same as Fig. 6, but calculated for the Paris potential.

a typical asymmetric behavior with a shifted maximum.⁵ The momentum distribution differs from the Fermi gas distribution in a similar manner as in Ref. 5; the values at $p=0$ are smaller for the (hard-core) HJ potential (0.8) as compared with the ("soft") RSC potential (0.88). The $\Lambda^{(G)}$ version¹⁸—which has a poor theoretical justification [$\Lambda^{(G)}$ is obtained from $\Lambda^{(11)}$ by the replacement $\epsilon(p) \rightarrow p^2/2m$] and was treated for simplicity mainly—shows no significant difference compared with the other Λ approximations (see, for instance, Table I, Figs. 1–3, and 5–7). The main objection against the $\Lambda^{(G)}$ propagator in comparison with the $\Lambda^{(11)}$ propagator is its "wrong" energy dependence due to the neglect of the dressing of the particles in matter. The outcome, it seems, indicates that the ground state properties are not extremely sensitive with respect to the energy dependence of the intermediate propagator.

The overall comparison of the Green's function results with the Brueckner treatments exhibits no essential differences. Of course, in detail, there are still deviations. For

FIG. 8. Comparison of the off-shell mass operator $V(p; \hat{\epsilon}(p))$ for the RSC potential using $\Lambda^{(00)}$ and $\Lambda^{(10)}$ approximations.FIG. 9. k mass ($\tilde{m}/m$) (full curve) and E mass ($\bar{m}/m$) (dashed curve) versus momentum for the HJ potential in the $\Lambda^{(11)}$ approximation.

instance, the curvature of the single particle potential seems to be smaller in our treatment, and also the E -mass peak is less pronounced.⁵ However, such effects are difficult to compare, since in the latter case the calculation was done for the hard-core Reid potential. Therefore it seems fair to conclude that in the frame of the nonrelativistic potential models of nuclear matter, the Λ scheme of the G functions theory describes a useful alternative. It can be extended as well as the Brueckner theory^{5,36} by adding field-theoretical ingredients.

The numerical calculations were performed by utilizing matrix inversion for the correlated wave function. For the center of mass momentum we used the expressions given by Sprung.⁶ The numerical procedure was tested for the HJ potential by comparison with Ref. 16.

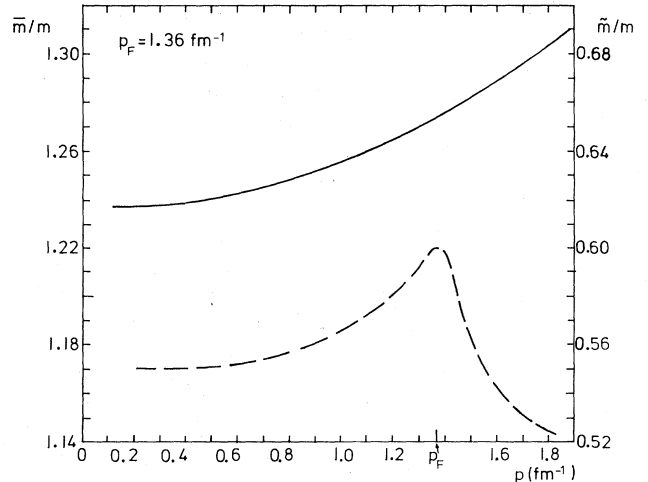


FIG. 10. Same as Fig. 9, but for the Paris potential.

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