NN-πNN equations and the chiral bag model

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(Received 26 July 1985)

The NN-πNN equations that describe, in a unified framework, pion production in nucleon-nucleon scattering, and pion-deuteron and nucleon-nucleon elastic scattering, have been extended to include the N(939) and Δ(1232) on an equal footing. This extension, motivated by the quark models of hadrons, has the bare N and Δ as three quark states with the same spacial wave function, but different spin isospin states. The final equations, referred to as the BB-πBB equations, are consistent with the chiral bag models to the extent that the πNN, πNΔ, and $\pi\Delta\Delta$ coupling constants and form factors are related, and can be taken from bag models. The resultant equations satisfy two- and three-body unitarity, and are derived by exposing the lowest unitarity cuts in the n-body Green's function. These equations retain important contributions missing from the NN-πNN equations. For pion production and N-N scattering they include the contribution of backward pions in the NN$\rightarrow$NΔ transition potential, which may overcome the problem of small pp$\rightarrow\pi$d cross section as predicted by the NN-πNN equations. For π-d elastic scattering they include an additional N$\Delta\rightarrow$NΔ tensor force that can influence the tensor polarization.

I. INTRODUCTION

It is well established that the constituents of nuclei are quarks and gluons whose dynamics are described by quantum chromodynamics (QCD). One therefore expects that a traditional description of nuclei in terms of nucleons, pions, . . . , should break down at some level. However, at the energies of intermediate energy nuclear physics, the need for quark degrees of freedom has not been unequivocally demonstrated as most failures of the traditional description can also be attributed to bad approximations, missing mechanisms, etc. Thus current research in this area has tended to separate into two different approaches. One has been the continual refinement of traditional models, the other has been in building quark models of nucleons. In this paper we attempt, in some sense, to amalgamate these two approaches by incorporating quark model ideas (originating from chiral bag models) into a multiple scattering theory for the NN-πNN system.

There has been a very strong interest in the NN-πNN system during the last few years. At intermediate energies, the observation of unusual structures in the measurement of N-N and π-d polarization observables has presented a challenge for theories that use conventional degrees of freedom (nucleons and mesons).[1] One of the most sophisticated of such theories has been the so-called unitary model,[2-7] named after the essential role that unitarity plays in the derivation of the equations. The fundamental input to the model is the (undressed) πNN vertex function f_0. Interactions are then generated through multiple pion exchanges and, typically, processes contributing to four- or more-body unitarity are either approximated with potentials or neglected. One is then able to obtain a simultaneous description of the processes

$$\begin{aligned} &\mathrm{N}+\mathrm{N}\rightarrow\pi+\mathrm{d}\ ,\\ &\pi+\mathrm{d}\rightarrow\pi+\mathrm{d}\ ,\\ \text{and}&\\ &\mathrm{N}+\mathrm{N}\rightarrow\mathrm{N}+\mathrm{N}\ , \end{aligned} \tag{1}$$

the amplitudes being given in terms of solutions of linear, coupled integral equations—we shall refer to them as the NN-πNN equations. The input into these equations is the πNN vertex function f_0, the π-N t matrix (with the nucleon-pole term removed), and the N-N t matrix below pion production threshold. An important feature of the equations is the fact that they obey two- and three-body unitarity and they effectively sum the whole multiple scattering series. Despite the sophistication of this model, a number of calculations[8-11] have indicated the inadequacy of the present model in describing, simultaneously, all the intermediate energy data that is now available for the reactions of Eq. (1). Unfortunately we cannot yet conclude that quark degrees of freedom are needed as some, possibly important, mechanisms are missing from the model. One of these is the "backward going pion" contribution to the NN$\rightarrow$NΔ one-pion exchange (OPE) potential, illustrated in Fig. 1. In the unitary model, the Δ is interpreted as arising from a multiple scattering of pions from a nucleon. Thus the diagram with the backward going pion [Fig. 1(b)] contributes to at least four-body unitarity and is neglected in the theory in order to obtain tractable equations. The neglect of this mechanism may be responsible for the small pp$\rightarrow\pi$d cross sections found in the latest calculations.[8-10,12] Similarly the OPE contribution to the N-Δ force, illustrated in Fig. 2, is missing from the model. This mechanism is expected to make a significant contribution to the tensor force and therefore

FIG. 1. Contributions to the one-pion exchange NN→NΔ potential. The forward going pion contribution (a) is included in the NN-πNN equations, while the backward going pion contribution (b) is not.

FIG. 3. The (bare) vertex functions f^0_{NN}, $f^0_{\Delta N}$, $f^0_{N\Delta}$, and $f^0_{\Delta\Delta}$ included in the cloudy bag model. We take these as the starting point in developing the BB-πBB equations.

to the tensor observables in πd→πd.[13]

Although we may argue that the best conventional calculation is not yet available, we cannot escape our conviction that the correct underlying description should take into account the quark degrees of freedom. It is with this in mind that we have endeavored to include some features of quark models while retaining the elegant multiple scattering formalism of the unitary model. Specifically, we appeal to the success of chiral bag models[14,15] in describing a baryon (B) in terms of confined quark wave functions interacting with an elementary pion field. One such model, the cloudy bag model (CMB) (Ref. 15) has had particular success at describing the static properties of the nucleon as well as π-N scattering in the P_{33} channel.[16,17] This model, discussed briefly in Sec. II, leads to the nucleon and delta being treated on an equal footing. This means that, before pionic renormalization, the nucleon and delta have related masses, wave functions, and πBB′ vertex functions (illustrated in Fig. 3), the only parameter being the bag radius. It is evident that π-N scattering in the P_{33} channel has contributions from both the delta-pole term [Fig. 4(a)] as well as multiple pion exchanges [Figs. 4(b) and (c)]. In fact it has been found[16] that the pole term provides the major contribution in the CBM. This is in stark contrast to what was assumed in the NN-πNN equations, namely that all the contribution comes from processes like that of Fig. 4(b). It is therefore tempting to start with the vertices of Fig. 3 in deriving the unitary model. By doing this, we can effectively describe that part of the scattering of two chiral bags that is due to pion exchange. At a later stage, it might be possible to include into the formalism the contribution of overlapping bags. There is another benefit from following this procedure in addition to the fact that it is consistent with a successful quark model. Since the nucleon and the "elementary" delta of Fig. 4(a) are included on an equal footing, we obtain contributions from all the BB→BB OPE [and two-pion exchange (TPE) (Ref. 18)] potentials. Thus, in the new formalism, both the backward going pion contribution to NN→ΔN, Fig. 1(b), and the N-Δ force of Fig. 2 are included for the case of an elementary delta. If we accept that the elementary delta dominates the contribution to the P_{33} resonance, then our new equations, which we shall call the BB-πBB equations,[18] will provide a way of including most of the major processes missing from the NN-πNN equations.

To derive the new equations, we use a diagrammatic method to classify all possible diagrams (contributing to a given process) according to their irreducibility.[19] Similar methods have been previously used to derive the NN-πNN equations.[3-5] Having specified the asymptotic states (Sec. III), the procedure initially involves the dressing of baryon propagators (Sec. IV) in order that we may work with diagrams having amputated legs. This is important as otherwise there would be disconnected diagrams contributing to off-shell processes, and in turn, this could lead to a disconnected kernel for our integral equations.

To classify our diagrams according to their irreducibility requires that we expose the $(n-1)$ particle unitarity cut before the n-particle cut. This is achieved in Secs. V and VI for the two- and three-body cuts, and results in expressions for the 2→2 and 2→3 amplitude. In a similar way we derive the 3→3 amplitude in Sec. VII. We can then write the two-body equation for B-B scattering and show that the potential includes not only one-pion exchange but also the full crossed two-pion exchange. In Sec. VIII we proceed to write a set of coupled linear integral equations, the BB-πBB equations, for the amplitudes corresponding to the reactions in Eq. (1). To get a computationally viable set of equations, we show in Sec. IX how the antisymmetry is included in the case where the π-B and B-B subamplitudes are separable. Finally in Sec. X we present our concluding remarks.

II. THE LAGRANGIAN

In this section we want to consider Lagrangians that describe a system of nucleons and pions, and that are consistent with the quark description of the nucleon. In particular, we will consider Lagrangians that arise from the imposition of chiral invariance on the MIT bag model.[20] Invariably, these involve the introduction of a massless pseudoscalar field often associated with the pion. This pseudoscalar field, which guarantees the continuity of the axial vector current at the bag surface, is coupled to the

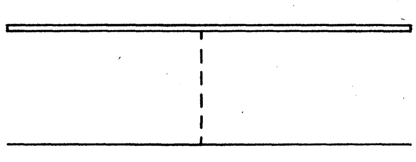

FIG. 2. The one-pion exchange contribution to the N-Δ force. It is neglected in the NN-πNN equations.

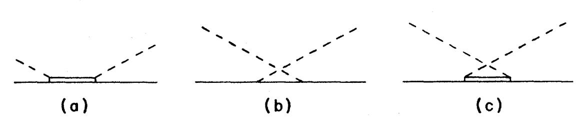

FIG. 4. Contributions to π-N scattering in the P_{33} channel from the delta-pole term (a) and multiple pion exchange (b) and (c).

quark field. The coupling, which is nonlinear in the pion field,[21] is not uniquely determined by the requirement of chiral invariance. The pion field in these models may[15] or may not[14] penetrate the region inside the bag,[22] depending on the particular model under consideration. Since our final results do not depend on the detailed form of the Lagrangian, we will only specify its general structure to be

$$L = L_{\rm MIT} + L_\pi + L_I \ , \qquad (2)$$

where $L_{\rm MIT}$ is the Lagrangian for the MIT bag,[20] and L_π is the Lagrangian for the free pion, which at a later stage might include the pion mass (the presence of the pion mass will break the chiral invariance of the Lagrangian). The interaction Lagrangian L_I is taken (at this stage) to be nonlinear in the pion field.

We can generate the corresponding Hamiltonian and then project onto the baryon space. This has been achieved in the case of the cloudy bag model[15,16] to get a static Hamiltonian for the π-N system of the form

$$H = H_0 + H_1 \ , \qquad (3)$$

where

$$H_0 = \sum_{\rm B} m_{\rm B}^{(0)} a_{\rm B}^\dagger a_{\rm B} + \int d^3k\, \omega(k) b_k^\dagger b_k \qquad (4)$$

and

$$H_1 = \sum_{\rm BB'} \int d^3k [v_{\rm BB'}(k) a_{\rm B}^\dagger a_{\rm B'} b_k + v_{\rm BB'}^\dagger(k) a_{\rm B} a_{\rm B'}^\dagger b_k^\dagger] \ . \qquad (5)$$

Here $a_{\rm B}^\dagger$ ($a_{\rm B}$) is the creation (annihilation) operator for the baryon B (B=N,Δ), and $m_{\rm B}^{(0)}$ is the bare mass for this baryon. In Eqs. (4) and (5) $b_k^\dagger$ (b_k) is the corresponding creation (annihilation) operator for the pion with $\omega(k) = (k^2 + m_\pi^2)^{1/2}$. The form factors $v_{\rm BB'}(k)$ for πB$\rightarrow$B are then given in terms of the quark wave function in the bag.

The above Hamiltonian is similar to that used by Chew-Low[23] for p-wave π-N scattering. One difference is the fact that the B$\rightarrow\pi$B form factors are present naturally in the Hamiltonian and are related to the internal quark structure of the baryon and size of the bag. The second and more important difference is the fact that the nucleon and Δ resonance in their bare form have identical spacial wave functions, and differ only in their spin-isospin wave function. As a result of this, their form factors are related. The final mass and form factor of the physical N and Δ are determined by renormalization and dressing.

There are several problems with the above Hamiltonian which one would like to overcome. First, the baryons are treated in the static approximation which might be valid for low energy pion scattering but not above the Δ resonance. More seriously is the fact that one cannot study nucleon-nucleon scattering in this approximation, not at 1 GeV. This problem may be overcome if one boosts the bag and its content when projecting onto the baryon space. This has been partly accomplished for the MIT bag,[24] and more recently for the soliton bag,[25] in calculating the electromagnetic properties of baryons. The second problem is the uncertainty in the choice of coupling between the pion and the quark. In particular do we have a surface coupling[16] or a volume coupling?[26] Since we need to truncate our interaction (if we are to have a renormalizable theory), the final form of the coupling might rely on the success of the numerical results for the different models. Finally, in projecting onto the baryon states, we have neglected the possible formation of six-quark configurations. At a later stage such configurations might be included by the addition of extra terms in the Hamiltonian.

III. CHOICE OF ASYMPTOTIC STATES

To discuss the coupling between the BB and πBB channels, we need to specify the quark structure of our asymptotic states in these two channels. For the BB part of the Hilbert space we take

$$|{\rm B}(1),{\rm B}'(2)\rangle = |(qqq){\rm B},(qqq){\rm B}'\rangle \ , \qquad (6)$$

where B,B$'$=N or Δ. In defining the above space we have not included exotic states such as $|(qqqq\bar{q}){\rm B},(qqq){\rm B}'\rangle$. We have also neglected the effect of antisymmetry between the quarks in the two bags. This antisymmetry is expected to be important when the bags overlap, or at short distances between the two baryons. Here again the domain of importance of this antisymmetry will depend on the chiral bag model used. If we restrict our baryon space to the N and Δ, then the baryon states are

$$|{\rm B}(1),{\rm B}'(2)\rangle = |a\rangle \ , \qquad (7)$$

where $a = 1, \ldots, 4$ refers to the sequential order NN, NΔ, ΔN, and $\Delta\Delta$. We now turn to the πBB part of our space. Here again we can write our space in terms of basic quark configurations such as

$$|{\rm B}(1),{\rm B}'(2),\pi(3)\rangle = |(qqq){\rm B},(qqq){\rm B}',\pi\rangle \ , \qquad (8)$$

where the pion is taken as an elementary field, i.e., Goldstone boson. Alternatively one could take the πBB part of the space as

$$|{\rm B}(1),{\rm B}'(2),\pi(3)\rangle = |(qqq){\rm B},(qqq){\rm B}',(q\bar{q})\pi\rangle \ , \qquad (9)$$

where the pion is taken as a $q\bar{q}$ pair. In this case the pion can achieve a size and its propagator can get dressed. We also can include the coupling to the BBm where m is any other meson (e.g., $\omega,\rho,\ldots$); however, we restrict our baryon to N or Δ and the mesons to the π. In this case our basis states are

$$|{\rm B}(1),{\rm B}'(2),\pi(3)\rangle = |L\rangle \ , \qquad (10)$$

and $L = 1, \ldots, 4$ in correspondence with Eq. (7). In the above basis all operators reduce to 4×4 matrices coupling the different channels in our BB and πBB space. There are three distinct operators defined in the above basis for which we need to write equations. They are

(a) $T^{(i)}$ the amplitude for BB$\rightarrow$BB,
(b) $F^{(i)}$ the amplitude for πBB$\rightarrow$BB, and
(c) $M^{(i)}$ the amplitude for πBB$\rightarrow\pi$BB.

Here the superscript (i) indicates the minimum number of pions in every intermediate state. The above operators are defined in the BB-πBB space. Our final equations will relate these operators to those defined in the π-B or B-B subspace of the πBB space. The corresponding operators in the subspace will be indicated by $t^{(i)}$, $f^{(i)}$, and $m^{(i)}$ and they correspond to BB→BB, πBB→B, and πB→πB amplitudes. These in turn should be defined in terms of the interaction in the chiral bag model.

IV. PROPAGATOR DRESSING

Before we can proceed to a full derivation of our equations, we need to carry out the dressing for the baryon propagators. For the present, we will not include any of the dressing for the pion propagators. Such dressings will involve the quark degrees of freedom, and these are not explicitly included at this stage.

The method used to derive our equations is based on the classification of all the diagrams contributing to a given amplitude, according to their irreducibility. The method was first developed on the basis of old fashioned time ordered field theory by Zachariasen,[27] and used by Thomas[28] to study the contribution of real absorption to π-d elastic scattering. More recently Thomas and Rinat[4] used the method for N-N scattering above the threshold for pion production, and Afnan and Blankleider[5] to derive the NN-πNN equations. A similar method based on covariant field theory has been developed by Taylor,[19] and used by Avishai and Mizutani[3] to derive the NN-πNN equations. Both methods give the same final equations for the NN-πNN system.

In the present paper we will follow the procedure used by Afnan and Blankleider,[5] yet maintain consistency with the basic tenet of the covariant formulation of Taylor.[19] We consider the Green's function, in momentum space, for the process with n initial momenta $p_1,\dots,p_n$, and l final momenta $q_1,\dots,q_l$. For the case of scalar particles, this is given by[29]

$$G^{(l,n)}(q_1,\dots,q_l,p_1,\dots,p_n)(2\pi)^4\delta^4(p_1+\cdots+p_n-q_1\cdots-q_l)$$
$$=\int d^4x_1\cdots d^4x_n d^4y_1\cdots d^4y_l \exp\left[i\sum_{k=1}^{l} q_k y_k - i\sum_{k=1}^{n} p_k x_k\right]\langle 0\,|\,T(\phi(y_1)\cdots\phi(y_l)\phi(x_1)\cdots\phi(x_n))\,|\,0\rangle\ , \tag{11}$$

where $\phi(x)$ is the field at the space-time point x. This Green's function can be taken as the sum of all topologically distinct connected diagrams that contribute to that process. The baryon propagators used to write a given diagram are initially undressed. We will show explicitly how this dressing is achieved. The corresponding amplitudes are obtained by employing Lehmann-Symanzik-Zimmermann (LSZ) (Ref. 29) reduction, i.e., taking the amplitude for a given reaction as the residue of the corresponding Green's function at the physical masses of all initial and final particles. For the case of scalar particles the S matrix is given by

$$\langle q_1,\dots,q_l;\text{out}\,|\,p_1,\dots,p_n;\text{in}\rangle=(2\pi)^4\delta^4(p_1+\cdots+p_n-q_1-\cdots-q_l)\langle q_1,\dots,q_l\,|\,T\,|\,p_1,\dots,p_n\rangle\ , \tag{12}$$

where the amplitude T is given in term of the Green's function

$$\langle q_1,\dots,q_l\,|\,T\,|\,p_1,\dots,p_n\rangle=(-iZ^{-1/2})^{l+n}\prod_{k=1}^{l}(q_k^2-m^2)G^{(l,n)}(q_1,\dots,q_l,p_1,\dots,p_n)\prod_{k=1}^{n}(p_k^2-m^2)\ . \tag{13}$$

Here Z is the wave function renormalization, and results from the fact that $\phi(x)$ is the interacting field. Since our initial and final baryons may be composite (e.g., the deuteron), we employ the generalization of LSZ to composite particles.[30]

To classify the diagrams that contribute to a given amplitude we need to introduce some basic definitions: (i) A k cut is an arc that separates initial from final states in a given diagram and cuts k particle lines with at least one internal line. An internal k cut is one that cuts internal lines only. In Fig. 5 we show a two- and three-particle cut. (ii) An amplitude is said to be r irreducible if all diagrams that contribute to the amplitude do not admit any k cuts with $k\le r$. (iii) The last cut lemma states that for a given amplitude that is $(r-1)$ irreducible there is a unique way of getting an internal r cut closest to either the initial or final state for all diagrams contributing to the amplitude. These definitions have previously been used by Taylor[19] for the three-body problem,[31] and by Avishai and Mizutani[3] for the NN-πNN problem.

To examine how the baryon propagator dressing arises, let us consider the B-B reaction

$$a+b\to c+d\ , \tag{14}$$

where $a,b,c,d=\text{N},\Delta,\dots$, are the bare particles. The corresponding amplitude is given by $\langle f(cd)\,|\,T\,|\,i(ab)\rangle$. All the connected diagrams that contribute to this amplitude can be divided into two classes:

(i) Those for which we can draw a self-energy contribution on one external leg (e.g., baryon a). These are of the general form

$$\langle f(cd)\,|\,T\,|\,i(ab)\rangle d_a^{(0)}\Sigma_a\ , \tag{15}$$

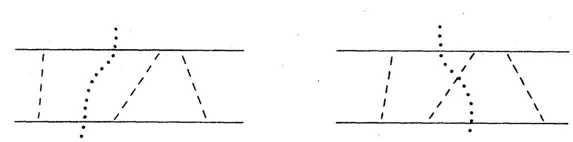

FIG. 5. Example of internal two- and three-particle cuts.

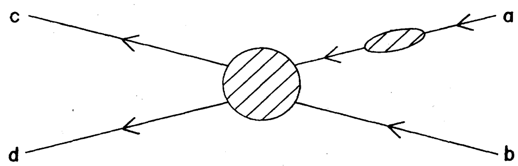

FIG. 6. Diagram that contributes to the initial state propagator dressing.

where $d_a^{(0)}$ and Σ_a are the undressed propagator [e.g., $d^{(0)}=(\not p-m_0)^{-1}$ for fermions] and the self-energy of baryon a. This contribution is represented by the diagram in Fig. 6.

(ii) The rest of the diagrams not included in the first class. These we denote by $\langle f(cd) \mid T \mid i(\hat{a}b)\rangle$. We now can write

$$\langle f(cd) \mid T \mid i(ab)\rangle = \langle f(cd) \mid T \mid i(ab)\rangle d_a^{(0)}\Sigma_a + \langle f(cd) \mid T \mid i(\hat{a}b)\rangle \ . \tag{16}$$

This result can be rewritten as

$$\langle f(cd) \mid T \mid i(ab)\rangle d_a^{(0)} = \langle f(cd) \mid T \mid i(\hat{a}b)\rangle d_a \ , \tag{17}$$

where d_a is the dressed propagator for the baryon a, and is given by

$$d_a = (d_a^{(0)-1} - \Sigma_a)^{-1} = Z_a d_a^R \ , \tag{18}$$

where d_a^R is the renormalized propagator with unit residue at the pole corresponding to the physical mass of baryon a. Here Z_a is the wave function renormalization constant for the corresponding baryon. We now can repeat this procedure for all the external legs to get,

$$Z_c^{1/2}Z_d^{1/2}d_c^R d_d^R \langle \hat{f}(cd) \mid T \mid \hat{i}(ab)\rangle d_a^R d_b^R Z_a^{1/2}Z_b^{1/2} = d_c^{(0)}d_d^{(0)}\langle f(cd) \mid T \mid i(ab)\rangle d_a^{(0)}d_b^{(0)} \ , \tag{19}$$

where the amplitude $\langle \hat{f}(cd) \mid T \mid \hat{i}(ab)\rangle$ has included in it a factor of $Z^{1/2}$ for each external leg in agreement with Eq. (13). Although we have applied this dressing procedure to the initial and final baryons, it can be extended to baryons in intermediate states. To show this, we take a two-particle cut of the diagrams belonging to $\langle \hat{f} \mid T \mid \hat{i}\rangle$ in such a way that the final result is of the form,

$$\langle \hat{f} \mid T \mid n(ab)\rangle d_a^{(0)}d_b^{(0)}\langle n(\hat{a}\hat{b}) \mid T \mid \hat{i}\rangle \ . \tag{20}$$

Using the result of Eq. (17) to the left-hand part of this expression will allow us to replace $\mid n(ab)\rangle d_a^{(0)}d_b^{(0)}$ by $\mid n(\hat{a}\hat{b})\rangle d_a d_b$. In this way we have completely replaced the undressed propagators by the dressed ones.

Although the amplitudes $\langle f \mid T \mid i\rangle$ and $\langle \hat{f} \mid T \mid \hat{i}\rangle$ are identical, up to factors of $Z_a^{1/2}$, on the mass shell (assuming zero mass shift in the renormalization), the latter has the added advantage that it leads to integral equations with a connected kernel. This is most clearly illustrated by the fact that any two-body cut of a diagram belonging to $\langle \hat{f} \mid T \mid \hat{i}\rangle$ leads to two connected diagrams. In the next section we will use this feature in conjunction with the last cut lemma to derive integral equations for $\langle \hat{f} \mid T \mid \hat{i}\rangle$. In deriving the above result we have assumed a covariant structure for our diagrams (i.e., the d's are Feynman propagators); however, the procedure is similar to the nonrelativistic formulation.[5] Although in the nonrelativistic case questions regarding the consistency of the renormalization procedure can be raised,[32] by working with d_a^R, the relativistic propagator with unit residue, this problem is overcome. To obtain three-dimensional equations we could carry out the Blankenbecler-Sugar[33] reduction on products of d_a^R.

The above procedure of replacing $d_a^{(0)}$ by d_a^R in the BB part of the Hilbert space can be extended to the πBB space. The degree to which this dressing is included will be determined by the level at which we truncate our final equations. In our case, this truncation is imposed by strict application of unitarity. In the πBB Hilbert space the dressing includes states with one more intermediate pion than is the case in the BB Hilbert space. Thus, if we need only two- and three-body unitarity, then the baryon in the BB Hilbert space will have dressing that contributes to three-body unitarity; however, in the πBB Hilbert space the lowest baryon dressing contributes to four-body unitarity and might therefore be neglected.

In the lowest order to the MIT bag model, the nucleon and Δ have the same mass. The fact that the one-gluon exchange is spin dependent explains some of the observed mass splitting. However, for the Δ to acquire a width for the decay into π-N one needs to carry out the dressing. In this way Δ production is more of a virtual process. From this point on, all our baryon propagators are dressed and have unit residue. These will be denoted by d_a to simplify the notation. Similarly, the amplitude $\langle \hat{f} \mid T \mid \hat{i}\rangle$ will be written as $\langle f \mid T \mid i\rangle$ without the hats.

V. TWO-BODY UNITARITY

To expose n-particle unitarity we need to use the last-cut lemma on the $(n-1)$ irreducible class of diagrams. Thus the logical starting point is to take the lowest irreducible amplitude for a given reaction. For B-B scattering we commence with the class of one-particle irreducible diagrams. Since we have baryon number conservation, and we will not include any antibaryons, we will label our irreducible class of diagrams by the number of mesons in intermediate states. We now can divide the diagrams that contribute to BB$\rightarrow$BB into two classes.

(i) Those with at least one pion in every intermediate state (i.e., two-particle irreducible). These we denote by $T^{(1)}$.

(ii) Those diagrams not belonging to (i). These are two-particle reducible, and can be written, using the last-cut lemma, as

$$T^{(1)}G^{(0)}T^{(0)} = T^{(0)}G^{(0)}T^{(1)} \ .$$

Here $G^{(0)}$ is a 4×4 diagonal matrix of BB propagators, i.e., the diagonal elements are of the form $d_a d_b$ $(a,b=\text{N},\Delta)$. We observe in passing that $G^{(0)}$ is one-particle irreducible, and has zero number of pions, which is what the superscript indicates. Because our baryon propagators are dressed, both $T^{(1)}$ and $T^{(0)}$ have contribution from connected diagrams only. We now can write a set of coupled integral equations for B-B scattering of the form,

$$T^{(0)}_{l,m}=T^{(1)}_{l,m}+\sum_n T^{(1)}_{l,n}G^{(0)}_n T^{(0)}_{n,m}$$
$$=T^{(1)}_{l,m}+\sum_n T^{(0)}_{l,n}G^{(0)}_n T^{(1)}_{n,m}\ , \tag{21}$$

where $T^{(i)}_{l,m}=\langle l\,|\,T^{(i)}\,|\,m\,\rangle$. In matrix form, the above equations are

$$T^{(0)}=T^{(1)}+T^{(1)}G^{(0)}T^{(0)}$$
$$=T^{(1)}+T^{(0)}G^{(0)}T^{(1)}\ . \tag{22}$$

By defining the 2×2 matrices

$$d(i)=\begin{bmatrix} d_{\mathrm{N}}(i) & 0 \\ 0 & d_\Delta(i)\end{bmatrix}\ , \tag{23}$$

we may write the 4×4 matrix $G^{(0)}$ as

$$G^{(0)}=d(1)\otimes d(2)\ , \tag{24}$$

where $\otimes$ is the direct product. The use of this direct product allows us to relate the 4×4 matrices of the BB-πBB Hilbert space to the 2×2 matrices of the π-B Hilbert space (see Appendix B).

Below the threshold for pion production, $T^{(1)}$ is a real function that plays the role of a potential. Above the pion production threshold, $T^{(1)}$ is no longer real and includes information about inelasticity—particularly the BB→πBB channel. In addition to this source of inelasticity we have contribution to three-body unitarity through $G^{(0)}$. This arises from the fact that our baryon propagators have been dressed. At medium energies (below the two-pion production threshold) the contribution of $G^{(0)}$ to three-body unitarity is most important in this model as it incorporates the dressing of the Δ, and this is the Mandelstam mechanism[34] for pion production, i.e., the isobar model. At this stage we have completed the requirement of dressing for the baryon propagator $G^{(0)}$. The only restriction is that this dressing be consistent with that for π-B scattering as discussed in Appendix A.

Returning to $T^{(1)}$, we observe that in lowest order, $T^{(1)}$ is the one-pion exchange potential. Extending our meson multiplet to SU(3) will allow us to include K-meson exchange, and thus extend N-N scattering to the full baryon octet. On the other hand, the inclusion of the vector meson multiplet will extend the one-pion exchange to include the ρ and ω exchange (i.e, $T^{(1)}$ will be the one-boson exchange potential). If we restrict $T^{(1)}$ to one-pion exchange, then in Eq. (21) we have a Bethe-Salpeter equation in the ladder approximation. We note that the propagators in this Bethe-Salpeter equation are dressed and this dressing is important above the one-pion production threshold since it includes the isobar model.

VI. THREE-BODY UNITARITY

To establish the coupling to the π-d channel we need to examine $T^{(1)}$ beyond the lowest order. In particular we need to go beyond one-pion intermediate states to get the N-N interaction and thus the deuteron in the π-d channel. To achieve this we have to examine the three-body unitarity structure of $T^{(1)}$.

The diagrams that contribute to $T^{(1)}$, which includes all connected diagrams with at least one pion in every intermediate state (i.e., two-particle irreducible), can be divided into two classes:

(i) Those with at least two pions in every intermediate state, i.e., three-particle irreducible diagrams. These we denote by $T^{(2)}$ (it includes only connected diagrams). Here we should point out that $T^{(2)}$ is nonzero only in a theory with an interaction Lagrangian that is nonlinear in the pion field. In such a theory one can get contributions to $T^{(2)}$ from diagrams which we might want to approximate by ρ exchange (see Fig. 7), that is, if ρ exchange is not included by extending the meson multiplet to include the vector mesons.

(ii) Those diagrams not included in (i). These are three-particle reducible, or have intermediate states with at least one pion, and can be written as

$$(F^{(2)}G^{(1)}F^{(1)\dagger})_c=(F^{(1)}G^{(1)}F^{(2)\dagger})_c\ . \tag{25}$$

Here the subscript c implies that only connected diagrams are included. However, the $F^{(i)}$ do include disconnected contributions as illustrated in Fig. 8. The πBB propagator $G^{(1)}$ is, in our basis, a 4×4 matrix that is diagonal with elements

$$\langle L\,|\,G^{(1)}\,|\,L\,\rangle=d_a(1)d_b(2)d_\pi\quad (a,b=\mathrm{N},\Delta) \tag{26a}$$

or

$$G^{(1)}=[d(1)\otimes d(2)]d_\pi\ . \tag{26b}$$

We now can write $T^{(1)}$ as,

$$T^{(1)}=T^{(2)}+(F^{(2)}G^{(1)}F^{(1)\dagger})_c$$
$$=T^{(2)}+(F^{(1)}G^{(1)}F^{(2)\dagger})_c\ . \tag{27}$$

To obtain a set of coupled equations for the BB-πBB system, we need to examine the amplitude for πBB→BB, i.e., $F^{(1)}$. The amplitude $F^{(i)}$ included in Eq. (27) has a connected and disconnected part and can be written as

$$F^{(i)}=F^{(i)}_c+F^{(i)}_d \tag{28}$$

with the disconnected contribution given by

$$F^{(i)}_d=f^{(i)}(1)\otimes d^{-1}(2)+d^{-1}(1)\otimes f^{(i)}(2)$$
$$=\sum_{jk}{}' f^{(i)}(j)\otimes d^{-1}(k)\bar{\delta}_{jk}\ , \tag{29a}$$
$$=\sum_j F^{(i)}_d(j)\ , \tag{29b}$$

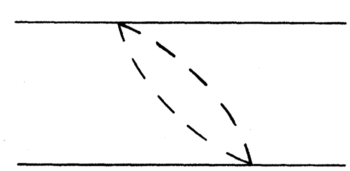

FIG. 7. Example of a diagram, contributing to $T^{(2)}$, that arises from nonlinear terms in the interaction Lagrangian.

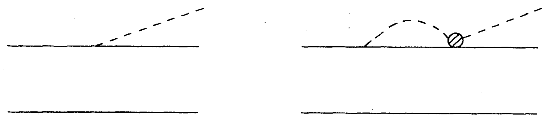

FIG. 8. Example of disconnected diagrams that contribute to $F_d^{(1)}$.

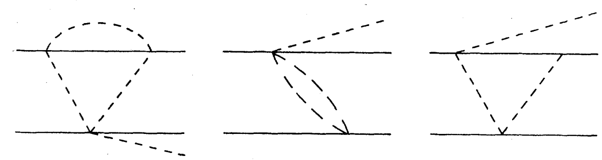

FIG. 9. Example of diagrams that contribute to $F_c^{(2)}$ due to the nonlinearity of the interaction Lagrangian in the pion field.

where $\overline{\delta}_{jk}=1-\delta_{jk}$ and j,k run over the two baryons. The prime on the sum indicates that we have to maintain the order of the factors in the direct product. In Eq. (29) we have the disconnected part of $F^{(i)}$ given in terms of more basic πB$\rightarrow$B form factors $f^{(i)}$. The structure of $f^{(1)}$ is given in Appendix A, viz.,

$$f^{(1)}(i)=f^{(2)}(i)+f^{(2)}(i)g^{(1)}(i)m^{(1)}(i)\ . \tag{30}$$

Here $m^{(1)}$ is that part of the π-B amplitude that is one-particle irreducible. We now can write $F_d^{(1)}$ as (see Appendix B)

$$F_d^{(1)}=F_d^{(2)}+(F_d^{(2)}G^{(1)}M_d^{(1)})_d\ , \tag{31}$$

where $M_d^{(1)}$ is the disconnected part of the πBB$\rightarrow\pi$BB amplitude, and is given in terms of the more basic π-B and B-B amplitudes $m^{(1)}(3)$ and $t^{(0)}(i)$, respectively, i.e.,

$$M_d^{(1)}=\sum_{ij}{}' m^{(1)}(i)\otimes d^{-1}(j)\overline{\delta}_{ij}+t^{(0)}(3)d_\pi^{-1}\ , \tag{32a}$$

$$=\sum_\alpha M_d^{(1)}(\alpha)\ , \tag{32b}$$

where $m^{(1)}(i)$ gives the amplitude for the pion interacting with baryon i. We observe that the second term on the r.h.s. of Eq. (32a) does not contribute to the disconnected part $(F_d^{(2)}G^{(1)}M_d^{(1)})_d$. At this stage we note that $t^{(0)}(3)$, the B-B amplitude, is in lowest order just one-pion exchange. Furthermore, had we assumed the B-B amplitude $t^{(0)}(3)$, involved in the definition of $M_d^{(1)}$, to be identical to $T^{(0)}$, given in Eq. (22), we would have a bootstrap problem and the resultant equations would be nonlinear in $T^{(0)}$. The justification for breaking this bootstrap situation is that the energy at which we need the B-B amplitude $t^{(0)}(3)$, in the πBB Hilbert space, is at least m_π less than in $T^{(0)}$, i.e., $t^{(0)}(3)=T^{(0)}[s^{1/2}-\omega(p)]$ with $\omega(p)=(p^2+m_\pi^2)^{1/2}$. Thus, at medium energies we need the B-B amplitude predominantly below the pion production threshold. This we can describe by a real potential.

Turning to the connected part of the πBB$\rightarrow$BB amplitude, $F_c^{(1)}$, we can use our classification scheme for the diagrams in conjunction with the last-cut-lemma to obtain

$$F_c^{(1)}=F_c^{(2)}+(F^{(2)}G^{(1)}M^{(1)})_c\ . \tag{33}$$

Here $F_c^{(2)}$ gets contributions from diagrams that arise as a result of nonlinearity of the interaction Lagrangian in the pion field. This is clear from the fact that we need to go from an intermediate state of two pions to zero pions. Examples of such diagrams are illustrated in Fig. 9. Combining this result with that of Eq. (31) we can write the two-particle irreducible amplitude for πBB$\rightarrow$BB as

$$F^{(1)}=F^{(2)}+F^{(2)}G^{(1)}M^{(1)}\ . \tag{34}$$

The three-particle irreducible amplitude $F^{(2)}$ consists of two parts: (i) A disconnected part given by Eq. (29), and related to the bare πB$\rightarrow$B form factor as given by the chiral bag model. (ii) A connected part $F_c^{(2)}$ which arises from the nonlinearity of the interaction Lagrangian in the pion field. In Eqs. (33) and (34), $M^{(1)}$ is the πBB$\rightarrow\pi$BB amplitude, and also includes both the connected and disconnected parts, i.e.,

$$M^{(1)}=M_d^{(1)}+M_c^{(1)}\ . \tag{35}$$

We will show in the next section that this amplitude is in fact the Faddeev amplitude for the πBB system, if the contribution of four-body unitarity is neglected.

With the above result we can write the B-B potential $T^{(1)}$ in terms of the 3$\rightarrow$3 amplitude $M^{(1)}$ as

$$T^{(1)}=T^{(2)}+(F^{(2)}G^{(1)}F^{(2)\dagger})_c+(F^{(2)}G^{(1)}M^{(1)}G^{(1)}F^{(2)\dagger})_c\ . \tag{36}$$

Here we note that the procedure of dressing all external and internal baryon propagators, before applying the last-cut-lemma to $T^{(1)}$, was to guarantee the connectedness of $T^{(1)}$.

VII. THE 3$\rightarrow$3 AMPLITUDE

To complete the definition of the B-B amplitude as given in Eq. (36), we need to get an equation for the connected part of $M_c^{(1)}$. Since all the diagrams that contribute to $M_c^{(1)}$ are by definition connected, we can employ the last-cut-lemma to write

$$\begin{aligned}M_c^{(1)}&=M_c^{(2)}+M_c^{(2)}G^{(1)}M_d^{(1)}+M_d^{(2)}G^{(1)}M_c^{(1)}\\&\quad+M_c^{(2)}G^{(1)}M_c^{(1)}+(M_d^{(2)}G^{(1)}M_d^{(1)})_c\\&=M_c^{(2)}+(M^{(2)}G^{(1)}M^{(1)})_c\\&=M_c^{(2)}+(M^{(1)}G^{(1)}M^{(2)})_c\ .\end{aligned} \tag{37}$$

On the other hand the disconnected part of $M^{(1)}$ is given by

$$\begin{aligned}M_d^{(1)}&=M_d^{(2)}+(M_d^{(2)}G^{(1)}M_d^{(1)})_d\\&=M_d^{(2)}+(M_d^{(1)}G^{(1)}M_d^{(2)})_d\ ,\end{aligned} \tag{38}$$

which results from taking the π-B amplitude $m(i)$ $i=1,2$ and the B-B amplitude $t^{(0)}(3)$ to satisfy the two-body equations. Combining the results of Eqs. (37) and (38) we get

$$\begin{aligned}M^{(1)}&=M^{(2)}+M^{(1)}G^{(1)}M^{(2)}\\&=M^{(2)}+M^{(2)}G^{(1)}M^{(1)}\ .\end{aligned} \tag{39}$$

This is basically the Lippmann-Schwinger equation for the three-body πBB system where the potential term $M^{(2)}$ is given by

$$M^{(2)}=M_c^{(2)}+\sum_{ij}{}' m^{(2)}(i)\otimes d^{-1}(j)\bar{\delta}_{ij}+t^{(1)}(3)d_\pi^{-1} \ . \quad (40)$$

The connected part $M_c^{(2)}$ plays the role of a three-body force and gets contributions from diagrams as those illustrated in Fig. 10. In Fig. 10(a) we have a diagram that arises from the terms in the interaction Lagrangian that are nonlinear in the pion field, while the diagram in Fig. 10(b) is present if the Lagrangian has terms linear in the pion field (e.g., the cloudy bag model with a surface coupling). The disconnected part of $M^{(2)}$ plays the role of the π-B and B-B interaction in the subsystem. The part involving the 2×2 π-B amplitude matrix $m^{(2)}$ gets contributions from diagrams such as that shown in Fig. 11(a). The part involving the 4×4 amplitude matrix $t^{(1)}$ includes one-pion exchange in lowest order as illustrated in Fig. 11(b).

Since we will not be investigating the four-body unitarity structure of our amplitudes, we can replace the three-particle irreducible amplitudes $M^{(2)}$ by potentials. Furthermore, to establish the connection with Faddeev-type analysis, we will use spectator particle notation, i.e.,

$$M^{(2)}=\sum_{\alpha=0}^{4} V_\alpha \ , \quad (41)$$

with

$$\begin{aligned} V_\alpha &=0 \ \text{ for } \ \alpha=0 \\ &=\sum_{j=1}^{2}{}' m^{(2)}(j)\otimes d^{-1}(i)\bar{\delta}_{ij} \ \text{ for } \ \alpha=i=1,2 \\ &=t^{(1)}(3)d_\pi^{-1} \ \text{ for } \ \alpha=3 \\ &=M_c^{(2)} \ \text{ for } \ \alpha=4 \ . \end{aligned} \quad (42)$$

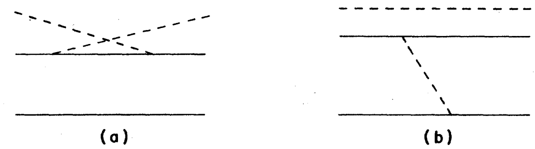

FIG. 11. The disconnected part of M has contributions from the π-B (a) and B-B (b) interaction.

In this case Eq. (39) is just the equation for the three-body problem where all operators are 4×4 matrices in our Hilbert space. We could continue our analysis with the inclusion of three-body forces; however, for practical calculations we will have to drop any contribution from V_4. In that case $M^{(1)}$, as given in Eq. (39), can be written as

$$\begin{aligned} M^{(1)} &=\sum_{\alpha\beta} M_{\alpha\beta} \\ &=\sum_{\alpha\beta}\left[M_d^{(1)}(\alpha)\delta_{\alpha\beta}+M_d^{(1)}(\alpha)G^{(1)}U_{\alpha\beta}^{(1)}G^{(1)}M_d^{(1)}(\beta)\right] , \end{aligned} \quad (43)$$

where the Alt, Grassberger, and Sandhas (AGS) (Ref. 35) amplitudes $U_{\alpha\beta}$ are 4×4 matrices and satisfy the equations

$$U_{\alpha\beta}^{(1)}=\bar{\delta}_{\alpha\beta}G^{(1)-1}+\sum_{\gamma}\bar{\delta}_{\alpha\gamma}M_d^{(1)}(\gamma)G^{(1)}U_{\gamma\beta}^{(1)} \ , \quad (44a)$$

$$=\bar{\delta}_{\alpha\beta}G^{(1)-1}+\sum_{\gamma}U_{\alpha\gamma}^{(1)}G^{(1)}M_d^{(1)}(\gamma)\bar{\delta}_{\gamma\beta} \ . \quad (44b)$$

Using the results of Eqs. (43) and (44) we can rewrite the B-B potential in terms of the dressed $\pi B\rightarrow B$ form factor $f^{(1)}$ or $F_d^{(1)}$. After some algebra we get

$$\begin{aligned} T^{(1)}=T^{(2)} &+\sum_{ij}F_d^{(1)}(i)\bar{\delta}_{ij}G^{(1)}F_d^{(1)\dagger}(j)+\sum_{\alpha\beta}\sum_{ij}F_d^{(1)}(i)\bar{\delta}_{i\alpha}G^{(1)}M_{\alpha\beta}^{(1)}G^{(1)}\bar{\delta}_{\beta j}F_d^{(1)\dagger}(j) \\ &+F_d^{(1)}G^{(1)}F_c^{(2)\dagger}+F_c^{(2)}G^{(1)}F_d^{(1)\dagger}+F_c^{(2)}G^{(1)}F_c^{(2)\dagger}+\sum_{\alpha\beta}\sum_{i}F_d^{(1)}(i)\bar{\delta}_{i\alpha}G^{(1)}M_{\alpha\beta}^{(1)}G^{(1)}F_c^{(2)\dagger} \\ &+\sum_{\alpha\beta}\sum_{j}F_c^{(2)}G^{(1)}M_{\alpha\beta}^{(1)}G^{(1)}\bar{\delta}_{\beta j}F_d^{(1)\dagger}(j)+F_c^{(2)}G^{(1)}M^{(1)}G^{(1)}F_c^{(2)\dagger} \ , \end{aligned} \quad (45)$$

where $i,j=1,2$ and $\alpha,\beta=1,2,3$. Within the framework of the cloudy bag model[15,16] with a surface coupling, the only nonzero contributions are from the second and third terms on the r.h.s. of Eq. (45), i.e., we have to take $F_c^{(2)}$ and $T^{(2)}$ to be zero. In this case Eq. (45) simplifies to

$$T_{\rm CBM}^{(1)}=\sum_{ij}F_d^{(1)}(i)\bar{\delta}_{ij}G^{(1)}F_d^{(1)\dagger}(j)+\sum_{\alpha\beta}\sum_{ij}F_d^{(1)}(i)\bar{\delta}_{i\alpha}G^{(1)}M_{\alpha\beta}^{(1)}G^{(1)}\bar{\delta}_{\beta j}F_d^{(1)\dagger}(j) \ . \quad (46)$$

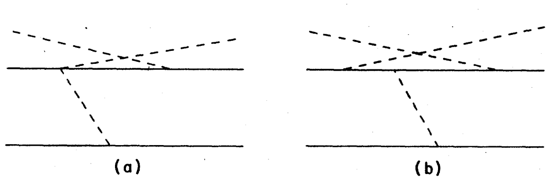

FIG. 10. Examples of contributions to $M_c^{(2)}$ from the nonlinear (a) and linear (b) terms in the pion field.

Here, the first term is the one-pion exchange potential with the added feature that the πBB vertex is dressed, and thus is energy and momentum dependent. This dressing removes the need for any arbitrary subtractions or cutoffs commonly used in one-boson exchange potentials. The form factors $f^{(1)}$ or $F_d^{(1)}$ are determined by the π-B scattering data and the intrinsic properties of the baryons. In particular, the rms radius and the charge form factor of the proton may be used to constrain $f^{(1)}$ off shell. The second term on the r.h.s. of Eq. (46) includes the crossed two-pion exchange mechanism as well as the multiple scattering of the pion off the two baryons. To see how the two-pion exchange comes in[18] let us consider the lowest order contribution to the second term on the r.h.s. of Eq. (46), namely,

$$\sum_{\alpha}\sum_{ij} F_d^{(1)}(i)\bar{\delta}_{i\alpha}G^{(1)}M_d^{(1)}(\alpha)G^{(1)}\bar{\delta}_{\alpha j}F_d^{(1)\dagger}(j)\ ,$$

which is represented diagrammatically in Fig. 12. If we recall that $M_d^{(1)}(3)=t^{(0)}(3)d_\pi^{-1}$ has for its lowest order the one-pion exchange amplitude, then in Fig. 12(a) for $i\neq j$ we have a crossed two-pion exchange contribution to N-N scattering. These are normally represented by the time ordered diagrams as in Fig. 13. Here, the first diagram was present in the original NN-πNN equations[2–7] while the additional three diagrams are the result of treating the N and Δ on equal footing. The other time ordered crossed two-pion exchange diagrams are included in Fig. 12(b), and are the result of the fact that the π-B amplitude $m^{(i)}$ $(i=1,2)$, in lowest order, includes the crossed pion diagrams of Figs. 4(b) and (c).

The advantage of the above method of including the two-pion exchange is that: (a) The Δ has been properly dressed (i.e., the equations satisfy three-body unitarity), and thus the threshold for pion production in N-N scattering via the Δ is properly included. (b) The P_{33} amplitude is adjusted to fit the π-N data, and thus the threshold has the proper energy dependence. In this way we have included the dominant mechanism for pion production with the correct energy dependence. Finally, by including the N and Δ on equal footing we have included most of the one- and two-pion exchange diagrams included in the Bonn potential.[39]

Since we have already included the Δ as an explicit channel, the contribution from Fig. 12(b) is due to the smaller π-N partial waves, and the nonpole part of the amplitudes in the P_{33} and P_{11} channels. Since these amplitudes are in general small, we do not expect a large contribution from these diagrams. However, when the coupling to the π-d channel is included, these diagrams give rise to multiple scattering of the pion by the two nucleons. Furthermore, Fig. 12(a) gives the coupling of the N-N channel to the π-d channel, which leads to true absorption in π-d elastic scattering.

VIII. THE BB-πBB EQUATIONS

So far we have concentrated on the BB sector of the problem. In particular we have examined the baryon-baryon potential $T^{(1)}$, and the coupling of the BB channel to intermediate πBB states. In its present form, the potential $T^{(1)}$ is written to explicitly illustrate the different mechanisms that contribute to its overall strength. In this way, we can compare the content of this potential with other descriptions of N-N scattering above the pion production threshold.[36–40] From a computational point of view, it is more advantageous to write a set of coupled equations for all the physical $2\to 2$ amplitudes. To achieve this aim, we need to examine the other $2\to 2$ channels, i.e., BB$\to\pi$d and π-d elastic scattering.

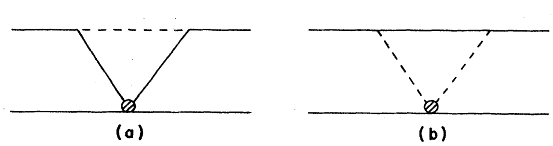

FIG. 12. The baryon-baryon (a) and pion-baryon (b) multiple scattering diagrams that give rise to the full two-pion exchange potential in our model.

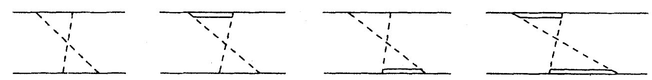

FIG. 13. The contribution of the crossed two-pion exchange to the N-N potential. These arise by replacing the N-N amplitude in Fig. 12(a) by OPE.

Consider the class of all diagrams that are connected and contribute to the process BB$\to\pi$BB. These can be divided into two groups:

(i) Those that are two-particle irreducible, and thus have at least one pion in every intermediate state. These we denote by $F_c^{(1)}$.

(ii) Those diagrams that are two-particle reducible. These can be written using the last cut lemma as $F^{(1)\dagger}G^{(0)}T^{(0)}$.

The total amplitude for pion production can now be written as

$$F_c^{(0)\dagger}=F_d^{(1)\dagger}G^{(0)}T^{(0)}+F_c^{(1)\dagger}(1+G^{(0)}T^{(0)})\ , \tag{47}$$

where $F_c^{(1)}$ is given in Eq. (33) in terms of the bare B$\to\pi$B form factor.

For the present analysis, we will assume the interaction Lagrangian is linear in the pion field, and that three-body forces can be ignored. In this case, we can rewrite $F_c^{(1)}$ in terms of the dressed B$\to\pi$B form factor $F_d^{(1)}$ as

$$F_c^{(1)}=\sum_{i\beta} F_d^{(1)}(i)G^{(1)}U_{i\beta}^{(1)}G^{(1)}M_d^{(1)}(\beta)\ . \tag{48}$$

In writing Eq. (48), we have made use of Eqs. (43) and (44) to iterate Eq. (33) and thus complete the dressing. We now can write Eq. (47) as

$$F_c^{(0)\dagger}=F_d^{(1)\dagger}G^{(0)}T^{(0)} +\sum_{\beta i} M_d^{(1)}(\beta)G^{(1)}U_{\beta i}^{(1)}G^{(1)}F_d^{(1)\dagger}(i)(1+G^{(0)}T^{(0)})\ . \tag{49}$$

To get the $2\to 2$ amplitudes for the reactions

$$\text{BB}\to\pi+(\text{BB})$$
$$\to\text{B}+(\pi\text{B})$$

we need to take the left-hand residue of the above amplitude at the (BB) or (πB) poles. In Appendix C we discuss the bound state wave function for the B-B system and in particular we show that

$$M_d^{(1)}(3)\equiv t^{(0)}(3)d_\pi^{-1}$$
$$\underset{D\to 0}{\simeq} d_\pi^{-1}\,|\,\phi(3)\rangle\,|\,\chi_\pi\rangle\frac{1}{D}\langle\chi_\pi\,|\,\langle\phi(3)\,|\,d_\pi^{-1}\ , \tag{50}$$

where $|\,\phi(3)\rangle$ is a column matrix of antisymmetric B-B form factors related to the bound state wave function $|\,\psi\rangle_d$, $|\,\chi_\pi\rangle$ is the wave function for a free pion, and D is a function that goes to zero as kinematical variables approach the on-shell values specifying the state $|\,\psi\rangle_d\,|\,\chi_\pi\rangle$. Using Eq. (50) in Eq. (49) and taking residues at the (BB) pole, we obtain an expression for the physical BB$\to\pi+$(BB) amplitude $X_{3\mathrm{B}}^{\text{physical}}$ of the form

$$X_{3\mathrm{B}}^{\text{physical}}=\langle\phi(3)\,|\,\langle\chi_\pi\,|\,d_\pi^{-1}G^{(1)}T_{3\mathrm{B}}\,|\,\psi\rangle\ , \tag{51}$$

where $|\,\psi\rangle$ is a column vector specifying the initial B-B wave function. In practice the only such physical channel will be NN$\to\pi$d for which

$$|\,\phi(3)\rangle=(\,|\,\Phi_{\mathrm{NN}}\rangle\ 0\ 0\ \ |\,\Phi_{\Delta\Delta}\rangle)^T$$

and

$$|\,\psi\rangle=(\,|\,\psi_{\mathrm{NN}}\rangle\ 0\ 0\ 0)^T\ .$$

Similar expressions give the BB$\to$B + (πB) amplitudes $X_{1\mathrm{B}}^{\text{physical}}$ and $X_{2\mathrm{B}}^{\text{physical}}$ in terms of the operators $T_{\lambda\mathrm{B}}$ where

$$T_{\lambda\mathrm{B}}=\sum_i U_{\lambda i}^{(1)}G^{(1)}F_d^{(1)\dagger}(i)(1+G^{(0)}T^{(0)})\ . \tag{52}$$

We now need to write a set of coupled equations for the amplitudes $T_{\lambda\mathrm{B}}$ and $T_{\mathrm{BB}}=T^{(0)}$ for the reactions

$$\begin{aligned}\mathrm{B}+\mathrm{B}&\to\mathrm{B}+\mathrm{B}\\ &\to\pi+(\mathrm{BB})\\ &\to\mathrm{B}+(\pi\mathrm{B})\ .\end{aligned} \tag{53}$$

This is achieved by iterating Eq. (49) using the AGS equations [Eq. (44)] to obtain

$$T_{\lambda\mathrm{B}}=\sum_i\bar{\delta}_{\lambda i}F_d^{(1)\dagger}(i)(1+G^{(0)}T_{\mathrm{BB}})$$
$$+\sum_\gamma\bar{\delta}_{\lambda\gamma}M_d^{(1)}(\gamma)G^{(1)}T_{\gamma\mathrm{B}}\ . \tag{54a}$$

To close the set of coupled equations, we have to make use of the BB potential as given in Eq. (46), in conjunction with Eqs. (22) and (52), to get

$$\mathrm{T}_{\mathrm{BB}}=V_{\mathrm{OPE}}(1+G^{(0)}T_{\mathrm{BB}})$$
$$+\sum_{i\lambda}F_d^{(1)}(i)\bar{\delta}_{i\lambda}G^{(1)}M_d^{(1)}(\lambda)G^{(1)}T_{\lambda\mathrm{B}}\ . \tag{54b}$$

The one-pion-exchange potential V_{OPE} is given in terms of the dressed B$\to\pi$B form factor $F_d^{(1)}$ by the relation

$$V_{\mathrm{OPE}}=\sum_{ij}F_d^{(1)}(i)\bar{\delta}_{ij}G^{(1)}F_d^{(1)\dagger}(j)\ . \tag{55}$$

In Eq. (54) we have a set of coupled integral equations for the amplitudes that describe the reactions in Eq. (53). The input to these equations is the B$\to\pi$B form factor and the π-B and B-B subsystem amplitudes. In structure these equations are similar to those derived for the NN-πNN system[2–7] with the exception that T_{BB} and $T_{\lambda\mathrm{B}}$ are 4×4 matrices that allow the coupling to the $\Delta\Delta$ channel, and thus the inclusion of additional physical processes that will be discussed in the next section.

To complete our description of the BB-πBB system, we need to write the equations for pion-deuteron scattering and the coupling to the pion absorption channel. Here we consider the class of all connected diagrams for πBB$\to\pi$BB. These can be divided into two groups:

(i) Those that are two-particle irreducible, which we have considered previously and denoted by $M^{(1)}$.

(ii) The rest of the diagrams are two-particle reducible, i.e., they have intermediate states of zero pions. These we can write using the last-cut-lemma as

$$(F^{(1)\dagger}G^{(0)}F^{(0)})_c=(F^{(0)\dagger}G^{(0)}F^{(1)})_c\ .$$

Here we should note that $F^{(0)}=F_c^{(0)}+F_d^{(0)}$, and $F_d^{(0)}=F_d^{(1)}$ since the two-particle reducible part of $F_d^{(0)}$, given in Fig. 14, has already been included in the dressing of the baryon propagators. We now can write the amplitude for πBB$\to\pi$BB as

$$M^{(0)}=M^{(1)}+F^{(1)\dagger}G^{(0)}F_c^{(0)}$$
$$+F_c^{(1)\dagger}G^{(0)}F_d^{(1)}+(F_d^{(1)\dagger}G^{(0)}F_d^{(1)})_c\ . \tag{56}$$

To get the $2\to 2$ amplitude for πd$\to\pi$d and B(πB)$\to$B(πB), we have to take the right and left residue of $M^{(0)}$. Since $F_d^{(1)}$ is just the πB$\leftarrow$B form factor, it has no (πB) pole. This means the only contribution to the $2\to 2$ amplitude is from the first two terms on the r.h.s. of Eq. (56), and this gives

$$T_{\alpha\beta}=U_{\alpha\beta}^{(1)}+\sum_i U_{\alpha i}^{(1)}G^{(1)}F_d^{(1)\dagger}(i)G^{(0)}T_{\mathrm{B}\beta}\ . \tag{57}$$

Using the AGS equations we can iterate this equation and eliminate $U_{\alpha\beta}^{(1)}$ to get

$$T_{\alpha\beta}=\bar{\delta}_{\alpha\beta}G^{(1)-1}+\sum_i\bar{\delta}_{\alpha i}F_d^{(1)\dagger}(i)G^{(0)}T_{\mathrm{B}\beta}$$
$$+\sum_\gamma\bar{\delta}_{\alpha\gamma}M_d^{(1)}(\gamma)G^{(1)}T_{\gamma\beta}\ . \tag{58a}$$

To obtain the equation for $T_{\mathrm{B}\beta}$ and close this set of coupled equations, we need to carry out the above procedure on the amplitude for πBB$\to$BB, i.e., $F_c^{(0)}$. In this case we obtain

$$T_{\mathrm{B}\beta}=V_{\mathrm{OPE}}G^{(0)}T_{\mathrm{B}\beta}+\sum_i F_d^{(1)\dagger}(i)\bar{\delta}_{i\beta}$$
$$+\sum_{i\alpha}F_d^{(1)}(i)\bar{\delta}_{i\alpha}G^{(1)}M_d^{(1)}(\alpha)G^{(1)}T_{\alpha\beta}\ . \tag{58b}$$

In Eqs. (54) and (58) we have a set of equations that

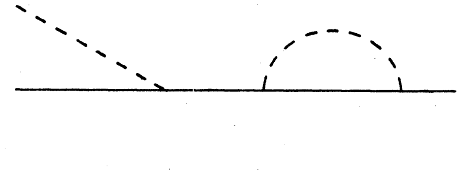

FIG. 14. Diagrams excluded from $F_d^{(0)}$ because they are included in the dressing of the baryon propagator.

describe the BB-πBB system for the case where the interaction Lagrangian is linear in the pion field and where we have neglected three-body forces.

IX. ANTISYMMETRY OF THE BB-πBB EQUATIONS

Before we proceed to the antisymmetry of the full set of coupled equations, let us consider baryon-baryon scattering in the BB part of the Hilbert space. In particular, we want to concentrate on the question of the N-Δ channel. In the NN-πNN (Refs. 2–7) equations the Δ was considered a π-N resonance and it was required that the nucleon in the Δ be antisymmetrized with respect to the spectator nucleon. This antisymmetry will still be required in the πBB part of the Hilbert space. Our discussion will lead us to the problem of undercounting in the NN-πNN problem. In particular, we find that the NN-πNN equations include only one of the time ordered diagrams for NN$\to$NΔ and N$\Delta\to$NΔ via one-pion exchange (OPE). This undercounting is the result of treating the Δ as a pure π-N resonance, and restricting the Hilbert space to one-pion intermediate states only.

Let us consider the problem of B-B scattering in the OPE approximation, i.e.,

$$T_{\rm BB}=V_{\rm OPE}(1+G^{(0)}T_{\rm BB})\ , \tag{59}$$

where we have neglected the coupling to the πBB channels in Eq. (54b). Here $V_{\rm OPE}$ is given by Eq. (55). To antisymmetrize the above equations we need to take the matrix elements of the equations with respect to antisymmetric states (AS). For the N-N and Δ-Δ, these states are given by

$$|\Psi^{\rm AS}_{\rm NN}\rangle=\frac{1}{\sqrt{2}}[\,|\psi_{\rm NN}(\sigma_1,\sigma_2;\mathbf{p})\rangle-|\psi_{\rm NN}(\sigma_2,\sigma_1;-\mathbf{p})\rangle]\ , \tag{60a}$$

$$|\Psi^{\rm AS}_{\Delta\Delta}\rangle=\frac{1}{\sqrt{2}}[\,|\psi_{\Delta\Delta}(s_1,s_2;\mathbf{p})\rangle-|\psi_{\Delta\Delta}(s_2,s_1;-\mathbf{p})\rangle]\ , \tag{60b}$$

where $\mathbf{p}$ is the relative momentum, while σ and s are the spin isospin coordinates of the nucleon and Δ, respectively. The matrix element of the OPE operator [Eq. (55)] for NN$\to$NN between the antisymmetric states gives

$$\begin{aligned}\langle\Psi^{\rm AS}_{\rm NN}|V_{\rm NN,NN}|\Psi^{\rm AS}_{\rm NN}\rangle&=\sum_{ij=1}^{2}\langle\Psi^{\rm AS}_{\rm NN}|f^{(1)}_{\rm N,N}(i)\bar{\delta}_{ij}d_\pi f^{(1)}_{\rm N,N}(j)|\Psi^{\rm AS}_{\rm NN}\rangle\\&=V^{\rm AS}_{\rm NN,NN}\ .\end{aligned} \tag{61}$$

As expected this is the difference of the two Feynman diagrams in Fig. 15(a). Note that each Feynman diagram is the sum of two different time ordered diagrams as illustrated in Fig. 15(b).

In a similar manner we can define the antisymmetrized amplitudes for NN$\to$NN and NN$\to\Delta\Delta$ as

$$\begin{aligned}T^{\rm AS}_{\rm NN,NN}&=\langle\Psi^{\rm AS}_{\rm NN}|T_{\rm NN,NN}|\Psi^{\rm AS}_{\rm NN}\rangle\ ,\\T^{\rm AS}_{\Delta\Delta,\rm NN}&=\langle\Psi^{\rm AS}_{\Delta\Delta}|T_{\Delta\Delta,\rm NN}|\Psi^{\rm AS}_{\rm NN}\rangle\ .\end{aligned} \tag{62}$$

To obtain the equations for the antisymmetric amplitudes we take matrix elements of Eq. (59) between N-N antisymmetric states. This gives

$$\begin{aligned}T^{\rm AS}_{\rm NN,NN}=&\,V^{\rm AS}_{\rm NN,NN}(1+\tfrac{1}{2}G^{(0)}_{\rm NN}T^{\rm AS}_{\rm NN,NN})+\langle\Psi^{\rm AS}_{\rm NN}|V_{\rm NN,N\Delta}G^{(0)}_{\rm N\Delta}T_{\rm N\Delta,NN}|\Psi^{\rm AS}_{\rm NN}\rangle\\&+\langle\Psi^{\rm AS}_{\rm NN}|V_{\rm NN,\Delta N}G^{(0)}_{\Delta\rm N}T_{\Delta\rm N,NN}|\Psi^{\rm AS}_{\rm NN}\rangle+V^{\rm AS}_{\rm NN,\Delta\Delta}\tfrac{1}{2}G^{(0)}_{\Delta\Delta}T^{\rm AS}_{\Delta\Delta,\rm NN}\ ,\end{aligned} \tag{63}$$

where the factor of $\frac{1}{2}$ in the NN and $\Delta\Delta$ propagator is a result of using the completeness relation

$$\sum_{s_1s_2}\int d^3p\,|\Psi^{\rm AS}_{\alpha\alpha}(s_1,s_2;\mathbf{p})\rangle\tfrac{1}{2}\langle\Psi^{\rm AS}_{\alpha\alpha}(s_1,s_2;\mathbf{p})|=1\quad(\alpha={\rm N},\Delta)\ , \tag{64}$$

where s_1, s_2 are the spins of the two baryons, and $\mathbf{p}$ the relative momentum. In Eq. (63) $V^{\rm AS}_{\rm NN,\Delta\Delta}$ is given by

$$V^{\rm AS}_{\rm NN,\Delta\Delta}=\langle\Psi^{\rm AS}_{\rm NN}|V_{\rm NN,\Delta\Delta}|\Psi^{\rm AS}_{\Delta\Delta}\rangle\ . \tag{65}$$

We observe here that this coupling (i.e., NN$\to\Delta\Delta$) was not present in the NN-πNN equations, and only arises as a result of treating the N and Δ on equal footing.

To simplify the second and third terms on the r.h.s. of Eq. (63), we introduce the complete set of N-Δ intermediate states, i.e.,

$$\sum_{\sigma s}\int d^3p\,|\psi_{\rm N\Delta}(\sigma\mathbf{p};s-\mathbf{p})\rangle\langle\psi_{\rm N\Delta}(\sigma\mathbf{p};s-\mathbf{p})|=1\ . \tag{66}$$

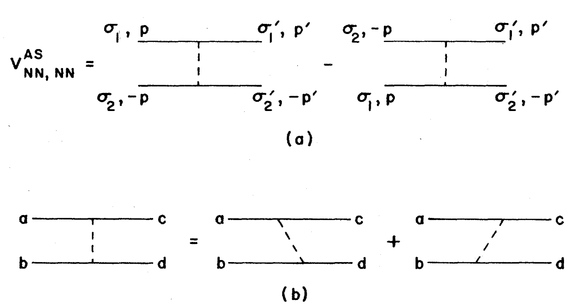

FIG. 15. Diagrammatic representation of the antisymmetrized OPE potential.

It is then simple to show that

$$\langle \Psi_{NN}^{AS} \mid V_{NN,\Delta N} \mid \psi_{\Delta N}(s-\mathbf{p};\sigma\mathbf{p})\rangle = -\langle \Psi_{NN}^{AS} \mid V_{NN,N\Delta} \mid \psi_{N\Delta}(\sigma\mathbf{p};s-\mathbf{p})\rangle \quad (67)$$

because of the antisymmetry of the N-N states. We now can write Eq. (63) as

$$T_{NN,NN}^{AS} = V_{NN,NN}^{AS}(1+\tfrac{1}{2}G_{NN}^{(0)}T_{NN,NN}^{AS}) + V_{NN,N\Delta}^{AS}G_{N\Delta}^{(0)}T_{N\Delta,NN}^{AS} + V_{NN,\Delta\Delta}^{AS}\tfrac{1}{2}G_{\Delta\Delta}^{(0)}T_{\Delta\Delta,NN}^{AS} \quad (68)$$

with

$$V_{NN,N\Delta}^{AS} = \langle \Psi_{NN}^{AS} \mid V_{NN,N\Delta} + V_{NN,\Delta N} \mid \Psi_{N\Delta}^{AS}\rangle \quad (69a)$$

and

$$T_{N\Delta,NN}^{AS} = \langle \Psi_{N\Delta}^{AS} \mid T_{N\Delta,NN} + T_{\Delta N,NN} \mid \Psi_{NN}^{AS}\rangle , \quad (69b)$$

where the antisymmetric N-Δ state is given by

$$\mid \Psi_{N\Delta}^{AS}\rangle = \frac{1}{\sqrt{2}}[\mid \psi_{N\Delta}(\sigma\mathbf{p};s-\mathbf{p})\rangle - \mid \psi_{\Delta N}(s-\mathbf{p};\sigma\mathbf{p})\rangle] , \quad (70)$$

and we have made use of the fact that $T_{\ldots,BB'} \mid \psi_{B'B}\rangle = 0$ for $B \neq B'$. This is identical to the antisymmetric N-Δ state used in the NN-πNN equations[8] where the Δ was a π-N resonance. This in turn implies that the antisymmetry of the N-Δ channel in the B-B space is the same as the $(B\pi)_\Delta$N channel in the πBB space.

Here we observe that $V_{NN,N\Delta}^{AS}$ has contributions from both time order diagrams in Fig. 1, while in the NN-πNN equations we had the contribution from Fig. 1(a) only. A similar problem of undercounting was present in the bound state model of the NN-πNN equations[41] where $V_{NN,NN}$ obtained contributions only from the first diagram of Fig. 15(b). This undercounting could lead to difficulties in the description of N-N scattering in the NN-πNN equations. It also reduces the distortion in the N-N channel for pp$\to\pi$d, since that distortion is generated self-consistently.

The above procedure of obtaining the integral equations for the antisymmetrized amplitudes can be carried out for the other channels in the B-B Hilbert space. This antisymmetry reduces Eq. (59) from being a set of 4×4 equations to a set of 3×3 equations, and results from the antisymmetry of the N and Δ. We can achieve this antisymmetry at the matrix level by introducing the 3×4 matrix

$$L^{\dagger} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (71)$$

and the 4×4 diagonal matrix of asymptotic wave functions $\mid \psi\rangle$ with elements $\mid \Psi_{NN}^{AS}\rangle$, $\mid \psi_{N\Delta}\rangle$, $\mid \psi_{\Delta N}\rangle$, and $\mid \Psi_{\Delta\Delta}^{AS}\rangle$. Then the matrix of antisymmetrized amplitudes is given by

$$X_{B,B} = L^{\dagger}\langle \psi \mid T_{BB} \mid \psi\rangle L \quad (72)$$

and satisfies the equations

$$X_{B,B} = Z_{B,B}(1+G_{AS}^{(0)}X_{B,B}) \quad (73)$$

with

$$Z_{B,B} = L^{\dagger}\langle \psi \mid V_{OPE} \mid \psi\rangle L \quad (74)$$

and $G_{AS}^{(0)}$ a diagonal 3×3 matrix with elements given by

$$(G_{AS}^{(0)})_{BB'} = \tfrac{1}{2}d_B d_B \text{ for } B=B'$$
$$= d_B d_{B'} \text{ for } B \neq B' . \quad (75)$$

It is then simple to show that Eq. (68) is the (1,1) element of the matrix Eq. (73), and $Z_{B,B}$ is a generalization of Eqs. (65) and (69a).

So far we have considered the B-B equations with no coupling to the πBB channel. We now have to extend our analysis to include this coupling by considering either Eqs. (54) or (58), depending on the initial and final states. As is the case with the standard three-body problem,[42] to obtain a closed set of equations for the physical amplitudes we need to assume separability of the two-body amplitudes in the πBB Hilbert space. This approximation will also reduce the dimensionality of the integral equations. To carry out the antisymmetry as well, we need to define antisymmetric asymptotic states in terms of the form factors. For this purpose we divide our two-body asymptotic wave functions in the πBB Hilbert space into two parts:

(i) The part where the B-B subsystem is interacting. It is now convenient to define the 4×4 diagonal matrix of asymptotic wave functions

$$\mid \psi_{(BB')_d\pi}\rangle = G^{(0)} \mid h_d, \chi_\pi\rangle = G^{(1)}d_\pi^{-1} \mid h_d, \chi_\pi\rangle , \quad (76)$$

where the B-B form factor h_d is introduced in Appendix C and χ_π is the wave function of a free pion. We note that the above wave function is assumed to be antisymmetric under the interchange of the two baryons and, as discussed in Appendix C, needs to be further multiplied by a column matrix **F** in order to yield the properly normalized bound state wave function.

(ii) The part where the π-B subsystem is interacting. Here the matrix of asymptotic wave functions is

$$\mid \psi_{(B\pi)_R B'}\rangle \equiv d(1)d_\pi \mid h_R(1)\rangle \otimes \mid \chi(2)\rangle \text{ if B is baryon 1}$$
$$= G^{(1)}[\mid h_R(1)\rangle \otimes d^{-1}(2) \mid \chi(2)\rangle]$$
$$\equiv \mid \chi(1)\rangle \otimes d(2)d_\pi \mid h_R(2)\rangle \text{ if B is baryon 2}$$
$$= G^{(1)}[d^{-1}(1) \mid \chi(1)\rangle \otimes \mid h_R(2)\rangle] , \quad (77)$$

where $\mid h_R(i)\rangle$ and $\mid \chi(j)\rangle$ are 2×2 diagonal matrices of π-B form factors and spectator baryon wave functions, respectively. In writing the above form for the asymptotic state it is essential that $G^{(1)}$ have the correct clustering behavior, otherwise our asymptotic states will not be of the form of a product of two cluster wave functions. In Eqs. (76) and (77) we have used the subscripts d and R to label B-B and π-B states, respectively.

We next have to write $M_d^{(1)}(\alpha)$ in terms of the above form factors. From Eq. (32) we have that

$$M_d^{(1)}(\alpha)=m^{(1)}(1)\otimes d^{-1}(2) \text{ for } \alpha=1$$
$$=d^{-1}(1)\otimes m^{(1)}(2) \text{ for } \alpha=2$$
$$=t^{(0)}(3)d_\pi^{-1} \text{ for } \alpha=3\ , \tag{78}$$

where we recall that $m(i)$ is the 2×2 matrix of amplitudes for the pion interacting with the ith baryon while $t^{(0)}(3)$ is the 4×4 matrix of B-B amplitudes which reduces to a 3×3 matrix after antisymmetry. The assumption of separability, at the partial wave level, allows us to write these two-body amplitudes as

$$m^{(1)}(i)=|h_R(i)\rangle\tau_R(i)\langle h_R(i)| \tag{79a}$$

and

$$t^{(0)}(3)=|h_d(3)\rangle\tau_d(3)\langle h_d(3)|\ . \tag{79b}$$

Furthermore, we can write the baryon propagators in terms of their spectral representation as

$$d^{-1}(i)=|\chi(i)\rangle\delta^{-1}(i)\langle\chi(i)|\ . \tag{80}$$

In writing Eq. (80) we have put the spectator baryon on a mass shell. Combining the results of Eqs. (79) and (80) in Eq. (78) we get

$$M_d^{(1)}(\alpha)=|H_n(\alpha)\rangle\mathscr{T}_n(\alpha)\langle H_n(\alpha)|\ , \tag{81}$$

where $n=R$ or d depending on the value of α.

$$\mathscr{T}_n(\alpha)=\tau_R(1)\otimes\delta(2)=P[\delta(2)\otimes\tau_R(1)]P \text{ for } \alpha=1$$
$$=\delta(1)\otimes\tau_R(2) \text{ for } \alpha=2$$
$$=\delta_\pi\tau_d(3) \text{ for } \alpha=3 \tag{82}$$

and

$$|H_n(\alpha)\rangle=|h_R(1)\rangle\otimes d^{-1}(2)|\chi_{\rm B}(2)\rangle$$
$$=P[d^{-1}(2)|\chi_{\rm B}(2)\rangle\otimes|h_R(1)\rangle]P \text{ for } \alpha=1$$
$$=d^{-1}(1)|\chi_{\rm B}(1)\rangle\otimes|h_R(2)\rangle \text{ for } \alpha=2$$
$$=d_\pi^{-1}|\chi_\pi\rangle|h_d\rangle \text{ for } \alpha=3\ . \tag{83}$$

In the above, P is a unitarity 4×4 matrix that interchanges the second and third row (column), and is introduced to interchange the $(N\pi)\Delta$ and $(\Delta\pi)N$ states. With the above definition of P we can write the antisymmetric $[(B\pi)_R B]$ state as

$$|\Psi^{\rm AS}_{(B\pi)_R B}\rangle=\frac{1}{\sqrt{2}}[G^{(1)}|H_R(1)\rangle-PG^{(1)}|H_R(2)\rangle P]\ . \tag{84}$$

We now can define the antisymmetric amplitudes for the reactions $BB\rightarrow(B\pi)_R B$ and $BB\rightarrow(BB)_d\pi$ as,

$$X_{R,\rm B}=\frac{1}{\sqrt{2}}[\langle H_R(1)|G^{(1)}T_{1\rm B} -P\langle H_R(2)|G^{(1)}T_{2\rm B}]|\psi\rangle L \tag{85}$$

and

$$X_{d,\rm B}=\underline{L}^\dagger\langle H_d(3)|G^{(1)}T_{3\rm B}|\psi\rangle L\ . \tag{86}$$

Here $\underline{L}$ is the 4×3 matrix that results from taking the adjoint of $L^\dagger$ after removing the $(-)$ sign from the second row. The removal of the $(-)$ sign is a result of the fact that $|H_d(3)\rangle$ corresponds to the baryons being in a given partial wave that satisfies the Pauli principle. In other words the wave function or form factor changes sign under the exchange of the coordinates of the two baryons, and we need only take a linear combination of the $N\Delta$ and ΔN states to get the antisymmetry. These definitions are consistent with those used by Avishai and Mizutani[2] and Blankleider[8] for the NN-πNN equations. Using these definitions in conjunction with Eq. (54) we get the final equations for the antisymmetrized amplitudes for the reactions

$$B+B\rightarrow B+B$$
$$\rightarrow B+(B\pi)_R$$
$$\rightarrow\pi+(BB)_d\ , \tag{87}$$

as

$$X_{\rm B,B}=Z_{\rm B,B}(1+G^{(0)}_{\rm AS}X_{\rm B,B}) +\sum_R Z_{{\rm B},R}\mathscr{T}_R(1)X_{R,\rm B}+\sum_d Z_{{\rm B},d}\mathscr{T}^{\rm AS}_d(3)X_{d,\rm B}\ , \tag{88a}$$

$$X_{R,\rm B}=Z_{R,\rm B}(1+G^{(0)}_{\rm AS}X_{\rm B,B}) +\sum_{R'} Z_{R,R'}\mathscr{T}_{R'}(1)X_{R',\rm B}+\sum_d Z_{R,d}\mathscr{T}^{\rm AS}_d(3)X_{d,\rm B}\ , \tag{88b}$$

$$X_{d,\rm B}=Z_{d,\rm B}(1+G^{(0)}_{\rm AS}X_{\rm B,B})+\sum_R Z_{d,R}\mathscr{T}_R(1)X_{R,\rm B}\ , \tag{88c}$$

where $\mathscr{T}^{\rm AS}_d(3)=\underline{L}^\dagger\mathscr{T}_d(3)\underline{L}$ is a 3×3 matrix for the $(BB)\pi$ propagator in the πBB space, while $Z_{{\rm B},R}$, $Z_{{\rm B},d}$, $Z_{R,R'}$, and $Z_{R,d}$ are given by

$$Z_{{\rm B},R}=\sqrt{2}L^\dagger\langle\psi|F_d^{(1)}(2)G^{(1)}|H_R(1)\rangle\ , \tag{89a}$$

$$Z_{{\rm B},d}=L^\dagger\langle\psi|\sum_{i=1}F_d^{(1)}(i)G^{(1)}|H_d(3)\rangle\underline{L}\ , \tag{89b}$$

$$Z_{R,R'}=-P\langle H_R(1)|G^{(1)}|H_R(2)\rangle\ , \tag{89c}$$

$$Z_{R,d}=\frac{1}{\sqrt{2}}[\langle H_R(1)|-P\langle H_R(2)|]G^{(1)}|H_d(3)\rangle\underline{L}$$
$$=\sqrt{2}\langle H_R(2)|G^{(1)}|H_d(3)\rangle\underline{L}\ , \tag{89d}$$

where in the last equation we have made use of the relation

$$P\langle H_R(2)|G^{(1)}|H_d(3)\rangle P=-\langle H_R(1)|G^{(1)}|H_d(3)\rangle\ . \tag{90}$$

Similarly we define the antisymmetrized amplitudes for the reactions $\pi d\rightarrow\pi d$, $\pi d\rightarrow(B\pi)_R B$, and $\pi d\rightarrow BB$ as

$$X_{d,d}=\underline{L}^{\dagger}\langle H_d(3)\,|\,G^{(1)}T_{33}G^{(1)}\,|\,H_d(3)\rangle\underline{L}\ , \qquad (91a)$$

$$X_{R,d}=\frac{1}{\sqrt{2}}[\langle H_R(1)\,|\,G^{(1)}T_{13}-P\langle H_R(2)\,|\,G^{(1)}T_{23}]\times G^{(1)}\,|\,H_d(3)\rangle\underline{L}\ , \qquad (91b)$$

$$X_{\mathrm{B},d}=L^{\dagger}\langle\psi\,|\,T_{\mathrm{B}3}G^{(1)}\,|\,H_d(3)\rangle\underline{L}\ . \qquad (91c)$$

We then make use of Eqs. (58) and (90) to write the integral equations for the antisymmetrized amplitudes for the reactions

$$\pi+\mathrm{d}\rightarrow\pi+\mathrm{d}$$
$$\rightarrow\mathrm{B}+(\mathrm{B}\pi)$$
$$\rightarrow\mathrm{B}+\mathrm{B} \qquad (92)$$

as

$$X_{d,d}+Z_{d,\mathrm{B}}G_{\mathrm{AS}}^{(0)}X_{\mathrm{B},d}+\sum_R Z_{d,R}\mathscr{T}_R(1)X_{R,d}\ , \qquad (93a)$$

$$X_{R,d}=Z_{R,d}+Z_{R,\mathrm{B}}G_{\mathrm{AS}}^{(0)}X_{\mathrm{B},d}+\sum_{R'}Z_{R,R'}\mathscr{T}_{R'}(1)X_{R',d}+\sum_{d'}Z_{R,d'}\mathscr{T}_{d'}^{\mathrm{AS}}(3)X_{d',d}\ , \qquad (93b)$$

$$X_{\mathrm{B},d}=Z_{\mathrm{B},d}+Z_{\mathrm{B},\mathrm{B}}G_{\mathrm{AS}}^{(0)}X_{\mathrm{B},d}+\sum_{R}Z_{\mathrm{B},R}\mathscr{T}_{R}(1)X_{R,d}+\sum_{d'}Z_{\mathrm{B},d'}\mathscr{T}_{d'}^{\mathrm{AS}}(3)X_{d',d}\ . \qquad (93c)$$

Finally we note that the physical (on-shell) amplitudes are obtained by contracting the above $X_{\alpha,\beta}$ amplitudes (4×4 matrices) with column vectors as illustrated by Eq. (51) in conjunction with Eq. (C15).

X. CONCLUSION

The final equations, although similar to the NN-πNN equations[2–7] in form, include a number of new features that arise as a result of treating the N and Δ on equal footing, as dictated by chiral bag models.[14,15]

(i) In the N-N channel, we now have the full contribution from two-pion exchange by including all four diagrams in Fig. 13, while previously we included only the first one. In particular we have ΔΔ intermediate states in our N-N potential. The advantage of our method for including two-pion exchange is that the Δ resonance is built into the theory to be consistent with π-N data. This should give rise to the proper energy dependence in the amplitude above the pion production threshold ensuring that the resonancelike behavior from the N-Δ threshold[43] is properly included.

(ii) For pion production, the inclusion of both diagrams in Fig. 1 might be crucial for getting the correct differential cross section. In any event the exclusion of the diagram in Fig. 1(b) should be considered a serious undercounting problem that could affect the energy dependence as well as the magnitude of the cross section.[44]

(iii) Because of the coupling between the π-N and π-Δ channels in the π-B subsystem, our π-N amplitude includes the contribution due to inelasticity. Since this coupled channel approach gives a good description of the π-N data,[43] we will be able to resolve some of the uncertainty in the NN-πNN result due to inelasticity in the π-N amplitude.[44,45] We also can extend the calculations to higher energies, since the input π-N amplitude fits the higher energy data.

(iv) Because we have included the πΔΔ vertex on equal footing with the πNN vertex, we get a contribution from the diagram in Fig. 2. This N-Δ potential has a tensor component that could contribute to the π-d tensor polarization,[12,13] and might possibly explain some of the discrepancy between theory and experiment for this observable.

On the more formal side, one of the main features of the present formulation is that it attempts to bridge the gap between the quark models of a hadron and the multiple scattering formalism of nuclear theory. Here one can get the πBB coupling constants and form factors from quark models and use them to calculate $A=2$ observables. In principle this can be a zero parameter calculation to the extent that all off-shell behavior is predetermined by the quark model. Furthermore, this procedure predetermines the relative strength of the pole and nonpole parts of the π-N amplitude in both the P_{11} and P_{33} partial waves, and thus overcomes one of the main problems with the NN-πNN equations.[44] The success of CBM (Refs. 15 and 17) for π-N scattering is evidence for the possible success of such an approach.

The advantage of the present approach for deriving our equations is its independence of the specific form of the Lagrangian and in particular the coupling of the pion to the baryon. Although in the final analysis we ignored the contribution from three-body type forces and terms nonlinear in the pion field, the theory is very specific as to how these terms should be included. This facilitates the use of perturbation theory to include the contribution of three-body forces, and terms nonlinear in the pion field.

ACKNOWLEDGMENTS

One of the authors (I. R. A.) would like to acknowledge many informative discussions with A. T. Stelbovics. He is also grateful for the hospitality of the Medium Energy Theory Group at Los Alamos and the Nuclear Theory Center at Indiana University where part of the work was done. We would also like to thank G. E. Walker for numerous fruitful discussions. The authors would like to thank the Australian Research Grant Scheme and the U.S. National Science Foundation for their financial support.

APPENDIX A

Since the π-B amplitude will be used as input into the BB-πBB equations, we need to maintain a consistency in our formulation for the two systems. To that extent we need to derive the π-B equations using the same techniques. Furthermore, we need to define the π-Δ channel within the framework of the chiral bag model.[14,15] This is particularly important as the Δ pole is in the complex energy plane, i.e., the Δ is a resonance.

In this case our basis states are $|\pi N\rangle$ and $|\pi\Delta\rangle$, and our operators are basically 2×2 matrices. The diagrams that contribute to the π-B amplitude can be divided into two classes:

(i) Those which are one-particle irreducible and can be denoted by $m^{(1)}$.

(ii) Those not belonging to (i). This class of diagrams gives amplitudes which can be written, using the last-cut lemma, as

$$f^{(1)\dagger}d^{(0)}f^{(0)}=f^{(0)\dagger}d^{(0)}f^{(1)} ,$$

where $d^{(0)}$ is the undressed baryon propagator and is of the form given in Eq. (23). In the above, we have used the superscript in m and f to denote the minimum number of pions in intermediate states. We now can write the π-B amplitude as

$$\begin{aligned} m^{(0)}&=m^{(1)}+f^{(1)\dagger}d^{(0)}f^{(0)}\\ &=m^{(1)}+f^{(0)\dagger}d^{(0)}f^{(1)} . \end{aligned} \tag{A1}$$

We now examine the $\pi B\rightarrow B$ amplitude $f^{(0)}$. We can divide the diagrams that contribute to $f^{(0)}$ into two classes:

(i) Those that are one-particle irreducible, which we denote by $f^{(1)}$.

(ii) The diagrams not belonging to (i). They can be written as

$$\Sigma^{(1)}d^{(0)}f^{(0)} ,$$

where $\Sigma^{(1)}$ is the self-energy of the baryon.

We now can write

$$f^{(0)}=f^{(1)}+\Sigma^{(1)}d^{(0)}f^{(0)} \tag{A2}$$

or

$$d^{(0)}f^{(0)}\equiv df^{(1)} , \tag{A3}$$

where the dressed propagator is given by

$$d_a^{-1}=d_a^{(0)-1}-\Sigma_a^{(1)} \quad (a=\mathrm{N},\Delta) . \tag{A4}$$

In a similar manner we can show that

$$f^{(0)\dagger}d^{(0)}=f^{(1)\dagger}d . \tag{A5}$$

With this result we can write the amplitude for π-B scattering as

$$m^{(0)}=m^{(1)}+f^{(1)\dagger}df^{(1)} . \tag{A6}$$

We now have to determine the structure of $f^{(1)}$, the amplitude for $\pi B\rightarrow B$ which is one-particle irreducible. The diagrams that contribute to $f^{(1)}$ can be divided into two classes:

(i) Those that are two-particle irreducible, which we denote by $f^{(2)}$.

(ii) Those that are two-particle reducible. These can be written, using the last-cut lemma, as

$$f^{(2)}g^{(1)}m^{(1)} ,$$

where $g^{(1)}$ is the 2×2 π-B propagator matrix that is diagonal with elements $d_a d_\pi$ $(a=\mathrm{N},\Delta)$. By taking $g^{(1)}$ to have the dressed baryon propagator it is guaranteed that any cut through diagrams belonging to $m^{(1)}$ leads to connected diagrams. We now can write

$$f^{(1)}=f^{(2)}+f^{(2)}g^{(1)}m^{(1)} . \tag{A7}$$

Diagrammatically this equation can be illustrated as in Fig. 16. The Δ in the intermediate state has to have its full dressing, i.e., it is a physical Δ. Here $f^{(2)}$ is the two-particle irreducible $\pi B\rightarrow B$ amplitude. If we do not want to go beyond two-body unitarity, then we need not proceed any further in exposing unitarity cuts, and we take $f^{(2)}$ from the chiral bag model.

We now turn to $m^{(1)}$ which is one-particle irreducible. The diagrams contributing to $m^{(1)}$ can be divided into two classes:

(i) Those that are two-particle irreducible, which we denote by $m^{(2)}$.

(ii) Those that are two-particle reducible. These can be written, using the last-cut lemma, as

$$m^{(2)}g^{(1)}m^{(1)}=m^{(1)}g^{(1)}m^{(2)} .$$

Here again $g^{(1)}$ is the dressed π-B propagator. We now can write

$$\begin{aligned} m^{(1)}&=m^{(2)}+m^{(2)}g^{(1)}m^{(1)}\\ &=m^{(2)}+m^{(1)}g^{(1)}m^{(2)} , \end{aligned} \tag{A8}$$

which is the standard two-body equation. Using the result of Eqs. (A7) and (A8) in Eq. (A6), we obtain the structure of the π-B amplitude in the N and Δ channels.

APPENDIX B

In this appendix we would like to demonstrate how we obtain the operators in the BB-πBB Hilbert space from those in the π-B subspace. In the π-B space our basis states are $|\pi\text{-N}\rangle$ and $|\pi\text{-}\Delta\rangle$. The corresponding operators are 2×2 matrices. In the BB-πBB space our states are $|\mathrm{N}(1),\mathrm{N}(2),\pi\rangle$, $|\mathrm{N}(1),\Delta(2),\pi\rangle$, $|\Delta(1),\mathrm{N}(2),\pi\rangle$, and $|\Delta(1),\Delta(2),\pi\rangle$, where 1 and 2 refer to the two baryons. In this case the operators are 4×4 matrices. To achieve this transition we introduce a direct product[46] of two 2×2 matrices as

$$\begin{bmatrix} a & b\\ c & d\end{bmatrix}\otimes E=\begin{bmatrix} aE & bE\\ cE & dE\end{bmatrix} , \tag{B1}$$

where E is a 2×2 matrix. Then it is easy to show that,

$$[A(1)B(1)]\otimes[C(2)D(2)] =[A(1)\otimes C(2)][B(1)\otimes D(2)] , \tag{B2}$$

where A, B, C, and D are 2×2 matrices.

To illustrate the application of the direct product defined above, we will present the derivation of Eq. (31). We have for the disconnected part of the $\pi BB\rightarrow BB$ am-

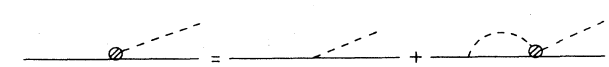

FIG. 16. Diagrammatic representation of the dressed $\pi B\rightarrow B$ vertex as given in Eq. (A7).

plitude [see Eq. (29)] that

$$F_d^{(1)}=f^{(1)}(1)\otimes d^{-1}(2)+d^{-1}(1)\otimes f^{(1)}(2) = \sum_{i=1}^{2}{}' F_d^{(1)}(i) \ . \quad \text{(B3)}$$

However, $f^{(1)}$ is given in terms of the more basic πB$\rightarrow$B form factor $f^{(2)}$ by relation [see Eq. (A7)]

$$f^{(1)}(i)=f^{(2)}(i)+f^{(2)}(i)g^{(1)}(i)m^{(1)}(i) \ . \quad \text{(B4)}$$

Employing Eq. (B4) in Eq. (B3) we get

$$F_d^{(1)}(1)=F_d^{(2)}(1)+f^{(2)}(1)g^{(1)}(1)m^{(1)}(1)\otimes d^{-1}(2) \ . \quad \text{(B5)}$$

But using Eq. (B2) we obtain

$$\begin{aligned} f^{(2)}(1)g^{(1)}(1)m^{(1)}(1)\otimes d^{-1}(2) &= [f^{(2)}(1)\otimes d^{-1}(2)][\{g^{(1)}(1)m^{(1)}(1)\}\otimes\{d(2)d^{-1}(2)\}] \\ &= [f^{(2)}(1)\otimes d^{-1}(2)][g^{(1)}(1)\otimes d(2)][m^{(1)}(1)\otimes d^{-1}(2)] \\ &= F_d^{(2)}(1)G^{(1)}M_d^{(1)}(1) \ , \end{aligned} \quad \text{(B6)}$$

where

$$M_d^{(1)}(1)=m^{(1)}(1)\otimes d^{-1}(2) \ . \quad \text{(B7)}$$

We now can write Eq. (B5) as

$$F_d^{(1)}(1)=F_d^{(2)}(1)+F_d^{(2)}(1)G^{(1)}M_d^{(1)}(1) \ . \quad \text{(B8)}$$

In a similar manner we can prove that

$$F_d^{(1)}(2)=F_d^{(2)}(2)+F_d^{(2)}(2)G^{(1)}M_d^{(1)}(2) \ . \quad \text{(B9)}$$

Combining the results of Eqs. (B8) and (B9) with that of Eq. (B3) gives us

$$F_d^{(1)}=F_d^{(2)}+(F_d^{(2)}G^{(1)}M_d^{(1)})_d \quad \text{(B10)}$$

which is Eq. (31).

APPENDIX C

An important feature of the BB-πBB equations is that they describe the nucleon and delta on the same footing. It is therefore important for consistency to construct both the input π-B and B-B interactions using a coupled channels scheme. For the π-B system such a scheme was described in Appendix A. In this appendix we shall consider the description of the input interactions for the B-B system. Here we are especially interested in the construction of bound state wave functions—in particular we note that our deuteron wave function has, in principle, $\Delta\Delta$ components.

The input B-B amplitudes have been represented by the 4×4 *matrix* $t^{(0)}(3)$. It is equivalent to consider $t^{(0)}(3)$ as an *operator* in the Hilbert space $\mathscr{H}$(BB) where

$$\mathscr{H}(\mathrm{BB})=\mathscr{H}_{\mathrm{NN}}\oplus\mathscr{H}_{\mathrm{N}\Delta}\oplus\mathscr{H}_{\Delta\mathrm{N}}\oplus\mathscr{H}_{\Delta\Delta} \ . \quad \text{(C1)}$$

Each orthogonal subspace $\mathscr{H}_{\mathrm{BB}'}$ is spanned by the basis states $\{\,|\,\mathbf{p};\sigma_{\mathrm{B}}m_{\mathrm{B}}\sigma_{\mathrm{B}'}m_{\mathrm{B}'}\rangle\}$ where $\mathbf{p}$ is the relative momentum, σ_{B} represents the spin-isospin quantum numbers for baryon B, and m_{B} are the corresponding z projections. The matrix interpretation of $t^{(0)}(3)$ may then be obtained through the definition

$$t^{(0)}_{cd,ab}\equiv\sum_{m's}|\,\sigma_c m_c\sigma_d m_d\rangle\langle\sigma_c m_c\sigma_d m_d\,| \times t^{(0)}\,|\,\sigma_a m_a\sigma_b m_b\rangle\langle\sigma_a m_a\sigma_b m_b\,| \ . \quad \text{(C2)}$$

The scattering equation that $t^{(0)}$ satisfies is essentially given by Eq. (22) except that, as discussed after Eq. (32), $t^{(1)}$ is approximated by a potential v, i.e.,

$$t^{(0)}=v+vG^{(0)}t^{(0)} \ . \quad \text{(C3)}$$

In a similar way we may write the bound state wave equation in the Hilbert space $\mathscr{H}$(BB) as

$$|\,\psi(1,2)\rangle_d=G^{(0)}(1,2)v(1,2)\,|\,\psi(1,2)\rangle_d \ , \quad \text{(C4)}$$

which again may be considered as a matrix equation through the definitions

$$|\,\psi_{\mathrm{BB}'}(1,2)\rangle_d\equiv\sum_{m's}|\,\sigma_{\mathrm{B}}m_{\mathrm{B}}\sigma_{\mathrm{B}'}m_{\mathrm{B}'}\rangle\langle\sigma_{\mathrm{B}}m_{\mathrm{B}}\sigma_{\mathrm{B}'}m_{\mathrm{B}'}\,|\,\psi(1,2)\rangle_d \ . \quad \text{(C5)}$$

In the last two equations we have specified particle labeling by the argument (1,2). We could equally well have assigned the labels (2,1). The antisymmetry of the wave function $|\,\psi_{\mathrm{BB}'}(1,2)\rangle_d$ follows immediately from the realization that

$$\langle\,\mathbf{p};\sigma_a m_a\sigma_b m_b\,|\,v(1,2)\,|\,\mathbf{p}';\sigma_c m_c\sigma_d m_d\,\rangle=\langle\,\mathbf{p};\sigma_a m_a\sigma_b m_b\,|\,v(2,1)\,|\,\mathbf{p}';\sigma_c m_c\sigma_d m_d\,\rangle \quad \text{(C6)}$$

and that all particles are fermions. In particular we note that

$$\langle\,\mathbf{p};\sigma_{\mathrm{B}}m_{\mathrm{B}}\sigma_{\mathrm{B}'}m_{\mathrm{B}'}\,|\,\psi_{\mathrm{BB}'}\rangle_d=-\langle\,-\mathbf{p};\sigma_{\mathrm{B}'}m_{\mathrm{B}'}\sigma_{\mathrm{B}}m_{\mathrm{B}}\,|\,\psi_{\mathrm{B}'\mathrm{B}}\rangle_d \ . \quad \text{(C7)}$$

Using Eq. (C7) we may write the bound state equation as

$$\langle\,\mathbf{p};\sigma_a m_a\sigma_b m_b\,|\,\psi_{ab}\,\rangle_d^{\mathrm{AS}}=\sum_{\substack{\sigma_c\sigma_d\\ m_c m_d}}\int d\mathbf{p}'\,G_{ab}^{\mathrm{AS}}(E_d)\,{}^{\mathrm{AS}}\langle\,\mathbf{p};\sigma_a m_a\sigma_b m_b\,|\,v\,|\,\mathbf{p}';\sigma_c m_c\sigma_d m_d\,\rangle^{\mathrm{AS}}\,\langle\,\mathbf{p};\sigma_c m_c\sigma_d m_d\,|\,\psi_{cd}\,\rangle_d^{\mathrm{AS}} \ , \quad \text{(C8)}$$

where the sum extends over NN, NΔ, and ΔΔ states only, $G_{ab}^{AS} \equiv (G_{AS}^{(0)})_{ab}$ is given by Eq. (75), and

$$|\psi_{BB'}\rangle^{AS} = |\psi_{BB'}\rangle_d \quad \text{if } B=B'$$
$$= 2|\psi_{BB'}\rangle_d \quad \text{if } B\neq B' \; . \tag{C9}$$

The antisymmetrized basis states in Eq. (C8) are defined by

$$|\mathbf{p};\sigma_B m_B \sigma_{B'} m_{B'}\rangle^{AS} \equiv \frac{1}{\sqrt{2}}(|\mathbf{p};\sigma_B m_B \sigma_{B'} m_{B'}\rangle - |-\mathbf{p};\sigma_{B'} m_{B'} \sigma_B m_B\rangle) \; . \tag{C10}$$

In deriving the BB-πBB equations it is necessary to take residues at bound state (or resonance) poles of two-body amplitudes. Having described the bound state wave function as above, the pole structure of the t matrix follows from writing the solution using Fredholm theory[47] as

$$t^{(0)}(E) = \sum_n \frac{|\phi_n\rangle\langle\phi_n|}{1-\lambda_n(E)} \; , \tag{C11}$$

where $\lambda_n(E)$ and $|\phi_n\rangle$ are the eigenvalues and eigenvectors of the kernel of the integral equation (C3) at energy E. At a bound state (or resonance) one of the eigenvalues $\lambda_d(E)\to 1$ as $E\to E_d$ with $t^{(0)}(E)$ having a pole. The corresponding eigenvector or form factor $|\phi_d\rangle$ is related to the wave function by

$$|\phi_d\rangle = v\,|\psi\rangle_d \; . \tag{C12}$$

As implied by Eq. (C5), both $|\phi_d\rangle$ and $|\psi\rangle_d$ may be viewed as column vectors, and as such we shall write them as $|\boldsymbol{\phi}_d\rangle$ and $|\boldsymbol{\psi}\rangle_d$ to distinguish them from square matrices.

If v is separable then we may write

$$v_{cd,ab} = |h_{cd}\rangle\lambda_{cd,ab}\langle h_{ab}| \tag{C13}$$

or

$$v = |h\rangle\lambda\langle h| \; , \tag{C14}$$

where $|h\rangle$ is a 4×4 diagonal matrix with elements $|h_{BB'}\rangle$. Using Eq. (C14) in Eq. (C12) gives

$$|\boldsymbol{\phi}\rangle = |h\rangle\mathbf{F} \; , \tag{C15}$$

where

$$\mathbf{F} = \lambda\langle h|\boldsymbol{\psi}\rangle_d \; . \tag{C16}$$

Finally we note that $|h_{BB'}\rangle$ is antisymmetric in the same sense as the wave function in Eq. (C7).

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