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# Probability current conservation imposed on nucleon knockout amplitudes

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We construct nucleon knockout amplitudes which exactly satisfy probability current conservation. The derivation is first produced for a nucleon bound to an inert core and is then generalized to the case of realistic nuclei. Numerical tests are presented for a nucleon bound in a square well potential for which exact results are compared with several approximated ones; the need of imposing probability current conservation is also numerically demonstrated.

## I. INTRODUCTION

The amplitude for a nuclear reaction in general, and for nucleon knockout in particular, is the matrix element of a transition operator $T$ taken between initial and final nuclear states. In the latter type of reactions it is not uncommon that different approximations are used for the nuclear many-body Hamiltonian in initial and final channels. For example, in a nucleon knockout reaction, the final state interaction (FSI) of the knocked-out nucleon and the residual nucleus is either neglected (plane-wave approximation), or is approximated by a complex optical potential. The latter need not necessarily generate the initial state and, consequently, wave functions for a bound $\phi_0$ and knocked-out nucleon $\psi_n$ are not orthogonal. An immediate consequence may be inferred if the knock-out mechanism is weak. The amplitude then appears proportional to the inelastic target form factor $F_{0n}(q)$ with $q$ the momentum transfer. The latter ought to go to zero for momentum transfer $q \to 0$, yet it will not if $\langle \phi_0, \psi_n \rangle \neq 0$. For $q \neq 0$ there is a more general constraint on $F_{0n}(q)$, namely the conservation of the probability current (PCC) of which orthogonality is a particular case.[1-3] Implementation of PCC is the topic of the present investigation. For an electromagnetic knockout process one clearly deals with the conservation of the em current or, equivalently, gauge invariance.[4]

In the present paper we consider nucleon knockout by a weak scalar probe, for which the nuclear part of the amplitude is just the inelastic form factor. New expressions for the knockout amplitudes which respect PCC are derived. In Sec. II we do so for a nucleon bound to an inert core. Apart from the exact expressions, we also focus on the Born approximation (BA) applied to a weak interaction, and on the eikonal approximation in the case of a fast, ejected nucleon. Section III contains generalizations to realistic nuclei.

In Sec. IV we present results of a numerical analysis for a nucleon bound to a square well. It is shown that approximations constrained by PCC are not only desirable on principal grounds, they are by and large closer to the exact answer than their standard counterparts.

## II. KNOCKOUT AMPLITUDES IN A SINGLE-PARTICLE NUCLEAR MODEL

### A. Exact expressions

In this subsection we derive exact formulae for transition amplitudes which have the desirable property of satisfying PCC in any approximation.

We start with a simple nuclear model in which a nucleon is bound to an inert core by a local, Hermitian potential $V_N$. Consider a weakly interacting scalar probe without internal degrees of freedom, exciting the nucleon from its ground state $\phi_0$ (energy $\epsilon_0 < 0$) to a continuum state $\psi_{\mathbf{p}}(E_{\mathbf{p}} = \mathbf{p}^2/2m)$ with asymptotic momentum $\mathbf{p}$ (Fig. 1). With $\mathbf{k},\mathbf{k}'$ the initial and final momenta of the projectile, and $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ the momentum transfer, one finds to lowest order

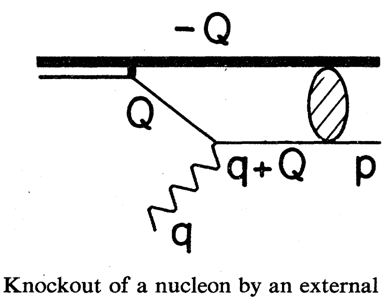


FIG. 1. Knockout of a nucleon by an external probe. The ellipse symbolizes scattering of the nucleon by the residual nucleus.

 

$$\langle \mathbf{k}'\psi_{\mathbf{p}}^{(-)} | T | \mathbf{k}\phi_0 \rangle \cong \langle \mathbf{k}'\psi_{\mathbf{p}}^{(-)} | U | \mathbf{k}\phi_0 \rangle = f_{x\mathrm{N}}(q) F_{0,\mathbf{p}}(\mathbf{q}) \ . \quad (2.1)$$

The factors in (2.1) are, respectively, the elementary projectile-nucleon amplitude $f_{x\mathrm{N}}$ and the inelastic form factor

$$F_{0,\mathbf{p}}(\mathbf{q}) = \int d\mathbf{r} \psi_{\mathbf{p}}^{(-)*}(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} \phi_0(r) \ . \quad (2.2)$$

The latter reads in the momentum representation

$$F_{0,\mathbf{p}}(\mathbf{q}) = \int \frac{d\mathbf{Q}}{(2\pi)^3} \psi_{\mathbf{p}}^{(-)*}(\mathbf{Q}+\mathbf{q}) \phi_0(Q) \ . \quad (2.3)$$

Consider now the operators for probability current and density $\hat{j}(\mathbf{r})$, $\hat{\rho}(r)$. For $\hat{V}^\dagger = \hat{V}$ these satisfy

$$\nabla \cdot \hat{\mathbf{j}}(\mathbf{r}) = i[\hat{\rho}(\mathbf{r}), \hat{H}]$$

or

$$\mathbf{q} \cdot \hat{\mathbf{j}}(\mathbf{q}) = -[\hat{\rho}(\mathbf{q}), \hat{H}] \ , \quad (2.4)$$

which expresses the conservation of probability current density. Taking matrix elements of (2.4) between $\phi_0$, $\psi_{\mathbf{p}}^{(-)}$ one has

$$\mathbf{q} \cdot \mathbf{j}_{0,\mathbf{p}}(\mathbf{q}) = \omega \rho_{0,\mathbf{p}}(\mathbf{q}) \ , \quad (2.5)$$

with

$$\omega = E_{\mathbf{p}} + |\epsilon_0| \ . \quad (2.6)$$

By definition one sees that

$$\rho_{0,\mathbf{p}}(\mathbf{q}) = \int d\mathbf{r} \psi_{\mathbf{p}}^{(-)*}(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} \phi_0(r) = \int \frac{d\mathbf{Q}}{(2\pi)^3} \psi_{\mathbf{p}}^{(-)*}(\mathbf{Q}+\mathbf{q}) \phi_0(\mathbf{Q}) \quad (2.7)$$

is just the transition form factor (2.2). Likewise,

$$j_{0,\mathbf{p}}(\mathbf{q}) = \int \frac{d\mathbf{Q}}{(2\pi)^3} \psi_{\mathbf{p}}^{(-)*}(\mathbf{Q}+\mathbf{q}) \frac{2\mathbf{Q}+\mathbf{q}}{2m} \phi_0(Q) \ . \quad (2.8)$$

Notice that the special case $\mathbf{q}=0$ in (2.7) expresses orthogonality

$$\rho_{0,\mathbf{p}}(\mathbf{q}=0) = \langle \psi_{\mathbf{p}}^{(-)} | \phi_0 \rangle = 0 \ . \quad (2.9)$$

Substituting Eqs. (2.7) and (2.8) into (2.5) one obtains

$$\int \frac{d\mathbf{Q}}{(2\pi)^3} \psi_{\mathbf{p}}^{(-)*}(\mathbf{Q}+\mathbf{q}) \left[ \mathbf{q} \cdot \frac{(2\mathbf{Q}+\mathbf{q})}{m} \right] \phi_0(Q) = \omega \int \frac{d\mathbf{Q}}{(2\pi)^3} \psi_{\mathbf{p}}^{(-)*}(\mathbf{Q}+\mathbf{q}) \phi_0(Q) \ . \quad (2.10)$$

In momentum representation the scattering state $\psi_{\mathbf{p}}^{(-)}$ reads

$$\psi_{\mathbf{p}}^{(-)*}(\mathbf{k}) = (2\pi)^3 \delta(\mathbf{k}-\mathbf{p}) + \frac{t(\mathbf{p},\mathbf{k})}{E_{\mathbf{p}} - E_{\mathbf{k}} + i\eta} \ . \quad (2.11)$$

Here

$$t(\mathbf{p},\mathbf{k}) \equiv \langle \mathbf{k} | t^{(-)}(E_{\mathbf{p}} - i\eta) | \mathbf{p} \rangle^* = \langle \mathbf{p} | t^+(E_{\mathbf{p}} + i\eta) | \mathbf{k} \rangle \quad (2.12)$$

is the half-off shell matrix element of the transition operator which satisfies a standard Lippmann-Schwinger equation $t = V + VG_0 t$. One now substitutes (2.11) into (2.1). Corresponding to the two terms of $\psi^{(-)}$ in Eq. (2.11), the inelastic form factor is decomposed into plane wave (PW) and final state interaction (FSI) parts

$$F_{0,\mathbf{p}}(q) = \phi_0(\mathbf{p}-\mathbf{q}) + \int \frac{d\mathbf{Q}}{(2\pi)^3} \frac{\phi_0(Q) t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} \ . \quad (2.13)$$

Similarly, using Eq. (2.11) in (2.8) gives

$$\mathbf{j}_{0,\mathbf{p}}(\mathbf{q}) = \frac{2\mathbf{p}-\mathbf{q}}{2m} \phi_0(\mathbf{p}-\mathbf{q}) + \int \frac{d\mathbf{Q}}{(2\pi)^3} \phi_0(Q) \frac{2\mathbf{Q}+\mathbf{q}}{2m} \frac{t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} \ . \quad (2.14)$$

Now, substituting Eqs. (2.13) and (2.14) into (2.5), the PCC relation may be rewritten as

$$\frac{\mathbf{q}\cdot(2\mathbf{p}-\mathbf{q})}{2m} \phi_0(\mathbf{p}-\mathbf{q}) + \int \frac{d\mathbf{Q}}{(2\pi)^3} \phi_0(Q) \frac{\mathbf{q}\cdot(2\mathbf{Q}+\mathbf{q})}{2m} \frac{t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} = \omega \phi_0(\mathbf{p}-\mathbf{q}) + \omega \int \frac{d\mathbf{Q}}{(2\pi)^3} \phi_0(Q) \frac{t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} \ . \quad (2.15)$$

It is easily seen that by making approximations for the FSI part only, one might violate PCC. In order to express quantitatively the degree of such a violation (see also Sec. IV) one may use the dimensionless ratio

$$R_{0,\mathbf{p}}(\mathbf{q}) = \frac{\omega \rho_{0,\mathbf{p}}(\mathbf{q}) - \mathbf{q} \cdot \mathbf{j}_{0,\mathbf{p}}(\mathbf{q})}{\omega \rho_{0,\mathbf{p}}(\mathbf{q})} \quad (2.16)$$

which ought to be zero if PCC is respected. However, taking as an example $t=0$, which is usually called the plane wave approximation (PW), we find using Eqs. (2.13), (2.14), and (2.16),

$$R_{0,\mathbf{p}}(\mathbf{q})_{\mathrm{PW}} = \frac{E_{\mathbf{p}-\mathbf{q}} + |\epsilon_0|}{E_{\mathbf{p}} + |\epsilon_0|} \neq 0 \ . \quad (2.17)$$

Results (2.17) show that the PW approximation does not respect PCC. This is due to the substitution $t \to 0$ which is applied to the FSI, but not simultaneously to the PW part.

Proceeding with our derivation, we obtain from (2.15) the following integral equation for $\phi_0$

$$\phi_0(\mathbf{p}-\mathbf{q}) = -\frac{1}{\omega + E_{\mathbf{p}-\mathbf{q}} - E_{\mathbf{p}}} \times \int \frac{d\mathbf{Q}}{(2\pi)^3} (\omega + E_{\mathbf{Q}} - E_{\mathbf{Q}+\mathbf{q}}) \times \phi_0(Q) \frac{t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} \ . \quad (2.18)$$

Equation (2.18) is now substituted into the first term of (2.13) resulting in

$$F_{0,\mathbf{p}}(\mathbf{q})=\frac{1}{E_{\mathbf{p}-\mathbf{q}}-\epsilon_0}\int\frac{d\mathbf{Q}}{(2\pi)^3}\frac{\mathbf{q}\cdot(\mathbf{Q}+\mathbf{q}-\mathbf{p})}{m}\phi_0(Q)\times\frac{t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}}-E_{\mathbf{Q}+\mathbf{q}}+i\eta}\ . \tag{2.19}$$

Also, substitution of Eq. (2.18) into (2.14) gives

$$\mathbf{j}_{0,\mathbf{p}}(\mathbf{q})=\int\frac{d\mathbf{Q}}{(2\pi)^3}\left[\frac{\mathbf{q}\cdot(\mathbf{Q}+\mathbf{q}-\mathbf{p})}{m(E_{\mathbf{p}-\mathbf{q}}+|\epsilon_0|)}\frac{2\mathbf{p}-\mathbf{q}}{2m}+\frac{\mathbf{Q}+\mathbf{q}-\mathbf{p}}{m}\right]\times\phi_0(Q)\frac{t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}}-E_{\mathbf{Q}+\mathbf{q}}+i\eta}\ . \tag{2.20}$$

Note that Eqs. (2.19) and (2.20) are exact and equivalent to, respectively, Eqs. (2.13) and (2.14). However, only the former set satisfies PCC even if $t(\mathbf{p},\mathbf{Q}+\mathbf{p})$ is approximated. This is a desirable property and we expect that these new expressions are preferable to the standard ones [Eqs. (2.13) and (2.14)] when approximations are introduced.

### B. Weak FSI interaction

We already saw from Eq. (2.17) that what is usually called the plane wave approximation (PW) is inconsistent with PCC. Consider the transition form factor (2.13) and its PCC counterpart (2.19) in the PW approximation $t\rightarrow 0$. Thus

$$F^{\mathrm{PW}}_{0,\mathbf{p}}(\mathbf{q})\nearrow \mathrm{Eq.}(2.13)\rightarrow\phi_0(|\mathbf{p}-\mathbf{q}|)\ , \tag{2.21a}$$

$$F^{\mathrm{PW}}_{0,\mathbf{p}}(\mathbf{q})\searrow \mathrm{Eq.}(2.19)\rightarrow 0\ . \tag{2.21b}$$

As has been remarked before that the difference between (2.21a) and (2.21b) is due to the fact that in the former the $t\rightarrow 0$ limit has not been applied to $\phi_0$. Had one done so, binding would not be possible, i.e., $\phi_0\rightarrow 0$, and one obtains Eq. (2.21b). Thus there is no *consistent* PW approximation for the inelastic form factor (2.2).

One may, however, consider the Born approximation $t\rightarrow V$. Equation (2.19) then becomes

$$F^{\mathrm{BA}}_{0,\mathbf{p}}(\mathbf{q})\underset{t\rightarrow V}{\rightarrow}\frac{1}{\epsilon_0-E_{\mathbf{p}-\mathbf{q}}}\int\frac{d\mathbf{Q}}{(2\pi)^3}\frac{\mathbf{q}\cdot(\mathbf{q}+\mathbf{Q}-\mathbf{p})}{\frac{\mathbf{q}+\mathbf{Q}+\mathbf{p}}{2}\cdot(\mathbf{q}+\mathbf{Q}-\mathbf{p})}\phi_0(Q)V(\mathbf{Q}+\mathbf{q}-\mathbf{p})=\frac{2\mathbf{q}\cdot\hat{\mathbf{n}}}{(\mathbf{q}+\mathbf{p}+\langle\mathbf{Q}\rangle)\cdot\hat{\mathbf{n}}}\cdot\phi_0(\mathbf{p}-\mathbf{q})\ , \tag{2.22}$$

where we have used the Schrödinger equation in momentum representation

$$\int\frac{d\mathbf{Q}}{(2\pi)^3}\phi_0(\mathbf{Q})V(\mathbf{Q}+\mathbf{p}-\mathbf{q})=(\epsilon_0-E_{\mathbf{p}-\mathbf{q}})\phi_0(\mathbf{p}-\mathbf{q})\ ; \tag{2.23}$$

$\langle\mathbf{Q}\rangle$ is some average of $Q$, determined by pronounced maxima of $\phi_0(Q)$ and $V(\mathbf{Q}+\mathbf{q}-\mathbf{p})$ in (2.22) and will be specified later;

$$\hat{\mathbf{n}}\equiv(\mathbf{q}-\mathbf{p}+\langle\mathbf{Q}\rangle)/(|\mathbf{q}-\mathbf{p}+\langle\mathbf{Q}\rangle|)\ .$$

Notice that in view of the singularity in the energy denominator of (2.19) care should be taken in the extraction of $\langle\mathbf{Q}\rangle$. If in the integrand of (2.22) $\phi_0$ peaks strongly at $\mathbf{Q}\sim 0$ (weak binding), then

$$F^{\mathrm{BA}}_{0,\mathbf{p}}(\mathbf{q})\underset{\langle Q\rangle\rightarrow 0}{\rightarrow}\frac{2\mathbf{q}\cdot(\mathbf{q}-\mathbf{p})}{\mathbf{q}^2-\mathbf{p}^2}\phi_0(\mathbf{p}-\mathbf{q})\ . \tag{2.24}$$

The estimate above makes sense only if $|\mathbf{p}|\neq|\mathbf{q}|$. If, however, $\mathbf{p}||\mathbf{q}$, Eq. (2.24) becomes

$$F^{\mathrm{BA}}_{0,\mathbf{p}}(\mathbf{q}||\mathbf{p})\sim\frac{2q}{p+q}\phi_0(\mathbf{p}-\mathbf{q}) \tag{2.25}$$

which is also defined for $p=q$.

In the alternative strong binding regime, the maxima of $\phi_0(Q)$ and $V(Q)$ are about equally pronounced, in which case $\langle\mathbf{Q}\rangle$ lies somewhere between $\mathbf{0}$ and $\mathbf{p}-\mathbf{q}$. In this case we do not have a simple expression for $F_{0,\mathbf{p}}$.

### C. Eikonal approximation

Consider next the case of a fast exiting nucleon to which we may apply the eikonal approximation. In this approximation the matrix element $\langle\mathbf{Q}+\mathbf{q}|G_0t|\mathbf{p}\rangle$ is given by (Ref. 5) ($v=p/m$)

$$\frac{t(\mathbf{p},\mathbf{Q}+\mathbf{q})}{E_{\mathbf{p}}-E_{\mathbf{Q}+\mathbf{q}}+i\eta}=\int d\mathbf{r}\,e^{i(\mathbf{p}-\mathbf{q}-\mathbf{Q})\cdot\mathbf{r}}\left(e^{(-i/v)\int_{-\infty}^{z}V(\mathbf{b},z')dz'}-1\right)\ , \tag{2.26}$$

where $\mathbf{r}=(\mathbf{b},z)$, and the $z$ axis is taken to be parallel to $\mathbf{p}$.

Substituting Eq. (2.26) into Eq. (2.19) we find

$$F^{\mathrm{eik}}_{0,\mathbf{p}}(\mathbf{q})=\frac{1}{\epsilon_0-E_{\mathbf{p}-\mathbf{q}}}\int d\mathbf{r}\,\phi_0(\mathbf{r})e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}}\times\frac{\mathbf{q}\cdot\nabla}{m}e^{(-i/v)\int_{-\infty}^{z}V(\mathbf{b},z')dz'}\ . \tag{2.27}$$

In particular if $\mathbf{p}||\mathbf{q}$ one may eliminate the derivative in the integrand of Eq. (2.27). The result is

$$F^{\mathrm{eik}}_{0,\mathbf{p}}(\mathbf{q})=\frac{1}{\epsilon_0-E_{\mathbf{p}-\mathbf{q}}}\int d\mathbf{r}\,\phi_0(r)e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}}\frac{q}{p}V(\mathbf{r})\times e^{(-i/v)\int_{-\infty}^{z}V(\mathbf{b},z')dz'}\ . \tag{2.28}$$

For comparison the standard eikonal approximation obtained upon substitution of (2.26) into (2.13) is given by[6]

$$F^{\mathrm{eik}}_{0,\mathbf{p}}(\mathbf{q})=\int d\mathbf{r}\,\phi_0(r)e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}}e^{(-i/v)\int_{-\infty}^{z}V(\mathbf{b},z')dz'}\ . \tag{2.29}$$

This concludes our discussion of the inelastic form factor (2.2) needed in the description of knockout of a nucleon bound to an inert core. In the following section we shall attempt to make generalizations to realistic nuclei.

## III. AMPLITUDES FOR NUCLEON KNOCKOUT FROM A MANY-BODY NUCLEAR SYSTEM

Under the conditions stated in Sec. II A, the amplitude for removing a nucleon from a $A$-nucleon system is proportional to the inelastic form factor.

$$F_{0,\mathbf{p}n}(\mathbf{q}) \equiv F_{0,\mathbf{p}}^{A;(A-1)_n}$$
$$= \langle \psi_{\mathbf{p}}^{(A-1)_n} | e^{i\mathbf{q}\cdot\mathbf{r}_1} | \phi_0^A \rangle \ . \tag{3.1}$$

Here $\phi_0^A$ is the target ground state wave function and $\psi_{\mathbf{p}}^{(A-1)_n}$ a scattering state of a nucleon (labeled "1") with asymptotic momentum $\mathbf{p}$ and a residual nucleus with $A-1$ nucleons in a state $n$.

For the $A$-body system one may define a single particle density and current density operators by

$$\hat{\rho}(\mathbf{r}) = \sum_{j=1}^{A} \delta(\mathbf{r}-\mathbf{r}_j) \ ,$$
$$\hat{\mathbf{j}}(\mathbf{r}) = \frac{1}{2mi} \sum_{j=1}^{A} \{ \nabla_j \delta(\mathbf{r}-\mathbf{r}_j) + \delta(\mathbf{r}-\mathbf{r}_j)\nabla_j \} \ . \tag{3.2}$$

As in Sec. II, transition matrix elements of the operators above, will then for a Hermition $\hat{H}$, satisfy

$$\langle \psi_{\mathbf{p}}^{(A-1)_n} | \mathbf{q}\cdot\mathbf{j}(\mathbf{q}) | \phi_0^A \rangle = \omega \langle \psi_{\mathbf{p}}^{(A-1)_n} | \rho(\mathbf{q}) | \phi_0^A \rangle \ . \tag{3.3}$$

Here

$$\omega = E_{\mathbf{p}} + \Delta_{0n} \ , \tag{3.4}$$

with $\Delta_{0n} = \epsilon_n^{A-1} - \epsilon_0^A$, the nucleon separation energy. The scattering state $| \psi_{\mathbf{p}}^{(A-1)_n} \rangle$ satisfies

$$| \psi_{\mathbf{p}}^{(A-1)_n} \rangle = | \mathbf{p} \otimes \phi_n^{A-1} \rangle + G_1 \bar{V}_1 | \psi_{\mathbf{p}}^{(A-1)_n} \rangle \ . \tag{3.5}$$

Here

$$\bar{V}_1 = \sum_{i=2}^{A} v_{1i} \ ,$$
$$G_1(E) = (E - H_{A-1} - h_1^0 + i\eta)^{-1} \ , \tag{3.6}$$

with $h_1^0$ the kinetic energy operator for nucleon 1. Next we define a transition operator by

$$T_1 = \bar{V}_1(1 + G_1 T_1) \ . \tag{3.7}$$

Equation (3.5) may then be rewritten as

$$| \psi_{\mathbf{p}}^{(A-1)_n} \rangle = (1 + G_1 T_1) | \mathbf{p} \otimes \phi_n^{A-1} \rangle \ . \tag{3.8}$$

Substitution of Eq. (3.8) into (3.1) leads to the standard result

$$F_{0,\mathbf{p}n}(\mathbf{q}) = \chi_n(\mathbf{p}-\mathbf{q}) + \sum_{n'} \int \frac{d\mathbf{Q}}{(2\pi)^3} \chi_{n'}(\mathbf{Q}) \frac{t_{n'n}(\mathbf{p},\mathbf{q}+\mathbf{Q})}{\omega - \Delta_{0n'} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} \ . \tag{3.9}$$

In Eq. (3.9),

$$t_{n'n}(\mathbf{p},\mathbf{q}+\mathbf{Q}) \equiv \langle (\mathbf{q}+\mathbf{Q}) \otimes \phi_{n'}^{A-1} | T_1 | \mathbf{p} \otimes \phi_n^{A-1} \rangle \tag{3.10}$$

and

$$\chi_n(\mathbf{p}) = \langle \phi_0^A | \mathbf{p} \otimes \phi_n^{A-1} \rangle \ . \tag{3.11}$$

Note that in Eq. (3.11) $\chi_n(\mathbf{p})$, the probability amplitude to find in the $A$ particle ground state, a $(A-1)$ particle core in state $n$ and a nucleon with relative momentum $\mathbf{p}$, plays the role of wave function $\phi_0$ in (2.13). Proceeding as in Sec. II one can show that parallel to (2.18),

$$\chi_n(\mathbf{p}-\mathbf{q}) = \frac{1}{-\Delta_{0n} - E_{\mathbf{p}-\mathbf{q}}} \int \frac{d\mathbf{Q}}{(2\pi)^3} [\omega + E_{\mathbf{Q}} - E_{\mathbf{Q}+\mathbf{q}}] \sum_{n'} \chi_{n'}(\mathbf{Q}) \frac{t_{n'n}(\mathbf{p},\mathbf{Q}+\mathbf{q})}{\omega - \Delta_{0n'} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} \ . \tag{3.12}$$

When substituted into (3.11) there results a many-channel generalization of (2.19)

$$F_{0,\mathbf{p}n}(\mathbf{q}) = \frac{1}{E_{\mathbf{p}-\mathbf{q}} + \Delta_{0n}} \int \frac{d\mathbf{Q}}{(2\pi)^3} \frac{\mathbf{q}\cdot(\mathbf{Q}+\mathbf{q}-\mathbf{p})}{m} \sum_{n'} \chi_{n'}(\mathbf{Q}) \frac{t_{n'n}(\mathbf{p},\mathbf{Q}+\mathbf{q})}{\omega - \Delta_{0n'} - E_{\mathbf{Q}+\mathbf{q}} + i\eta} \ . \tag{3.13}$$

Disregarding excitations $n' \neq n$, one finds a quasi-one-particle result with $t_{nn}$ directly calculated from an optical potential $V_{\rm opt}$, which is in general complex.

Next come the generalizations of the approximations discussed in Secs. II B and II C. We start with what is usually called the PW approximation

$$F_{0,\mathbf{p}n}^{\rm PW} = \chi_n(\mathbf{p}-\mathbf{q}) \ . \tag{3.14}$$

The critical remarks made after Eq. (2.21) hold here as well.

There is no difficulty to generalize the BA respecting PCC. A weak elementary NN force leads of course to $T_1 \to \bar{V}_1 = \sum_{j=2}^{A} v_{1j}$ and thus

$$F_{0,\mathbf{p}n}^{\rm BA} \sim \frac{2\mathbf{q}\cdot\hat{\mathbf{n}}}{(\mathbf{q}+\mathbf{p}+\langle \mathbf{Q} \rangle)\cdot\hat{\mathbf{n}}} \chi(\mathbf{p}-\mathbf{q}) \ . \tag{3.15}$$

Without further difficulty we write down the eikonal approximation which respects PCC for $F_{0,\mathbf{p}n}$,

$$F^{\rm eik}_{0,\mathbf{p}n}=(E_{\mathbf{p}-\mathbf{q}}+\Delta_{0n})^{-1}\int\cdots\int d\mathbf{r}_1\cdots d\mathbf{r}_A\phi_0^A(\mathbf{r}_2\cdots r_A)\phi_n^{A-1*}e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}}$$
$$\times\frac{q}{p}\sum_{j=2}^{A}[v_{1j}(\mathbf{r}_1-\mathbf{r}_j)]e^{-(i/v)}\sum_{j=2}^{A}\int_{-\infty}^{z_l}v_{1j}(\mathbf{b}_1-\mathbf{b}_j,z_1'-z_j)dz_1' \tag{3.16}$$

where $\mathbf{r}_i=(\mathbf{b}_i,z_i)$, and $\mathbf{p}||\mathbf{q}$ has been assumed.

## IV. NUMERICAL RESULTS FOR A SQUARE WELL

In this section we present results for a nucleon bound in the lowest $S$ state of a square well. Bound and scattering state wave functions can be easily obtained.[7]

Two sets of potential parameters have been studied:

$$\text{Strong binding: } V_0=-40\text{ MeV};\ a_0=3\text{ fm};\quad \epsilon_0=-25.62\text{ MeV}, \tag{4.1}$$

$$\text{Weak binding: } V_0=-33\text{ MeV};\ a_0=1.5\text{ fm};\quad \epsilon_0=-2.16\text{ MeV}. \tag{4.2}$$

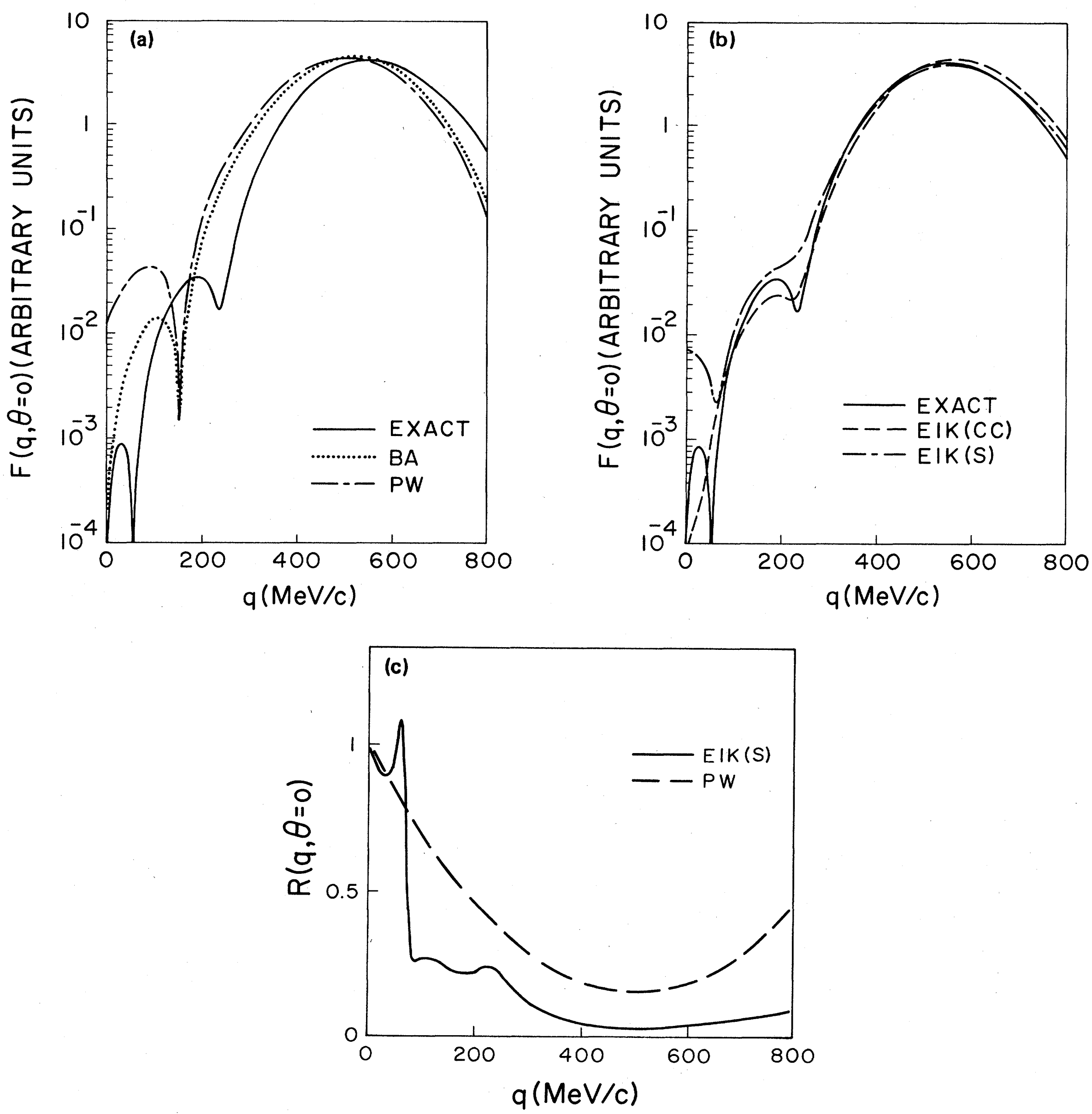


FIG. 2. (a) Comparison of $|F^{\rm ex}|$, $|F^{\rm PW}|$, and $|F^{\rm BA}|$ for the strong-binding parameter set (4.1); $p=500$ MeV/$c$, $\mathbf{p}||\mathbf{q}$. (b) Comparison of $|F^{\rm ex}|$, $|F^{\rm eik}(S)|$, and $|F^{\rm eik}(CC)|$, with the same parameters as in Fig. 2(a). (c) Function $R(q)$, which measures the extent of PCC violation in a given approximation, is plotted for PW and EIK($S$). Parameters are the same as in (a).

For these model parameters above and with fixed $p=500$ MeV, we calculated $[\theta=\cos^{-1}(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})]$,

$$F(q,\theta)=|F_{0,\mathbf{p}}(\mathbf{q})| \tag{4.3}$$

and compare

(i) $F^{\text{ex}}$: exact result, Eq. (2.3) or (2.19);

(ii) $F^{\text{PW}}$: plane wave approximation, Eq. (2.21a);

(iii) $F^{\text{BA}}$: PCC conserving Born approximation, Eq. (2.24);

(iv) $F^{\text{eik}}(S)$: standard eikonal approximation, Eq. (2.27);

(v) $F^{\text{eik}}(CC)$: PCC respecting eikonal approximation, Eq. (2.29).

We first discuss results for parameters (4.1) leading to strong binding. Figure 2(a) shows that, due to the strong distorting potential, PW and BA are clearly not adequate.

It is seen from Fig. 2(b) that $F^{\text{eik}}(CC)$ fits the overall shape of $F^{\text{ex}}$ better than $F^{\text{eik}}$ does. Because of lack of orthogonality $F^{\text{eik}}(S)$ does not vanish at $q=0$, consequently it is unreliable for small $q$. Both $F^{\text{eik}}(S)$ and $F^{\text{eik}}(CC)$ are very close to $F^{\text{ex}}$ in the region of the quasi-elastic peak $q\sim p$. In fact here $F^{\text{eik}}(S)$ seems to do somewhat better than $F^{\text{eik}}(CC)$. This can be attributed to the fact that, in order to maintain PCC in a given approximation, one must also approximate $\phi_0$ accordingly [see Eq. (2.18)]. This seems to have an adverse effect in the quasi-elastic region where violation of PCC by $F^{\text{eik}}$ is minimal. [See Fig. 2(c).]

A comparison between Figs. 2(a) and (b) clearly shows the superiority of the eikonal approximation over PW and BA as expected. The same observation can also be made for the angular distributions shown in Figs. 3(a) and (b).

As mentioned before, the function $R_{0,\mathbf{p}}(\mathbf{q})$ of Eq. (2.16) measures the extent of PCC violation in a given approximation. This function is plotted for PW and EIK($S$) in Fig. 2(c), with $\mathbf{q}||\mathbf{p}$, and $p=500$ MeV/$c$. We see that PW, and to a lesser extent EIK($S$), violate PCC badly around $q=0$. However, these violations are minimal around the quasi-elastic peak ($q\sim p$).

Next we discuss the weak binding case for which BA is expected to be adequate. That this is indeed borne out, can clearly be seen in Fig. 4(a). $F^{\text{PW}}$ is not reliable for small $q$, again due to lack of orthogonality. Figure 4(b) shows that $F^{\text{eik}}(CC)$ is much closer to $F^{\text{ex}}$ than is $F^{\text{eik}}(S)$ for small $q$, while the latter does better in the quasi-elastic region. Figure 4(c) is qualitatively similar to Fig. 2(c), indicating that PCC violations are larger when $q$ is small, and minimal near the quasi-elastic peak.

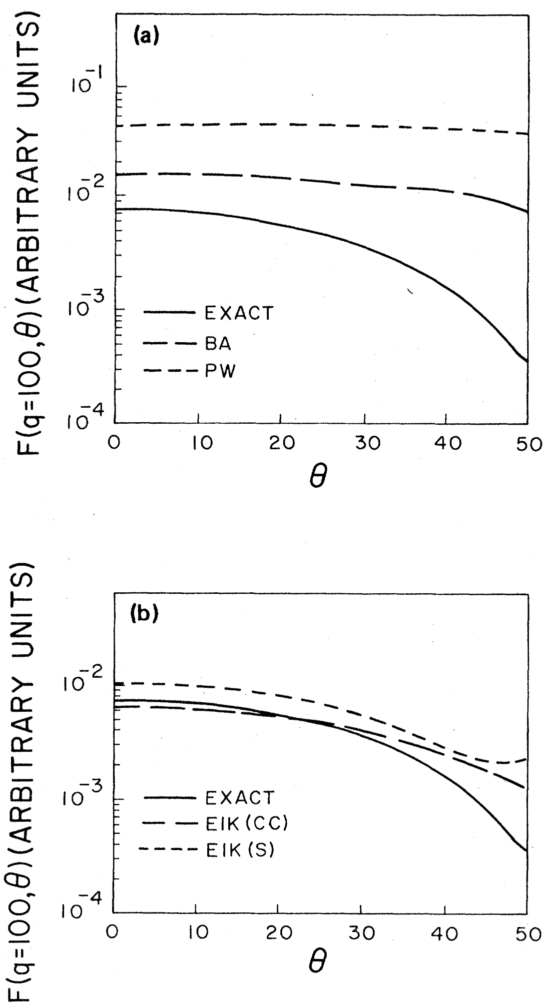


FIG. 3. (a) Comparison of angular dependence of $|F^{\text{ex}}|$, $|F^{\text{PW}}|$, and $|F^{\text{BA}}|$, with parameter set (4.1), $p=500$ MeV/$c$, and $q=100$ MeV/$c$. (b) Comparison of angular dependence of $|F^{\text{ex}}|$, $|F^{\text{eik}}(S)|$, and $|F^{\text{eik}}(CC)|$, with parameters the same as in (a).

## V. SUMMARY

We have studied above various versions of, and approximations for nucleon removal amplitudes by imposing the condition of probability current conservation (PCC). A relation has been established between the transition matrix element of the probability density (transition form factor) and that of the current density. This relation is generally violated when one invokes approximations in the calculation of final state interactions.

We have derived above new, exact expressions, (2.19) and (2.20), for amplitudes describing nucleon knockout by a weak scalar probe. These have the merit of respecting PCC, even if the final state interaction is not calculated exactly. In particular, the orthogonality constraint is manifestly satisfied in any approximation and appears as a special case of PCC in the limit $q\rightarrow 0$. PCC has first been implemented in a model with a nucleon bound to an inert core and has subsequently been generalized to nucleon knockout from a genuine nuclear target.

For the case of a particle bound in a square well, numerical calculations demonstrate the expectation that, wherever current conservation is significantly violated, PCC respecting approximations resemble exact answers more than do their standard PCC violating analogs. This is, for instance, the case for the FSI treated in the eikonal approximation. Our results clearly show that for the

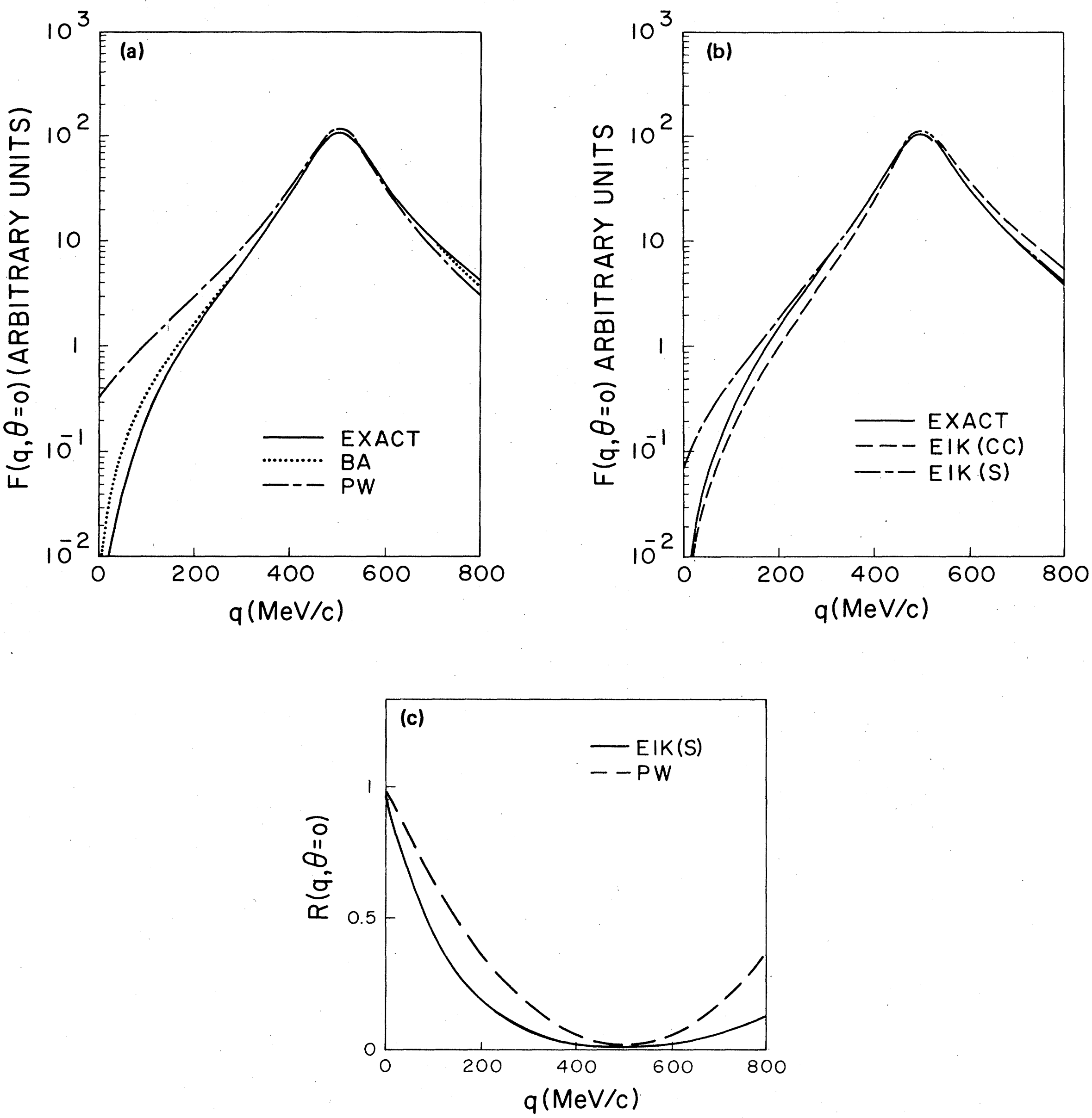


FIG. 4. (a) Same as Fig. 2(a) for weak-binding parameter set (4.2). (b) Same as Fig. 2(b) for set (4.2). (c) Same as Fig. 2(c) for set (4.2).

$q \sim 0$ region, only PCC respecting approximations approach the exact results. However, it has been found that PCC violations by PW and EIK($S$) are minimal in the region of the quasi-elastic peak ($\mathbf{p} \sim \mathbf{q}$). There usage of our new expressions does not improve the quality of the PCC violating results. It is reasonable to expect that these conclusions will also hold in the case of nucleon knockout from a realistic many-body target.[8]

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