Validity of the coupled-channel method for the study of Δ excitation in intermediate-energy NN scattering

T.-S. H. Lee
Argonne National Laboratory, Argonne, Illinois 60439

A. Matsuyama
Swiss Institute for Nuclear Research, Villigen, Switzerland

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Within the unitary πNN theory with Δ excitation, the effect of πd and πNN production channels on NN elastic scattering is examined. It is shown that the two-body coupled-channel method, which has been widely employed to study the Δ excitation in intermediate-energy NN scattering, is a valid approximation. Our results indicate that the difficulty encountered by the meson-exchange theory in understanding the NN data up to 1 GeV cannot be resolved within the model based on the one-pion-exchange Δ excitation mechanism.

Intermediate-energy nucleon-nucleon (NN) scattering has been extensively studied with the coupled-channel model.[1–5] The model is constructed by extending the conventional meson-exchange theory of NN force to include the Δ excitation. The propagation of Δ is described according to the πN P_{33} resonant dynamics which can be most conveniently described by a πN↔Δ vertex interaction [Fig. 1(a)]. No interaction in the πNN intermediate state is included. Because of this simplicity, it is straightforward to carry out an extensive calculation in practice.

The coupled-channel model has achieved impressive successes in describing the NN scattering phase-shift data. However, problems are encountered[3] in understanding the energy dependences of the NN total cross sections with various spin orientations of either the target or the projectile. The NN results and many studies[6] of πd spin observables have been very suggestive of the possibility that the short-range quark dynamics has exhibited its genuine signature even in the intermediate-energy region. One of the interesting developments motivated by these results is the exploration of the possible existence of dibaryon resonances.

Before any major step is taken to implement the much less understood quark dynamics into the theory, it is important to recognize the fact that the NN scattering can couple to πd and πNN production channels and can only be properly described in a unitary approach to the problem. Within the unitary formulation[7,8] of πNN interactions, the existing coupled-channel description of Δ excitations is incomplete even on the level of one-pion approximation. Under the NN and πNN unitarity conditions, all of the Δ excitation mechanisms shown in Fig. 1 are necessary consequences of the one-pion intermediate state. The mechanism shown in Fig. 1(a) is the familiar free Δ propagation. In Fig. 1(b), the Δ↔πN decay mechanism can generate the exchange of an *on-mass-shell* pion between two NΔ states. Figure 1(c) shows that the intermediate πNN state can be influenced by the NN scattering. If the NN subsystem is in the deuteron state, this is the lowest order effect due to the coupling to the πd channel. The existing coupled-channel studies based on meson theory only consider the free Δ propagation mechanism [Fig. 1(a)]. In this paper, we want to examine the extent to which discrepancies between the NN data and the existing coupled-channel calculations are due to the incomplete treatment of this "long-range" πNN dynamics. Clearly, this is an important step before any serious search of quark dynamics can be taken.

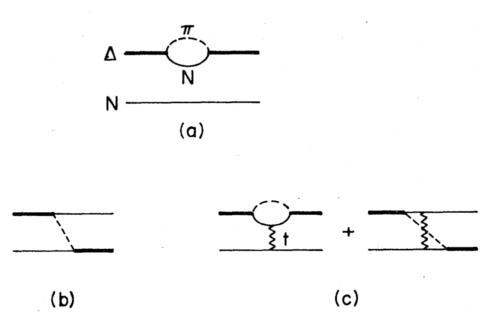

FIG. 1. Δ excitation mechanisms in the one-pion approximation. See the text for the meaning of each term.

Our starting point is the πNN formulation of Ref. 8, which is most convenient for illustrating the approximation employed in deriving the coupled-channel model. The essence of the theory is to introduce a systematic procedure for extending the simplest Δ excitation meson theory to include the effects due to the nonresonant πN interaction and possible existence of dibaryon resonances originating from the short-range quark dynamics. In this general formulation, the existing coupled-channel model for NN scattering can be derived as follows. Keeping only the Δ excitation terms in Sec. III A of Ref. 8, the NN scattering t matrix equation [summarized in Eqs. (3.27) and (3.38) of Ref. 8] can be cast exactly into the following form:

$$T(E)=U(E)+U(E)\frac{P_{\rm NN}}{E-H_0+i\epsilon}T(E)\ , \tag{1}$$

where P_{NN} is the projection operator for the NN state and H_0 is the sum of kinetic energy operators. The effective NN potential consists of three terms,

$$U(E)=V_{NN,NN} + V_{NN,N\Delta}\frac{P_{N\Delta}}{E-H_0-\Sigma_\Delta(E)}V_{N\Delta,NN}+U_c(E) \quad (2)$$

with

$$U_c(E)=V_{NN,N\Delta}\frac{P_{N\Delta}}{E-H_0-\Sigma_\Delta(E)} \times T_c(E)\frac{P_{N\Delta}}{E-H_0-\Sigma_\Delta(E)}V_{N\Delta,NN} . \quad (3)$$

In the above equations, $P_{N\Delta}$ is the projection operator for the NΔ state. $V_{NN,NN}$ and $V_{NN,N\Delta}$ are, respectively, the NN$\rightarrow$NN and NN$\rightarrow$NΔ transition potential constructed from the meson theory. All of the Δ resonant effects due to the coupling to πd and πNN production channels are described by the Δ self-energy $\Sigma_\Delta(E)$ and the connected NΔ scattering matrix $T_c(E)$. Both are related to the πN$\leftrightarrow\Delta$ vertex interaction h,

$$\Sigma_\Delta(E)=\sum_{i=1}^{2}h_i^+\frac{P_{\pi NN}}{E-H_0+i\epsilon}h_i , \quad (4)$$

$$T_c(E)=V_c(E)+V_c(E)\frac{P_{N\Delta}}{E-H_0-\Sigma_\Delta(E)}T_c(E) , \quad (5)$$

with

$$V_c(E)=\sum_{i\neq j}^{2}h_i^+\frac{P_{\pi NN}}{E-H_0+i\epsilon}h_j + \sum_{i,j}^{2}h_i^+\frac{P_{\pi NN}}{E-H_0+i\epsilon}t(E-E_\pi)\frac{P_{\pi NN}}{E-H_0+i\epsilon}h_j . \quad (6)$$

The first term of Eq. (6) is the interaction due to the exchange of an *on-mass-shell* pion shown in Fig. 1(b) (one-pion-exchange N$\Delta$$\rightarrow$N$\Delta$ interaction generated by a $\Delta\leftrightarrow\pi\Delta$ is considered to be a 2π contribution and is neglected). The second term describes the NN scattering in the presence of a spectator pion [Fig. 1(c)]. Clearly, this πNN interaction term contains the effect due to the coupling to the πd state since $t(E-E_\pi)$ has a deuteron pole in the $^3S_1+{}^3D_1$ channel. The higher-order effects which can be generated by these two "long-range" mechanisms are described by the integral Eq. (5) of T_c. Equations (1)—(6) are equivalent to the coupled NN$\oplus$NΔ equation employed by Betz and Lee[9] in their study of NN scattering using a separable πNN model.

We want to stress here that the above equations contain all of the possible *one*-pion-exchange Δ excitation mechanisms within the constraint imposed by the NN and πNN unitary conditions. If the connected NΔ interaction U_c is set to zero, Eqs. (1) and (2) then define the existing coupled-channel model. The validity of the coupled-channel model can therefore be tested by examining the effect of U_c in NN scattering.

To calculate the NN scattering amplitude, we need to solve Eqs. (1)—(6) in each NN eigenchannel. Applying the standard angular momentum projection, Eq. (1) can be reduced to a one-dimensional integral equation in momentum space. The integral equation can be solved by using the standard matrix method,[10] since the matrix element of the effective NN potential $U(E)$ remains a bounded function of the NN relative momentum p even when the connected NΔ interaction U_c is included.

Formalisms needed to calculate the matrix elements of $\Sigma_\Delta(E)$ and the first two terms of $U(E)$ [Eq. (2)] have been given in Ref. 3. We focus our discussion on the calculation of $U_c(E)$, which contains the following complication. The πNN propagator in the driving term V_c of the integral Eq. (5) for T_c becomes singular when the collision energy E is above the pion production threshold ($E\geq 280$ MeV in the laboratory frame). The standard matrix method is not applicable for calculating the matrix element of T_c from Eq. (5). Our procedure is developed by using the property that the singularities of T_c and the NΔ propagator are all located in the upper-half of the complex p plane. We therefore can evaluate the desired partial-wave matrix element by the standard contour rotation method[11] (we suppress all eigenchannel quantum numbers)

$$U_c(p',p)=\int dp_c' p_c'^2 V_{NN,N\Delta}(p',p_c')\frac{1}{E-E_N(p_c')-E_\Delta(p_c')-\Sigma_\Delta(E,p_c')} \times T_c(p_c',p_c,E)\frac{1}{E-E_N(p_c)-E_\Delta(p_c)-\Sigma_\Delta(E,p_c)}V_{N\Delta,NN}(p_c,p)p_c^2dp_c , \quad (7)$$

where $p_c,p_c'=pe^{i\theta}$ with $\theta<0$. On the complex p_c and p_c' axis, the driving term $V_c(p_c',p_c,E)$ is bounded everywhere; the matrix element $T_c(p_c',p_c,E)$ needed to evaluate Eq. (7) can be easily generated from Eq. (5) by the matrix inversion method. It is found that with the choice $\theta=15°$, 24 Gaussian points are sufficient for the integration Eq. (7). In calculating T_c from Eqs. (5) and (6), we use only the $^3S_1+{}^3D_1$NN interaction to generate the intermediate NN t matrix $t(E-E_\pi)$. This is a good approximation as found in Ref. 9.

Calculation of $T_c(p_c',p_c,E)$ can be further simplified by using the property that if the separable representation of the NN scattering matrix $t(E-E_\pi)$ is used in evaluating Eq. (6), Eqs. (5) and (6) can be cast exactly into the form of the quasiparticle Faddeev-type equation employed extensively in πd calculations with Δ excitation.[12,13] It is then easy to calculate $T_c(p_c',p_c,E)$ in a complex momentum axis by using a standard three-body program developed by one of the authors.[13]

We now turn to the discussion of our results. The start-

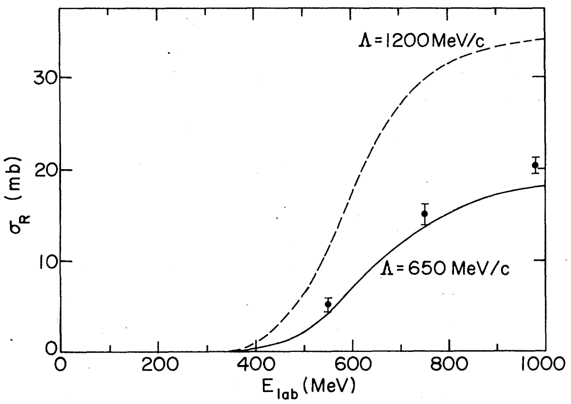

FIG. 2. Comparison of total pion production cross sections calculated with $\Lambda=650$ and $\Lambda=1200$ MeV/c, within the coupled-channel model of Ref. 3.

ing point of our study is the meson-exchange coupled-channel model of Ref. 3 which was constructed by employing Eqs. (1) and (2) with the approximation $U_c=0$. The main ingredients of the meson theory of Ref. 3 are the following: (a) the $\pi N \leftrightarrow \Delta$ vertex interaction h and the bare mass of Δ are determined by fitting the πN P_{33} scattering phase shifts; (b) $V_{NN,N\Delta}$ is parametrized according to the sum of static one-pion and one-rho exchange with dipole form factors; (c) $V_{NN,NN}$ is constructed by using a momentum-dependent procedure to subtract from the Paris potential (or any low-energy NN potential) the contribution due to the intermediate Δ excitation. In this construction, the cutoff parameter Λ of the form factor of $V_{NN,N\Delta}$ is the only free parameter. It was found

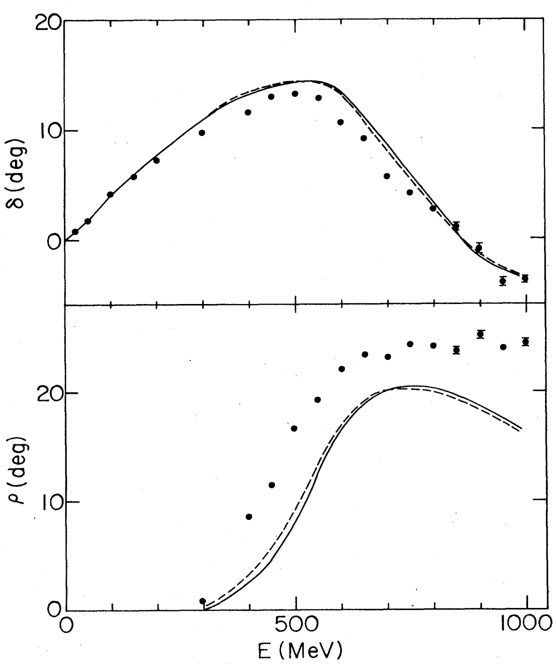

FIG. 3. Comparison of NN phase shifts in the $^1D_2$ channel calculated with Eqs. (1)–(6) (solid curve) and with the $U_c=0$ approximation (dashed curve).

TABLE I. Effect of T_c on $^1D_2$ NN phase shifts. We use the conventions of Ref. 15.

	δ (deg)		ρ (deg)	
E	$T_c=0$	$T_c\neq 0$	$T_c=0$	$T_c\neq 0$
		$\Lambda=650$		
362	12.65	12.46	1.73	1.31
442	13.86	13.68	5.24	3.81
522	14.39	14.33	10.96	9.71
602	14.01	13.10	14.12	13.47
682	8.95	9.42	19.56	19.67
762	4.78	5.15	20.2	20.28
842	0.67	0.93	19.34	19.68
922	−2.09	−1.91	17.63	18.03
		$\Lambda=1200$		
362	18.02	17.6	2.99	2.12
442	23.87	23.23	10.05	7.15
522	30.15	29.95	22.59	20.02
602	28.35	29.68	34.29	33.45
682	16.04	18.18	37.74	38.11
762	4.16	5.87	37.32	37.65
842	−3.84	−2.93	34.77	35.48
922	−6.07	−5.51	32.15	32.93

that the NN scattering data up to 1 GeV can be best described[14] when $\Lambda=650$ MeV/c is chosen. To illustrate the severe constraint imposed by the data in determining Λ, we compare in Fig. 2 the NN reaction cross sections (which measure the strength of the $NN \rightarrow N\Delta \rightarrow NN\pi$ transition) calculated with $\Lambda=650$ and 1200 MeV/c. It is clear that within the model, the data favor a small cutoff parameter.

It is expected that the effect of U_c is most visible in the NN $^1D_2$ channel, because it couples to an s-wave $N\Delta$ $^5S_2$ state. In Table I and Fig. 3 we show our results for the $^1D_2$ phase shifts. It is seen that the effect of the connected $N\Delta$ interaction U_c does not significantly change the calculated results. The differences between the solid (including U_c) and dashed ($U_c=0$) curves are less than 1°. The calculations for all other $T=1$ channels with $l\leq 3$ have also been carried out. These channels cannot couple to an $N\Delta$ s wave (see Table III of Ref. 9). It is found that the effect of U_c in these channels is completely negligible. All of the results in these channels presented in Ref. 3 are not changed by the connected $N\Delta$ interaction T_c.

To see the model dependence of our result, we also show in Table I the $^1D_2$ result calculated with $\Lambda=1200$ MeV/c. As illustrated in Fig. 2, a large cutoff parameter greatly enhances the $NN \rightarrow N\Delta \rightarrow \pi NN$ transition strength and hence the contribution of U_c is also correspondingly increased. However, even with such an unphysical cutoff parameter, the effect of U_c is still small. We, therefore, conclude that *within the meson theory the coupled-channel method is valid in describing the Δ excitation in NN scattering*. In searches for parameters of meson theory, it is a good approximation to neglect the coupling to the πd and πNN production channels through Δ excitation. To end this paper, we want to emphasize that the calculation based on Eqs. (1)–(6) is a necessary treatment of one-pion-exchange Δ excitation mechanisms within the πNN unitarity condition. The present lengthy calculation of U_c and the result of Ref. 3 has essentially shown that the NN data cannot be satisfactorily described by only extending the conventional meson theory to include the "long-range" Δ excitation. The next step for developing the πNN theory is to investigate the importance of the nonresonant pion-exchange mechanism and to consider the short-range mechanism, as suggested by the quark dynamics. Our study in these two directions will be published elsewhere.

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