Effect of shell structure on the nucleon transfer contribution to the imaginary part of the heavy ion optical potential

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We calculate the imaginary part of the heavy ion optical potential at the strong absorption radius assuming that the absorption is essentially given by the depopulation of the entrance channel due to nucleon transfer. The transfer probabilities of individual nucleons between specific quantum states are calculated with a simple formula. The method is applied to several pairs of nuclei and the results are compared to the experiment and other previous calculations.

I. INTRODUCTION

In the present work we calculate the imaginary part of the heavy ion optical potential at the strong absorption radius. As pointed out by Broglia, Pollarolo, and Winther,[1] the nucleon transfer process is the dominant mechanism for describing the tail of the imaginary part.

Using such an argument in a previous paper,[2] we calculated the imaginary part W_{trans} of the optical potential between various pairs by using the proximity method and a Fermi gas model for describing the interacting nuclei. In this way we could calculate the flux of nucleons tunneling through the barrier formed between the single particle wells. All Coulomb effects were neglected. More recently[3] the use of the proximity method was formally justified by deriving the definition of W_{trans} used in Ref. 2 from the transfer probabilities between specific quantum states. The major approximations in relating W_{trans} to the sum of the transition probabilities were that the nuclei are leptodermous and that their relative motion can be described in a semiclassical approximation by a straight line trajectory.

In the present work we study explicitly the effect of the nuclear structure on the nucleon transfer contribution to the imaginary part. The study is very similar to that of Ref. 4 for W_{trans}, where the shell structure of the interacting nuclei is taken into account but the method is simpler for numerical calculations. The disadvantage is that it is valid only for large separations between nuclei. On the other hand, it can be easily applied to any pair of nuclei. The simplification is due to two main assumptions: (i) The transfer is a peripheral process (the implications of this assumption are discussed in the next section). (ii) The protons and neutrons give identical contributions, as in the proximity method.[2] This will allow us to compare the present results with those of Ref. 2, which gave good agreement with data.

In the next section we shall derive the formula for W_{trans}. In Sec. III we present numerical results for various pairs and make a comparison with experiment and other theoretical estimates. Section IV is devoted to discussion and conclusions.

II. THE FORMALISM

We assume that the origin of coordinates is located in the center of the target, called nucleus 2. The projectile, nucleus 1, moves on a straight line along the y axis with uniform velocity v. The distance of closest approach $z=d$ is reached at $t=0$. The transition amplitude $A(2f,1i)$ of a nucleon from a bound state ψ_{1i} of the projectile to a bound state ψ_{2f} of the target is given perturbatively by[5]

$$A(2f,1i)\simeq\frac{1}{i\hbar}\int_{-\infty}^{\infty}\langle\psi_{2f},V_1\psi_{1i}\rangle dt\ , \tag{2.1}$$

where V_1 is the single particle potential of nucleus 1. For a peripheral collision this amplitude can take a simpler form. If the separation distance is large enough one can find a plane surface Σ at $z=z_0$ between the nuclei where both the single particle potentials V_1 and V_2 can be neglected for all points (x,y,z_0) at any time t. In such a case $A(2f,1i)$ can be rewritten[6] in terms of a surface integral over Σ

$$A(2f,1i)\approx\frac{\hbar}{2mi}\int_{-\infty}^{\infty}dt\int_{\Sigma}d\mathbf{S}(\psi_{2f}^{*}\nabla\psi_{1i}-\psi_{1i}\nabla\psi_{2f}^{*})\ , \tag{2.2}$$

which does not contain single particle potentials any more. This is the basic formula used to evaluate the imaginary optical potential W_{trans} which can be related to the total transfer probability

$$P_{ba}=\sum_{i,f}|A(bf,ai)|^2\quad(a,b=1,2) \tag{2.3}$$

through the relation[1,3]

$$\frac{2}{\hbar}\int_{-\infty}^{\infty}W_{\text{trans}}[\mathbf{R}(t)]dt=P_{21}+P_{12}\ , \tag{2.4}$$

where $\mathbf{R}$ is a point on the trajectory

$$\mathbf{R}=\mathbf{d}+\mathbf{v}t \ . \tag{2.5}$$

In order to obtain an analytic expression for W_{trans} compatible with Eq. (2.4) we have to introduce explicitly the single particle wave functions ψ_{1i} and ψ_{2f} in Eq. (2.2). If φ_i and φ_f are the eigenstates of the nucleus 1 and 2 at rest corresponding to eigenvalues ϵ_i and ϵ_f, respectively, one has

$$\psi_{1i}=\varphi_i[\mathbf{r}-\mathbf{R}(t)]e^{(i/\hbar)[m\mathbf{v}\cdot\mathbf{r}-\epsilon_i t-(1/2)mv^2 t]} , \tag{2.6}$$

$$\psi_{2f}=\varphi_f(\mathbf{r})e^{-(i/\hbar)\epsilon_j t} . \tag{2.7}$$

At this stage we introduce the double Fourier transform

$$f_\alpha(k_x,k_y,z)=\int_{-\infty}^{\infty} e^{-i(k_x x+k_y y)}\varphi_\alpha(x,y,z)dx\,dy \tag{2.8}$$

for $\alpha=i,f$. It can be shown[7] that if z is outside the range of the single particle potentials, expression (2.8) becomes

$$f_i(z)=C_i e^{-\gamma(d-z)}\frac{2\pi}{\gamma}Y_{l_i m_i}(\hat{\mathbf{k}}_1) \quad \text{for } \alpha=i \tag{2.9}$$

and

$$f_f(z)=C_f e^{-\gamma z}\frac{2\pi}{\gamma}Y_{l_f m_f}(\hat{\mathbf{k}}_2) \quad \text{for } \alpha=f , \tag{2.10}$$

where C_α are normalization constants (see the Appendix) and

$$\gamma^2=k_x^2+k_1^2+\gamma_{0i}^2=k_x^2+k_2^2+\gamma_{0f}^2 , \tag{2.11}$$

$$\hat{\mathbf{k}}_1=\frac{1}{\gamma_{0i}}(-ik_x,-ik_1,\gamma); \quad \hat{\mathbf{k}}_2=\frac{1}{\alpha_{0f}}(-ik_x,-ik_2,\gamma) , \tag{2.12}$$

$$k_1=\frac{1}{\hbar v}(\epsilon_f-\epsilon_i-\tfrac{1}{2}mv^2); \quad k_2=\frac{1}{\hbar v}(\epsilon_f-\epsilon_i+\tfrac{1}{2}mv^2) , \tag{2.13}$$

$$\gamma_{0\alpha}^2=-\frac{2m}{\hbar^2}\epsilon_\alpha . \tag{2.14}$$

Returning to Eq. (2.2) one can see that only the z components of the gradient evaluated at $z=z_0$ contribute to the transition amplitude. By replacing φ_α with the inverse Fourier transform of (2.8) and making the derivative with respect to z of (2.9) and (2.10) one obtains at $z=z_0$

$$A(2f,1i)=\frac{i\hbar}{mv}\frac{1}{2\pi}\int_{-\infty}^{\infty}dk_x\gamma f_f^*(k_x,k_2,z_0) \times f_i(k_x,k_1,z_0-d) , \tag{2.15}$$

or using the explicit forms (2.9) and (2.10) the transition amplitude from 1 to 2 becomes

$$A(2f,1i)=i\frac{2\pi\hbar}{mv}C_i C_f^*\int_{-\infty}^{\infty}dk_x\frac{e^{-\gamma d}}{\gamma}Y_{l_f m_f}^*(\hat{\mathbf{k}}_2) \times Y_{l_i m_i}(\hat{\mathbf{k}}_1) . \tag{2.16}$$

A similar expression for the transfer of nucleons from the target to the projectile can be obtained by interchanging i and f in (2.16). We notice that γ is a function of k_x, as defined in Eq. (2.11). We introduce the quantity

$$\eta=(k_1^2+\gamma_{0i}^2)^{1/2}=(k_2^2+\gamma_{0f}^2)^{1/2} , \tag{2.17}$$

which is independent of k_x and which will be used below.

The transition probability from a specific orbit l_i in nucleus 1 to a specific orbit l_f in nucleus 2 is obtained by summing the square modulus of (2.16) over m_i and m_f. Making use of the addition theorem of spherical harmonics, we get

$$P_{1l_i,2l_f}=\left[\frac{2\pi\hbar}{mv}\right]^2\frac{2l_i+1}{4\pi}\frac{2l_f+1}{4\pi}|C_i|^2|C_f|^2 \times\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_x'\frac{e^{-(\gamma+\gamma')d}}{\gamma\gamma'} \times P_{l_i}(\hat{\mathbf{k}}_1^*\cdot\hat{\mathbf{k}}_1')P_{l_f}(\hat{\mathbf{k}}_2^*\cdot\hat{\mathbf{k}}_2') , \tag{2.18}$$

where

$$\hat{\mathbf{k}}_1^*\cdot\hat{\mathbf{k}}_1'=\frac{1}{\gamma_{0i}^2}(k_x k_x'+k_1^2+\gamma\gamma') ,$$
$$\hat{\mathbf{k}}_2^*\cdot\hat{\mathbf{k}}_2'=\frac{1}{\gamma_{0f}^2}(k_x k_x'+k_2^2+\gamma\gamma') . \tag{2.19}$$

We write the transition probability from nucleus 1 to nucleus 2 as

$$P_{12}=2\sum_{i,f}\frac{2j_i+1}{2l_i+1}\frac{2j_f+1}{2l_f+1}P_{1l_i,2l_f} . \tag{2.20}$$

This equation contains a number of approximations: (i) We assume that neutrons and protons contribute equally to P_{12}. The sum is taken only over neutron states and the factor 2 accounts for the proton contribution. (ii) Actually the transition takes place from levels $j_i l_i$ to levels $j_f l_f$. The factors $(2j_i+1)$ and $(2j_f+1)$ are the degeneracies of the j levels and occur because we assume that the initial levels j_i are full and the final levels j_f are empty. (iii) Expression (2.20) contains an implicit dependence on j_i and j_f through the binding energies ϵ_i and ϵ_f. We include this effect by allowing the normalization coefficients C_i and C_f and γ_{0i} and γ_{0f} to be j dependent. There are other spin-dependent effects which are neglected.

One can evaluate numerically the double integral in (2.18) but it is interesting to go a step further with the analytical expression. If we use the approximations

$$\gamma\simeq\eta+\frac{1}{2\eta}k_x^2, \quad \gamma'\simeq\eta+\frac{1}{2\eta}k_x'^2 , \quad \frac{1}{\gamma\gamma'}\simeq\frac{1}{\eta^2} \tag{2.21}$$

and make the change of variables

$$x=k_x-k_x'; \quad X=\tfrac{1}{2}(k_x+k_x')\sqrt{d/\eta} , \tag{2.22}$$

expression (2.18) becomes

$$P_{1l_i,2l_f}=\frac{\pi}{2}\left[\frac{\hbar}{mv}\right]^2(2l_i+1)(2l_f+1)\,|\,C_iC_f\,|^2 \times\frac{e^{-2\eta d}}{\eta d}M_{l_il_f}\,, \tag{2.23}$$

where

$$M_{l_il_f}=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}dX\,e^{-X^2}P_{l_i}(A_i+B_iX^2) \times P_{l_f}(A_f+B_fX^2)\,, \tag{2.24}$$

with

$$A_i=1+\frac{2k_1^2}{\gamma_{0i}^2};\quad B_i=\frac{2\eta}{d\,\gamma_{0i}^2}\,, \qquad A_f=1+\frac{2k_2^2}{\gamma_{0f}^2};\quad B_f=\frac{2\eta}{d\,\gamma_{0f}^2}\,. \tag{2.25}$$

Approximation (2.23) has been tested numerically and found to be accurate within 1%.

The interesting point about (2.23) is that it shows an explicit exponential decrease of the transition probability and it hints at a similar behavior for $W_{\rm trans}$. This helps us to find an expression for $W_{\rm trans}$ compatible with (2.4). Expression (2.23) suggests that each transition will give rise to an absorptive potential of the form

$$W_{\rm trans}(R)\sim W_0e^{-2\eta R}\,. \tag{2.26}$$

Around the point of the closest approach one can take

$$R\simeq d\left[1+\frac{v^2t^2}{2d^2}\right], \tag{2.27}$$

and using it together with (2.26) one can solve the integral on the left-hand side (lhs) of (2.4). This brings us to the following form for the absorptive potential at the strong absorption radius d:

$$W_{\rm trans}^{1i\to 2f}(d)=\frac{\hbar v}{2}\sqrt{\eta/\pi d}\,P_{1l_i,2l_f}\,, \tag{2.28}$$

where we have assumed

$$W_{\rm trans}^{1i\to 2f}(d)=W_0e^{-2\eta d}\,. \tag{2.29}$$

The quantity η depends on the specific transition, as we shall see in the next section. The imaginary part of the optical potential at the strong absorption radius $d=D_{1/2}$ will be calculated as

$$W_{\rm trans}(d)=\sum_{i,f}[W_{\rm trans}^{1i\to 2f}(d)+W_{\rm trans}^{2i\to 1f}(d)]\,. \tag{2.30}$$

III. RESULTS

The essential ingredients of our calculations are the single particle spectra and the normalization constants C_α of the asymptotic wave functions of the associated orbits in the initial $\alpha=i$ and the final $\alpha=f$ nuclei.

Each nucleus is described by a single particle potential of the form

$$V=V_0f(r)+V_{ls}\mathbf{l}\cdot\mathbf{S}r_0^2\frac{1}{r}\frac{d}{dr}f(r)\,, \tag{3.1}$$

where

$$f(r)=\left[1+\exp\frac{r-R}{a}\right]^{-1}. \tag{3.2}$$

In order to be able to make a comparison with the proximity results we choose the parameters in Eqs. (3.1)—(3.2) from Bohr and Mottelson[8] as in Ref. 2,

$$V_0=-50\text{ MeV},\quad V_{ls}=22\text{ MeV}\,; \qquad r_0=1.25,\quad a=0.65\text{ fm},\quad R=r_0A^{1/3}\,. \tag{3.3}$$

The single particle energies for ^{32}S, ^{60}Ni, and ^{208}Pb and the corresponding normalization constants C_α are given in Table I. The C_α can also be calculated in the approximation described in the Appendix.

Our results are summarized in Table II. The pairs of nuclei chosen for this study are common either with Ref. 2 or Ref. 4. As a general trend we notice that the present method gives values at the strong absorption radius for $W_{\rm trans}$ close to those of Pollarolo *et al.*[4] On the other hand, they are usually smaller than those obtained with the proximity method, where nuclei are treated in a Fermi gas model. This indicates that the shell structure plays a very important role. In order to see how this effect appears we give in Table III the detailed contribution of each transfer taking place between the pair ^{16}O + ^{40}Ca at $E_{\rm lab}=74.4$ MeV ($D_{1/2}=9.3$ fm) and $E_{\rm lab}=214.1$ MeV ($D_{1/2}=8.8$ fm). For each transfer we also indicate the value of η. Equation (2.23) for the transfer probability contains a factor $\exp(-2\eta d)$, where d is the separation distance between nuclei and η is given in Eq. (2.17). Using (2.13) and (2.14) one can rewrite (2.17) as

$$\eta^2=\frac{1}{\hbar^2v^2}[(\epsilon_f-\epsilon_i)^2+(\tfrac{1}{2}mv^2)^2]-\frac{m}{\hbar^2}(\epsilon_f+\epsilon_i)\,. \tag{3.4}$$

The transfer probability is very sensitive to the value of η. One can see that a small Q value ($Q=\epsilon_f-\epsilon_i$) at fixed v favors smaller values of η and, accordingly, larger values of the transfer probability. This is why levels closest to the Fermi surface usually contribute most to $W_{\rm trans}$. The quantity η also enters the integral (2.24) through the arguments (2.25) and can give a substantial modification to the exponential term in (2.23). The C_i and C_f in the same equation are sensitive to the binding energy and angular momentum of the corresponding orbits (Table V) and can amplify or diminish the transition probability.

Table II shows several examples of the energy dependence of $W_{\rm trans}$. We found that it increases in strength with increasing bombarding energy. The single transition values given in Table III show that more channels contribute at higher energies. The same pattern occurs in the other cases. Our examples are in the range where for each pair of levels η decreases with increasing energy. This is not always the case. Equation (3.4) shows that for high energies, η increases with v.

The dependence on η also implies a sensitivity of the nucleon transition probability on the single particle spec-

TABLE I. Neutron single particle spectra for ^{32}S, ^{60}Ni, and ^{208}Pb. ϵ_α (MeV) is the eigenvalue for the bound state α of the standard potential (3.1)–(3.3); C_α^{exact} (fm$^{-1/2}$) is the normalization constant of (A2) calculated from the exact radial wave function R_α. The gap separates the occupied from the unoccupied orbits. (See Table V for ^{16}O and ^{40}Ca).

^{32}S			^{60}Ni			^{208}Pb		
α	ϵ_α	C_α^{exact}	α	ϵ_α	C_α^{exact}	α	ϵ_α	C_α^{exact}
						$3d_{3/2}$	−2.7895	1.6087
						$4s_{1/2}$	−3.3560	5.8686
			$3s_{1/2}$	−0.7877	0.3119	$2g_{7/2}$	−3.3833	0.3746
$2p_{1/2}$	−0.0432	0.0049	$2d_{5/2}$	−1.3491	0.1483	$3d_{5/2}$	−4.4095	5.3064
$2p_{3/2}$	−1.6683	0.3924	$1g_{9/2}$	−4.7324	0.1908	$1j_{15/2}$	−5.2280	0.0488
$1f_{7/2}$	−3.9442	0.2095	$2p_{1/2}$	−7.9488	7.6725	$1i_{11/2}$	−6.3574	0.2326
$1d_{3/2}$	−8.8164	2.3397	$1f_{5/2}$	−8.1472	1.6389	$2g_{9/2}$	−6.9109	4.5960
$2s_{1/2}$	−11.0877	11.6815	$2p_{3/2}$	−10.0705	14.1180	$3p_{1/2}$	−10.5284	81.3012
$1d_{5/2}$	−14.8916	9.4355	$1f_{7/2}$	−14.3796	10.6020	$2f_{5/2}$	−11.5394	40.7710
$1p_{1/2}$	−22.5635	26.6880	$1d_{3/2}$	−19.9035	31.4978	$3p_{3/2}$	−11.6171	117.7670
$1p_{3/2}$	−25.7068	43.3700	$2s_{1/2}$	−19.9354	77.4060	$1i_{13/2}$	−12.3764	6.5715
$1s_{1/2}$	−36.0250	80.7050	$1d_{5/2}$	−23.6765	68.0470	$2f_{7/2}$	−14.1093	105.1920
			$1p_{1/2}$	−30.6912	130.9680	$1h_{9/2}$	−14.7793	19.2727
			$1p_{3/2}$	−32.4161	186.1800	$1h_{11/2}$	−19.1688	96.9650
			$1s_{1/2}$	−40.3615	275.8000	$3s_{1/2}$	−19.2613	687.9700
						$2d_{3/2}$	−19.6001	413.9300

trum associated with each reaction partner. In other words, the results are expected to be sensitive to ϵ_i and ϵ_f of each specific channel through the difference $Q=\epsilon_i-\epsilon_f$ as well as through the sum $\epsilon_i+\epsilon_f$, as seen from Eq. (3.4). In order to have an idea of the dependence of the results on the choice made for the single particle spectrum, we give in Table IV the values of the imaginary potential $W_{\text{trans}}^{\text{esps}}$ for $^{16}\text{O}+{}^{208}\text{Pb}$ obtained with the experimental single particle spectrum (esps) taken from Ref. 9. The separate contributions from O→Pb and Pb→O are also indicated. In these calculations the normalization constants C_α have been evaluated according to the approximation described in the Appendix and are based on the standard average potential (3.1)–(3.5) but taking $V_{ls}=0$. The remaining columns in Table IV show the corresponding quantities W^{app} calculated with theoretical single particle energies. To make a better comparison we have used C_α^{app} obtained by the approximate method. The same number of transitions are included in both cases. In contrast, the values in Table II included contributions from three deeper levels $1h_{11/2}$, $3s_{1/2}$, and $2d_{3/2}$, which are not known experimentally. One can see that for all energies the transfer Pb→O is by far the dominant contribution to $W_{\text{trans}}^{\text{esps}}$, while in $W_{\text{trans}}^{\text{app}}$ the O→Pb and Pb→O transfers contribute more similarly. This change is due to the fact that the experimental occupied single particle spectrum is higher in energy than the theoretical one in ^{208}Pb and lower in ^{16}O. The effect is especially pronounced at $E_{\text{lab}}=129.5$ MeV.

Finally the largest value of $W_{\text{trans}}^{\text{th}}$ in Table II is for $^{16}\text{O}+{}^{60}\text{Ni}$. This is because ^{60}Ni is not a closed shell nucleus. The shell effects are therefore less important and the result is closer to the proximity value than for all the other cases. The same holds for the system $^{32}\text{S}+{}^{32}\text{S}$.

IV. CONCLUSIONS

We have calculated the nucleon transfer contribution to the imaginary part of the heavy ion optical potential by a method which takes account of shell effects. Our results for the absorptive potential at the strong absorption radius are systematically smaller than those obtained with the proximity method[2] which uses a Fermi gas model. This comparison shows that shell structure plays an important role in the depopulation of the entrance channel.

Our potentials are also smaller than those fitted to experimental data. This is not surprising, as the contribution of inelastic channels has been neglected. The results of Ref. 4 show that this contribution can be important. For example, the inelastic and transfer channels are both significant for $^{16}\text{O}+{}^{208}\text{Pb}$ at $E_{\text{lab}}=192$ MeV.

Sometimes very few transitions contribute to W_{trans}. In the case of the pair $^{16}\text{O}+{}^{40}\text{Ca}$ at $E_{\text{lab}}=40$ MeV there is only one. At $E_{\text{lab}}=214.1$ MeV five transitions make a significant contribution (cf. Table III). This illustrates a general trend. At high energies more transitions are important. The energy dependence is influenced strongly by the factor $\exp(-2\eta d)$ in Eq. (2.23). This factor is a maximum when η is as small as possible. In all the cases we have studied η decreases with increasing energy. Typical values for the dominant transitions have η values lying in

TABLE II. The absorptive potential due to transfer channels. $D_{1/2}$—the strong absorption radius; $W_{\text{trans}}^{\text{exact}}$—the present result; $W_{\text{trans}}^{\text{PBW}}$—from Ref. 4; $W_{\text{trans}}^{\text{prox}}$—from Ref. 2; W^{exp}—the value of the phenomenological potential at the strong absorption radius taken from the references indicated in the last column.

System	E_{lab} (MeV)	$D_{1/2}$ (fm)	$W_{\text{trans}}^{\text{exact}}$ (MeV)	$W_{\text{trans}}^{\text{PBW}}$ (MeV)	$W_{\text{trans}}^{\text{prox}}$ (MeV)	W^{exp} $(D_{1/2})$ (MeV)	Ref.
$^{16}O + {}^{40}Ca$	40.0	9.7	0.0081	0.06	0.07	0.32	12
	55.6	9.5	0.0648		0.17	0.46	
	74.4	9.3	0.1605	0.18	0.31	0.66	
	103.6	9.1	0.3169	0.27	0.55	0.95	
	214.1	8.8	0.4993		1.30	1.64	
$^{16}O + {}^{60}Ni$	142.0	9.6	0.4693		0.86	1.13±0.17	13
$^{16}O + {}^{208}Pb$	129.5	12.8	0.1230	0.21	0.26	0.89	12
	192.0	12.4	0.2732	0.18	0.69	1.34	
	312.6	12.1	0.3143		1.23	1.53	
$^{32}S + {}^{32}S$	90.9	10.1	0.0989		0.11	0.37	12
$^{40}Ca + {}^{40}Ca$	143.6	10.75	0.0483		0.13	0.34	14
	186.0	10.72	0.1169		0.20	0.37	
	240.0	10.6	0.2071		0.34	0.48	

the range 0.7—1 fm^{-1}. This corresponds to a surface diffuseness $a=0.5$—0.7 fm, consistent with the proximity values in Ref. 2.

For a given ϵ_i the maximum value of η occurs when

$$\epsilon_f=\epsilon_i+\tfrac{1}{2}mv^2\ ,$$

showing that transitions to the continuum can be significant when the incident energy per nucleon at the Coulomb barrier ($\frac{1}{2}mv^2$) is of the order of or larger than the separation energy $|\epsilon_i|$ (≈ 10 MeV). For such energies it will not be a good approximation to neglect continuum levels. We note that the proximity method of Ref. 2 treats bound and continuum levels on the same footing. The proximity method should continue to give a reasonable approximation to W_{trans} even at high energies when the continuum is important.

TABLE III. Detailed contribution of the transitions considered in the calculation of the imaginary potential for $^{16}O + {}^{40}Ca$ at $E_{\text{lab}}=74.4$ and 214.1 MeV. For each transition we indicate the value of η—Eq. (3.4), and the contribution to the imaginary potential calculated with C_α^{exact}—Eq. (A2).

Transition	$E_{\text{lab}}=74.4$ MeV		$E_{\text{lab}}=214.1$ MeV	
O→Ca	η	$W_{\text{O-Ca}}$	η	$W_{\text{O-Ca}}^{\text{exact}}$
$1p_{1/2}$-$1f_{7/2}$	0.7786	0.059 206	0.7943	0.064 865
-$2p_{3/2}$	0.8544	0.003 532	0.7718	0.011 077
-$2p_{1/2}$	0.9302	0.000 126	0.7656	0.001 203
$1p_{3/2}$-$1f_{7/2}$	1.5944	0.048 211	0.9105	0.181 401
-$2p_{3/2}$	1.1666	0.000 248	0.9114	0.011 468
-$2p_{1/2}$	1.2652	0.000 005	0.9189	0.001 035
Ca→O	η	$W_{\text{Ca-O}}^{\text{exact}}$	η	$W_{\text{Ca-O}}^{\text{exact}}$
$1d_{3/2}$-$1d_{5/2}$	0.8710	0.024 842	0.7911	0.042 559
-$2s_{1/2}$	0.9390	0.000 226	0.7853	0.001 304
$2s_{1/2}$-$1d_{5/2}$	0.9501	0.013 273	0.8265	0.089 152
-$2s_{1/2}$	1.0254	0.000 047	0.8244	0.002 561
$1d_{5/2}$-$1d_{5/2}$	1.1913	0.006 515	0.9334	0.076 662
-$2s_{1/2}$	1.2817	0.000 010	0.9405	0.001 042
$1p_{1/2}$-$1d_{5/2}$	1.6826	0.000 002	1.1521	0.005 635
-$2s_{1/2}$	1.7885	4.3×10^{-10}	1.1716	0.000 026
$1p_{3/2}$-$1d_{5/2}$	1.8547	6.4×10^{-7}	1.2294	0.009 164
-$2s_{1/2}$	1.9638	8.3×10^{-12}	1.2521	0.000 031
$1s_{1/2}$-$1d_{5/2}$	2.4955	4.2×10^{-12}	1.5203	0.000 127
-$2s_{1/2}$	2.6125	1.2×10^{-16}	1.5518	0.000 001

TABLE IV. Comparison between results for the imaginary potential of $^{16}O + ^{208}Pb$ obtained with the experimental single particle spectrum W^{esps} and with the theoretical spectrum W^{app}. Separate contributions from transitions O→Pb and Pb→O are indicated (see the text for details). In all cases the normalization constants C_α^{app}—Eq. (A8)—are used.

E_{lab} (MeV)	$D_{1/2}$ (fm)	$W^{esps}_{O\to Pb}$	$W^{esps}_{Pb\to O}$	W^{esps}_{trans}	$W^{app}_{O\to Pb}$	$W^{app}_{Pb\to O}$	W^{app}_{trans}
129.5	12.8	0.0021	0.3248	0.3269	0.0336	0.0718	0.1054
192.0	12.4	0.0180	0.3710	0.3890	0.0778	0.1420	0.2198
312.6	12.1	0.0354	0.2552	0.2906	0.0864	0.1240	0.2104

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APPENDIX

In this appendix we recall the definition[7] of the asymptotic normalization constants C_α and derive an analytic approximation which can be used for estimating them with a pocket calculator. We introduce $R_\alpha(r)$ as the radial part of the eigenstate $\varphi_\alpha(\mathbf{r})$ normalized such as

$$\int_0^\infty |R_\alpha(r)|^2 dr = 1 . \tag{A1}$$

The quantity C_α is defined as the normalization constant of R_α outside the range of the single particle potential. In this region R_α can be represented by a spherical Bessel function $h_l^{(1)}$ of the third kind[10] and therefore we can write

$$R_\alpha \simeq -i^l C_\alpha \gamma_{0\alpha} r h_l^{(1)}(i\gamma_{0\alpha} r) \quad \text{for } V \simeq 0 . \tag{A2}$$

The asymptotic form of $h_l^{(1)}$ at very large r gives

$$R_\alpha \simeq C_\alpha e^{-\gamma_{0\alpha} r} . \tag{A3}$$

The function $-i^l h_l^{(1)}(i\gamma_{0\alpha} r)$ is always positive and we choose by convention to have $C_\alpha > 0$ always. The exact value of the normalization constant C_α^{exact} can be found by solving the Schrödinger equation numerically for R_α and then extracting it from (A2) at an appropriate range r. In this case the index α stands for the energy ϵ_α or equivalently the wave number $\gamma_{0\alpha}$, the angular momentum l_α, and the total angular momentum j_α of the orbit under consideration. Our approximate evaluation of the C_α is based on the Wentzel-Kramers-Brillouin (WKB) expression[11] of the wave function R_α. If a_α and b_α are the inner and outer turning points, one can write

$$R_\alpha(r) \sim \frac{\bar{C}_\alpha}{\sqrt{|\gamma_\alpha|}} \sin\left[\int_{b_\alpha}^r |\gamma_\alpha(r')| dr' + \frac{\pi}{4}\right], \quad a_\alpha < r < b_\alpha , \tag{A4}$$

TABLE V. Comparison between C_α^{exact}—Eq. (A2)—and C_α^{app}—Eq. (A8)—for theoretical single particle levels of ^{16}O and ^{40}Ca. We indicate also the binding energy ϵ_α, the wave number $\gamma_{0\alpha}$—Eq. (2.14), the outer turning point b_α, and the half-period τ_α—Eq. (A11).

Level	ϵ_α (MeV)	$\gamma_{0\alpha}$ (fm^{-1})	C_α^{exact} (fm$^{-1/2}$)	C_α^{app} (fm$^{-1/2}$)	b_α (fm)	τ_α (10^{-23} s)
			^{16}O			
$2s_{1/2}$	−2.9373	0.3749	1.0644	2.7440	4.9529	11.4265
$1d_{5/2}$	−4.9194	0.4852	0.5802	0.6350	3.7505	7.1141
$1p_{1/2}$	−12.2286	0.7649	4.6102	4.2778	3.6548	6.8580
$1p_{3/2}$	−17.2769	0.9092	10.0503	5.2666	3.3324	6.4469
$1s_{1/2}$	−29.9454	1.1970	26.8500	9.9422	2.8892	6.7085
			^{40}Ca			
$2p_{1/2}$	−2.1203	0.3185	0.5804	1.7171	5.9683	12.1040
$2p_{3/2}$	−4.2638	0.4517	1.9106	3.3520	5.5996	10.2596
$1f_{7/2}$	−7.6545	0.6052	1.2285	0.8697	4.5320	7.3001
$1d_{3/2}$	−12.8574	0.7843	6.5208	5.7509	4.5900	7.6115
$2s_{1/2}$	−14.2045	0.8244	22.9585	18.7093	4.8757	8.4500
$1d_{5/2}$	−18.1113	0.9309	19.3034	8.3067	4.2586	7.3629
$1p_{1/2}$	−25.6467	1.1077	46.3880	21.8710	4.0943	7.8970
$1p_{3/2}$	−28.2255	1.1621	71.2640	22.3218	3.9429	7.9719
$1s_{1/2}$	−37.7005	1.3430	121.2380	29.0376	3.5469	9.5376

$$\sim \frac{1}{2}\frac{\bar{C}_\alpha}{\sqrt{\gamma_\alpha}} e^{-w_\alpha(r)}, \quad r > b_\alpha , \tag{A5}$$

where

$$\gamma_\alpha = \left[\gamma_{0\alpha}^2 + \frac{2m}{\hbar^2} V(r) + \frac{(l+\frac{1}{2})^2}{r^2} \right]^{1/2} , \tag{A6}$$

$$w_\alpha(r) = \int_{b_\alpha}^{r} \gamma_\alpha(r')dr' . \tag{A7}$$

The normalization constants $\bar{C}_\alpha$ can be on one hand related to C_α. From (A3), (A5), and (A7) one obtains for large r

$$C_\alpha = \frac{1}{2}\frac{\bar{C}_\alpha}{\sqrt{\gamma_{0\alpha}}} e^{\gamma_{0\alpha} b_\alpha + K_\alpha} , \tag{A8}$$

where

$$K_\alpha = \int_{b_\alpha}^{r} [\gamma_{0\alpha} - \gamma_\alpha(r')]dr' ; \quad r \to \infty . \tag{A9}$$

On the other hand, $\bar{C}_\alpha$ can be expressed in terms of the period $2\tau_\alpha$ of the orbit α. By making the assumption that the main contribution to the lhs of (A1) comes from the interval (a_α, b_α), one has

$$\int_0^\infty R_\alpha^2 dr \simeq \tfrac{1}{2}\bar{C}_\alpha^2 \int_{a_\alpha}^{b_\alpha} \frac{dr}{|\gamma_\alpha|} = 1 , \tag{A10}$$

where we have averaged over the periodical function. By defining τ_α as

$$\tau_\alpha = \frac{m}{\hbar}\int_{a_\alpha}^{b_\alpha} \frac{dr}{|\gamma_\alpha|} , \tag{A11}$$

one obtains

$$\bar{C}_\alpha^2 = \frac{2m}{\hbar}\frac{1}{\tau_\alpha} . \tag{A12}$$

One can see now that in order to calculate C_α from the approximation (A8) one must know τ_α and K_α. Both these quantities depend on the single particle potential $V(r)$ through (A6). At this stage we neglect the spin-orbit part of (3.1). Then the j dependence of τ_α and K_α will come only through ϵ_α (or $\gamma_{0\alpha}$). The half-period τ_α is obtained by integrating (A11) numerically. In K_α we make a further approximation, by neglecting also the central part of $V(r)$. Extending the upper limit to ∞, the integral (A9) can be evaluated with the result

TABLE VI. Comparison between the imaginary potential calculated with $C_\alpha^{\rm app}$ and $C_\alpha^{\rm exact}$ for separate transitions O→Ca and Ca→O at $E_{\rm lab}=103.6$ MeV; $D_{1/2}=9.1$ fm.

Transfer	$W_{\rm trans}^{\rm app}$ (MeV)	$W_{\rm trans}^{\rm exact}$ (MeV)
O→Ca	0.0749	0.1965
Ca→O	0.0877	0.1204

$$K_\alpha \simeq [\gamma_{0\alpha}^2 b_\alpha^2 + (l+\tfrac{1}{2})^2]^{1/2} - (l+\tfrac{1}{2})\ln\frac{[\gamma_{0\alpha}^2 b_\alpha^2 + (l+\tfrac{1}{2})^2]^{1/2} + l + \tfrac{1}{2}}{\gamma_{0\alpha} b_\alpha} - \gamma_{0\alpha} b_\alpha . \tag{A13}$$

In Table V we show a comparison between $C_\alpha^{\rm exact}$ obtained from solving the Schrödinger equation for the orbit α and $C_\alpha^{\rm app}$ calculated from (A8) and (A11)–(A14) for ^{16}O and ^{40}Ca. Together with the results for C_α we also reproduce the values of the quantities $\gamma_{0\alpha}$, b_α, and τ_α used in (A8). We notice that the approximation (A8) takes into account quite well the effect of the barrier (i.e., large values of C_α for $l=0$ and smaller values with increasing l). The agreement between $C_\alpha^{\rm exact}$ and $C_\alpha^{\rm app}$ for orbits around the Fermi level is satisfactory. For deeper orbits the agreement worsens because the neglect of $V(r)$ is no longer a good approximation, but the transition probability between these levels is insignificant at the bombardment energies considered throughout this study. For nuclei where $C_\alpha^{\rm app}$ is sometimes larger or sometimes smaller than $C_\alpha^{\rm exact}$, a compensation effect can occur in adding up the contributions of various transitions. A typical example is the transfer Ni→O at $E_{\rm lab}=142$ MeV. But this is not always the case. In Table VI we show results for $W_{\rm trans}$ of ^{16}O + ^{40}Ca at $E_{\rm lab}=103.6$ MeV obtained on one hand with $C_\alpha^{\rm exact}$ and on the other hand with $C_\alpha^{\rm app}$. We notice that $W_{\rm Ca\to O}$ does not change much but $W_{\rm O\to Ca}$ becomes more than two times larger when $C_\alpha^{\rm exact}$ are used. The approximation (A8) is useful when an exact solution of the Schrödinger equation is not known. For example, transitions between experimental single particle levels can be studied by using experimental energies ϵ_α and by estimating b_α and τ_α from the standard single particle potential (3.1)–(3.3). Values of the imaginary potential $W^{\rm esps}$ in Table IV are calculated in this way.

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