Spectroscopic properties of highly excited states

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Numerical calculations based on the continuum shell model with a unified description of long-lived and short-lived resonance states are performed for ^{16}O. The results show that the spectroscopic properties of the individual states depend on the degree of overlapping with other resonance states.

It is believed, generally, that at high level density the scattering amplitude can be written as a sum of contributions from many levels each given by a Breit-Wigner amplitude.[1] It is usual to assume that all the widths of the long-lived resonances are, in a certain energy region, about the same and that the decay amplitudes have a random distribution of amplitudes and phases. The analysis of the data then effectively determines the mean level width, and the real and imaginary parts of the scattering amplitude have Gaussian distributions with mean zero.

Careful experimental investigations called into question this simple picture. As early as 1963 Lee and Schiffer[2] found that the assumption of complete randomness, which is basic to the statistical model, is invalid in the strong cross section fluctuations observed in the excitation functions $^{58}Ni+p$. Kanter *et al.*[3] tested these results by measuring the time evolution of compound elastic scattering by crystal blocking. They found average compound nucleus widths which are substantially smaller than those of the observed structures in the excitation functions. Further, the mean compound nucleus lifetime is significantly longer at the higher bombarding energy, contrary to expectations of the purely statistical theory. Nonstatistical effects have been found also in other experiments. For example, recent careful attempts to find a Gaussian distribution of decay amplitudes failed.[4] That is interpreted as evidence of a strong direct contribution to the excitation of resonances.[5] In Ref. 6 these experimental findings are considered as arising from the error caused by the finite number of analyzed resonances. But the deviations from the complete randomness, observed, e.g., in Refs. 2 and 3, are explained by short-lived resonances which exist simultaneously with the long-lived resonances. According to this statement it is difficult to find, in a realistic nucleus, a sufficiently large number of long-lived resonances well separated from short-lived ones as it is claimed, on principle, in Ref. 6. One of the results obtained from an experimental investigation of heavy ion scattering is the presence in the excitation functions of both narrow and broad structures. This indicates that there is an interplay of various interaction times, ranging from the lifetime of the compound nucleus to the time associated with shape resonances in the ion-ion potentials.[7] This result sets the task of finding a unified description of long-lived and short-lived resonances. The basic assumptions of statistical description for long-lived resonances can then be proved.

A model allowing a unified description of long-lived and short-lived resonances is the continuum shell model (CSM) formulated in Refs. 8 and 9. In the present paper, results of numerical calculations performed on the basis of this model are given. The aim is to investigate the spectroscopic properties of excited states and the behavior of the cross section, both in dependence on the degree of overlapping of the resonance states. The degree of overlapping has been varied by changing the distance between the shell model states being an input for the coupled channels calculations. In such a procedure all the effect observed can be traced back to the changes of the degree of overlapping since all other parameters and the shell model wave functions of the states are kept constant.

The calculations are performed for the reaction $^{15}N+p$ with excitation of 1^- resonances. In a first step, the shell model Hamiltonian

$$(E_R^{SM}-H_{QQ})\phi_R=0 \tag{1}$$

is diagonalized where $H\equiv QHQ$, $H=H_0+V$, and Q is the projection operator onto the subspace of discrete shell model states constructed from bound and quasibound single particle states. These shell model states are called quasibound states embedded in the continuum[8,9] (QBSEC). They differ from the bound states embedded in the continuum (BSEC) introduced by Mahaux and Weidenmüller[10] by the contribution of the single particle resonances. The potential is of Woods-Saxon type with standard parameters.[11] The configuration space is $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$. The basic wave functions are of 1p-1h and 2p-2h nuclear structure corresponding to $1\hbar\omega$ and $3\hbar\omega$ excitations. Altogether, there are 76 states of such a type with $J^\pi=1^-$. The shell model wave functions of the target nucleus ^{15}N are the two $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states of $1h$ structure. In a second step, the eigenfunctions $\tilde{\phi}_R$ and eigenvalues

$$\tilde{E}_R-\frac{i}{2}\tilde{\Gamma}_R$$

of the operator

$$H_{QQ}^{\text{eff}}=H_{QQ}+H_{QP}G_P^{(+)}H_{PQ} \tag{2}$$

[$G_P^{(+)}=P(E-H_{PP})^{-1}P$ is the Green's function for the motion of the particle in the P subspace of scattering

states, $P+Q=1$] as well as the elastic and inelastic cross sections ^{15}N+p are calculated with 29 out of the 76 resonance states in an energy region where the $d_{3/2}$ single particle resonance is not important. The eigenvalues of H_{QQ}^{eff} definitely determine the energies E_R and widths Γ_R of the resonance states as long as the QBSEC but not the BSEC are used[8] as basic states in diagonalizing H_{QQ}^{eff}. In addition to the internal of configuration mixing contained in the shell model calculations (1), the second term of the operator (2) describes the external mixing of the resonance states via the continuum. It includes coupled channel effects described by the solutions of

$$(E-H_{PP})\xi_E^c=0 \tag{3}$$

in the P subspace.

The dependence of the inelastic cross section and of the widths on the degree of overlapping $\langle\Gamma\rangle/\langle D\rangle$ of the resonance states ($\langle\Gamma\rangle$—mean width, $\langle D\rangle$—mean distance) is shown in Figs. 1 and 2. The overlapping has been varied by reducing the differences between the energies E_R^{SM} of the shell model states [solutions of Eq. (1)] six times by a factor 2. The wave functions ϕ_R of the shell model states thereby remain unchanged as well as the parameters of the Woods-Saxon potential and of the residual interaction. Such a procedure to vary the degree of overlapping is justified because the eigenfunctions $\tilde{\phi}_R$ and eigenvalues

$$\tilde{E}_R-\frac{i}{2}\tilde{\Gamma}_R$$

of the operator (2) depend only weakly on energy. The sum of the widths of all the 29 resonance states is equal to

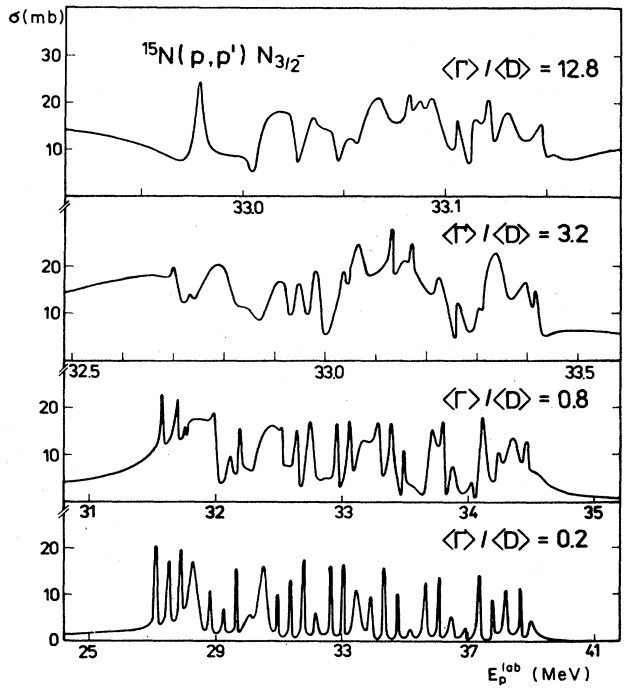

FIG. 1. The inelastic cross section ^{15}N(p,p') in dependence on the degree of overlapping $\langle\Gamma\rangle/\langle D\rangle$.

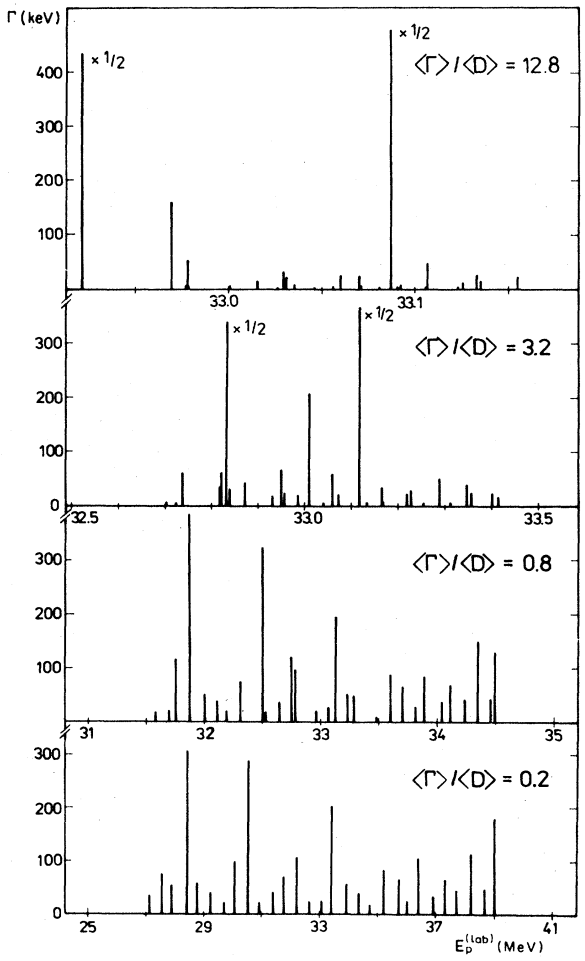

FIG. 2. The widths Γ_R of the individual resonance states R in dependence on the degree of overlapping $\langle\Gamma\rangle/\langle D\rangle$.

the sum of the widths of the isolated resonance states as has been checked numerically. Thus, $\langle\Gamma\rangle$ remains unchanged while $\langle D\rangle$ decreases altogether by a factor 64 from the lowest curves in Figs. 1 and 2 (every other is shown) to the uppermost ones.

Some remarks are necessary here in order to clarify how we reduce shell model state energy differences by factors of 2 without changing the Woods-Saxon potential parameters. In a first step, the shell model state energies E_R^{SM} and wave functions ϕ_R have been calculated by solving the standard shell model problem (1) without any coupling to the continuum with a Woods-Saxon potential (instead of a harmonic oscillator). The values E_R^{SM} and ϕ_R as well as the matrix elements $\langle\phi_R\,|\,H_{QP}\,|\,\xi_E^c\rangle$ are input values for the coupled channels equations (3) without source term and

$$(E-H_{PP})\omega_R=H_{PQ}\phi_R \tag{4}$$

with source term. The two systems of equations (3) and (4) are solved then in the subspace of scattering wave functions.[8,9,11] The input values E_R^{SM} can be varied by

hand in solving the coupled channels equations (3) and (4) while the wave functions ϕ_R and matrix elements $\langle \phi_R \mid H_{QP} \mid \xi_E^c \rangle$ remain those which follow from Eq. (1). The energies $E_R^{\rm SM}$ are shifted in our calculations altogether by no more than 5 MeV, which is still comparable with the usual accuracy of standard shell model calculations. The shell model diagonalization (1) has been performed with a complete set of states within the chosen shell model configuration subspace Q. The operator (2) acting in the whole function space is diagonalized with the subset of discrete shell model states (29 out of 76 states) within an energy region around 33 MeV. Effects arising from the diagonalization of (2) by neglecting states beyond a limited energy region were investigated by us earlier (e.g., see Ref. 12). They arise from the external mixing of all the resonance states and are shown to play a role only at the borderline. They are not important in the case considered here due to the large number of resonance states used in the diagonalization procedure.

The inelastic proton scattering cross section (Fig. 1) shows at high level density intermediate-like structures although no strongly absorbed channels were open originally and all the resonance states are of comparable lifetime at low level density. Further, the widths of a few resonance states increase at the cost of the widths of the remaining ones (Fig. 2). However, the 1p-1h contributions to the nuclear structure of these few resonance states do not increase sufficiently in order to explain the large widths by simple nuclear structure arguments. The six basic states $(1p_{3/2})^{-1}2s_{1/2}$, $(1p_{3/2})^{-1}1d_{5/2}$, and $(1p_{1/2})^{-1}2s_{1/2}$ with isospin $T=0$ and 1 are present in the wave functions of our 29 resonance states in most cases with amplitudes less than 0.1. The largest components are five out of 174 with values between 0.20 and 0.36 in the case $\langle \Gamma \rangle / \langle D \rangle = 0.2$ while there are altogether 11 of such a type in the real parts of the wave function and four in the imaginary parts in the case $\langle \Gamma \rangle / \langle D \rangle = 6.4$.

At high level density, the wave functions and partial widths of the resonance states are complex. The consequence is that only the whole of the resonance states has a physical meaning. The cross section cannot be represented by a sum of contributions from many levels each of which has nuclear structure properties which can be described by a standard nuclear structure model as, e.g., the shell model, Eq. (1). The width of the individual resonance states are changed due to the external mixing of the states via the continuum. Intermediate-like structures in the cross section may appear also if standard nuclear structure calculations do not give any hint of a short-lived state. It is impossible, in such a case , to find the counterpart of the intermediate structure at low level density as, e.g., in the parent nucleus for an isobaric analog resonance.

Thus, the results obtained show that deviations from the complete randomness appear. They are caused by the fact that most excited states of a really existing nucleus decay by particle emission. This gives rise to an external mixing of the resonance states via the continuum which is described by the second term in the Hamiltonian operator (2). The interaction produces an unequal distribution of the ensemble: short-lived resonances exist apart from long-lived ones which all must be considered together as a whole. Consequently, the number of long-lived resonances used as data can hardly be sufficiently extended in order to verify the statistical assumptions experimentally for a really existing nucleus as it is claimed in Ref. 6. This conclusion is in accordance with experience as stated also in Ref. 6.

According to general mathematical properties, the sum of the widths of all the resonance states considered in the calculation (imaginary part of the sum of the eigenvalues of $H_{QQ}^{\rm eff}$) is equal to the sum of the widths of the isolated resonance states (QBSEC) used as basic states in the diagonalization of $H_{QQ}^{\rm eff}$ (imaginary part of the sum of the diagonal matrix elements).[9] In an analogous manner, the sums of the partial widths with and without external mixing are the same.[9] Due to these relations, the nonstatistical effects considered above are not important for values averaged over some energy interval as is done, e.g., in the Hauser-Feshbach theory. Difficulties may appear, however, in an attempt to simultaneously reproduce the mean level widths observed and the averaged cross sections due to the results shown in Fig. 2.

Summarizing, it can be stated that deviations from the complete randomness appear in real nuclei at high level density which can be described by the external mixing of the resonance states via the continuum. They lead to changes in the spectroscopic properties of the individual resonances but may be neglected, to a good approximation, for values averaged over energy.

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