Mixed-symmetry interpretation of some low-lying bands in deformed nuclei

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An alternative interpretation of the second excited $K^\pi=0^+$ band in rare earth nuclei, as a mixed-symmetry state in the neutron-proton interacting boson model, is compared with the conventional interpretation as a two-quasiparticle band. It is shown that, on the basis of electromagnetic decay probabilities and particle transfer amplitudes alone, it is difficult to discriminate between the two interpretations. Only on the level of form factors and transition densities do major differences occur.

I. INTRODUCTION

In most medium heavy nuclei the low-lying collective states are symmetric in the neutron and proton degrees of freedom. This implies that neutrons and protons contribute in a fixed ratio to all matrix elements of a given operator. In order to extract information on the neutron and proton contributions separately it is important to measure also the properties of states that are of mixed symmetry (sometimes also called antisymmetric) in the neutron and proton degrees of freedom. The energy of these states, for example, gives an indication of the difference of the interaction between like particles and the neutron-proton interaction.

The occurrence of the mixed-symmetry modes has been predicted in various geometrical models[1,2] and in the version of the interacting boson model where neutron and proton degrees of freedom are explicitly taken into account (IBA-2).[3–6] In electron scattering experiments on deformed nuclei indeed such mixed-symmetry states have been observed, in particular, the bandhead of a $K^\pi=1^+$ band at $E_x\simeq 3$ MeV in various rare earth nuclei.[7] This had led to the conclusion, based on a particular choice of the IBA-2 Hamiltonian, that in deformed nuclei *all* mixed-symmetry states lie at an excitation energy of at least 3 MeV.[8,9] In this paper we show that with an alternative choice of the Hamiltonian, which is equally plausible as the conventional choice, it is possible to reproduce all existing data. However, this alternative choice introduces a low-lying mixed-symmetry $K^\pi=0^+$ band near the energy of the β and γ bands. In energy, this state lies close to an experimentally well-known[10] second excited $K^\pi=0^+$ band (denoted by $K^\pi=0_3^+$ in the following) which cannot be explained using the conventional choice for the IBA-2 Hamiltonian.[11] This $K^\pi=0_3^+$ band is assumed to be based on a two-quasiparticle (2 qp) excitation. In this paper we will investigate the possibility that this observed band is instead the mixed-symmetry band. For this reason we will present here a detailed comparison of the properties of a 2 qp band with those of a $K^\pi=0^+$ mixed-symmetry band.

II. THE IBA-2 MODEL

In the IBA-2 model,[5,6] the structure of the collective states in even-even nuclei is calculated by considering a system of interacting neutron and proton s and d bosons. The lowest lying states correspond predominantly to the fully symmetric SU(6) representation $[N]$, where N is the total number of neutron and proton s and d bosons. States belonging to the mixed-symmetry representations, such as $[N-1,1]$ are also present in the spectrum. The different neutron-proton symmetries can conveniently be labeled by introducing an F-spin quantum number.[12] Its interpretation is analogous to that of isospin for fermions. A boson is an object with F spin equal to $\frac{1}{2}$ and $F_z=-\frac{1}{2}$ for neutrons and $F_z=+\frac{1}{2}$ for proton bosons. The states with a maximal value of the F spin, $F_0=(N_\nu+N_\pi)/2$, thus belong to the maximally symmetric representation, $[N]$, of SU(6). The less symmetric states with $F=F_0-1$ belong to the $[N-1,1]$ representation. Most of the low-lying states have $F=F_0$. The aim of the present paper is to investigate the possibility of the occurrence of low-lying states in deformed nuclei with $F=F_0-1$. In spherical nuclei the properties of the lowest lying mixed-symmetry states have been investigated in Refs. 9, 13, and 14. In this latter case the states of principal interest are 2^+ levels, while, as will be shown in the following, in deformed nuclei the levels of interest belong to a $K^\pi=0^+$ rotational band.

The boson Hamiltonian used in most phenomenological IBA-2 calculations can be written as[5,6]

$$H=\epsilon_d(n_{d_\nu}+n_{d_\pi})+\kappa Q_\nu^{(2)}\cdot Q_\pi^{(2)}+M_{\nu\pi}\ , \qquad (1)$$

where

$$Q_\rho=(s_\rho^\dagger\tilde{d}_\rho+d_\rho^\dagger s_\rho)^{(2)}+\chi_\rho(d_\rho^\dagger\tilde{d}_\rho)^{(2)},\quad(\rho=\nu,\pi)\tag{2}$$

and

$$M_{\nu\pi}=\xi_2(s_\nu^\dagger d_\pi^\dagger-d_\nu^\dagger s_\pi^\dagger)^{(2)}\cdot(s_\nu\tilde{d}_\pi-\tilde{d}_\nu s_\pi)^{(2)}-2\sum_{k=1,3}\xi_k(d_\nu^\dagger d_\pi^\dagger)^{(k)}\cdot(\tilde{d}_\nu\tilde{d}_\pi)^{(k)}\,.\tag{3}$$

An additional interaction between like bosons is often added.[5] In this paper we will consider only a like boson interaction of the form

$$V_{\rho\rho}=\kappa_{\rho\rho}Q_\rho^{(2)}\cdot Q_\rho^{(2)},\quad \rho=\nu,\pi\,,$$

where $Q^{(2)}$ is defined in Eq. (2). This particular form is chosen since for $\kappa_{\nu\nu}=\kappa_{\pi\pi}=\frac{1}{2}\kappa$ the amount of F-spin breaking is minimal. The Majorana force serves to raise the excitation energy of the states that are not fully symmetric in the neutron and proton degrees of freedom (those having $F<F_0$). This term has been added to the Hamiltonian for purely phenomenological reasons. Without it the density of low-lying levels would be far greater than indicated by experiment. This Majorana force contains three parameters, ξ_1, ξ_2, and ξ_3. In most phenomenological applications, as for example given in Refs. 8 and 14, for simplicity, these parameters have been chosen equal, in which case the Majorana force pushes up the mixed-symmetry states by an amount depending only on their F-spin value,[8,9,14] i.e., this force is a true Majorana force.

While for the ϵ, κ, and $\chi_{\nu,\pi}$ parameters extensive microscopic calculations exist,[15,16] the microscopic origin of the Majorana force is at best only partially understood.[16,17] In particular, there is very little, or no, evidence that the three parameters ξ_1, ξ_2, and ξ_3 should be chosen equal.[17] On the basis of recent shell-model calculations for the very neutron-deficient even-even Pd (Ref. 14) isotopes, one finds that the parameters ξ_1 and ξ_3 can be chosen equal. Also from the point of view of the microscopic picture behind the IBA model, the term multiplying ξ_2 is of a completely different nature than the other two terms in the Majorana force. This term namely changes the number of neutron and proton d bosons (although the total number of d bosons is conserved by the Majorana force). In terms of the usual microscopic picture of the bosons in terms of particle pairs, the ξ_2 term corresponds to a matrix element in which the seniority in both the neutron and proton sector of the space changes while the ξ_1 and ξ_3 terms correspond to seniority-conserving matrix elements.

For the above reasons we made a phenomenological investigation of the structure of the calculated spectra choosing in the Majorana force $b=\xi_1=\xi_3$ and $\xi_2=0.0$. The extreme choice $\xi_2=0$ has been made in order to present a case that is complementary to the traditional choice $a=\xi_1=\xi_3=\xi_2$. The difference between the two choices for the Majorana force is most intriguing in the SU(3) limit of the IBA model on which we will therefore focus our attention.

In the deformed rare earth isotopes a probable candidate for the collective 1^+ state has been observed[7] near an excitation energy of $E_x\simeq 3$ MeV. In the IBA model a 1^+ state is necessarily of a mixed-symmetry character.[3,4] This state can therefore be used to determine the strength of the Majorana force.

As an example of a typical deformed rare earth nucleus we will discuss in some more detail the nucleus ^{156}Gd. The parameters of the IBA-2 Hamiltonian have been chosen as to reproduce the low-lying levels.[18] The strength of the Majorana force, as determined from the position of the 1^+ level near $E_x\cong 3$ MeV, can be taken as $a=\xi_1=\xi_2=\xi_3=0.15$ MeV (calculation I) or as $b=\xi_1=\xi_3=0.3$ MeV and $\xi_2=0$ (calculation II). In Fig. 1 the calculated excitation energies of the bandheads using these two different choices for the Majorana force are compared with some known bandheads in the experimental spectrum[10] of ^{156}Gd. Even though in both calculations the strength of the Majorana force is chosen such that the $K^\pi=1^+$ band lies near $E_x\simeq 3$ MeV, the spectrum of mixed-symmetry states in the two calculations is totally different. In calculation I the $K^\pi=1^+$ band is the lowest mixed-symmetry band, while in calculation II the lowest mixed-symmetry band lies near the position of the β and γ bands and all other mixed-symmetry states occur at the position of the $K^\pi=1^+$ bandhead or higher. The lowest mixed-symmetry $K^\pi=0^+$ band in calculation II results in energy very near an experimentally well-known $K^\pi=0_3^+$ band. This experimental $K^\pi=0^+$ band has hitherto been interpreted as being based on a 2 qp configuration.[19] In the IBA framework, on the other hand, this band has been explained through the introduction of s' and d' bosons.[11] Since these primed bosons can be seen as corresponding to the degrees of freedom related to cross-closed-shell excitations, this interpretation is equivalent to that of a 2-qp excitation. The present calculation suggests, however, that it could also be interpreted as a mixed-symmetry state and as such has escaped attention thus far. This would open the possibility of studying in some detail the properties of mixed-symmetry states, since the levels lie in a region of the spectrum where the level density is relatively low.

It is interesting to note that the calculation shows that the moment of inertia of the mixed-symmetry band is larger than that of the ground state band, while that of the calculated β band is smaller than that of the ground

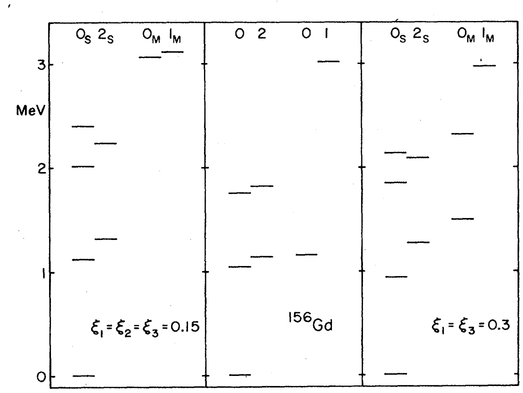

FIG. 1. A comparison between calculated bandhead energies and those observed in ^{156}Gd. In the calculation presented on the left-hand side (calculation I) the Majorana force is parametrized (see the text) by $a=0.15$ MeV while in the calculation on the right-hand side (II) $b=0.3$ MeV is used.

state band. Experimentally the moment of inertia of the $K^\pi=0_2^+$ band is somewhat larger than that of the ground state band. This even could be seen as an argument to interpret the observed *second* $K^\pi=0^+$ as the mixed-symmetry band. This could be realized in the calculation by choosing ξ_2 negative in the Majorana operator. In this paper we will, however, not explore this possibility any further. In the next section, calculated properties of this mixed-symmetry $K^\pi=0^+$ band are compared with those expected for a 2 qp band.

III. ELECTROMAGNETIC PROPERTIES

In this section, we discuss $E2$, $M1$, $E0$, and $M3$ matrix elements between the $K^\pi=0_3^+$ band and the ground state, β, and γ bands. The calculated values for the various quantities will be compared with estimates following from a 2 qp interpretation of this band.

A. $E2$

A 2 qp deformed state $|(\Omega_1,\Omega_2)\rangle$, projected on a fixed angular momentum IM,

$$|(\Omega_1,\Omega_2)I,M\rangle = P_M^I\,|(\Omega_1,\Omega_2)\rangle \;,$$

can, using the spherical decomposition of a Nilsson orbital,

$$|\Omega\rangle = \sum_j C_{j,\Omega}\,|j,\Omega\rangle \;,$$

be written as

$$|(\Omega_1,\Omega_2)I,M\rangle = \sum_{j_1,j_2} C_{j_1,\Omega_1}C_{j_2,\Omega_2} \times\langle j_1\Omega_1,j_2\Omega_2\,|\,IM\rangle\,|\,j_1j_2,IM\rangle \;,$$

where now, $|j_1j_2IM\rangle$ is a spherical two-quasiparticle state. For $I=2$ a sum over all possible spherical single particle configurations results in a $B(E2)$ value for the deformed state which is almost equal to the $B(E2)$ value for a pure (j^2) configuration, i.e., of the order of 1 single particle unit (s.p.u.)$\simeq 150\ e^2\mathrm{fm}^4$ in the mass region of interest. This value is of the order of magnitude of the experimental values (see Ref. 10, Table 5).

In a 2 qp picture one thus expects that the $E2$ transition strength from the $K^\pi=0_3^+$ band to the other low-lying bands will be of the order of 1 s.p.u.,[10,20] strongly hindered compared to the collective $2_1^+\to 0_1^+$ transition in the ground state band. The $E2$ transitions within the $K^\pi=0_3^+$ band are expected to be of the same order of magnitude, although slightly smaller, as those within the g.s. band.

In the IBA-2 model, $E2$ transitions are calculated using the operator

$$T^{E2}=e_\pi Q_\pi^{(2)}+e_\nu Q_\nu^{(2)} \;, \tag{4}$$

where $Q_\pi^{(2)}$ and $Q_\nu^{(2)}$ are given by Eq. (2). In phenomenological calculations one usually assumes that the neutron and proton boson effective charges are equal, $e_\pi=e_\nu$. While in the study of purely symmetric states there is no sensitivity in the calculation on the difference of the neutron and proton effective charges, this is no longer the case when studying mixed-symmetry states. In fact, if $e_\nu=e_\pi$, all $E2$ transitions leading from symmetric to mixed-symmetry states would vanish identically in the case of an unbroken symmetry. Shell-model calculations in this mass region indicate that the contribution to the fermion charge, as a result of core polarization, is of the order of $0.8e$,[21] giving $e_p=1.8e$, $e_n=0.8e$. To obtain boson effective charges one has to include also the effects of radial integrals, which are larger for neutrons than for protons. Renormalizations of the effective charge due to model space truncations, specifically the omission of G pairs from the model space, tend to make the neutron and proton effective charge more equal. Some recent phenomenological calculations[13,22] in which special attention is paid to the difference in the boson effective charges tend to favor a value of $1.5e_\nu\le e_\pi\le 2.0e_\nu$. For these reasons we present in Table I some $B(E2)$ values for the mixed-symmetry states, calculated for two different choices for the boson effective charges, $e_\pi=2e_\nu$ and $e_\pi=e_\nu$. Even though the neutron and proton boson effective charges differ by a factor 2, the transitions from the $K^\pi=0_3^+$ band to the other low-lying bands is only a fraction of the $2_1^+\to 0_1^+$ transition and of the order of a few s.p.u. The transitions within the band are strong but somewhat weaker than the transitions within the g.s. band. In this sense the interpretations of the $K=0_3^+$ band as a mixed-symmetry or as a 2 qp band are indistinguishable.

Even for the case of equal boson effective charges the transitions leading to the mixed-symmetry (MS) states do not vanish. This is an indication that there is a considerable amount of F-spin breaking in the spectrum. The origin of this lies in the fact that in the Hamiltonian only a quadrupole-quadrupole interaction between neutrons and protons has been assumed. Since in ^{156}Gd the number of neutron and proton bosons ($N_\pi=7$, $N_\nu=5$) is not equal, this introduces a strong F-spin breaking term in the Hamiltonian. In the present case the interaction matrix element between the pure 0_β^+ and 0_{MS}^+ is of the order of 200

TABLE I. Calculated $B(E2)$ values for various transitions involving members of the mixed-symmetry $K^\pi=0^+$ band for two different choices for the boson effective charges. In calculation A, no quadrupole interaction between like particles has been included, while in calculation B an almost (see the text) F-spin symmetric Hamiltonian has been used.

	A		B	
Transition	$e_\nu=e_\pi/2$	$e_\pi=e_\nu$	$e_\nu=e_\pi/2$	$e_\pi=e_\nu$
$2_1^+\to 0_1^+$	100.	100.	100.	100.
$0_{\mathrm{MS}}^+\to 2_1^+$	0.36	0.86	0.53	0.002
$0_{\mathrm{MS}}^+\to 2_\beta^+$	0.85	1.11	0.52	0.001
$0_{\mathrm{MS}}^+\to 2_\gamma^+$	2.13	3.38	0.74	0.006
$2_{\mathrm{MS}}^+\to 0_1^+$	0.005	0.12	0.33	0.0004
$2_{\mathrm{MS}}^+\to 0_\beta^+$	0.10	0.05	0.27	0.0002
$2_{\mathrm{MS}}^+\to 0_{\mathrm{MS}}^+$	78.15	70.86	69.4	67.7

keV. In order to test the effects of symmetry breaking we have done an additional calculation (calculation B) in which F-spin mixing is minimized, by adding to the Hamiltonian (1) a quadrupole-quadrupole interaction between like particles with half the strength of the neutron-proton interaction. A minor amount remains since $\chi_\pi=-1.0$ is not equal to $\chi_\nu=-1.1$ [see Eq. (2)]. The strength of the quadrupole interaction has been adjusted in order to keep the positions of the β and γ bands unchanged. The strength of the Majorana force was increased to $b=0.55$ MeV to keep the 1^+ level at $E_x\simeq3$ MeV. The position of the 0^+_{MS} state is somewhat lower in the spectrum, at $E_x=1.3$ MeV, near the γ band. From column B in Table I, it can be seen that, although now the absolute magnitude of the $\Delta F=1$ transitions is strongly dependent on the ratio of the neutron and proton effective charges, the basic features have remained unchanged.

B. $M1$

One might expect a large difference in the prediction of the $M1$ strength for a 2 qp and a collective band. The $M1$ transitions are expected to be of the order of less than 0.1 of a single particle unit for a 2 qp band (Ref. 23, p. 748). On the other hand, for a purely collective band, where neutrons and protons move in a coherent fashion, one expects that all $M1$ transitions vanish, and the observed small $M1$ transitions have to be explained via a band-mixing calculation.[24,25]

In the IBA-2 model, the $M1$ transition operator is written as

$$T^{M1}=\sqrt{30/4\pi}[g_\pi(d^\dagger_\pi\tilde{d}_\pi)^{(1)}+g_\nu(d^\dagger_\nu\tilde{d}_\nu)^{(1)}]\,, \tag{5}$$

where g_π and g_ν are the boson g factors. On the basis of the collectivity of the bosons one can argue that the spin contributions to g_π and g_ν essentially cancel and that therefore the g factors are equal to the orbital nucleon g factors, $g_\pi\simeq1.0\,\mu_{\mathrm{N}}$ and $g_\nu\simeq0.0\,\mu_{\mathrm{N}}$. Analysis of magnetic moments of low-lying states[26] basically supports this conclusion but deviations of the order of 0.3 μ_{N} could occur.

Because of the similarity with the angular momentum operator, the matrix elements of the $M1$ operator between two purely symmetric states vanish exactly and only due to small admixtures of mixed-symmetry components, nonzero $M1$ transition rates can be obtained.[25] The $M1$ operator has relatively strong matrix elements connecting symmetric and mixed-symmetry states.[3,4,14] It was in fact this feature that helped to identify the $K^\pi=1^+$ band observed in electron scattering[7] as the mixed-symmetry band predicted by the IBA model. In Table II we present a number of calculated $M1$ matrix elements between the $K^\pi=0^+_3$ band and the ground state and γ bands. As expected on the basis of the above arguments these transition rates are relatively large and of the same order of magnitude as can be expected for a 2 qp band. This implies that also $M1$ transitions are precluded from distinguishing between a 2 qp and a mixed-symmetry interpretation of the observed $K^\pi=0^+_3$ band.

TABLE II. Some calculated $B(M1)$ values in units of μ^2_{N}, using $g_\pi=1\,\mu_{\mathrm{N}}$ and $g_\nu=0$ in Eq. (5) for $\Delta K\leq1$ transitions involving mixed-symmetry states. The difference between calculations A and B is explained in the text.

Transition	A	B
$0^+_1\to1^+_1$	1.68	0.90
$0^+_1\to1^+_2$	0.55	0.27
$0^+_1\to1^+_3$	0.05	0.38
$2^+_{\mathrm{MS}}\to2^+_1$	0.046	0.10
$2^+_{\mathrm{MS}}\to2^+_\beta$	0.001	0.01
$2^+_{\mathrm{MS}}\to2^+_\gamma$	0.0	0.0004
$0^+_2\to1^+_1$	0.24	0.07

In Table II also the $M1$ transition strength leading to the lowest 1^+ states has been given, for completeness. It is seen that the 1^+_2 state which lies at $E_x=3.8$ MeV still carries a considerable $M1$ strength. The present calculation thus also explains some of the experimentally observed splitting[7] of the collective $M1$ strength.

If it would be possible to measure the difference between the spin and convection current contribution to these $M1$ transitions, one might be able to distinguish the two interpretations. In the decay of the 2 qp band the contribution of the spin part of the operator will be large. For mixed-symmetry states, the spin contribution is negligible, since for the neutron and proton bosons separately it is small.[26,27] However, since there are no allowed $M1$ transitions from a $K^\pi=0^+$ band to the ground state, this will be hard to determine experimentally.

C. $E0$

Monopole decays of various 0^+ states have been observed experimentally. In general, the $E0$ transition probability from a 2 qp state to the ground state is hindered as compared to the decay of the β bandhead.

In the IBA-2 model the $E0$ transition operator can be put in the form

$$T^{E0}=e_{0,\nu}\hat{n}_{d\nu}+e_{0,\pi}\hat{n}_{d\pi}\,. \tag{6}$$

The s-boson contribution has been eliminated, using boson number conservation. The calculated $E0$ transition probabilities are given in Table III for two different choices of the monopole effective charges. Depending on the choice, for which there exists essentially no phenomenological information, the $E0$ transition probability to the mixed-symmetry state is a factor 2 to 100 smaller than that to the β band.

TABLE III. Calculated $B(E0)$ values in units of $e^2_{0,\pi}$. The difference between calculations A and column B is explained in the text.

	A		B	
Transition	$e_{0,\pi}=e_{0,\nu}$	$e_{0,\nu}=0$	$e_{0,\pi}=e_{0,\nu}$	$e_{0,\nu}=0$
$0^+_1\to0^+_\beta$	0.49	0.40	0.66	0.22
$0^+_1\to0^+_{\mathrm{MS}}$	0.19	0.002	0.0003	0.14
$0^+_\beta\to0^+_{\mathrm{MS}}$	0.096	0.43	0.0	0.0003

TABLE IV. Calculated values for $B(M3\uparrow)$ strength, in units of Ω_π^2 for two different choices of the boson octupole moments, leading to different 3^+ levels, specified by their excitation energy. The difference between calculations A and B is explained in the text.

	A			B	
E_x	$\Omega_\nu=-\Omega_\pi$	$\Omega_\nu=0$	E_x	$\Omega_\nu=-\Omega_\pi$	$\Omega_\nu=0$
1.4	0.86	3.13	1.4	0.14	1.71
2.2	0.49	0.10	2.3	1.13	0.22
2.5	1.32	0.11	2.3	0.08	0.10
2.9	1.21	0.31	3.0	0.06	0.01
3.1	0.86	0.31	3.1	3.13	1.07
3.2	3.20	0.75	3.4	0.00	0.00
3.5	0.00	0.00			

D. $M3$

It has recently been suggested[28] that $B(M3\uparrow)$ values would give important additional information on the position of mixed-symmetry states since in the boson $M3$ operator

$$T^{M3}=\left[\frac{35}{8\pi}\right]^{1/2}[\Omega_\pi(d_\pi^\dagger \tilde{d}_\pi)^{(3)}+\Omega_\nu(d_\nu^\dagger \tilde{d}_\nu)^{(3)}]$$

the boson octupole moments Ω_π,Ω_ν have opposite signs. For this reason the octupole transitions would predominantly excite mixed-symmetry states. The microscopic calculations presented in Ref. 28 indicate that $\Omega_\nu\sim-\frac{1}{5}\Omega_\pi$. For this reason, in Table IV we present calculations for two different choices of the boson octupole moments. If the operator is purely antisymmetric, $\Omega_\pi=-\Omega_\nu$, predominantly the 3^+ level of the mixed-symmetry $K^\pi=1^+$ band is excited, although in calculation A an appreciable amount of strength is concentrated in the lower states due to the F-spin nonconserving terms in the Hamiltonian. If in the $M3$ operator $\Omega_\nu=0$ is used, the calculations predict that the 3_γ^+ is by far the strongest state in the spectrum. This result deviates significantly from the results reported in Ref. 28 where the standard choice for the Majorana force was used.

IV. NONELECTROMAGNETIC PROBES

In two-nucleon stripping one expects to be able to excite a 2 qp band with a strong selectivity. For nuclei in this mass region the excitation probability is of the order of 20% of that of the ground state.[29] In two-nucleon pickup the excitation of a 2 qp state is essentially forbidden.

If the bosons under consideration correspond to particle pairs, i.e., the major shell is less than half filled, the operator for $L=0$ two-neutron pickup is given by

$$T_{\nu-}^{(0)}=A_\nu s_\nu\ , \tag{7a}$$

while $L=0$, two-neutron stripping is described by the operator

$$T_{\nu+}^{(0)}=A_\nu s_\nu^\dagger\ . \tag{7b}$$

TABLE V. Amplitudes for $L=0$ two-neutron transfer (in arbitrary units). The difference between calculations A and B is explained in the text.

	A		B	
	(p,t)	(t,p)	(p,t)	(t,p)
0_1^+	1.82	1.54	1.93	1.68
0_β^+	0.15	0.61	0.06	0.97
$0_{\rm MS}^+$	0.15	0.79	0.16	0.41

For $L=2$ transfer the expressions are more complicated and it is therefore impossible to make *a priori* predictions of excitation probabilities since the IBA model operators contain several adjustable parameters.

The calculated two-neutron transfer probabilities in units of A_ν [Eq. (7)] are given in Table V. Although the calculated transition strengths are nucleus dependent, it is clear from Table V that the strength leading to the mixed-symmetry state in IBA-2 obeys the same selection rules as would be expected for a 2 qp state. This implies that also on the basis of two-nucleon transfer strength, it is not possible to distinguish a 2 qp and a mixed-symmetry interpretation of this band.

Since the $L=0$ two-nucleon transfer operators in IBA-2 are parametrized with only a single coefficient (in contrast to the $E2$ operator where there are two, e and χ) it is expected that all 0^+ states are populated with the same form factor. The mixed-symmetry 0^+ state should thus be excited with the same angular distribution (neglecting Q-value effects) as the ground state. For a 2 qp state one would in general expect a form factor which is different from that of the ground state. The angular distributions for two-nucleon stripping might thus give a clue to the correct interpretation of this band.

Another interesting possibility to distinguish between the 2 qp and the mixed-symmetry interpretation may lie in measuring the neutron and proton component of the transition matrix element separately. For a pure mixed-symmetry state one expects the neutron and proton transition matrix elements to the ground state to be equal in magnitude but opposite in phase. A 2 qp excitation on the other hand has in most cases a character that is either neutronlike or protonlike. A combination of an isoscalar and an isovector probe (inelastic α scattering and $E2$ transitions or inelastic π^+/π^- scattering[30]) could thus distinguish between the two pictures. Core polarization effects, which are predominantly isoscalar in character, might obscure some of the effects.

V. CONCLUSIONS

In this paper we have suggested an alternative choice for the Majorana force in the IBA-2 model. This choice has the peculiar feature of producing in the SU(3) limit a spectrum in which there appears a mixed-symmetry $K^\pi=0^+$ band at approximately the same energy as that of the β and γ bandheads. All other mixed-symmetry bands appear at or above the energy of the $K^\pi=1^+$ band.

In the spectra of the deformed rare earth nuclei an additional $K^\pi=0^+$ band has been observed experimentally near the energy of the β and γ bands. This band has been interpreted as based on a 2 qp excitation.[11,19] We propose here an alternative explanation in terms of a mixed-symmetry $K^{\pi}=0^{+}$ band. Several experimental observables for such a mixed-symmetry band are calculated. A qualitative comparison of these values with the predictions for a typical 2 qp band shows that on the basis of transition rates only, the two interpretations are indistinguishable. Only on the level of more detailed properties such as transition densities and form factors, one might expect to be able to differentiate between the two interpretations. Also a detailed comparison of the excitation probabilities for different probes may yield data on the neutron and proton matrix elements separately, and as such can also lead to a differentiation between the different pictures.

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