Relativistic treatment of spin observables in the excitation of the 1^+ $T=0$ state in ^{12}C

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In the present paper we extend earlier work on spin-dominated transitions using a plane wave formalism to a full distorted wave treatment. We perform a distorted wave calculation using a relativistic approach to nucleon-nucleus inelastic scattering. We focus on the 1^+, $T=0$ transition in ^{12}C using 500 MeV protons with special emphasis on spin observables. We obtain sizable effects for spin observable differences without invoking exchange processes.

I. INTRODUCTION

Current relativistic models of nucleon-nucleus scattering, using the Dirac equation, have been very successful in describing elastic scattering data and especially spin observables.[1-3] The Dirac formalism incorporates spin very naturally into the picture, and the study of spin dominated transitions becomes both interesting and important. In the present paper we calculate the transition amplitude, for nucleon-nucleus inelastic scattering, in the framework of a relativistic distorted wave impulse approximation (RDWIA). Our main objective is to assess the importance of distortion effects, which were completely ignored in our previous relativistic plane wave treatment.[4] In that work we developed a relativistic plane wave formalism which focused exclusively on the transition potential and allowed the determination of the scattering amplitude in an exact and completely analytic way. This relativistic plane wave impulse approximation (RPWIA) formalism revealed the fundamental role played by upper to lower transitions in the determination of spin observables, and particularly spin observable differences like $(P-A_y)$ or more generally the spin difference function.[4,5]

The additional complication in the present RDWIA treatment arises solely from the presence of distorted waves. This fact will render our previous analytic treatment no longer feasible and will make us resort to numerical computations, where the extraction of physically meaningful results is not always straightforward. Fortunately, however, we obtained a great deal of physical insight from the RPWIA calculation done before and this will, hopefully, put our conclusions on a firm footing.

II. THE TRANSITION POTENTIAL

We consider the excitation of a nucleus, originally in a 0^+ state, to a final JM state with parity Π_f by means of its interaction with a proton of initial momentum $\mathbf{k}$ and final momentum $\mathbf{k}'$. In the present formulation we only consider excitations to nuclear states by means of single particle transitions. More complicated modes of excitations (e.g., two particle excitations) are not contemplated in the present approach. The transition amplitude, in a relativistic distorted wave impulse approximation treatment, is given, according to Ref. 6, by

$$A_{JM}=\int d\mathbf{r}\,\psi^{\dagger(-)}_{\mathbf{k}'s'}(\mathbf{r})\times\left\langle JM\left|\sum_{n=1}^{A}\gamma^0\gamma^0(n)t_n(|\mathbf{r}-\mathbf{r}_n|)\right|0^+\right\rangle\psi^{(+)}_{\mathbf{k}s}(\mathbf{r}),\quad(1)$$

where the Dirac gamma matrices are defined using the Bjorken and Drell convention;[7] $t_n(|\mathbf{r}-\mathbf{r}_n|)$ is proportional to the Fourier transform of the Lorentz invariant parametrization of the NN interaction

$$t_n=t_S+t_V\gamma^\mu\gamma_\mu(n)+t_T\sigma^{\mu\nu}\sigma_{\mu\nu}(n)+t_P\gamma^5\gamma^5(n)+t_A\gamma^5\gamma^\mu\gamma^5(n)\gamma_\mu(n)\quad(2)$$

written in terms of scalar (S), vector (V), tensor (T), pseudoscalar (P), and axial vector (A) amplitudes, and $\psi^{\dagger(-)}_{\mathbf{k}'s'}(\mathbf{r})$ and $\psi^{(+)}_{\mathbf{k}s}(\mathbf{r})$ are eikonal distorted waves given by[8]

$$\psi^{(\pm)}_{\mathbf{k}s}(\mathbf{r})=\sqrt{E+m/2m}\begin{bmatrix}1\\ \dfrac{1}{\mathscr{E}+\mathscr{M}}(\boldsymbol{\sigma}\cdot\mathbf{P})\end{bmatrix}e^{i\mathbf{k}\cdot\mathbf{r}}e^{iS^{\pm}(\mathbf{r})}\chi_s\,,$$
$$\mathscr{M}\equiv m+S,\quad\mathscr{E}\equiv E-V\,,\quad(3)$$

where the Dirac eikonal phase

$$S^{\pm}(\mathbf{r})=-\frac{m}{K}\int_{\mp\infty}^{z}dz'\{V_c(r')+V_{so}(r')[\boldsymbol{\sigma}\cdot(\mathbf{b}\times\mathbf{K})-iKz']\}\,,$$
$$\mathbf{K}=\tfrac{1}{2}(\mathbf{k}+\mathbf{k}')\quad(4)$$

is written in terms of equivalent central and spin-orbit effective Schrödinger potentials which depend on the strong scalar S and timelike vector V Dirac potentials, i.e.,

$$V_c(r)=\left[S(r)+\frac{E}{m}V(r)\right]+\frac{1}{2m}[S^2(r)-V^2(r)]\,,$$
$$V_{so}(r)=\frac{1}{2m}\frac{1}{\mathscr{E}+\mathscr{M}}\frac{1}{r}\frac{d}{dr}[V(r)-S(r)]\,.\quad(5)$$

The algebraic manipulations of the transition potential are identical to those developed in Ref. 6. We then start, by separating the spin contribution to the NN amplitude, from the four component Dirac contribution

$$\gamma^0\gamma^0(n)t_n(|\mathbf{r}-\mathbf{r}_n|)=\sum_{v=1}^{4}[f^v(|\mathbf{r}-\mathbf{r}_n|)+g^v(|\mathbf{r}-\mathbf{r}_n|)\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}(n)]\Gamma_v\Gamma_v(n) \tag{6}$$

in terms of the structure matrices Γ_v defined by

$$\Gamma_1=1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \Gamma_2=\gamma^0=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},$$
$$\Gamma_3=\gamma^5=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\quad \Gamma_4=\gamma^0\gamma^5=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}, \tag{7}$$

and the eight amplitudes

$$\begin{aligned} &f^1=t_V,\ \ g^1=-t_A\ ,\\ &f^2=t_S,\ \ g^2=2t_T\ ,\\ &f^3=t_A,\ \ g^3=-t_V\ ,\\ &f^4=t_P,\ \ g^4=2t_T\ . \end{aligned} \tag{8}$$

By expanding these amplitudes in spherical harmonics and the inner product $\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}(n)$ in terms of its spherical components, we can write

$$\gamma^0\gamma^0(n)t_n(|\mathbf{r}-\mathbf{r}_n|)=\sum_{v=1}^{4}\sum_{jmls}(-1)^{j+l+s}h_{ls}^v(r,r_n)[Y_l(\hat{\mathbf{r}}_n)\sigma_s(n)]_{jm}[Y_l(\hat{\mathbf{r}})\sigma_s]_{jm}^*\Gamma_v(n)\Gamma_v\ , \tag{9}$$

where

$$h_{ls}^v(r,r_n)=\left[\frac{2}{\pi}\right]\int_0^\infty q^2dq\, h_s^v(q)j_l(qr)j_l(qr_n) \tag{10}$$

and

$$\begin{aligned} h_s^v(q)&=f^v(q)\ \text{for}\ s=0\ ,\\ &=g^v(q)\ \text{for}\ s=1\ . \end{aligned}$$

We continue, by writing the one particle target space operator in a second quantized formalism, and at the end obtain the most general form for the single-step inelastic transition amplitude

$$A_J=\sum_{v=1}^{4}\sum_{ls}\int d\mathbf{r}\,\psi_{\mathbf{k}'s'}^{\dagger(-)}(\mathbf{r})G_{Jls}^v(r)\Omega_{Jls}(\hat{\mathbf{r}})\Gamma_v\psi_{\mathbf{k}s}^{(+)}(\mathbf{r})\ , \tag{11}$$

where

$$G_{Jls}^v(r)=\sum_{jij_f}(-1)^{J+l+s+1}\frac{\hat{j}_f}{\hat{J}}A_{J(j_i,j_f)}\langle\psi_f||h_{ls}^v(r,r_1)[Y_l(\hat{\mathbf{r}}_1)\sigma_s(1)]_J\Gamma_v(1)||\psi_i\rangle\ . \tag{12}$$

$A_{J(j_i,j_f)}$ are nuclear structure amplitudes, $\hat{j}=\sqrt{2j+1}$, and $\Omega_{Jls}(\hat{\mathbf{r}})$ is a rotational invariant operator

$$\begin{aligned} \Omega_{J=1,ls}&=\sqrt{3/4\pi}(\boldsymbol{\Sigma}\cdot\hat{\mathbf{r}}),\ \ (l=1,s=0)\ ,\\ &=\frac{1}{\sqrt{4\pi}}(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}),\ \ (l=0,s=1)\ ,\\ &=\sqrt{3/8\pi}\,i\boldsymbol{\Sigma}\cdot(\hat{\mathbf{r}}\times\boldsymbol{\sigma}),\ \ (l=1,s=1)\ ,\\ &=\frac{1}{\sqrt{8\pi}}[(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma})-3(\boldsymbol{\Sigma}\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{r}})],\ \ (l=2,s=1)\ , \end{aligned} \tag{13}$$

defined in terms of the polarization (axial) vector of the target

$$\Sigma_M \equiv |1^+ M\rangle\langle 0^+| \ . \tag{14}$$

We readily observe, that in the limit of distorted waves given by their asymptotic plane wave behavior, we immediately recover the plane wave result.[4] The selection rules established in the PWIA treatment allowing values of l and s, that make Ω_{Jls} a pseudoscalar operator, to contribute to the transition amplitude, as long as they appear in conjunction with the structure matrices Γ_3 and Γ_4 which generate terms linear in the lower components, are still valid in the present case.

III. THE TRANSITION AMPLITUDE

As we mentioned previously, the only complication arises from the presence of distorted waves. These distorted waves were obtained by solving a Schrödinger-type equation for the upper component and subsequently determining the lower component, in terms of the upper, by means of the relation

$$w_{\mathbf{k},s}^{(+)}(\mathbf{r}) = \frac{1}{\mathcal{E}+\mathcal{M}}(\sigma\cdot\mathbf{P})u_{\mathbf{k},s}^{(+)}(\mathbf{r}) \ .$$

So far, we have been able to avoid letting the gradient operator act on the complicated eikonal phase (3), by either integrating by parts, in the case of the elastic scattering amplitude,[8] or by using the differential equation satisfied by the upper component, in the case of simple Tassie-type collective excitations.[9] In the present case of microscopic transitions, neither of these two approaches is possible due to the complicated form of the transition potential and all the different types of coupling between components. Our only recourse then, is to proceed in the spirit of the eikonal treatment and write, to leading order in the approximation, the following expression for the lower components

$$w_{\mathbf{k},s}^{(+)}(\mathbf{r}) \simeq \frac{1}{\mathcal{E}+\mathcal{M}}(\sigma\cdot\mathbf{k})u_{\mathbf{k},s}^{(+)}(\mathbf{r}) \ ;$$

$$w_{\mathbf{k}',s'}^{(-)}(\mathbf{r}) \simeq \frac{1}{(\mathcal{E}+\mathcal{M})^*}(\sigma\cdot\mathbf{k}')u_{\mathbf{k}',s'}^{(-)}(\mathbf{r}) \ ,$$

which in turn make the transition amplitude take the following form:

$$A_J = \sum_{v=1}^{4}\sum_{ls}\int d\mathbf{r}\, G_{Jls}^{v}(r)e^{i\mathbf{q}\cdot\mathbf{r}}e^{-\chi_c(b)}e^{imb\left[\int_z^\infty dz' V_{\rm so}\right](\sigma\cdot\hat{\varphi})} \times\left\{\left[\frac{\mathcal{E}+\mathcal{M}}{2m}\right]\begin{bmatrix}1, & \dfrac{\sigma\cdot\mathbf{k}'}{\mathcal{E}+\mathcal{M}}\end{bmatrix}\Omega_{Jls}\Gamma_v\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{k}}{\mathcal{E}+\mathcal{M}}\end{bmatrix}\right\}e^{imb\left[\int_{-\infty}^z dz' V_{\rm so}\right](\sigma\cdot\hat{\varphi})} \ , \tag{15}$$

where in the above expression

$$\chi_c(b) = i\frac{m}{K}\int_{-\infty}^{\infty} dz\, V_c(r) \tag{16}$$

is the central part of the profile function, $\mathbf{q}=(\mathbf{k}-\mathbf{k}')$ is the momentum transfer to the nucleus, $\hat{\varphi}\equiv(\hat{\mathbf{K}}\times\hat{\mathbf{b}})$, and we have already performed the algebra associated with the Darwin term.[9]

It has been shown[10,11] that for moderate to large values of q, i.e., $qc \gg 1$, where c is the nuclear radius, the dominant contribution to the integral comes from a region in b space close to a singularity of the profile function $\chi_c(b)$ which, for a Woods-Saxon form, is localized at $b_0 = c + i\pi\beta$, where β is the diffusivity parameter. This is simply the statement that the dominant contribution to the integral comes from the surface since only there do the rapid oscillations of the integrand not cancel themselves out. This implies that the most important contribution of the Thomas-type spin orbit potential $V_{\rm so}(r)$, proportional to the derivative of the nuclear ground state density, comes from values of b close to the nuclear surface, or equivalent from small values of z. This fact, together with the evenness of the spin orbit potential, allows us to write the above spin orbit integrals as

$$\int_z^\infty dz' V_{\rm so} \simeq \int_0^\infty dz' V_{\rm so} = \tfrac{1}{2}\int_{-\infty}^{\infty} dz' V_{\rm so}$$

and similarly,

$$\int_{-\infty}^z dz' V_{\rm so} \simeq \int_{-\infty}^0 dz' V_{\rm so} = \tfrac{1}{2}\int_{-\infty}^{\infty} dz' V_{\rm so} \ ,$$

and to rewrite the transition amplitude in the following form

$$A_J = \sum_{v=1}^{4}\sum_{ls}\int d\mathbf{r}\, G_{Jls}^{v}(r)e^{i\mathbf{q}\cdot\mathbf{r}}e^{-\chi_c(b)}e^{i[\chi_{\rm so}(b)/2](\sigma\cdot\hat{\varphi})}\left\{\left[\frac{\mathcal{E}+\mathcal{M}}{2m}\right]\begin{bmatrix}1, & \dfrac{\sigma\cdot\mathbf{k}'}{\mathcal{E}+\mathcal{M}}\end{bmatrix}\Omega_{Jls}\Gamma_v\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{k}}{\mathcal{E}+\mathcal{M}}\end{bmatrix}\right\}e^{i[\chi_{\rm so}(b)/2](\sigma\cdot\hat{\varphi})} \ , \tag{17}$$

where we have defined the spin orbit contribution to the profile function as

$$\chi_{\rm so}(b) \equiv mb\int_{-\infty}^{\infty} dz\, V_{\rm so}(r) \ . \tag{18}$$

The expression in curly braces, aside from the substitution $(E+m)\rightarrow(\mathcal{E}+\mathcal{M})$, is precisely the matrix element evaluated in the plane wave calculation.[4] In the present case, the spin algebra has not yet been exhausted due to the presence of the spin operator $e^{i[\chi_{\rm so}(b)/2]}(\sigma\cdot\hat{\varphi})$. Its contribution however is not difficult to obtain. As an operator in the spin of the projectile, the expression in curly braces is at most linear in the Pauli matrices σ, and we can therefore write it as

$$\left\{\left[\frac{\mathscr{E}+\mathscr{M}}{2m}\right]\left[1,\frac{\sigma\cdot\mathbf{k}'}{\mathscr{E}+\mathscr{M}}\right]\Omega_{Jls}\Gamma_v\left[\begin{array}{c}1\\ \dfrac{\sigma\cdot\mathbf{k}}{\mathscr{E}+\mathscr{M}}\end{array}\right]\right\}=\alpha^v_{Jls}+\sigma\cdot\beta^v_{Jls}\ .$$

The spin independent term then simply gives

$$e^{i[\chi_{so}(b)/2](\sigma\cdot\hat{\varphi})}\alpha^v_{Jls}e^{i[\chi_{so}(b)/2](\sigma\cdot\hat{\varphi})}=\alpha^v_{Jls}\{\cos[\chi_{so}(b)]+i\sin[\chi_{so}(b)](\sigma\cdot\hat{\varphi})\}\ ,$$

while the spin dependent one yields

$$e^{i[\chi_{so}(b)/2](\sigma\cdot\hat{\varphi})}(\beta^v_{Jls}\cdot\sigma)e^{i[\chi_{so}(b)/2](\sigma\cdot\hat{\varphi})}=\beta^v_{Jls}\cdot\sigma+i\sin[\chi_{so}(b)](\beta^v_{Jls}\cdot\hat{\varphi})-2\sin^2\left[\frac{\chi_{so}(b)}{2}\right](\sigma\cdot\hat{\varphi})(\beta^v_{Jls}\cdot\hat{\varphi})\ .$$

Let us for the purpose of illustration evaluate the $v=3$, spin independent contribution to the 1^+ state which clearly has no nonrelativistic counterpart. In this case

$$\alpha^{v=3}_{110}=0;\ \ \beta^{v=3}_{110}=\sqrt{3/4\pi}\left[\frac{K}{m}\right](\Sigma\cdot\hat{\mathbf{r}})\hat{\mathbf{K}}\ ,$$

and since $(\hat{\mathbf{K}}\cdot\hat{\varphi})=0$, the contribution from this term to the amplitude is given by

$$A^{v=3}_{110}=\sqrt{3/4\pi}\left[\frac{K}{m}\right](\sigma\cdot\hat{\mathbf{K}})\int d\mathbf{r}\,G^v_{1ls}(r)e^{i\mathbf{q}\cdot\mathbf{r}}e^{-\chi_c(b)}(\Sigma\cdot\hat{\mathbf{r}})\ . \tag{19}$$

The difficulty in evaluating the above expression analytically stems from the fact that $G^v_{Jls}(r)$ is proportional to the Fourier transform of the fundamental NN interaction which is only given in numerical form, and is not suitable for a simple Gaussian- or Yukawa-type parametrization. The spatial dependence of $G^v_{Jls}(r)$ is however only on $r=\sqrt{b^2+z^2}$, and we can therefore proceed to perform the azimuthal φ integration

$$\int_0^{2\pi}d\varphi(\Sigma\cdot\hat{\mathbf{r}})e^{i\mathbf{q}\cdot\mathbf{r}}=-\frac{i}{r}(\Sigma\cdot\nabla_{\mathbf{q}})\int_0^{2\pi}d\varphi\,e^{i\mathbf{q}\cdot\mathbf{r}}=2\pi i\left[\frac{b}{r}\right]J_1(qb)(\Sigma\cdot\hat{\mathbf{q}})$$

and finally obtain

$$A^{v=3}_{110}(q)=\left[\sqrt{3\pi}i\left[\frac{K}{m}\right]\int_0^\infty b\,db\,J_1(qb)e^{-\chi_c(b)}\int_{-\infty}^{\infty}dz\left[\frac{b}{r}\right]G^v_{1ls}(r)\right](\Sigma\cdot\hat{\mathbf{q}})(\sigma\cdot\hat{\mathbf{K}})\ . \tag{20}$$

Note that this yields a contribution to the cross term amplitude A_{qK},[4] essential in generating spin observable differences.

This is as far as we can go without having to resort to numerical computations. All the remaining contributions to the amplitude can be handled in a completely analogous way. As we have mentioned before all results that we have established for the plane wave case hold, obviously, in this case as well. The presence of distorted waves however, makes the extraction of simple and transparent results no longer possible in this case. It is only through the detailed and careful comparison between theoretical predictions and experimental measurements that we can establish any kind of meaningful results. Unfortunately, a broad range of data, which include all possible spin observables and especially spin observable differences, have yet to emerge. Although polarization and analyzing power differences in the excitation of the 1^+ state in ^{12}C by protons have already been measured, these measurements were done at a beam energy of $T_{lab}=150$ MeV where the use of the impulse approximation is highly questionable.[12] The full set of spin observables has been measured, also for the 1^+ state in ^{12}C, at a higher beam energy of $T_{lab}=500$ MeV,[13] where the impulse approximation is expected to work. Unfortunately, only a few data points exist in this case. Nonetheless, we compare our theoretical predictions to these data in the absence of a more prolific source. We concentrate on the 1^+ (12.71 MeV $T=0$) state and present three different calculations, all of which assume a pure single particle transition from

TABLE I. Strengths and ranges used to fit the elastic spin observables.

		Strength (MeV)	C (fm)	Beta (fm)	W
Scalar	real	−390.06	2.074	0.817	−0.09
	imag	102.60	2.821	0.311	−0.09
Vector	real	282.25	2.019	0.845	−0.09
	imag	−102.56	2.735	0.352	−0.09

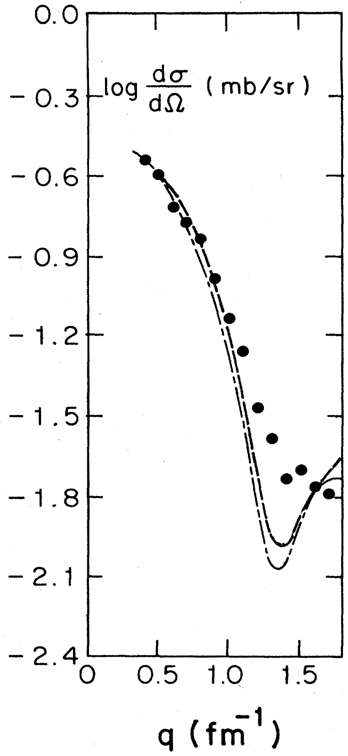

FIG. 1. Cross section for p-^{12}C at $T_{lab}=500$ MeV. Experimental cross section is an average of the 400 and 600 MeV data. Theoretical (RDWIA) calculations were done with a single geometry (dash) and with geometries chosen to fit elastic scattering (dash-dot).

a $p^{3/2}$ to a $p^{1/2}$ single particle state. We include for comparison our previous plane wave calculation.[4] We also include two different relativistic distorted wave impulse approximation calculations which only differ in the form of the distortion. One uses a single underlying nuclear density approach with a harmonic oscillator parametrization given by

$$\rho(r)=\left[1+\frac{4}{3}\left[\frac{r}{c}\right]^2\right]e^{-(r/c)^2}, \quad (21)$$

with $c=1.64$ fm and scalar and vector strengths determined from the impulse approximation[14]

$$S_0=(-303+i73)\text{ MeV},$$

$$V_0=(+191-i86)\text{ MeV},$$

while the other approach uses instead a three parameter Woods-Saxon form and scalar and vector strengths explicitly chosen to fit the elastic spin observable measurements.[15] (See Table I.) Finally, to determine the transition potential we use the relativistic transformation developed in Ref. 16, and bound state upper components given by nonrelativistic harmonic oscillator wave functions, with lower components determined from the upper components by using the Dirac equation, i.e.,

$$w_{Ejl'}(r)=\frac{1}{(\mathscr{E}+\mathscr{M})}\left[\frac{d}{dr}+\frac{(1+\kappa)}{r}\right]u_{Ejl}(r), \quad (22)$$

where

$$l=j\pm\tfrac{1}{2};\quad l'=j\mp\tfrac{1}{2};\quad \text{for } \kappa=\pm(j+\tfrac{1}{2})$$

and with bound state potential strengths $S_0=-420$ MeV; $V_0=328$ MeV, obtained from Ref. 17.

In calculating spin observables we adopt the standard definition given by

$$\left[\frac{d\sigma}{d\Omega}\right]D_{\alpha\beta}=\tfrac{1}{2}T_r(\sigma_\alpha A\sigma_\beta A^\dagger), \quad (23)$$

where $\alpha,\beta=(0,n,q,K)$; $\mathbf{n}=\mathbf{q}\times\mathbf{K}$, $\sigma_0\equiv 1$, and $D_{00}\equiv 1$, so the unpolarized cross section is

$$\left[\frac{d\sigma}{d\Omega}\right]=\tfrac{1}{2}T_r(AA^\dagger) \quad (24)$$

and the spin difference function[4,5]

$$\Delta_s\equiv(D_{qK}+D_{Kq})+i(D_{n0}-D_{0n})$$
$$=(Q-B)+i(P-A_y). \quad (25)$$

In Fig. 1 we show, in the absence of published 500 MeV cross section data, an experimental cross section given by the average of the 398 and 597 MeV data, which show very similar shapes.[18] We note that both RDWIA calcu-

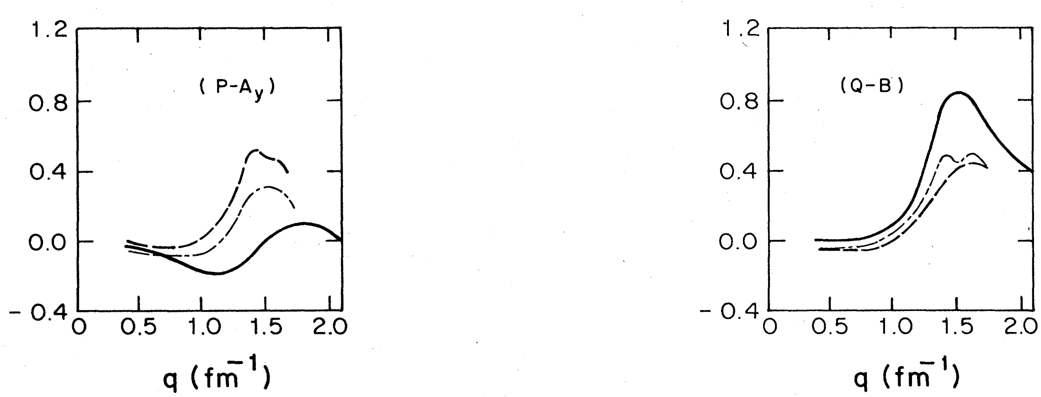

FIG. 2. Spin difference function for p-^{12}C at $T_{lab}=500$ MeV. Theoretical (RDWIA) calculations were done with a single geometry (dash) and with geometries chosen to fit elastic scattering (dash-dot). Also shown is the (RPWIA) (Ref. 4) result (solid).

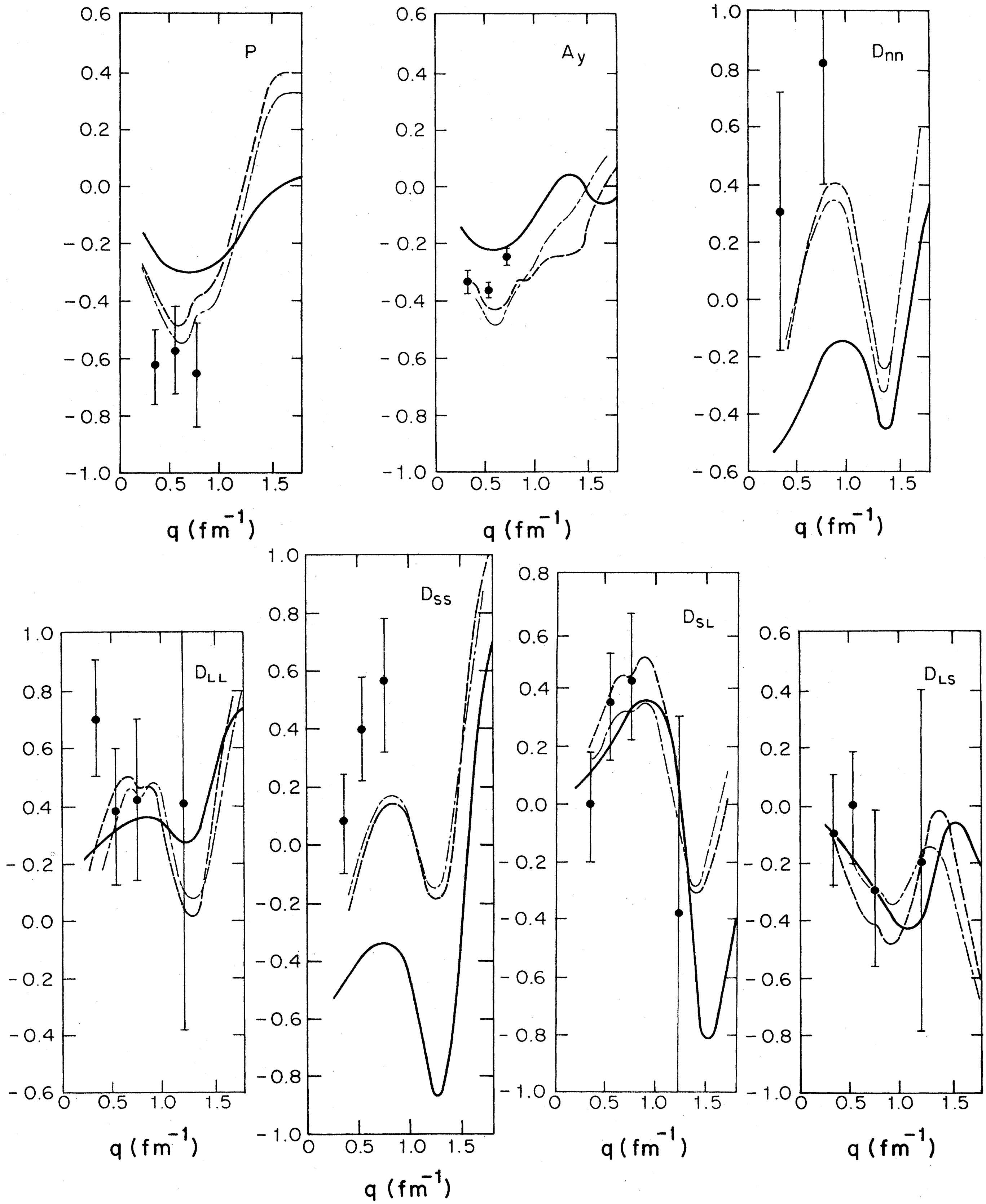

FIG. 3. Remaining spin observables for p-^{12}C at $T_{\rm lab}=500$ MeV. Theoretical (RDWIA) calculations were done with a single geometry (dash) and with geometries chosen to fit elastic scattering (dash-dot). Also shown is the (RPWIA) (Ref. 4) result (solid).

lations have very deep minima and both had to be renormalized to about 75 percent of their values in order to reproduce the small q behavior of the experimental cross section. Since we know that all remaining spin observables are divided by the cross section, we prevent this anomalous behavior from affecting the value of the spin observables, by multiplying the observables by the theoretical cross section and dividing by the experimental one, i.e., we effectively substitute the experimental one for our theoretical one.

In Figs. 2 and 3 we observe considerable structure in the spin observables. We also note that the addition of distortion and of strong potentials does not qualitatively change the spin observables but does lead to modest improvement with the data. Presumably, a calculation which includes configuration mixing will further enhance these results. Unfortunately, a relativistic shell model has yet to emerge. We note that although we predict $(P-A_y)$ and $(Q-B)$ to be sizable, especially for $q \sim 1.5$ fm^{-1}, the data are too sparse to check that prediction. We stress that in the Dirac treatment these spin observable differences arise in a very natural way without having to invoke exchange processes.

As far as we can tell the spin difference function[4,5] provides a clear signature of the importance of lower components, and therefore of the relativistic approach, and more experimental work should be motivated to test this prediction. More data, at energies where the impulse approximation should be valid, and for states with very simple configuration (e.g., stretched states on closed shell nuclei) are essential to confront all available theoretical models. Furthermore, because of the importance of distortion in detailed studies and because distortion is largely a geometric effect, these experimental studies should include elastic and inelastic cross sections along with the spin observable measurements.

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