Anomalous projectile fragments as nuclear molecules

K. Langanke and O. S. van Roosmalen*
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
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The explanation of anomalous mean free paths, observed in relativistic heavy-ion collisions in emulsions, by the excitation of molecular configurations in projectile fragments, is discussed. The γ-decay half-life of molecular states in ^{24}Mg and ^{32}S are calculated in a microscopic cluster model and compared with the inferred mean lifetimes of anomalous projectile fragments. While the half-life of the $^{16}O+^{16}O$ molecular states might be compatible with this explanation, an identification as anomalous projectile fragments can be ruled out for the $^{12}C+^{12}C$ molecular states. It is argued that because of nuclear structure effects, molecular configurations in neighboring nuclei have lifetimes similar to the $^{12}C+^{12}C$ molecular states.

I. INTRODUCTION

Secondary fragments of relativistic nuclear projectiles show an anomalous mean free path in emulsions which is significantly smaller than for the primary fragments.[1] The fraction of secondary fragments believed to be responsible for this still unexplained effect is sometimes termed "anomalons." Recently Bayman *et al.* made an attempt to explain anomalous projectile fragments (APF's) in terms of standard nuclear physics.[2] Specifically, they argued that the anomalous mean free paths for fragments with charges $Z \approx 10$—20 might be due to the formation of long-lived quasimolecular states which are excited during the collision process. This hypothesis can be tested by a comparison of the lifetime of such quasimolecular states with the mean lifetimes of unstable APF's ($\tau \gtrsim 5\times 10^{-11}$ s, Ref. 3). Since no experimental information about the lifetimes of molecular states in heavy-ion systems exists, such a comparison has to rely on theoretical modeling. In this paper, we test the hypothesis of Bayman *et al.* by estimating an upper limit for the lifetime of the lowest molecular states in ^{24}Mg and ^{32}S. These nuclei are chosen because the existence of the long-lived $^{12}C+^{12}C$ and $^{16}O+^{16}O$ molecular states or sometimes called shape isomeric states is predicted on convincing theoretical grounds[4] and the latter have been used as illustrative examples.[2] An estimate of the lifetimes of these states is also relevant as it may support their experimental detection.

Possible decay mechanisms of the $^{12}C+^{12}C$ and $^{16}O+^{16}O$ molecular states are electromagnetic transitions or light particle emission. Since the structure of states in the open light particle channels is very different from that of the molecular configurations, a particle decay of these states can be assumed suppressed. Hence, in the following we restrict ourselves only to electromagnetic decay and hence will derive an upper limit for the mean lives of the molecular states. Furthermore, since the molecular states, as well as possible final states of the decay, are expected to have total isospin quantum number $T=0$, an $E1$ transition will be forbidden and the decay has to be of the $E2$-type. The $E2$ transition rate between two $T=0$ states is given by

$$R=\frac{4\pi}{75\hbar}\left[\frac{E_\gamma}{\hbar c}\right]^5\frac{1}{(2l+1)}\,|\langle\Psi_f||\hat{Q}_2||\Psi_{\rm mol}\rangle|^2\,, \qquad (1)$$

where $\hat{Q}$ is the isoscalar quadrupole operator, $\Psi_{\rm mol}$ is the initial molecular state (with angular momentum l), and Ψ_f the final state. E_γ is the energy of the emitted photon. For a reasonably accurate evaluation of the matrix element in (1) it is essential to consider the microscopic character of the initial and final states, including appropriate antisymmetrization and angular momentum projection.

One of the main features of nuclear molecules is that they form rotational bands with an approximately constant moment of inertia. Hence, an upper limit for the lifetime of molecular states with angular momenta $l \geq 2$ (for systems with identical fragments) is given by the rate for intraband γ decay within this rotational band which can be estimated from (1) using the well-known Bohr-Mottelson formula[5]

$$B(E2,l\rightarrow l-2)=\frac{15}{32\pi}\frac{l(l-1)}{(4l^2-1)}e^2Q_0^2\,. \qquad (2)$$

Making reasonable assumptions for the rotational constant and the internal quadrupole moment of the rotational bands [$\Delta E/\Delta(l(l+1))\approx 120$ keV, $Q_0\approx 1.2$ b for the $^{12}C+^{12}C$ band (Ref. 6) and $\Delta E/\Delta(l(l+1))\approx 100$ keV, $Q_0\approx 2$ b for the $^{16}O+^{16}O$ band (Ref. 7)], one deduces upper limits for the lifetimes of the molecular states with $l \geq 4$ ($\tau \leq 3\ 10^{-13}$ s) which are not compatible with the identification of APF's as nuclear molecules. Even the lifetimes of the 2^+ members of the $^{12}C+^{12}C$ and $^{16}O+^{16}O$ bands ($\tau\approx 1.5\times 10^{-11}$ s and $\tau\approx 2.6\times 10^{-11}$ s, respectively) are smaller than the mean lifetimes of APF's.

We conclude from these considerations that only the band heads of the molecular bands with angular momenta $l=0$ in the present cases (and maybe the lowest excited members of the bands) can have a lifetime which allows their identification as APF's. Therefore, we assume that the initial state in (1) is a molecular state with angular momentum $l=0$. Guided by the energy dependence of

the transition strength R, the final state is chosen as the first excited 2^+ state in the compound nuclei ^{24}Mg and ^{32}S.

II. THE LIFETIME OF THE $^{16}O+^{16}O$ STATES

The state at $E^*=8.51$ MeV in ^{32}S has been suggested[8] as an experimental candidate for the band head of the $^{16}O+^{16}O$ molecular band in ^{32}S. The energies and spins of this suggested band[8] are well reproduced by the bound states of an $^{16}O+^{16}O$ potential[7] which is derived from the expectation value of a microscopic Hamiltonian in the basis of antisymmetrized $^{16}O+^{16}O$ cluster wave functions.[9] Hence, we are motivated to approximate our $^{16}O+^{16}O$ molecular state using the same ansatz:

$$\Psi^{I=0}_{\rm mol}=\sum_N \frac{1}{\sqrt{\mu_N}}\langle u_{Nl=0}\,|\,g_{l=0}\rangle \mathscr{A}(\Phi_O\Phi_O u_{Nl=0}) \quad (3)$$

where Φ_O is the intrinsic wave function of the ^{16}O fragment in its harmonic oscillator shell model ground state. The u_{Nl} are spherical harmonic oscillator wave functions with principal quantum number $N=(2n+l)$, and $\mathscr{A}$ is the antisymmetrizer. The overlap kernels

$$\mu_N=\langle \Phi_O\Phi_O u_{Nl=0}\,|\,\mathscr{A}\,|\,\Phi_O\Phi_O u_{Nl=0}\rangle \quad (4)$$

ensure that many-body states with different, orthogonal, relative motion wave functions are appropriately orthonormalized. The relative motion wave function $g_{l=0}$ is calculated as the lowest bound state of the microscopically derived $^{16}O+^{16}O$ potential $V(r)$ of Ref. 9 by solving the Schrödinger equation

$$\Lambda\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{d\mathbf{r}^2}+V(r)-E\right]g_{l=0}(\mathbf{r})=0\ . \quad (5)$$

The projector Λ ensures that the many-body state (3) has no components which violate the Pauli principle. For the $^{16}O+^{16}O$ system these Pauli forbidden states of relative motion are given by the u_{NL} with $N<24$.

Expanding $\Psi^{I=0}_{\rm mol}$ in terms of oscillator shell model eigenstates, one finds that the component with minimal number of oscillator quanta has $N_{\rm tot}=48$ (the total number of quanta in the internal wave function of both clusters and the relative motion). When we compare this with the number of quanta in the lowest shell model states of ^{32}S $[(s)^4(p)^{12}(sd)^{16}$, which has $N_{\rm tot}=44]$, we see that such shell model states in ^{32}S cannot be factored into product wave functions with both ^{16}O clusters in their shell model ground states. However, the $J=2^+$ state at $E=2.23$ MeV in ^{32}S which is the most likely final state of the $E2$ decay of the $J=0^+$ molecular $^{16}O+^{16}O$ state has a strong overlap with the $(s)^4(p)^{12}(sd)^{16}$ harmonic oscillator shell model configuration.[10] To include this shell model configuration in the molecular model space to describe the final state of the $E2$ decay of the molecular states in ^{32}S we make use of the suggestion that the lowest ^{32}S shell model states might be factorized into a product in which one of the ^{16}O fragments is excited by a 4p-4h configuration:[11]

$$\Psi^{l=2}_f=\sum_N \frac{1}{\sqrt{\tilde{\mu}_{Nl=2}}}\langle u_{Nl=2}\,|\,h_{l=2}\rangle \mathscr{A}(\Phi_O\Phi^{\text{4p-4h}}_{O,I=0}u_{Nl=2})\ . \quad (6)$$

For simplicity we assume identical oscillator parameters for the fragments in (3) and (6) ($b=1.58$ fm, Ref. 9). The overlap kernels $\tilde{\mu}_{Nl}$ which, in contrast to the ones given in (4), are now dependent on both quantum numbers N and l are given by

$$\tilde{\mu}_{Nl}=\langle \Phi_O\Phi^{\text{4p-4h}}_{O,I=0}u_{Nl}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{\text{4p-4h}}_{O,I=0}u_{Nl}\rangle\ . \quad (7)$$

Following Ref. 12, we identify the 4p-4h configuration with the first excited 0^+ state in ^{16}O at $E^*=6.06$ MeV, which can be well approximated by an $\alpha+^{12}$C cluster wave function

$$\Phi^{\text{4p-4h}}_{O,I=0}=\mathscr{A}(\Phi_\alpha\Phi^{I=0}_C u_{N=8,l=0})\ , \quad (8)$$

where Φ_α and $\Phi^{I=0}_C$ denote the α particle and the ^{12}C nucleus ($spin\ I=0$) in the harmonic oscillator wave functions with highest spatial symmetry. The relative wave function $h_{l=2}$ in (6) can now be calculated from an $^{16}O+^{16}O$ (4p-4h) potential by solving the Schrödinger equation

$$\tilde{\Lambda}\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{d\mathbf{r}^2}+V(r)-E'\right]h_{l=2}(\mathbf{r})=0\ , \quad (9)$$

where $E'=E-6.06$ MeV is the relative energy in the $^{16}O+^{16}O^*$ channel. Note that the Pauli projector $\tilde{\Lambda}$ is different from the projector Λ of (5). This reflects the difference in internal structure of the many-body wave functions (3) and (6). $\tilde{\Lambda}$ projects off the u_{Nl} with $N<16$ which violate the Pauli principle in (6). The potential $V(r)$ is assumed to be given by the Coulomb potential of a homogeneously charged sphere (radius 3.8 fm) and a nuclear potential $V_n(r)$ of Gaussian form

$$V_n(r)=V_0\exp\left[\frac{-r^2}{a^2}\right]\ . \quad (10)$$

The two parameters of the nuclear potential are determined from the conditions that $V(r)$ has a bound state with (within experimental uncertainty) the same energy and quadrupole moment as the first excited 2^+ state in ^{32}S. This requirement is fulfilled by a Gaussian potential with the parameters $V_0=-934.8$ MeV and $a=0.4$ fm which has a bound state at $E'=-20.4$ MeV with a quadrupole moment of $Q=-9.0\ e\,\text{fm}^2$. These numbers have to be compared with the experimental values $E'=-20.37$ MeV (Ref. 13) and $Q=-9\pm4\ e\,fm^2$ (Ref. 14). The aforementioned quadrupole moment has been calculated from the many-body matrix element

$$Q=\left[\frac{16\pi}{5}\right]^{1/2}\langle\Psi_f\,|\,\hat{Q}_{20}\,|\,\Psi_f\rangle$$
$$=\left[\frac{16\pi}{5}\right]^{1/2}\sum_{N,N'}(\tilde{\mu}_{Nl}\tilde{\mu}_{N'l})^{-1/2}\langle u_{N'l=2}\,|\,h_{l=2}\rangle\langle h_{l=2}\,|\,u_{Nl=2}\rangle\langle\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{Nl=2}\,|\,\mathscr{A}\hat{Q}_{20}\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{N'l=2}\rangle\ , \tag{11}$$

where $\hat{Q}_{20}$ is the $m=0$ component of the isoscalar part of the quadrupole operator. The many-body matrix elements in (11) can be evaluated along the lines of Ref. 15. Inserting the unit operator in the model space spanned by the $^{16}O+^{16}O$ cluster wave functions between the antisymmetrizers and the quadrupole operator and using the fact that $\mathscr{A}$ commutes with $\hat{Q}_{20}$ and conserves both oscillator energy and angular momentum we obtain

$$Q=\left[\frac{16\pi}{5}\right]^{1/2}\sum_{N,N'}(\tilde{\mu}_{Nl}\tilde{\mu}_{N'l})^{-1/2}\langle u_{N'l=2}\,|\,h_{l=2}\rangle\langle h_{l=2}\,|\,u_{Nl=2}\rangle$$
$$\times(\tilde{\mu}_{Ml}\langle u_{Nl=2}\,|\,\hat{Q}^{\,rel}_{20}\,|\,u_{N'l=2}\rangle+\langle\Phi^{2p\text{-}2h}_{O,I=2}\,|\,\hat{Q}^{\,int}_{20}\,|\,\Phi^{4p\text{-}4h}_{O,I=0}\rangle\langle\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{Nl=2}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{2p\text{-}2h}_{O,I=2}u_{N'l=2}\rangle$$
$$+\langle\Phi^{4p\text{-}4h}_{O,I=0}\,|\,\hat{Q}^{\,int}_{20}\,|\,\Phi^{2p\text{-}2h}_{O,I=2}\rangle\langle\Phi_O\Phi^{2p\text{-}2h}_{O,I=2}u_{Nl=2}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{N'l=2}\rangle$$
$$+\langle\Phi^{4p\text{-}4h}_{O,I=0}\,|\,\hat{Q}^{\,int}_{20}\,|\,\Phi^{4p\text{-}4h}_{O,I=2}\rangle\langle\Phi_O\Phi^{4p\text{-}4h}_{O,I=2}u_{Nl=2}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{N'l=2}\rangle)\tag{12}$$

with $M=\min(N,N')$. $\hat{Q}^{\,rel}_{20}$ and $\hat{Q}^{\,int}_{20}$ are those parts of the many particle operator $\hat{Q}_{20}$ which act only on the relative coordinate and the internal coordinates of the second fragment, respectively. If we make the approximation

$$\Phi^{2p\text{-}2h}_{O,I=2}=\mathscr{A}(\Phi_\alpha\Phi^{I=0}_C u_{N=6,l=2}) \tag{13}$$

we can calculate the contributions of $\hat{Q}^{\,int}_{20}$ as outlined in Ref. 15. Evaluating of the overlap kernels is discussed in the Appendix.

Having defined the microscopic description of the final and initial states for the $E2$ decay of the $^{16}O+^{16}O$ molecular states, the many-body matrix element of Eq. (1) can be put in a form similar to Eq. (11). We find

$$\langle\Psi^{I=0}_{mol}\,|\,Q_2\,|\,\Psi^{I'=2}_f\rangle=\sum_{N,N'}(\mu_N\tilde{\mu}_{N'l'})^{-1/2}\langle u_{N'l'=2}\,|\,h_{l'=2}\rangle\langle g_{l=0}\,|\,u_{Nl=0}\rangle\langle\Phi_O\Phi_O u_{Nl=0}\,|\,\mathscr{A}Q_2\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{N'l'=2}\rangle$$
$$=\sum_{N,N'<N-4}(\mu_N\tilde{\mu}_{N'l'})^{-1/2}\langle g_{l=0}\,|\,u_{Nl=0}\rangle\langle u_{N'l'=2}\,|\,h_{l'=2}\rangle$$
$$\times\Big[\langle\Phi_O\,|\,\hat{Q}^{\,int}_2\,|\,\Phi^{2p\text{-}2h}_{O,I=2}\rangle\langle\Phi_O\Phi^{2p\text{-}2h}_{O,I=2}u_{Nl=0}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{N'l'=2}\rangle\delta_{N,N'+2}$$
$$+\sum_M\langle u_{Nl=0}\,|\,\hat{Q}^{\,rel}_2\,|\,u_{Ml=2}\rangle\langle\Phi_O\Phi_O u_{Ml=2}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{N'l'=2}\rangle\delta_{M,N'+4}\Big]$$
$$+\sum_{N,N'\geq N-4}(\mu_N\tilde{\mu}_{N'l'})^{-1/2}\langle g_{l=0}\,|\,u_{Nl=0}\rangle\langle\mu_{N'l'=2}\,|\,h_{l'=2}\rangle$$
$$\times\Big[\langle\Phi^{4p\text{-}4h}_{O,I=2}\,|\,\hat{Q}^{\,int}_2\,|\,\Phi^{4p\text{-}4h}_{O,I=0}\rangle\langle\Phi_O\Phi_O u_{Nl=0}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=2}u_{N'l'=2}\rangle\delta_{N,N'+4}$$
$$+\langle\Phi^{2p\text{-}2h}_{O,I=2}\,|\,\hat{Q}^{\,int}_2\,|\,\Phi^{4p\text{-}4h}_{O,I=0}\rangle\langle\Phi_O\Phi_O u_{Nl=0}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{2p\text{-}2h}_{O,I=2}u_{N'l'=2}\rangle\delta_{N,N'+2}$$
$$+\sum_M\langle u_{Ml=0}\,|\,\hat{Q}^{\,rel}_2\,|\,u_{N'l'=2}\rangle\langle\Phi_O\Phi_O u_{Nl=0}\,|\,\mathscr{A}\,|\,\Phi_O\Phi^{4p\text{-}4h}_{O,I=0}u_{Ml=0}\rangle\delta_{N,M+4}\Big]\ .\tag{14}$$

The internal wave function $\Phi^{4p\text{-}4h}_{O,I=2}$ is set to

$$\Phi^{4p\text{-}4h}_{O,I=2}=\mathscr{A}(\Phi_\alpha\Phi^{I=0}_C u_{N=8l=2})\ . \tag{15}$$

Then the matrix elements of the internal part of $\hat{Q}_2$ termed $\hat{Q}^{\,int}_2$, can be taken from Ref. 15. The required overlap kernels are given in the Appendix. Since the final and initial states are both bound states, the sum in (14) is restricted to a few values of N and N' and can be easily

evaluated numerically.

We find a mean life of $\tau = R^{-1} \approx 2 \times 10^{-11}$ s for the molecular $^{16}O + ^{16}O$ state (3) to make a transition into the final ^{32}S state (6). This is slightly smaller than the mean lifetime of the APF's. To test how sensitive our result is to the choice of the final state we have varied the parameters of the Gaussian potential (10) to generate other 32-particle states which agree with the experimental binding energy and quadrupole moment of the first 2^+ state in ^{32}S. We find lifetimes of $\tau \approx 1.5$–3×10^{-11} s for the $E2$ decay of the molecular $^{16}O + ^{16}O$ state into ^{32}S states whose quadrupole moments varied between -8 and -13 $e\,\mathrm{fm}^2$ and hence were compatible with the experimental value $Q = -9 \pm 4$ $e\,\mathrm{fm}^2$. The energy of all investigated states agreed with the experimental value to better than 20 keV.

Since the two quantities (energy difference and quadrupole moment), which are expected to influence the $E2$ transition strength R most are either in agreement with experiment or, if experimentally unknown, of a reasonable magnitude backed by theoretical models, one can have some confidence at least in the order of magnitude of the calculated lifetime of the molecular $^{16}O + ^{16}O$ state under γ decay. Considering the energy difference between these states ($E_\gamma = 6.2$ MeV) the lifetime of the molecular configuration is extremely long and reflects a very small $E2$ coupling between the final and initial states. This small $B(E2)$ matrix element can be understood from the different internal structure of the final and initial states; the molecular state cannot decay into the lowest harmonic oscillator shell model component of the final state via $E2$ transition.

Although the calculated lifetime of the ($l = 0$) molecular $^{16}O + ^{16}O$ state due to γ decay is smaller than the mean lifetime of the APF's by approximately a factor of 2, this deviation is not large enough to rule out the interpretation of APF's as molecular $^{16}O + ^{16}O$ states. It should, however, be emphasized that the lifetime of the $^{16}O + ^{16}O$ molecular states might be smaller due to particle decay into the α-^{28}Si channel[8] which is open at the suggested energy of the molecular states.

III. THE LIFETIME OF THE $^{12}C + ^{12}C$ MOLECULAR STATES

To estimate the lifetime of the $^{12}C + ^{12}C$ molecular states in ^{24}Mg we describe the initial states as an antisymmetrized product similar to (3)

$$\Psi_{\mathrm{mol}}^{I=0} = \sum_N \langle u_{Nl} \mid g_{l=0} \rangle \frac{1}{\sqrt{\mu_{Nl}}} \mathscr{A} (\Phi_C^{I=0} \Phi_C^{I=0} u_{Nl=0}) \,, \tag{16}$$

where the wave function $\Phi_C^{I=0}$ which describes the internal degrees of freedom of the ^{12}C nucleus with spin $I = 0$ is the shell model ground state with highest spatial symmetry [(0,4) configuration in SU(3) classification]. The oscillator width of the fragments is set to $b = 1.7$ fm. The normalization kernels are defined as

$$\mu_{Nl} = \langle \Phi_C^{I=0} \Phi_C^{I=0} u_{Nl} \mid \mathscr{A} \mid \Phi_C^{I=0} \Phi_C^{I=0} u_{Nl} \rangle \tag{17}$$

and can be found in Ref. 6. A Gaussian nuclear $^{12}C + ^{12}C$ potential that yields a reasonable description of the molecular resonances in this system was found in Ref. 6 ($V_0 = -190.0$ MeV and $a = 3.3$ fm). The wave function $g_{l=0}$ in (16) is the first excited 0^+ bound state in this potential (binding energy $E = -5.5$ MeV) and is calculated by solving the Schrödinger-type equation of motion (5) with a Pauli operator Λ which now projects off the spherical harmonic oscillator wave functions $u_{Nl=0}$ (width $\beta = b/\sqrt{\mu}$) with $N < 12$, the Pauli forbidden ones for the $^{12}C + ^{12}C$ system.

In contrast to the case for ^{32}S, the lowest ^{24}Mg shell model configuration with $N_{\mathrm{tot}} = 28$ quanta $[(s)^4(p)^{12}(sd)^8]$ is contained in the space of cluster wave functions with both of the ^{12}C nuclei in their shell model ground state. Hence we can make the following approximation for the final ^{24}Mg state:

$$\Psi_f^{l=2} = \sum_N \frac{1}{\sqrt{\mu_{Nl}}} \langle u_{Nl} \mid h_{l=2} \rangle \mathscr{A} (\Phi_C^{I=0} \Phi_C^{I=0} u_{Nl}) \,. \tag{18}$$

Unfortunately the final ^{24}Mg state cannot be chosen as the lowest 2^+ state of the potential of Ref. 6 as it overbinds this state by a few MeV. Therefore we determine the final state as the lowest bound state of a Gaussian potential whose parameters have been fixed from the requirement that the energy and quadrupole moment of this state agree with the experimental values for the first excited 2^+ state in ^{24}Mg. These conditions are fulfilled when we use the parameters $V_0 = -196.0$ MeV and $a = 3.0$ fm in which case Ψ_f has a binding energy of $E = -12.56$ MeV and a quadrupole moment of $Q = -20.5$ $e\,\mathrm{fm}^2$. This is to be compared to the experimental values $E = -12.565$ MeV (Ref. 13) and $Q = -18 \pm 2$ $e\,\mathrm{fm}^2$ (Ref. 14).

The quadrupole moment of the many-body state (17) can be evaluated using

$$\begin{aligned}
\langle \Psi_f^{l=2} \mid \hat{Q}_{20} \mid \Psi_f^{l=2} \rangle = & \sum_{N,N' \le N} (\mu_{Nl} \mu_{N'l})^{-1/2} \langle h_{l=2} \mid u_{Nl} \rangle \langle \mu_{N'l} \mid h_{l=2} \rangle \\
& \times (2 \langle \Phi_C^{I=0} \mid \hat{Q}_{20}^{\mathrm{int}} \mid \Phi_C^{I=2} \rangle \langle \Phi_C^{I=2} \Phi_C^{I=0} u_{Nl=2} \mid \mathscr{A} \mid \Phi_C^{I=0} \Phi_C^{I=0} u_{N'l=2} \rangle \delta_{N,N'} \\
& + \langle u_{Nl=2} \mid \hat{Q}_{20}^{\mathrm{rel}} \mid u_{N'l=2} \rangle \mu_{N'l}) \\
& + \sum_{N,N'>N} (\mu_{Nl} \mu_{N'l})^{-1/2} \langle h_{l=2} \mid u_{Nl} \rangle \langle u_{N'l} \mid h_{l=2} \rangle \langle u_{Nl=2} \mid \hat{Q}_{20}^{\mathrm{rel}} \mid u_{N'l=2} \rangle \mu_{Nl}
\end{aligned} \tag{19}$$

and the many-body matrix element occurring in the $E2$ transition of the $^{12}C+^{12}C$ molecular state into the ^{24}Mg final state can be obtained from[6]

$$\langle \Psi^{I=0}_{\text{mol}} | \hat{Q}_2 | \Psi^{I=2}_f \rangle = \sum_{N,N'} (\mu_{Nl=2}\mu_{N'l'=0})^{-1/2} \langle g_{l=0} | u_{N'l'} \rangle \langle u_{Nl} | h_{l=2} \rangle \times (2\langle \Phi^{I=0}_C | \hat{Q}^{\text{int}}_2 | \Phi^{I=2}_C \rangle \langle \Phi^{I=2}_C \Phi^{I=0}_C u_{N'l'=0} | \mathscr{A} | \Phi^{I=0}_C \Phi^{I=0}_C u_{Nl=2} \rangle \delta_{NN'} + \langle u_{N'l'=0} | \hat{Q}^{\text{rel}}_2 | u_{Nl=2} \rangle \mu_{ML}) \tag{20}$$

with

$$\mu_{ML} = \begin{cases} \mu_{N'l'} \text{ if } N' \leq N \\ \mu_{Nl} \text{ if } N' > N \end{cases} .$$

In Eqs. (19) and (20) $\Phi^{I=2}_C$ denotes the SU(3) description of the lowest shell model ^{12}C state with $I=2$ which we identify with the 2^+ state in ^{12}C at $E^*=4.43$ MeV. The matrix elements of $\hat{Q}^{\text{int}}_2$ are derived from the experimental lifetime of the 2^+ state. Their sign is adopted in accordance with the discussion in Ref. 16.

From Eqs. (1) and (20) we calculate the γ-decay mean life of the $^{12}C+^{12}C$ molecular states to be $\tau \approx 0.8$ fs, which is smaller than the mean lifetime of APF's by more than four orders of magnitude. Note that a generator coordinate method (GCM) study of the $^{12}C+^{12}C$ system[17] yields a result for the same $B(E2)$ matrix element as we have considered here, $B(E2)=50\ e^2\ \text{fm}^4$ which agrees very nicely with the present result, $B(E2)=55\ e^2\ \text{fm}^4$. Reference 17 predicts an even smaller mean life, since the binding energy of the molecular states is smaller than in our study.

We do not expect that a more elaborate description of the initial and final states in (1) will change the $E2$ transition strength between these states by four orders of magnitude or more. Therefore, we consider it highly unlikely that $^{12}C+^{12}C$ molecular states can be identified with APF's of charge $Z=12$.

IV. CONCLUSION

We have tested the hypothesis, proposed in Ref. 2, that the anomalously small mean free paths observed in relativistic heavy-ion collisions in emulsions are due to the excitation of quasimolecular states in the projectile fragments. This was done by calculating the mean lifetimes of molecular states in ^{24}Mg and ^{32}S under $E2$ decay and comparing them to the mean lifetime of the APF's. To obtain a reasonable estimate for the $E2$ transition rates, both the initial molecular states and the final states were described in a microscopic cluster model which accounts for antisymmetrization and angular momentum projection exactly. We find that the mean lives of the $^{12}C+^{12}C$ molecular states are much too small ($\tau \approx 0.8$ fs) for them to be possible APF candidates with $Z=12$. On the other hand, the lifetimes of the $^{16}O+^{16}O$ molecular states (with $l=0$ and $l=2$) are approximately equal to the lifetimes of APF's.

The different behaviors of $^{12}C+^{12}C$ and $^{16}O+^{16}O$ molecular states under γ decay are a consequence of the Pauli principle. If the fragment nuclei are described by their shell model ground states, the Pauli principle requires that the $^{16}O+^{16}O$ molecular states are at least a 4p-4h excited configuration compared to the lowest ^{32}S shell model states. Consequently an $E2$ transition from an $^{16}O+^{16}O$ molecular state into the lowest ^{32}S shell model configuration is not possible. This reduces the $B(E2)$ matrix element between the molecular initial and the final state in ^{32}S very strongly since the lowest shell model component is the strongest contribution to the final state. The situation for the $^{12}C+^{12}C$ system is totally different. Here the lowest ^{24}Mg shell model wave function is included in the model space for the antisymmetrized molecular wave functions and, hence, an $E2$ coupling between the molecular state and the shell model component is allowed.

The influence of the Pauli principle on the lifetime of molecular states in neighboring systems with $Z \approx 10$–20 (like $^{13}C+^{13}C$ and $^{12}C+^{14}C$ in ^{26}Mg, $^{12}C+^{16}O$ in ^{28}Si) is similar to that of the $^{12}C+^{12}C$ system. We therefore expect photon decay lifetimes of the molecular states in these nuclei which are compatible to that of the $^{12}C+^{12}C$ molecular states in ^{24}Mg. These lifetimes would also not be in agreement with the explanation of anomalous mean free paths in relativistic heavy ion collision with the excitation of molecular states in the projectile fragments.

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APPENDIX: NORMALIZATION KERNELS

In this appendix we want to derive the normalization kernels needed to evaluate the electromagnetic transition matrix elements. Since the kernels for the $^{12}C+^{12}C$ system, as defined in Eq. (17), have already been given in Ref. 6, we will restrict our discussion to the $^{16}O+^{16}O$ kernels needed to calculate the matrix elements (12) and (14).

To be able to compute normalization kernels we have assumed in the previous sections that both the $^{16}O+^{16}O$ and the $^{16}O^*+^{16}O$ configurations can be well approximated by $\alpha+^{12}C+^{16}O$ cluster wave functions with each of the clusters in its shell-model ground state (a universal oscillator parameter is assumed). The low lying 4p-4h O^+ state in ^{16}O is thought of as an $\alpha+^{12}C$ cluster configuration with eight-oscillator quanta in the relative motion

and with $l=0$ while the ^{12}C system is in its shell model configuration labeled with SU(3) quantum numbers (04) and intrinsic spin $I=0$ ["weak coupling" of relative motion and intrinsic SU(3)$\supset$O(3) classified states]. The ^{16}O shell-model ground state with SU(3) quantum numbers (00) and angular momentum $J=0^+$ is obtained from an $\alpha+^{12}$C cluster configuration with four quanta in the relative motion and zero angular momentum in both the relative motion and the ^{12}C cluster. After antisymmetrization only the (00) component of the product representation (40)$\otimes$(04) survives, and the shell model ground state of ^{16}O is consistently obtained.

In Fig. 1 the $\alpha+^{12}$C$+^{16}$O system is pictorially represented with the relative coordinates ξ_1 and ξ_2 we will use in the following discussion. We can write these coordinates in terms of the c.m. coordinates X_i of the clusters and their mass number A_i as

$$\xi_1=X_3-X_1\ ,$$
$$\xi_2=X_2-\frac{A_1X_1+A_3X_3}{A_1+A_3}\ . \tag{A1}$$

Harmonic oscillator (HO) relative wave functions are denoted by $V_{N_il_i}(\xi_i,\gamma_i)$, where N_i is the total number of oscillator quanta and l_i the angular momentum. Upon making a transformation from the single particle coordinates to the cluster relative c.m. coordinates one has to replace in the HO wave functions the oscillator parameter $\nu=m\omega/2\hbar$ by

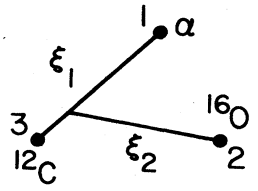

FIG. 1. Definition of the coordinates used to evaluate the ^{16}O$+^{16}$O normalization kernels.

$$\gamma_1=\frac{A_1A_3\nu}{A_1+A_3}\ ,$$
$$\gamma_2=\frac{(A_1+A_3)A_2\nu}{A_1+A_2+A_3}\ . \tag{A2}$$

In the following we denote the intrinsic shell-model ground state of the cluster i by $\varphi_{(0\Lambda_i)I_i}(i)$ (and we omit I_i if it is uniquely given by $I_i=0$). The kernels we need to calculate and their relation to kernels in an SU(3) coupled wave function representation discussed in Ref. 18 are given by (we closely follow the notation of Ref. 18)

$$\begin{aligned}\langle\varphi_{(00)}(1)\varphi_{(00)}(2)[V_{N_2l_2}(\xi_2,\gamma_2)V_{N_1l_1}(\xi_1,\gamma_1)]_J\varphi_{(04)I=0}(3)\,|\,\mathscr{A}\,|\,\varphi_{(00)}(1)\varphi_{(00)}(2)[V_{N_2'l_2'}(\xi_2,\gamma_2)V_{N_1'l_1'}(\xi_1,\gamma_1)]_J\varphi_{(04)I=0}(3)\rangle\\
=\sum_{\substack{(\sigma\tau)\kappa\\(\sigma'\tau')\kappa'\\(\lambda\mu)\epsilon}}\langle(N_2,0)l_2;(N_1,0)l_1||(\sigma\tau)\kappa J\rangle\langle(N_2',0)l_2';(N_1',0)l_1'||(\sigma'\tau')\kappa' J\rangle\\
\times\langle(\sigma\tau)\kappa J;(04)I=0||(\lambda\mu)\epsilon J\rangle\langle(\sigma'\tau')\kappa' J;(04)I=0||(\lambda\mu)\epsilon J\rangle\\
\times\langle\varphi_{(00)}\varphi_{(00)}[(V_{(N_2,0)}V_{(N_1,0)})_{(\sigma\tau)}\varphi_{(04)}]_{(\lambda\mu)\alpha}\,|\,\mathscr{A}\,|\,\varphi_{(00)}\varphi_{(00)}[(V_{(N_2',0)}V_{(N_1',0)})_{(\sigma'\tau')}\varphi_{(04)}]_{(\lambda'\mu')\alpha}\rangle\end{aligned} \tag{A3}$$

with

$$\begin{aligned}\langle\varphi_{(00)}\varphi_{(00)}[(V_{(N_2,0)}V_{(N_1,0)_{(\sigma\tau)}}\varphi_{(04)}]_{(\lambda\mu)\alpha}\,|\,\mathscr{A}\,|\,\varphi_{(00)}\varphi_{(00)}[(V_{(N_2',0)}V_{(N_1',0)})_{(\sigma'\tau')}\varphi_{(04)}]_{(\lambda'\mu')\alpha'}\\
=A^{N(\lambda\mu)}_{\sigma\tau,\sigma'\tau'}\delta_{NN'}\delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{\alpha\alpha'}=\sum_i X^{N(\lambda\mu)}_{(\sigma\tau)t}(i)X^{N(\lambda\mu)}_{(\sigma'\tau')t'}(i)\epsilon_i^{(\lambda\mu)}\end{aligned} \tag{A4}$$

and $N_1=\sigma+\tau-t$, $N_2=\tau+t$, and $N=N_1+N_2$. The entities $X^{N(\lambda\mu)}_{(\sigma\tau)t}(i)$ and $\epsilon_i^{(\lambda\mu)}$ are the ith eigenvector and ith eigenvalue of the matrix A, respectively. A method to calculate this matrix is extensively discussed in Ref. 18. We only state the result here.

We first determine a matrix B given by

$$\begin{aligned}B^{N(\lambda\mu)}_{\sigma t,\sigma' t'}=\sum_{m_1\sim m_9}I(m_1\sim m_9)(5-m_9)(6-m_9)\tfrac{1}{2}[(N-\tau-t)!(\tau+t)!(N-\tau'-t')!(\tau'+t')!]^{1/2}\\
\times\sum_{l,m}Y(m_1\sim m_9;l,m,\sigma,\tau,\sigma',\tau',\lambda,\mu)\ .\end{aligned} \tag{A5}$$

The restrictions on the summation as well as the function Y are explicitly given in Ref. 18. Y is essentially a sum over products of nine SU(3) symbols. The coefficients $I(m_1\sim m_9)$ are the crucial quantities in Eq. (A5). They can be obtained by expanding the generator coordinate kernel in powers of the generator coordinates

$$\hat{N}(\mathbf{R}_1,\mathbf{R}_2,\mathbf{R}_3^*,\mathbf{R}_1',\mathbf{R}_2',\mathbf{R}_3'^*)\equiv\exp\left[\frac{\nu}{2}A_1(\mathbf{T}_1^{*2}+\mathbf{T}_1'^2)+\frac{\nu}{2}A_2(\mathbf{T}_2^{*2}+\mathbf{T}_2'^2)\right]$$

$$\times\exp[-(A_1\mathbf{T}_1^*+A_2\mathbf{T}_2^*)(A_1\mathbf{T}_1'+A_2\mathbf{T}_2')/(A_1+A_2+A_3)]$$

$$\times\langle\psi_{(00)}(C_1;\mathbf{T}_1)\psi_{(00)}(C_2;\mathbf{T}_2)\psi_{(04)}^{\mathbf{R}_3^*}(C_3)\,|\,\mathscr{A}\,|\,\psi_{(00)}(C_1;\mathbf{T}_1')\psi_{(00)}(C_2;\mathbf{T}_2')\psi_{(04)}^{\mathbf{R}_3'^*}(C_3)\rangle$$

$$=\sum_{\substack{m_1\sim m_9=0\\ m_3+m_6+m_9=4\\ m_7+m_8+m_9=4}} I(m_1\sim m_9)(\nu\mathbf{T}_1^*\cdot\mathbf{T}_1')^{m_1}(\nu\mathbf{T}_1^*\cdot\mathbf{T}_2')^{m_2}(\sqrt{\nu}\mathbf{T}_1^*\cdot\mathbf{R}_3)^{m_3}$$

$$\times(\nu\mathbf{T}_2^*\cdot\mathbf{T}_1')^{m_4}(\nu\mathbf{T}_2^*\cdot\mathbf{T}_2')^{m_5}(\sqrt{\nu}\mathbf{T}_2^*\cdot\mathbf{R}_3)^{m_6}$$

$$\times(\sqrt{\nu}\mathbf{R}_3'^*\cdot\mathbf{T}_1')^{m_7}(\sqrt{\nu}\mathbf{R}_3'^*\cdot\mathbf{T}_2')^{m_8}(\mathbf{R}_3^*\cdot\mathbf{R}_3)^{m_9}\,. \tag{A6}$$

Here $\psi_{(0\Lambda)}(C_i,\mathbf{T}_i)$ are Brink-type cluster functions with the particles of cluster C_1 put in the lowest eigenstates of the harmonic oscillator (parameter ν) located at $\mathbf{S}_i$ and

$$\mathbf{T}_1=\mathbf{S}_1-\mathbf{S}_3\,,\quad \mathbf{T}_2=\mathbf{S}_2-\mathbf{S}_3\,,\quad \mathbf{T}_3=0\,. \tag{A7}$$

The $\mathbf{T}_i$ are the generator coordinates in the ^{12}C centered frame and are most convenient to work with. They are related to the relative motion generator coordinates $\mathbf{R}_i$ defined by

$$\frac{\mathbf{R}_1}{\sqrt{\gamma_1}}=\mathbf{S}_3-\mathbf{S}_1\,,\quad \frac{\mathbf{R}_2}{\sqrt{\gamma_2}}=\mathbf{S}_2-\frac{A_1\mathbf{S}_1+A_3\mathbf{S}_3}{A_1+A_3} \tag{A8}$$

in the following way:

$$\sqrt{\nu}\mathbf{T}=\sum_{j=1}^{2}\Omega_{ij}\mathbf{R}_j \tag{A9}$$

with

$$\Omega=\begin{bmatrix} -\left[\dfrac{A_1+A_3}{A_1A_3}\right]^{1/2} & 0 \\ -\left[\dfrac{A_1}{A_3(A_1+A_3)}\right]^{1/2} & \left[\dfrac{A_1+A_2+A_3}{(A_1+A_3)A_2}\right]^{1/2}\end{bmatrix}. \tag{A10}$$

The matrix element (A6) of the antisymmetrizer between the product of Brink-type cluster functions can be found by evaluating a simple Slater determinant;

$$\exp\left\{\frac{\nu}{2}[A_1(\mathbf{T}_1^{*2}+\mathbf{T}_1'^2)+A_2(\mathbf{T}_2^{*2}+\mathbf{T}_2'^2)]\right\}\langle\psi_{(00)}(C_1;\mathbf{T}_1)\psi_{(00)}(C_2;\mathbf{T}_2)\psi_{(04)}^{\mathbf{R}_3^*}(C_3)\,|\,\mathscr{A}\,|\,\psi_{(00)}(C_1;\mathbf{T}_1')\psi_{(00)}(C_2;\mathbf{T}_2')\psi_{(04)}^{\mathbf{R}_3^*}(C_3)\rangle$$

$$=\det{}^4\begin{bmatrix} G_{11} & G_{12} & t_{12\mu}G_{12} & 1 & T_{1\nu}^* \\ G_{21} & G_{22} & t_{22\mu}G_{22} & 1 & T_{2\nu}^* \\ -t_{21\rho}G_{21} & -t_{22\rho}G_{22} & G_{22}(\delta_{\rho\mu}-t_{22\mu}t_{22\rho}) & -T_{2\rho}^* & (\delta_{\nu\rho}-T_{2\nu}^*T_{2\rho}^*) \\ 1 & 1 & -T_{2\mu}' & 1 & 0 \\ T_{1\sigma}' & T_{2\sigma}' & (\delta_{\mu\sigma}-T_{2\mu}'T_{2\sigma}') & 0 & \delta_{\nu\sigma}\end{bmatrix}$$

$$=[(\mathbf{R}_3'^*\cdot\mathbf{R}_3)k_0+(\mathbf{T}_1^*\cdot\mathbf{R}_3)(\mathbf{R}_3'^*\cdot\mathbf{T}_1')\mathrm{k}_{11}+(\mathbf{T}_2^*\cdot\mathbf{R}_3)(\mathbf{R}_3'^*\cdot\mathbf{T}_2)\mathrm{k}_{22}+(\mathbf{T}_1'^*\cdot\mathbf{R}_3)(\mathbf{R}_3'^*\cdot\mathbf{T}_2')\mathrm{k}_{12}+(\mathbf{T}_2^*\cdot\mathbf{R}_3)(\mathbf{R}_3'^*\cdot\mathbf{T}_1')\mathrm{k}_{21}]^4\,. \tag{A11}$$

Here we have used the following abbreviations ($i,j=1,2$):

$$k_0=G_{22}[(1-G_{22})^2\det(Z'_{ij})-(\mathbf{T}_1^*\cdot\mathbf{T}_1)Z'^2_{22}(1-G_{22})\det(\delta_{ij}-G_{ij})$$
$$-Z_{22}G_{22}(1-G_{21})(1-G_{12})\det(\mathbf{T}_i^*\cdot\mathbf{T}'_j)-(\mathbf{T}_2^*\cdot\mathbf{T}'_2)G_{22}(1-G_{22})\det(Z_{ij})] ,$$
$$k_{11}=-(G_{12}-G_{22})(G_{21}-G_{22})[(\mathbf{T}_2^*-\mathbf{T}'_2)^2G_{22}-(1-G_{22})^2] ,$$
$$k_{22}=(1-G_{22})G_{22}\det(Z'_{ij})-(\mathbf{T}_1^*\cdot\mathbf{T}'_1)Z'^2_{22}G_{22}\det(\delta_{ij}-G_{ij}) ,$$
$$k_{12}=(G_{12}-G_{22})G_{22}[Z_{22}(1-G_{21})(\mathbf{T}_2^*\cdot\mathbf{T}'_1)-Z_{21}(1-G_{22})(\mathbf{T}_2^*\cdot\mathbf{T}'_2)] ,$$
$$k_{21}=(G_{21}-G_{22})G_{22}[Z_{22}(1-G_{12})(\mathbf{T}_1^*\cdot\mathbf{T}'_2)-Z_{12}(1-G_{22})(\mathbf{T}_2^*\cdot\mathbf{T}'_2)] ,$$

and

$$G_{ij}=\exp\{\nu\mathbf{T}_i^*\cdot\mathbf{T}'_j\} , \quad \mathbf{t}_{ij}=\mathbf{T}_i^*-\mathbf{T}'_j ,$$
$$Z_{ij}=G_{ij}-\delta_{ij}-\mathbf{T}_i^*\cdot\mathbf{T}'_j , \quad Z'_{ij}=G_{ij}-\delta_{ij}-\mathbf{T}_i^*\cdot\mathbf{T}'_jG_{ij} .$$

Performing a tedious but straightforward expansion of expression (A11) into powers of $(\mathbf{T}_i^*\cdot\mathbf{T}'_j)$ [cf. Eq. (6)] we obtain

$$I(m_1\sim m_9)=\begin{bmatrix}4\\m_9\end{bmatrix}\sum_{x=\max(0,m_8-m_3,m_6-m_7)}^{\min(m_6,m_8)}(-)^{4-m_9-m_6-m_8+x}\begin{bmatrix}4-m_9\\x\end{bmatrix}\begin{bmatrix}4-m_9-x\\m_6-x\end{bmatrix}\begin{bmatrix}m_3\\m_8-x\end{bmatrix}$$
$$\times\sum_{y=0}^{m_9}\sum_{\substack{a_1+a_2\\=m_3}}\sum_{\substack{b_1+b_2\\=m_7}}\sum_{\substack{c_1+c_2+c_3+c_4\\=m_9-y+x}}\sum_{\substack{d_1+d_2+d_3+d_4\\=y}}\sum_{e_1+e_2=m_6-x}\sum_{f_1+f_2=m_6-x}$$
$$\times(-)^{y+a_2+b_2+c_2+c_3+d_2+d_4+e_2+f_2}\begin{bmatrix}m_9\\y\end{bmatrix}\begin{bmatrix}m_3\\a_1\end{bmatrix}\begin{bmatrix}m_7\\b_1\end{bmatrix}\begin{bmatrix}m_6-x\\e_1\end{bmatrix}\begin{bmatrix}m_8-x\\f_1\end{bmatrix}$$
$$\times\frac{(m_9-y+x)!}{c_1!c_2!c_3!c_4!}\frac{y!}{d_1!d_2!d_3!d_4!}\sum_{\substack{q_1+q_2\\=4-m_6-m_8-m_9+x}}\begin{bmatrix}4-m_6-m_8-m_9+x\\q_1\end{bmatrix}(-1)^{q_2}$$
$$\times F(m_9+y+m_6+m_8-x+q_1\ ,m_9-y+a_1+b_1+c_1+c_2+c_3+d_3+d_4+e_2+f_2+2q_2\ ,$$
$$c_1+c_3+c_4,d_1+d_2+d_3+e_1+f_1,d_1+d_3+d_4+e_2+f_2+2q_1,m_5,\alpha_{22})$$
$$\times F(0,c_3,c_1,d_3,c_3+c_4+d_1,m_1,\alpha_{11})$$
$$\times F(0,a_2+c_4+d_1+d_2+e_1,c_2,d_4+e_2+d_2+e_1,m_2,\alpha_{12})$$
$$\times F(0,b_2+c_4+d_1+d_2+f_1,c_2,d_4+f_2,d_2+f_1,m_4,\alpha_{12}) \tag{A12}$$

with

$$F(a_1,a_2,a_3,a_4,a_5,n,\alpha)=\sum_{b_1=0}^{a_3}\sum_{b_2=0}^{a_4+a_3-b_1}\sum_{b_3=0}^{a_2+b_1+b_2}(-)^{a_4+a_3-b_1+b_3}\begin{bmatrix}a_3\\b_1\end{bmatrix}\begin{bmatrix}a_4+a_3-b_1\\b_2\end{bmatrix}\begin{bmatrix}a_2+b_1+b_2\\b_3\end{bmatrix}$$
$$\times\begin{cases}\dfrac{(a_1+b_3-\alpha)^{n-a_5-a_4-a_3+b_2}}{(n-a_5-a_4-a_3+b_2)!} & \text{if } n-a_5-a_4-a_3+b_2\geq 0\\ 0 & \text{if } n-a_5-a_4-a_3+b_2<0\end{cases} \tag{A13}$$

and $\alpha_{11}=\frac{1}{2}$, $\alpha_{22}=8$, and $\alpha_{12}=2$. Using Eqs. (A12) and (A13) the matrix B in Eq. (A5) can be calculated. Finally the kernel matrix A is obtained after a simple transformation on B involving the matrix Ω given in (A10)

$$A^{N(\lambda,\mu)}_{\sigma t,\sigma' t'}=\sum_{s=0}^{\sigma}\sum_{s'=0}^{\sigma'}B^{N(\lambda,\mu)}_{\sigma s,\sigma' s'}D^{(\sigma\tau)}_{s,t}(\Omega)D^{(\sigma',\tau')}_{s',t'}(\Omega) \tag{A14}$$

with

$$D^{(\sigma\tau)}_{s,t}(\Omega)=\left[\frac{(\sigma+\tau+1)!\tau!}{\sigma+1}\right]^{1/2}\sum_{m_1\sim m_4}{}' \Omega_{11}^{m_1}\Omega_{12}^{m_2}\Omega_{21}^{m_3}\Omega_{22}^{m_4}(-)^{(m_1-m_4)}\{(\sigma+1)/[(m_1+m_3)!(m_2+m_4)!]\}^{1/2}$$
$$\times\begin{pmatrix}\frac{m_1+m_2}{2} & \frac{m_3+m_4}{2} & \frac{\sigma}{2}\\ \frac{m_1-m_2}{2} & \frac{m_3-m_4}{2} & \frac{m_2-m_1+m_4-m_3}{2}\end{pmatrix}. \tag{A15}$$

The sum $\sum'$ is subject to the restrictions $m_1+m_2=\sigma+\tau-s$, $m_3+m_4=\tau+s$, $m_1+m_3=\sigma+\tau-t$, and $m_2+m_4=\tau+t$.

The seemingly huge numerical effort involved in determining the kernel matrix A can be severely reduced by a proper organization of the computer code. We needed about twenty hours of control processing unit (cpu) time on a VAX 11-750 to compute all the required kernels.

There are two relations we have used as a final check on our calculations. First one should reproduce the regular $^{16}O+^{16}O$ normalization kernels [Eq. (A11) of Ref. 9]

$$\mu'_N=\sum_{(\sigma\tau),(\sigma',\tau')}\frac{[d(\sigma\tau)d(\sigma'\tau')]^{1/2}}{d(N_2,0)d(4,0)}A^{N(N-4,0)}_{\sigma t,\sigma' t'} \tag{A16}$$

with $d(\mu\nu)=\frac{1}{2}(\mu+1)(\nu+1)(\mu+\nu+2)$ and

$$\mu'_N=\sum_{k=0}^{4}(-\tfrac{1}{64})^k\begin{bmatrix}4\\k\end{bmatrix}\frac{N!}{(N-2k)!}\sum_{r=0}^{7-k}(-)^r\begin{bmatrix}16-2k\\r\end{bmatrix}\left[1-\frac{k+r}{8}\right]^{N-2k}$$

for $N\geq 24$ and $\mu'_N=0$ otherwise. Second, the normalization kernel for $\alpha+^{12}C+^{16}O$ with the minimal number of quanta, four, in the $\alpha+^{12}C$ system

$$\mu_N\equiv\langle\varphi_{(00)}(1)\varphi_{(00)}(2)(V_{N,l_2=0}V_{N_1=4,l_1=0})_{J=0}\varphi_{(04)I=0}(3)\,|\,\mathscr{A}\,|\,\varphi_{(00)}(1)\varphi_{(00)}(2)(V_{N,l_2=0}V_{N_1=4,l_1=0})_{J=0}\varphi_{(04)I=0}(3)\rangle \tag{A17}$$

is related to the normalization kernels (A16) and those for the $\alpha+^{12}C$ system $\mu^N_{(\lambda\mu)}$ discussed in Ref. 19

$$\mu_N=\mu'_N\mu^{N_1=4}_{(\lambda\mu)=(00)}=4.44\mu'_N\ . \tag{A18}$$

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*Present address: A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, CT 06520.

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