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Inclusive angular distribution of α and Li fragments produced in Fe-C and Fe-Pb collisions at 1.88 GeV/nucleon

C. O. Kim and S. N. Kim
Department of Physics, Korea University, Seoul 132, Korea

I. K. Chae and D. H. Kim
Department of Physics, Sookmyung Women's University, Seoul 140, Korea

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From the measurements of laboratory system emission angles θ of 2188 α and 298 Li fragments, produced inclusively by relativistic Fe-C and Fe-Pb collisions, a target-independent differential frequency formula, $dN = \exp(a + b\cot\theta)d(\cot\theta)$, is obtained with the constant $b \cong -0.026$ at 1.88 GeV/nucleon, which seems also to be independent on the kinds of projectile fragments. The significance of this approximate formula is discussed in relation mainly with Kaplon's formula.

When nuclear interactions produced by high-energy hadrons in nuclear emulsion ("stars") are observed in the laboratory system (LS), heavy fragments of low velocity are easily distinguished from relativistic shower particles. Nevertheless, the identification of these very slow fragments is difficult, mainly because of their high ionization and short track lengths. Now, by virtue of the recent development of the technique of accelerating relativistic heavy ions, the "heavy fragments" in the projectile rest system (ALS; antilaboratory system) can be observed as "relativistic particles" in the LS. Especially, the relativistic α and Li fragments are easily indentified in nuclear emulsion, just by simple inspection through an optical microscope, since the former have about four times the grain density of minimum-ionizing tracks, and the latter about nine times. The present paper reports the experimental result using this technique of identification, and compares our result with some recent angular measurements of α fragments.[1-5]

The specific arrangements of the experiment to the ^{56}Fe-ion beam at the Lawrence Berkeley Laboratory (LBL) Bevalac are illustrated in Fig. 1. From the left, Fe ions were incident on the 3-mm thick target, either of carbon or lead. The detectors of fragments and incident Fe ions were nuclear emulsion stacks (made of 100 Fuji ET7B pellicles of sheet size $2\times10\times0.04$ cm^3), and were either placed just behind the target plate (S stacks) or 15 cm away from the back of the target plate (L stacks) in the downstream. Altogether, eight stacks (4 S stacks and 4 L stacks) were exposed primarily in order to confirm or reject the anomalous-particle-fragment (sometimes called "anomalon") effect.[6]

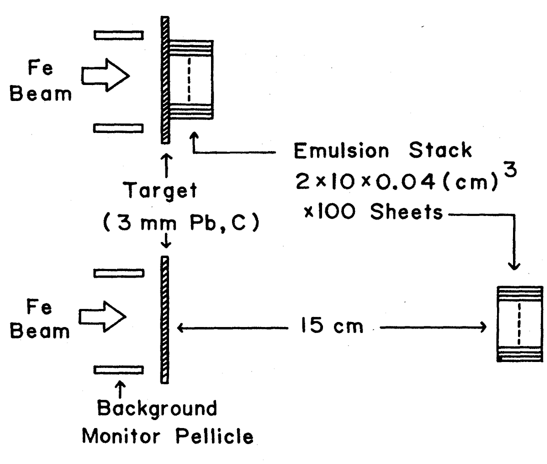

FIG. 1. Experimental setup at LBL Bevalac.

The shorter edges of pellicles were exposed horizontally to the Fe-ion beam with a track density of about 5×10^3 ions/cm^2. Our experiment differs from previous experiments[1-5] mainly in several aspects: First, the target for the incident Fe ions was either pure substance of carbon or lead. Secondly, by adopting the method of "relative scattering measurements" or of our reference-track method[7] in order to avoid the effect due to the distortion inherent in the processed nuclear emulsion, we have tried to improve the accuracy of measuring LS emission angles θ always *in reference to* nearby Fe beam tracks. Measured at ~ 1 mm downstream from the entrance edge of pellicles, where angular measurements of the fragments were usually performed with special care not to miss such α and Li fragments as were produced with large θ, the present version of the reference-track method gave the angular deviations of the incident Fe beam tracks as 2.3 ± 0.3 and 3.7 ± 0.3 milliradians, respectively, for the detectors behind the carbon and lead targets. Semiautomatic systems of three-coordinate digitalized Nikon OPTIPHOT 66 microscopes were employed for our measurements with, at least, 1-μm readout accuracy, whose precision in angular measurements exceeded the defining angular deviations of the incident Fe beam tracks by far.

Our results closely resemble those of Refs. 1–5, when the angular distributions (and the projection angle distributions as well) of α fragments are plotted against θ as shown separately for Fe-C collisions (with vacant circles) and for Fe-Pb collisions (with filled circles) in Fig. 2. The two plots each of S stacks and L stacks are indicated in the above and the third plot underneath is that of the combined data. There seems to be a tendency of more population of α fragments of $\theta \gtrsim 10°$ for Fe-C collisions than for Fe-Pb collisions, and the same holds for those of $\theta \lesssim 1°$. To show this effect vividly, the data of $0° < \theta \lesssim 3°$ and those of $\theta > 5°$ are fitted, respectively, with the Gaussian and the exponential regression functions whose results are also indicated as the curves and equations in Fig. 2.[8]

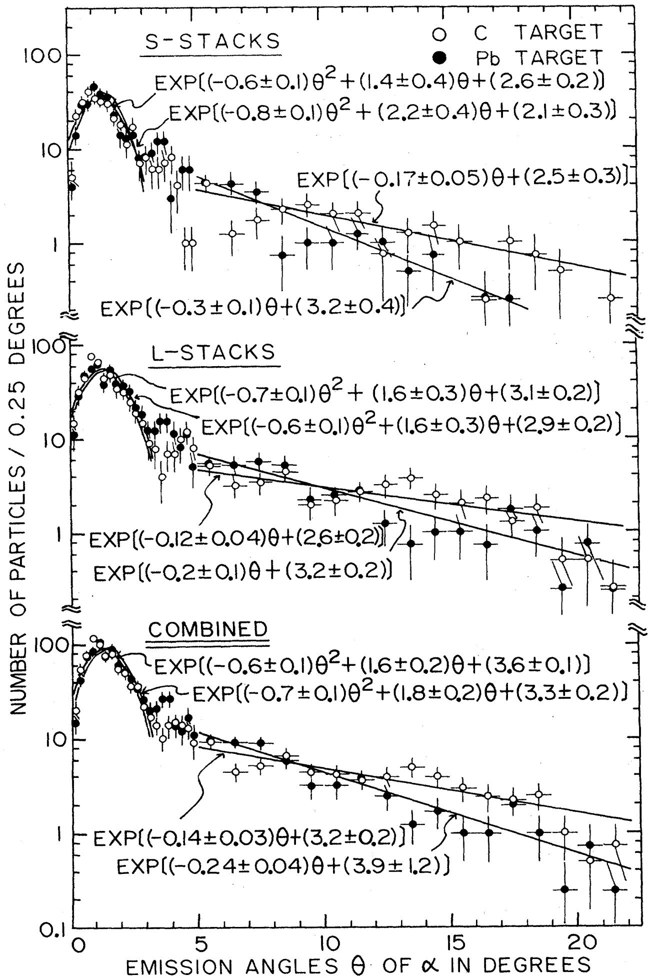

FIG. 2. The differential frequency of α fragments vs θ in degrees.

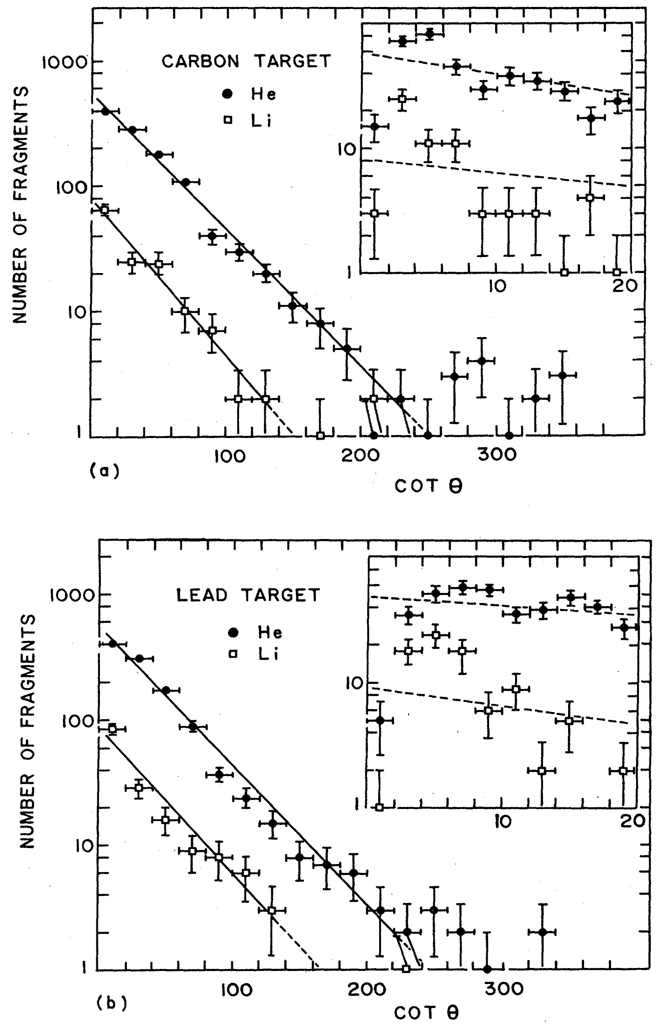

FIG. 3. The differential frequency vs cotθ, (a) for Fe-C collisions and (b) for Fe-Pb collisions.

We find it most reasonable and revealing to plot the differential frequency dN vs $d(\cot\theta)$ as shown in Fig. 3(a) for Fe-C collisions and 3(b) for Fe-Pb collisions. In the figures the filled circles represent the angular data from 2188 α fragments, and the vacant squares those from 218 Li fragments. Since no statistically significant differences between the angular data of S stacks and those of L stacks were detected, only the combined data are shown in the figures. For the least squares fits, we used the regression function,

$$dN = \exp(a + b\cot\theta)\,d(\cot\theta)\ , \tag{1}$$

with reasonable fits to most of the angular data, as seen from the best fitted values of a and b separately for Fe-C and Fe-Pb collisions in Table I. For the interval of $\cot\theta = 0\text{–}20$, the amplified versions of dN vs $d(\cot\theta)$ with one-tenth the interval in the main figures are shown as the inserts, respectively. The straight lines in the figures and the dotted lines in the inserts show the best fitted curves. Actually, sharp falloffs of dN for $\cot\theta \lesssim 4$ can be seen in the inserts and some detailed but *target-independent* structures seem to be common. But those portions of α and Li fragments, with extremely small and large θ, which deviate appreciably from the general trends represented by Eq. (1) constitute several percents.

Our immediate concern about Eq. (1) is related closely with Kaplon's formula,[9]

$$(\langle\theta^2\rangle)^{1/2} = \frac{\zeta}{u\gamma_p}\ , \tag{2}$$

with $\zeta = 0.056$, where θ is in radians and $u\gamma_p$ in GeV. The above equation has been a convenient tool for estimating the *unknown* primary energy of cosmic-ray primaries.[10] By use of our angular data of $\theta \leq 5°$,[11] the values of $(\langle\theta^2\rangle)^{1/2}$ have been calculated and listed in Table I. Thus, at $u\gamma_p = 1.88$ GeV, the calibrated values of ζ are 0.057 (0.056) and 0.067 (0.062) for the α (Li) fragments, respectively, for Fe-C and Fe-Pb collisions; our experiment generally confirms Kaplon's formula, Eq. (2), with $\zeta \cong 0.06$.

On the other hand, through integration of Eq. (1) with the use of the variable cotθ, we come to the *median-angle* formula

$$(\cot\theta)_{1/2} = \ln 2/|b|\ , \tag{3}$$

which, with the plausible assumption of $(\langle\theta^2\rangle)^{1/2}$

TABLE I. Some relevant data for α fragments (and for Li fragments inside the brackets) and the best fitted values to Eq. (1).

Target	Number of α fragments = number from S stacks + number from L stacks	$\sqrt{\langle\theta^2\rangle}$ [a]	a	b	χ^2/DF
Carbon	$1103=408+695$	0.037 ± 0.002	6.3 ± 0.05	-0.0248 ± 0.0009	$\frac{21}{9}$
	$(138=35+103)$	(0.032 ± 0.002)	(4.4 ± 0.1)	(-0.029 ± 0.003)	$(\frac{6}{5})$
Lead	$1085=408+677$	0.038 ± 0.001	6.3 ± 0.04	-0.0265 ± 0.0009	$\frac{27}{10}$
	$(160=62+98)$	(0.035 ± 0.001)	(4.4 ± 0.1)	(-0.030 ± 0.003)	$(\frac{11}{5})$

[a]The averages are taken only for the data of $\theta\leqslant 5°$ (Ref. 11).

$=\cot^{-1}[(\cot\theta)_{1/2}]$, gives us the values of ζ as 0.063 (0.074) and 0.067 (0.075), for the α (Li) fragments, respectively, for Fe-C and Fe-Pb collisions, i.e., from the use of the values of $|b|$ listed in Table I. This confirms the Kaplon's formula again.

This kind of excellent accord cannot be accidental. From the Lorentz transformation of $\cot\theta$ from the LS to the (double-barred) kinematic quantities of the ALS, we obtain the relation

$$\cot\theta=\frac{p_L}{p_T}=\frac{\gamma_p(\bar{\bar{p}}_L+\beta_p\bar{\bar{E}})}{\bar{\bar{p}}_T}=\frac{M_F\gamma_p\bar{\bar{\gamma}}(\bar{\bar{\beta}}\cos\bar{\bar{\theta}}+\beta_p)}{M_F\bar{\bar{\gamma}}\bar{\bar{\beta}}\sin\bar{\bar{\theta}}}\ , \tag{4}$$

where γ_p $[=(1-\beta_p^2)^{-1/2}$, and $u\gamma_p$ is the primary energy per nucleon] is the Lorentz factor of the transformation and $\bar{\bar{\beta}}[=(\bar{\bar{\gamma}}^2-1)^{1/2}/\bar{\bar{\gamma}}]$ and $\bar{\bar{\theta}}$ are the velocity and emission angle (measured with respect to the direction of the incident primary) of a fragment of the mass M_F in the ALS. Since $\bar{\bar{\gamma}}$ *cancels out* and $\bar{\bar{\beta}}_L=\bar{\bar{\beta}}\cos\bar{\bar{\theta}}\cong 0$,[12] we obtain an important relation,

$$\cot\theta\cong\gamma_p\beta_p/\bar{\bar{\beta}}_T\ . \tag{4'}$$

Thus, Eq. (1) suggests the following *approximate* differential frequency formula of the ALS transverse velocity, $\bar{\bar{\beta}}_T=\bar{\bar{\beta}}\sin\bar{\bar{\theta}}$:

$$dN\cong(\kappa/b)\exp(a+\kappa/\bar{\bar{\beta}}_T)d(1/\bar{\bar{\beta}}_T)\ , \tag{5}$$

which suggests a Lorentz-invariant constant κ, as

$$\kappa=\gamma_p\beta_p b\ , \tag{6}$$

since we believe in the limiting fragmentation shown by the experiments of Ref. 12. [Actually, the validity of Eq. (6) needs only the invariant character of the differential distribution concerning the ALS transverse velocity $\bar{\bar{\beta}}_T$, which may come from small and constant $\bar{\bar{p}}_T$ and from the uncertainty relation only.] Moreover, in case future experiments with much higher values of γ_p than that of the present experiment and those with different *kinds* of projectile fragments confirm the universality of κ, besides the limiting fragmentation, an improved method over Kaplon's formula of energy (actually momentum) estimation of cosmic rays will be offered through the use of Eq. (6).

Thus, through the good agreement in the general trends and the values of $b\cong-0.026$ in Eq. (1) both for Fe-C and Fe-Pb collisions, we are led to the conclusion of "limiting fragmentation" in the sense[12] that neither the inclusive angular distributions of α and Li fragments have anything to do with the *kinds* of the target nucleus for the majority of α and Li fragments, as shown in Fig. 3 and in Table I, nor with the primary energy of relativistic incident heavy ions.

On the other hand, it is remarked here that the emission of α and Li fragments of $\theta\geqslant 5°$ seems to be the products of "sequential breakup," which takes place far away from the Coulomb field of the target nucleus, considering the fact that the substantial increase of *large-angle scattering* from the Fe-Pb collisions over the Fe-C collisions is not detected from our experiment.[13]

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[11]When the whole data, inclusive of those of $\theta>5°$ are taken, ζ's are 0.269 (0.306) and 0.190 (0.239) for α (Li) fragments, respectively, for Fe-C and Fe-Pb collisions. Reference 9 recommended the restriction of $\theta\lesssim 5°$.

[12]D. E. Greiner *et al.*, Phys. Rev. Lett. **35**, 152 (1975). In this work, it has been found that $\langle\bar{\bar{p}}_L\rangle\cong-0.03\sim-0.04$ GeV/c both for α and Li fragments in the collisions of ^{12}C-ion collisions with various target nuclei at 1.05 and 2.1 GeV/nucleon, and $\sigma_{p_L}\cong 0.13-0.16$ GeV/c, i.e., $\bar{\bar{\beta}}\cos\bar{\bar{\theta}}\cong 0.035<<\beta_p\cong 1$.

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