PHYSICAL REVIEW C VOLUME 32, NUMBER 4 OCTOBER 1985

U(5)×SU(2) limits of the interacting boson fermion model, their associated supersymmetries, and their application to ^{76}Se and ^{75}As

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(Received 22 April 1985)

Boson-fermion symmetries and supersymmetries associated with the U(5)×SU(2) limits in the interacting boson fermion model with $j=\frac{3}{2}$, $\frac{5}{2}$ and $j=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ are developed in detail. Their relevance for the description of the available experimental data on the low-energy spectra of the nuclei ^{76}Se and ^{75}As is examined.

I. INTRODUCTION

In recent years, it has been shown that algebraic models provide a powerful tool to study the spectroscopy of low-lying states in medium and heavy mass nuclei. In the interacting boson model (IBM), rotational and vibrational degrees of freedom in even-even nuclei are united in a single framework.[1] Similarly, the extension of the IBM to odd-mass nuclei, called the interacting boson fermion model (IBFM), provides a unified description of the collective properties in these nuclei.[2] In addition, a set of closed analytic formulae for the energies and other observables of the levels can be obtained whenever the IBM (IBFM) Hamiltonian possesses a dynamical symmetry. Since, in the IBFM, both collective (boson) and single particle (fermion) degrees of freedom are present, the dynamical symmetries of the IBFM are called boson-fermion (BF) symmetries.[3] The concept of dynamical symmetries can be extended further to supersymmetries, in which both even-even and odd-mass nuclei are described in a single framework. Many different examples of BF symmetries and supersymmetries have been studied. The simplest cases are those in which the odd particle only occupies one shell model orbit with spin $j=\frac{1}{2}$ or $\frac{3}{2}$.[4—7] More recently, these symmetries have been extended to include several orbits, e.g., $j=\frac{1}{2},\frac{3}{2}$ (Ref. 8) or $j=\frac{3}{2},\frac{5}{2}$ (Ref. 7) or $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$ (Refs. 8—12).

In the present paper, we first describe three BF symmetries and their associated supersymmetries, wherein the boson part corresponds to the U(5) dynamical symmetry of the IBM,[1] and the odd-fermion occupies two orbits with $j=\frac{3}{2}$ and $\frac{5}{2}$, or three orbits with $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$. We stress, in particular, the relations between these various cases. We next examine to what extent the low-energy spectra of the nuclei ^{76}Se and ^{75}As can be described by these symmetries, for what concerns the excitation energies and the other observables of their levels, electromagnetic moments and transition probabilities, and one- and two-nucleon transfer amplitudes.

II. THEORETICAL ANALYSIS OF U(5)×SU(2) BF SYMMETRIES

In this section, we present an overview of the properties of three U(5)×SU(2) BF symmetries in odd-A nuclei, which occur when the bosons have a U(5) symmetry and for several combinations of the dominant single-particle (s.p.) orbits available to the odd fermion in the odd-A nuclei. In Ref. 7, a detailed description was given of the U(5)×SU(2) symmetries with s.p. orbits $j=\frac{1}{2}$ *or* $j=\frac{3}{2}$. The possibility of a U(5)×SU(2) symmetry based on the s.p. orbits $j=\frac{3}{2}$ *and* $\frac{5}{2}$ was also mentioned in Ref. 7, and the properties of the U(5)×SU(2) symmetry with $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ were discussed in some detail in Refs. 8 and 10. In this section, we analyze in more detail one U(5)×SU(2) symmetry with the s.p. orbits $j=\frac{3}{2}$ and $\frac{5}{2}$, and two U(5)×SU(2) symmetries with the s.p. orbits $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$. This analysis includes a discussion of the energy spectra and a classification of the states in the three symmetries, as well as the calculation of $B(E2)$ values and one- and two-nucleon transfer intensities.

We start by indicating the group chain in each of the above-mentioned symmetries. For the U(5)×SU(2) symmetry based on the s.p. orbits $j=\frac{3}{2}$ and $\frac{5}{2}$, this group chain (I) reads:

$$U^{(B)}(6)\times U^{(F)}(10)\supset U^{(B)}(5)\times U^{(F)}(5)\times SU^{(F)}(2)\supset U^{(B+F)}(5)\times SU^{(F)}(2)$$
$$\supset O^{(B+F)}(5)\times SU^{(F)}(2)\supset O^{(B+F)}(3)\times SU^{(F)}(2)\supset \mathrm{Spin}(3)\supset \mathrm{Spin}(2)\ , \quad (2.1a)$$

where the superscripts (B) or (F) refer to bosons or fermions, respectively. With the s.p. orbits $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, the following two chains can be considered:

$$\mathrm{U}^{(B)}(6)\times\mathrm{U}^{(F)}(12)\supset\mathrm{U}^{(B)}(6)\times\mathrm{U}^{(F)}(6)\times\mathrm{SU}^{(F)}(2)\supset\begin{bmatrix}\mathrm{U}^{(B)}(5)\times\mathrm{U}^{(F)}(5)\times\mathrm{SU}^{(F)}(2)\\ \mathrm{U}^{(B+F)}(6)\times\mathrm{SU}^{(F)}(2)\end{bmatrix}\supset\mathrm{U}^{(B+F)}(5)\times\mathrm{SU}^{(F)}(2)$$

$$\supset\mathrm{O}^{(B+F)}(5)\times\mathrm{SU}^{(F)}(2)\supset\mathrm{O}^{(B+F)}(3)\times\mathrm{SU}^{(F)}(2)\supset\mathrm{Spin}(3)\supset\mathrm{Spin}(2)\ , \tag{2.1b}$$

where the upper (lower) group chain between parentheses corresponds to limit II (II′).

In Eqs. (2.1a) and (2.1b), we have used the group reduction

$$\mathrm{U}^{(F)}(m)\supset\mathrm{U}^{(F)}(\tfrac{1}{2}m)\times\mathrm{SU}^{(F)}(2)$$

with $m=10$ and $m=12$, respectively. This corresponds to a decomposition of the fermion angular momenta into a pseudo-orbital part, $k=2$ (2.1a) or 0 and 2 (2.1b), and a pseudo-spin part, $s=\frac{1}{2}$. We note that the pseudo-orbital angular momentum does not necessarily coincide with the physical orbital angular momentum. The chains (2.1a) and (2.1b) will be referred to hereafter as chains I, II, and II′, respectively.

Neglecting terms which only contribute to the binding energy, the Hamiltonians corresponding to the group chains (2.1a) and (2.1b) are given by:

$$H^{(I)}=AC_1[\mathrm{U}^{(B)}(5)]+A'C_2[\mathrm{U}^{(B)}(5)]+A''C_1[\mathrm{U}^{(B)}(5)]C_1[\mathrm{U}^{(F)}(5)]$$
$$+BC_1[\mathrm{U}^{(B+F)}(5)]+B'C_2[\mathrm{U}^{(B+F)}(5)]+CC_2[\mathrm{O}^{(B+F)}(5)]+DC_2[\mathrm{O}^{(B+F)}(3)]+EC_2[\mathrm{Spin}(3)]\ , \tag{2.2a}$$

$$H^{(\mathrm{II})}=H^{(\mathrm{I})}\ , \tag{2.2b}$$

$$H^{(\mathrm{II}')}=A_0C_2[\mathrm{U}^{(B+F)}(6)]+B_0C_1[\mathrm{U}^{(B+F)}(5)]+B'C_2[\mathrm{U}^{(B+F)}(5)]$$
$$+CC_2[\mathrm{O}^{(B+F)}(5)]+DC_2[\mathrm{O}^{(B+F)}(3)]+EC_2[\mathrm{Spin}(3)]\ , \tag{2.2c}$$

where $C_n[G]$ denotes the linear $(n=1)$ or quadratic $(n=2)$ Casimir operator of the group G. With Eq. (2.2b), we only imply that Casimir operators of the same groups appear in both $H^{(\mathrm{II})}$. However, the realization of the Casimir operators in terms of fermion creation and annihilation operators may be different in the two cases. For the explicit form of the Casimir operators appearing in Eqs. (2.2a)–(2.2c), we refer to Ref. 10.

States can be classified by associating the appropriate quantum numbers to the various groups appearing in the chains I, II, and II′. In the one-fermion case $(M=1)$, these quantum numbers are

$$\mathrm{U}^{(B)}(6){:}[N],\quad \mathrm{U}^{(B+F)}(6){:}[N_1,N_2]\ ,$$
$$\mathrm{U}^{(B)}(5){:}\{n_B\},\quad \mathrm{U}^{(B+F)}(5){:}\{n_1,n_2\}\ ,$$
$$\mathrm{U}^{(F)}(12){:}[M=1],\quad \mathrm{O}^{(B+F)}(5){:}(\nu_1,\nu_2)\ ,$$
$$\mathrm{U}^{(F)}(10){:}[M=1],\quad \mathrm{O}^{(B+F)}(3){:}L\ ,$$
$$\mathrm{U}^{(F)}(6){:}[1],\quad \mathrm{Spin}(3){:}J\ ,$$
$$\mathrm{U}^{(F)}(5){:}\{n_F\},\quad \mathrm{Spin}(2){:}M_J\ ,$$
$$\mathrm{SU}^{(F)}(2){:}s=\tfrac{1}{2}\ . \tag{2.3}$$

Omitting the unnecessary labels, the basis states in each of the chains of Eq. (2.1) can be characterized by:

$$(\mathrm{I,II})\quad |N\{n_B\}\{n_F\}\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJM_J\rangle\ , \tag{2.4a}$$

where $n_1+n_2=n_B+n_F$ with $n_F=1$ for chain I and $n_F=0,1$ for chain II;

$$(\mathrm{II}')\quad |N[N_1,N_2]\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJM_J\rangle\ . \tag{2.4b}$$

In Eq. (2.4), an extra label α has been introduced, since the decomposition of the representations of $\mathrm{O}^{(B+F)}(5)$ into those of $\mathrm{O}^{(B+F)}(3)$ is not unique. The eigenvalues of the Hamiltonians (2.2a)–(2.2c) in the states (2.4a) and (2.4b) are given by:

$$E^{(\mathrm{I})}=An_B+A'n_B(n_B+4)+A''n_Bn_F$$
$$+B(n_1+n_2)+B'[n_1(n_1+4)+n_2(n_2+2)]$$
$$+C[\nu_1(\nu_1+3)+\nu_2(\nu_2+1)]$$
$$+DL(L+1)+EJ(J+1)\ , \tag{2.5a}$$

$$E^{(\mathrm{II})}=E^{(\mathrm{I})}\ , \tag{2.5b}$$

$$E^{(\mathrm{II}')}=A_0[N_1(N_1+5)+N_2(N_2+3)]$$
$$+B_0(n_1+n_2)+B'[n_1(n_1+4)+n_2(n_2+2)]$$
$$+C[\nu_1(\nu_1+3)+\nu_2(\nu_2+1)]$$
$$+DL(L+1)+EJ(J+1)\ . \tag{2.5c}$$

The energy eigenvalues (2.5a)–(2.5c), together with the reduction rules,[7,8,10] enable one to construct typical energy spectra for each of the limits. The states of limit II can be divided into two groups, which are characterized by the quantum number $n_F=0$ $(j=\frac{1}{2})$ and $n_F=1$ $(j=\frac{3}{2},\frac{5}{2})$, and whose relative energy is given by a linear combination of the coefficients A'', B, and B'. We note that the states of limit I are equivalent to a subset of those of limit II with $n_F=1$. This is illustrated in Fig. 1, where the addi-

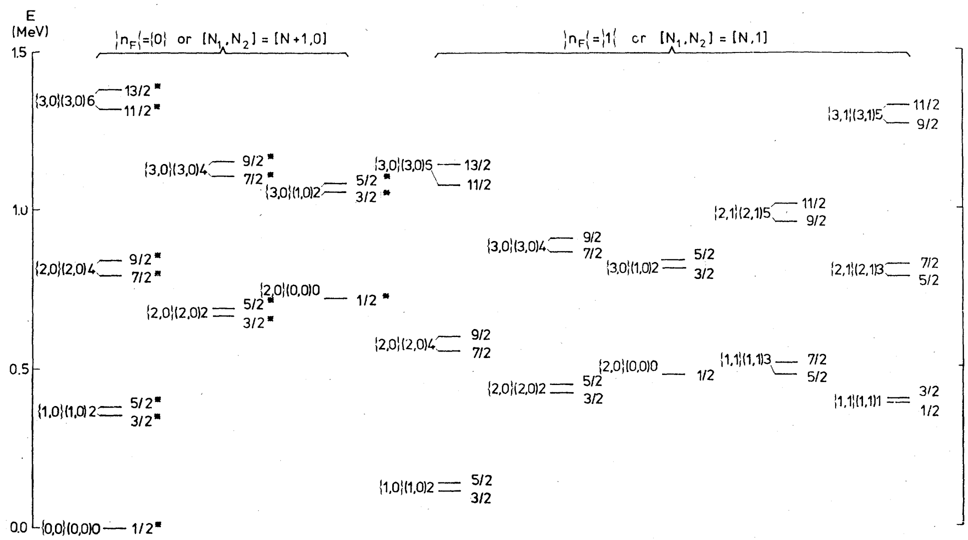

FIG. 1. Calculated energy spectrum in the limit II or II′. In the limit II, the expression (2.5b) is used, with $A=240$, $A'=A''=0$, $B=60$, $B'=-C=10$, and $D=E=5$. In the limit II′, the expression (2.5c) is used, with $A_0=20$, $B_0=300$, $B'=-C=10$, and $D=E=5$. All parameters are given in keV. The boson number is $N=5$. The nonstarred states also occur in limit I.

tional states of limit II with $n_F=0$ are marked with a star. The third limit II′ represents a different coupling scheme. Again, the eigenstates can be divided into two groups, which are now labeled by the quantum numbers $[N_1,N_2]=[N+1,0]$ and $[N_1,N_2]=[N,1]$, and whose relative energy is determined by the first term in the Hamiltonian $H^{(\mathrm{II}')}$ of Eq. (2.2c), i.e., depends on A_0. The spectra of limits II and II′ become *almost* identical through the following choice of parameters appearing in Eqs. (2.5b) and (2.5c): $A'=A''=0$; $A=-2(N+1)A_0$; $B=B_0-2(N+1)A_0$; equal values of B', C, D, and E in both limits. This is illustrated in Fig. 1, where the states in limit II with $n_F=0$ and $n_F=1$ have the same excitation energies as the corresponding states in limit II′ with $[N_1,N_2]=[N+1,0]$ and $[N_1,N_2]=[N,1]$, respectively. However, the excitation energies of the states with $\{n_1,n_2\}=\{N+1,0\}$, which have $n_F=1$ in limit II and $[N_1,N_2]=[N+1,0]$ in limit II′, will be different in this case. Thus, except for the states with $\{n_1,n_2\}=\{N+1,0\}$, the most general spectrum of limit II′ can be made to coincide with the spectrum of limit II for a particular choice of the parameters A, A', A'', and B.

An advantage of a group-theoretical classification of the eigenstates is that it enables one to calculate closed analytic expressions for all matrix elements of interest. In the present case, it is convenient to expand the coupled basis states of Eq. (2.4) into the direct product of the boson and fermion states. The matrix elements of a transition or transfer operator can then be reduced to a matrix element in the boson space, which has already been derived in Ref. 13, and a matrix element in the fermion space, which is trivial to calculate since we only consider states with one fermion. The basis state in Eq. (2.4a) for the limits I and II can be expanded as:

$$|N\{n_B\}\{n_F\}\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJM_J\rangle=\sum_{\substack{\nu_B\alpha_B L_B\\ \nu_F L_F}}\left\langle\begin{matrix}\{n_B\}\{n_F\}\\(\nu_B)(\nu_F)\end{matrix}\middle|\begin{matrix}\{n_1,n_2\}\\(\nu_1,\nu_2)\end{matrix}\right\rangle\left\langle\begin{matrix}(\nu_B)(\nu_F)\\\alpha_B L_B L_F\end{matrix}\middle|\begin{matrix}(\nu_1,\nu_2)\\\alpha L\end{matrix}\right\rangle$$

$$\times\sum_j(-1)^{L_B+J+1/2}\hat{L}\hat{j}\begin{Bmatrix}L_B & L_F & L\\ \frac{1}{2} & J & j\end{Bmatrix}\{|[N]\{n_B\}(\nu_B)\alpha_B L_B\rangle\times a_j^\dagger\}M_J^{(J)}\,, \tag{2.6a}$$

where $\hat{j}=\sqrt{2j+1}$ and, furthermore, $|[N]\{n_B\}(\nu_B)\alpha_B L_B\rangle$ denotes a "vibrational" state of a neighboring even-even nucleus.[13] The relation between the wave functions in limits II and II′ is

$$| N[N_1,N_2]\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJM_J\rangle = \sum_{n_B,n_F} \left\langle \begin{matrix} [N][1] \\ \{n_B\}\{n_F\} \end{matrix} \middle| \begin{matrix} [N_1,N_2] \\ \{n_1,n_2\} \end{matrix} \right\rangle | N\{n_B\}\{n_F\}\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJM_J\rangle \ . \tag{2.6b}$$

In Eq. (2.6a), the isoscalar factors appear, denoted by $\langle ::|:\rangle$, associated with the reductions U(5)⊃O(5) and O(5)⊃O(3), and in Eq. (2.6b), the U(6)⊃U(5) isoscalar factor appears. The U(6)⊃U(5) and U(5)⊃O(5) isoscalar factors are known in general.[10] The O(5)⊃O(3) isoscalar factor, which, for $\nu_2=0$, is related to the d-boson coefficient of fractional parentage, is known for high L values contained in (ν_1,ν_2). In Ref. 10, the method of their calculation was outlined, and a few cases were calculated explicitly. Additional O(5)⊃O(3) isoscalar factors, which can be derived from the results given in Refs. 13 and 14 and from orthonormality properties, have been calculated, and are available on request.

From Eq. (2.6a), we conclude that the structure of the states in limit I is identical to that of the *corresponding* states in limit II. Thus, we may henceforth restrict our analysis to the limits II and II′, since all the properties of limit I are contained in limit II. On the other hand, Eq. (2.6b) shows that, although the low-energy spectra may be identical in limits II and II′, the structure of the corresponding states is different, and, as a consequence, the two limits will have different $M1$, $E2$, etc., . . . properties.

To discuss the properties of limits II and II′, we introduce a notation with starred and nonstarred states, i.e.,

$$\begin{aligned} | N\{n_1,0\}(\nu_1,0)\alpha LJ\rangle^* &\equiv | N\{n_B\}\{0\}\{n_1=n_B,0\}(\nu_1,0)\alpha LJ\rangle \text{ in limit II ,} \\ &\equiv | N[N+1,0]\{n_1,0\}(\nu_1,0)\alpha LJ\rangle \text{ in limit II}' \ , \end{aligned} \tag{2.7a}$$

and

$$\begin{aligned} | N\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJ\rangle &\equiv | N\{n_B\}\{1\}\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJ\rangle \text{ in limit II ,} \\ &\equiv | N[N,1]\{n_1,n_2\}(\nu_1,\nu_2)\alpha LJ\rangle \text{ in limit II}' \ . \end{aligned} \tag{2.7b}$$

This notation corresponds to the one used in Fig. 1 and allows for a simultaneous discussion of limits II and II′.

It is now straightforward to calculate, with the help of the expressions (2.6a) and (2.6b) for the wave functions, $B(E2)$ values as well as one- and two-nucleon transfer strengths for the lowest states. First, we specify the form of the electromagnetic and transfer operators. As $E2$ operator, we take:

$$\begin{aligned} T(E2) = e_f[A^{(2)}(02)+A^{(2)}(20)] \\ +e_b(s^\dagger\tilde{d}+d^\dagger s)^{(2)}+e_f' A^{(2)}(22) \\ +e_b'(d^\dagger\tilde{d})^{(2)} \ . \end{aligned} \tag{2.8}$$

In limit II, the selection rules of the four terms in the $E2$ operator (2.8) are the following: $\Delta(n_B,n_F)=(0,\pm1)$, $(\pm1,0)$, (0,0), and (0,0), respectively. In limit II′, if we take $e_b=e_f$ and $e_b'=e_f'$ in Eq. (2.8), $T(E2)$ is the sum of two $U^{(B+F)}(6)$ generators, and therefore cannot connect states with different values of $[N_1,N_2]$. The one-nucleon transfer operator is of the form:

$$\begin{aligned} P_j^+ = \xi_j a_j^\dagger + \sum_{jj'} \xi_{jj'}(s^\dagger \tilde{d} a_{j'}^\dagger)^{(j)} \\ +\theta_j(a_j^\dagger s)^{(j)} + \sum_{j'} \theta_{jj'}(a_{j'}^\dagger \tilde{d})^{(j)} \ , \end{aligned} \tag{2.9}$$

for the reaction from an even-even to an odd-even nucleus, and the Hermitian conjugate of (2.9) for the inverse reaction. The first two terms in Eq. (2.9) describe the one-nucleon transfer from an even-even to an odd-even nucleus in which the number of bosons, N, is conserved, whereas the last two terms describe the transfer in which the number of bosons is changed by one unit. Finally, as a two-nucleon transfer operator, we use $s^\dagger$ or $d^\dagger$ (or the Hermitian conjugates).

In Table I, we give $B(E2)$ values obeying the selection rule $\Delta(n_1,+n_2)=\pm1$, and, in Table II, those with $\Delta(n_1+n_2)=0$. The following quantities have also been calculated, and are available on request: the electric quadrupole moments; the spectroscopic factors for one-nucleon transfer reactions; and the intensities for two-nucleon transfer reactions.

III. EXTENSION TO DYNAMICAL SUPERSYMMETRIES

An interesting extension of dynamical boson-fermion symmetries is that to dynamical supersymmetries in which states in both even-even and odd-mass nuclei are treated in a single framework. In the previous section, we have discussed dynamical symmetries in a mixed system of boson and fermion degrees of freedom for a fixed number of bosons, N, and one fermion, $M=1$ [see Eq. (2.3)]. Since the operators that generate the Lie algebra of the symmetry group $U^{(B)}(6)\times U^{(F)}(m)$ can only transform bosons into bosons and fermions into fermions, the numbers of bosons, N, and fermions, M, are both conserved quantities. If in addition to these operators, we introduce operators that can transform a boson into a fermion and *vice versa,* the enlarged set of operators forms a graded Lie algebra, which can be identified with the algebra of $U(6/m)$. The supergroup $U(6/m)$ can then be decomposed into a chain of subgroups:

$$U(6/m)\supset U^{(B)}(6)\times U^{(F)}(m)\supset \cdots \supset \text{Spin}(3)\supset \text{Spin}(2) \ , \tag{3.1}$$

where $m=10$ for chain I and $m=12$ for chains II and II′ of Eq. (2.1). All states of a supermultiplet can be labeled by the totally supersymmetric representation $[\mathcal{N}\}$ of

TABLE I. $B(E2)$ values for transitions with $\Delta(n_1+n_2)=\pm 1$ between the lowest states in the limits II and II′.

$$F_2 \equiv F_2(L_fJ_f;L_iJ_i)=(-1)^{L_f+J_i+1/2}\hat{L}_i\hat{J}_i\hat{J}_f \begin{Bmatrix} L_i & J_i & \frac{1}{2} \\ J_f & L_f & 2 \end{Bmatrix} .$$

Initial state	Final state	$B(E2;J_i\rightarrow J_f)\times(2J_i+1)$ Limit II	Limit II′
$\|N\{1,0\}(1,0)2J_i\rangle^*$	$\|N\{0,0\}(0,0)01/2\rangle^*$	$Ne_b^2F_2^2$	$\frac{1}{N+1}(e_f+Ne_b)^2F_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle^*$	$\|N\{1,0\}(1,0)2J_f\rangle^*$	$2(N-1)e_b^2F_2^2$	$\frac{2N}{(N+1)^2}(e_f+Ne_b)^2F_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle^*$	$\|N\{1,0\}(1,0)2J_f\rangle$	0	$\frac{2}{(N+1)^2}(e_f-e_b)^2F_2^2$
$\|N\{1,0\}(1,0)2J_i\rangle$	$\|N\{0,0\}(0,0)01/2\rangle^*$	$e_f^2F_2^2$	$\frac{N}{N+1}(e_f-e_b)^2F_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle$	$\|N\{1,0\}(1,0)2J_f\rangle^*$	$e_f^2F_2^2$	$\frac{(N-1)N}{(N+1)^2}(e_f-e_b)^2F_2^2$
$\|N\{1,1\}(1,1)L_iJ_i\rangle$	$\|N\{1,0\}(1,0)2J_f\rangle^*$	$e_f^2F_2^2$	$\frac{N}{N+1}(e_f-e_b)^2F_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle$	$\|N\{1,0\}(1,0)2J_f\rangle$	$Ne_b^2F_2^2$	$\frac{N-1}{(N+1)^2}[e_f-(N+2)e_b]^2F_2^2$
$\|N\{1,1\}(1,1)L_iJ_i\rangle$	$\|N\{1,0\}(1,0)2J_f\rangle$	$Ne_b^2F_2^2$	$\frac{1}{N+1}(e_f+Ne_b)^2F_2^2$

TABLE II. $B(E2)$ values for transitions with $\Delta(n_1+n_2)=0$ between the lowest states in the limits II and II′.

$$F_2 \equiv F_2(L_fJ_f;L_iJ_i)=(-1)^{L_f+J_i+1/2}\hat{L}_i\hat{J}_i\hat{J}_f \begin{Bmatrix} L_i & J_i & \frac{1}{2} \\ J_f & L_f & 2 \end{Bmatrix} ,$$

$$E_2 \equiv E_2(L_fJ_f;L_iJ_i)=\sqrt{5}\hat{L}_f \begin{Bmatrix} 2 & 2 & 2 \\ L_i & L_f & 2 \end{Bmatrix} F_2(L_fJ_f;L_iJ_i) .$$

Initial state	Final state	$B(E2;J_i\rightarrow J_f)\times(2J_i+1)$ Limit II	Limit II′
$\|N\{1,0\}(1,0)2J_i\rangle^*$	$\|N\{1,0\}(1,0)2J_f\rangle^*$	$e_b'^2F_2^2$	$\frac{1}{(N+1)^2}(e_f'+Ne_b')^2F_2^2$
$\|N\{1,0\}(1,0)2J_i\rangle^*$	$\|N\{1,0\}(1,0)2J_f\rangle$	0	$\frac{N}{(N+1)^2}(e_f'-e_b')^2F_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle^*$	$\|N\{2,0\}(\nu_f,0)L_fJ_f\rangle^*$	$4e_b'E_2^2$	$\frac{4}{(N+1)^2}(e_f'+Ne_b')^2E_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle^*$	$\|N\{2,0\}(\nu_f,0)L_fJ_f\rangle$	0	$\frac{2(N-1)}{(N+1)^2}(e_f'-e_b')^2E_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle^*$	$\|N\{1,1\}(1,1)L_fJ_f\rangle$	0	$\frac{2}{N+1}(e_f'-e_b')^2E_2^2$
$\|N\{1,0\}(1,0)2J_i\rangle$	$\|N\{1,0\}(1,0)2J_f\rangle$	$e_f'^2F_2^2$	$\frac{1}{(N+1)^2}(Ne_f'+e_b')^2F_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle$	$\|N\{2,0\}(\nu_f,0)L_fJ_f\rangle$	$(e_f'+e_b')^2E_2^2$	$\frac{1}{(N+1)^2}[(N-1)e_f'+(N+3)e_b']^2E_2^2$
$\|N\{2,0\}(\nu_i,0)L_iJ_i\rangle$	$\|N\{1,1\}(1,1)L_fJ_f\rangle$	$(e_f'-e_b')^2E_2^2$	$\frac{N-1}{N+1}(e_f'-e_b')^2E_2^2$
$\|N\{1,1\}(1,1)L_iJ_i\rangle$	$\|N\{1,1\}(1,1)L_fJ_f\rangle$	$(e_f'+e_b')^2E_2^2$	$(e_f'+e_b')^2E_2^2$

U(6/m) (Ref. 5), where $\mathcal{N}=N+M$ is the total number of bosons and fermions. Subsequently, these states can be divided into several groups of states which are labeled by a fixed value of the number of bosons, N, and the number of fermions, M. The allowed values of N and M are given by the branching rule:[5]

$$(N,M)=(N=\mathcal{N},M=0)+(N=\mathcal{N}-1,M=1)+\cdots$$
$$+(N=\mathcal{N}-m,M=m),\quad m\le\mathcal{N}$$
$$+(N=0,M=\mathcal{N}),\quad m\ge\mathcal{N}\,. \tag{3.2}$$

The terms on the right-hand side of Eq. (3.2) correspond to the collective states in an even-even nucleus with $N=\mathcal{N}$, $M=0$, the one quasiparticle states in an odd-mass nucleus with $N=\mathcal{N}-1$, $M=1$, the two quasiparticle states in an even-even nucleus with $N=\mathcal{N}-2$, $M=2$, etc., . . . Whenever the Hamiltonian is written in terms of Casimir invariants of the group chains (3.1), a dynamical supersymmetry arises. We note that, in the present case, the first term in Eq. (3.2) corresponds to the U(5) limit of the IBM (Ref. 13), while the second term corresponds to the U(5)×SU(2) limit of the IBFM, Sec. II.

The excitation energies of the states in the even-even nucleus with $N=\mathcal{N}$, $M=0$ are given explicitly, as functions of the parameters of Eq. (2.5), by:

$$E^{(\mathrm{I})}=(A+B)n_1+(A'+B')n_1(n_1+4)+C\nu_1(\nu_1+3)$$
$$+(D+E)L(L+1)\,, \tag{3.3a}$$
$$E^{(\mathrm{II})}=E^{(\mathrm{I})}\,, \tag{3.3b}$$
$$E^{(\mathrm{III})}=B_0n_1+B'n_1(n_1+4)+C\nu_1(\nu_1+3)$$
$$+(D+E)L(L+1)\,. \tag{3.3c}$$

Those of the states in the odd-mass nucleus with $N=\mathcal{N}-1$, $M=1$ are given by Eq. (2.5). Furthermore, in a supersymmetric scheme, all nuclei belonging to the same supermultiplet $[\mathcal{N}\}$ are described by the corresponding energy formulae with the *same* values of the parameters for all members of the supermultiplet. Similarly, the electromagnetic transition rates are described by one transition operator with the *same* values of the coefficients for all members of the supermultiplet $[\mathcal{N}\}$.

IV. POSSIBLE EXAMPLES OF U(5)×SU(2) BF SYMMETRIES AND THEIR ASSOCIATED SUPERSYMMETRIES: THE NUCLEI ^{76}Se AND ^{75}As

In this section, we examine to what extent the available experimental data on the nuclei ^{76}Se and ^{75}As—and, to some extent, ^{74}Se and ^{73}As, as reached by the (p,t) reaction on ^{76}Se and ^{75}As—can be reproduced by the U(5)×SU(2) BF symmetries and associated supersymmetries described in Secs. II and III. The data have been taken from the Nuclear Data Sheets,[15–18] supplemented by more recent results.[19,22,23]

In the BF symmetries of the IBFM, the odd-proton nuclei ^{73,75}As are described by coupling the odd proton to even-even cores. In a supersymmetric scheme, the nuclei $^{76}_{34}\mathrm{Se}_{42}$ ($N=7$, $M=0$) and $^{75}_{33}\mathrm{As}_{42}$ ($N=6$, $M=1$) belong to the same supermultiplet with $\mathcal{N}=7$, and so do $^{74}_{34}\mathrm{Se}_{40}$ ($N=6$, $M=0$) and $^{73}_{33}\mathrm{As}_{40}$ ($N=5$, $M=1$) with $\mathcal{N}=6$. This follows from the rules of supersymmetry, wherein the creation of a (proton) fermion is associated with the annihilation of a (proton) boson, both with respect to the ($Z=$) 28 magic number. The nuclei $^{74}_{32}\mathrm{Ge}_{42}$ ($N=6$, $M=0$) and $^{72}_{32}\mathrm{Ge}_{40}$ ($N=5$, $M=0$) belong to different supermultiplets, with $\mathcal{N}=6$ and 5, respectively.

The approximate validity of these BF symmetries and supersymmetries depends on the extent with which (i) the even-even nuclei can be described by the U(5) dynamical symmetry of the IBM,[13] and (ii) the odd nucleon of the odd-A nuclei can be confined to s.p. orbits with $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$. We shall show that the nucleus ^{76}Se approximately fulfills the first condition, for what concerns the energies of its excited states (Fig. 2) and the $B(E2)$'s between its low-lying levels (Fig. 3). The ^{74}Ge(^{3}He,d)^{75}As and ^{76}Se(d,^{3}He)^{75}As data[16,19] indicate that the lowest-lying orbits available to the 3rd proton of ^{75}As are $2p\frac{3}{2}^-$ (mainly concentrated in the ground state), $1f\frac{5}{2}^-$ (the 279.5 keV level), and $2p\frac{1}{2}^-$ (the 468.8 keV level); the $1g\frac{9}{2}^+$ orbit (the 303.9 keV level) has opposite parity, and the $1f\frac{7}{2}^-$ proton-hole orbit (partly the 821.6 keV level) plays a minor role at low excitation energy.

As pointed out in Sec. II, there are many similarities between the three U(5)×SU(2) symmetries considered in the present paper: the levels of limit I coincide with part of the spectrum in limit II; their properties are identical in limits I and II; the energy spectra predicted by limits II and II′ may be identical in the low energy region by appropriate choices of the parameters; and many of their electromagnetic properties have the same geometrical dependence (i.e., the factors F_2 and E_2) in limits II and II′ (Tables I and II), and only differ by factors which depend on N, e_b, e_f, e_b' and e_f'. There are some differences, however: the existence of other (starred) levels in limit II with respect to limit I (Fig. 1); the larger number of parameters in the energy formulae for limits I and II [eight in Eqs. (2.5a) and (2.5b)] than for limit II′ [six in Eq. (2.5c)]; and the existence of transitions which are forbidden in limit II and allowed in limit II′ (Tables I and II).

We compare, in Fig. 2, the experimental spectra of ^{76}Se and ^{75}As (Refs. 15 and 16) with those calculated for the following cases: the symmetry U(5) for ^{76}Se alone [parameters $A+B,A'+B',C,D+E$ for limits I and II in Eqs. (3.3a) and (3.3b) or, equivalently, $B_0,B',C,D+E$ for limit II′ in Eq. (3.3c)]; limits II *and* II′ of U(5)×SU(2) for ^{75}As alone [only the nonstarred levels with $n_1+n_2\le 2$ and the lowest $\frac{1}{2}^*$ state in Fig. 1; parameters $A+5A'+A'',B,B',C,D,E$ for limit II in Eq. (2.5b), or, equivalently, A_0,B_0,B',C,D,E for limit II′ in Eq. (2.5c)]; and the supersymmetry U(6/12) with limits II *or* II′ for ^{76}Se and ^{75}As together [parameters A,A',A'',B,B',C,D,E for limit II in Eqs. (2.5b) and (3.3b), parameters A_0,B_0,B',C,D,E for limit II′ in Eqs. (2.5c) and (3.3c)]. We have assumed that the experimental levels at 585, 617.7, 865, and 886.0 keV in ^{75}As correspond to the calculated $\frac{1}{2}^-\{2,0\}(0,0)0$, $\frac{3}{2}^-\{2,0\}(2,0)2$, $\frac{7}{2}^-\{1,1\}(1,1)3$, and $\frac{5}{2}^-\{2,0\}(2,0)2$ states, respectively; this is compatible with the experimental data on these levels,[16] but not im-

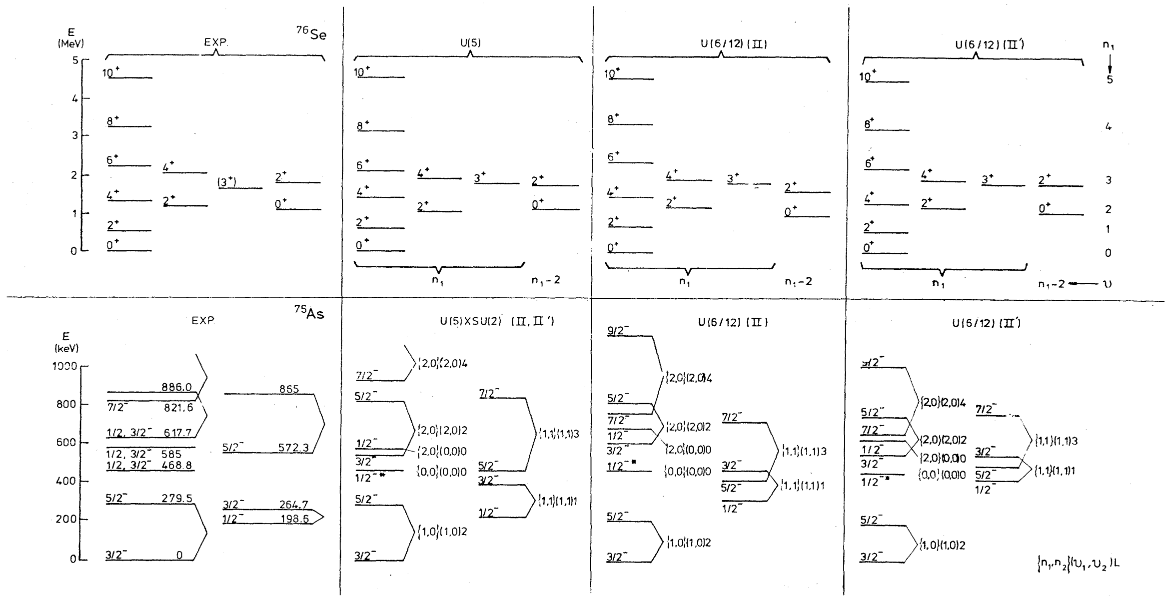

FIG. 2. Comparison between the experimental (Refs. 15 and 16) and calculated spectra of ^{76}Se and ^{75}As. The hypotheses underlying the calculations and the significance of the corresponding quantum numbers are outlined in Secs. II to IV. The parameters used in Eqs. (2.5b) and (3.3b) are the following (in keV): for U(5), $A+B=494$, $A'+B'=11$, $C=-6$, $D+E=15$; for U(5)×SU(2) (limit II), $A+5A'+A''=940$, $B=-823$, $B'=69$, $C=-8$, $D=-21$, $E=56$; for U(5)×SU(2) (limit II'), $A_0=67$, $B_0=117$, $B'=69$, $C=-8$, $D=-21$, $E=56$; for U(6/12) (limit II), $A=1381$, $A'=-79$, $A''=-63$, $B=-768$, $B'=69$, $C=-6$, $D=-26$, $E=44$; for U(6/12) (limit II'), $A_0=66$, $B_0=357$, $B'=31$, $C=-4$, $D=-25$, $E=38$. The experimental energies in ^{75}As are given in keV. The quantities ϕ (%), σ (keV), and $n-k$, defined in Sec. IV [Eqs. (4.1) and (4.2)], have the following values: for U(5), 2%, 73 keV, 10-4; for U(5)×SU(2) (limits II and II'), 9%, 109 keV, 10-6; for U(6/12) (limit II), 6%, 117 keV, 20-8; for U(6/12) (limit II'), 7%, 135 keV, 20-6. Only the calculated levels which are included in the fits are shown in the figure, with the exception of the calculated $\frac{9}{2}^-$ state in ^{75}As.

posed by them. The quality of the fits is quantified by the values of ϕ and σ:

$$\phi=[\sum |E_{\text{expt.}}-E_{\text{calc}}| / \sum E_{\text{expt.}}]\ (\%)\ , \qquad (4.1)$$

$$\sigma=[\sum (E_{\text{expt.}}-E_{\text{calc}})^2/(n-k)]^{1/2}\ (\text{keV})\ , \qquad (4.2)$$

where n is the number of levels included in the fit, k the number of parameters, and $n-k$ the number of degrees of freedom.

The comparison performed in Fig. 2 for ^{76}Se alone first justifies the above-mentioned statement on the approximate validity of the U(5) symmetry for the description of this nucleus. It further shows that all the experimental levels in ^{75}As below 1 MeV excitation energy, which do not have an experimentally established positive parity, can approximately be described by limits II and II' of the U(5)×SU(2) symmetries. All of them, except the $\frac{1}{2}^-$ state at 468.8 keV, correspond to the nonstarred levels below 0.6 MeV and with $n_1+n_2\leq 2$ in Fig. 1; as to the 468.8 keV state, it can be associated with the lowest starred $\frac{1}{2}^*$ level in Fig. 1, which can be brought higher in the calculated spectrum than shown in Fig. 1 by an appropriate choice of the parameters in Eqs. (2.5b) and (2.5c). There is just one missing level, the first $\frac{9}{2}^-$ state, predicted above 1 MeV. The symmetries U(5) and limits II and II' of U(5)×SU(2) are thus able to reproduce reasonably well, with $\phi\sim 2\%$ to 9% and $\phi\sim 73$ to 109 keV, the low-lying spectra of ^{76}Se and ^{75}As, considered separately. On the other hand, the fits to ^{76}Se and ^{75}As, considered separately and together, display comparable values of ϕ and σ, a fact which suggests that these two nuclei can be associated together in a supermultiplet of the U(6/12) supersymmetry, at least for what concerns the excitation energies of their low-lying levels. There are no striking differences between the fits obtained for limits II and II'.

It should be noticed, at this point, that the number of parameters entering into the calculated energies [Eqs. (2.5) and (3.3)] is rather large (between four and eight for the fits of Fig. 2). This is in contrast with the U(6/4) supersymmetry,[5] and is inherent to the U(5) dynamical symmetry of the IBM (Ref. 13) and its associated boson-fermion symmetries and supersymmetries.[7,10,20] Consequently, the reasonable agreement obtained for the energies could just be the result of a parameter game. It is thus very important to compare the experimental data and the theoretical predictions for other observables than the energies alone, such as the reduced $E2$ transition probabilities and the

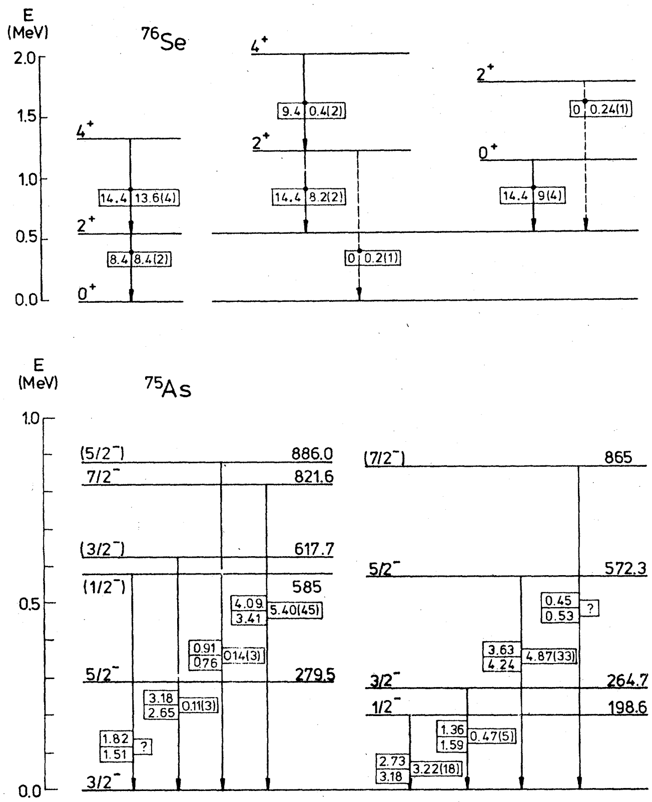

FIG. 3. Comparison between the experimental (Refs. 15 and 16) and calculated [limits II and II′ of U(5)×SU(2)] values of the $B(E2)$'s in ^{76}Se and ^{75}As. The latter are given to the left of each transition, and the former, to the right, with the experimental uncertainties indicated into parentheses as affecting the last digit(s). The upper (lower) calculated value for ^{75}As corresponds to limit II (II′). The experimental $B(E2)$'s are given in $10^{-2}\ e^2b^2$. The normalizations of the calculated $B(E2)$'s are defined in Sec. IV and correspond to $e_b=0.110$ and 0.087 e b for ^{76}Se and ^{75}As, respectively. The assignments of experimental to calculated levels is the one suggested in Fig. 2, and the assumed spins are indicated in parentheses.

one- and two-nucleon transfer intensities, which are more sensitive probes of the wave functions than the energies.

The situation with respect to the $B(E2)$'s is shown in Fig. 3 for ^{76}Se and ^{75}As separately. The calculated $B(E2)$'s with the $E2$ operator given in Eq. (2.8) have been normalized in the following way. The calculated $B(E2,2_1^+\to 0_1^+)$ in ^{76}Se is equal to Ne_b^2 for U(5); it has been made identical to the experimental value[15] $0.084\,e^2b^2$. The sum of the calculated $B(E2)$'s from the first and second $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ nonstarred levels with $n_1+n_2=2$ in Fig. 1, to the $\frac{3}{2}^-$ ground state in ^{75}As, is equal to $3Ne_b^2$ for limit II, and also for limit II′ if one takes $e_f=e_b$ (Table I); it has been normalized to the experimental value 0.136 e^2b^2 as measured in Coulomb excitation,[16] with the above-mentioned assignments of experimental levels to calculated states. The comparison performed in Fig. 3 for ^{76}Se strengthens the argument for the approximate applicability of the U(5) dynamical symmetry of the IBM to this nucleus, with the exception of the second 4^+ experimental level which is probably not the 4^+ $(n_B=3)$ calculated state. Concerning ^{75}As, limits II and II′ of the U(5)×SU(2) symmetries predict "strong" $B(E2)$'s between the $\frac{1}{2}^-\{1,1\}(1,1)1$, $\frac{3}{2}^-\{2,0\}(2,0)2$, $\frac{5}{2}^-\{1,1\}(1,1)3$, $\frac{7}{2}^-\{2,0\}(2,0)4$ levels and the $\frac{3}{2}^-\{1,0\}(1,0)2$ ground state, and "weak" $B(E2)$'s between the $\frac{1}{2}^-\{2,0\}(2,0)0$, $\frac{3}{2}^-\{1,1\}(1,1)1$, $\frac{5}{2}^-\{2,0\}(2,0)2$, $\frac{7}{2}^-\{1,1\}(1,1)3$ levels and the $\frac{3}{2}^-$ ground state (Table I). The experimental data on the $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ levels shown in Fig. 3 roughly confirm this prediction: there is a semiquantitative agreement for the "strong" $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ levels, at 198.6, 572.3, and 821.6 keV, respectively, and only a qualitative agreement for the "weak" $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ levels, at 585, 886.0, and 865 keV, respectively. On the other hand, the first and second excited $\frac{3}{2}^-$ levels, at 264.7 and 617.7 keV, respectively, are much less populated in Coulomb excitation from the $\frac{3}{2}^-$ ground state than predicted. It should be pointed out that the $B(E2)$'s for the transitions shown in Fig. 3, as calculated in limits II and II′, only involve *one* parameter, the effective boson charge e_b (if $e_f=e_b$ in limit II′). As a consequence, the comparison between the experimental and calculated $B(E2)$ *ratios* in Fig. 3 is parameter free.

The normalizations adopted in Fig. 3 allow one to determine the quantity Ne_b^2 (Table II) for ^{75}As $(N=6)$ and ^{76}Se $(N=7)$ separately, yielding 0.045(2) and 0.084(2) e^2b^2, respectively, which correspond to $e_b=0.087(2)$ and 0.110(1) e b, respectively. These results are in approximate agreement with the prediction of the U(6/12) supersymmetry that the effective boson charge e_b should be the same for the two members of the same supermultiplet ^{76}Se and ^{75}As, but that the number N of bosons in ^{75}As (6) is one unit smaller than in ^{76}Se (7). This remark fits into the general trend of $B(E2)$'s in odd-A and even-even nuclei pertaining to the same supermultiplet, as pointed out recently.[21]

The calculated quadrupole moment of the $\frac{3}{2}^-$ ground state of ^{75}As and the value of $B(E2)$ between the first $\frac{5}{2}^-$ level and the ground state, both characterized by $\{1,0\}(1,0)2$, only depend on the effective fermion charge e_f' for limit II, and for limit II′ if one takes $e_f'=e_b'$ (Table II). With $e_f'=0.273$ e b, the calculated values of these two quantities are the following: $Q(\frac{3}{2})=0.324$ e b, $B(E2,\frac{5}{2}^- - \frac{3}{2}^-)=0.0149\ e^2b^2$, in good agreement with the experimental values:[16] 0.29 e b, 0.0183(24) e^2b^2. This means that they can both be reproduced with *one* parameter, e_f'. The experimental value of

$$B(E2,\tfrac{1}{2}^* - \tfrac{3}{2}^-_{\rm g.s.})=0.66(8)\times 10^{-3}\ e^2b^2$$

(Ref. 16) is very small. It can be exactly predicted in limit II by a suitable choice of the parameter e_f (Table I), i.e., $e_f=0.000\,91(6)$ e b, and it is predicted to vanish in limit II′ if one takes $e_f=e_b$ (Table I), as chosen above.

The above discussion of the electric quadrupole properties in ^{76}Se and ^{75}As does not allow a distinction between limits II and II′ of the U(5)×SU(2) and U(6/12) symmetries.

In the (p,t) reaction on ^{76}Se and ^{75}As, the $L=0$ transfers to the 0^+ and $\frac{3}{2}^-$ ground states of ^{74}Se and ^{73}As, respectively, are the only ones allowed if ^{76,74}Se and ^{75,73}As can be described by the U(5) and U(5)×SU(2) (limits I and II) symmetries, respectively. Experimentally,[22,23] they are indeed much stronger than to the other 0^+ and $\frac{3}{2}^-$ levels, by factors between 17 and 67 (Table III). The $L=2$ transfers are only allowed to the first 2^+ level in ^{74}Se, forbidden to the $\frac{5}{2}^-\{1,0\}(1,0)2$ level and allowed to the first and second nonstarred $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ levels in ^{73}As for limits I and II. This is roughly confirmed by the experimental data[22,23] (Table III), if the $\frac{5}{2}^-$ state at 67.0 keV in ^{73}As is assigned to the above-mentioned $\frac{5}{2}^-$ level, as suggested by the ^{72}Ge(^{3}He,d)^{73}As and ^{74}Se(d,^{3}He)^{73}As results.[18,19]

The comparison between the experimental data and the theoretical predictions in limits II and II′ (nonstarred levels with $n_1+n_2\leq 2$ and lowest $\frac{1}{2}^*$ state in Fig. 1) for the one-proton transfer reactions (^{3}He,d) and (d,^{3}He) between ^{76}Se, ^{75}As, and ^{74}Ge is shown in Fig. 4. For these limits and these levels, the only allowed transitions are to the $\frac{3}{2}^-$ and $\frac{5}{2}^-\{1,0\}(1,0)2$, and $\frac{1}{2}^{-*}\{0,0\}(0,0)0$ levels of ^{75}As in the ^{76}Se(d,^{3}He)^{75}As and ^{74}Ge(^{3}He,d)^{75}As reactions; to the 0^+ ground state (limit II) and also to the first excited 2^+ level (limit II′) of ^{74}Ge in the ^{75}As(d,^{3}He)^{74}Ge reaction; to the 0^+ ground state and first excited 2^+ level (limit II) and also to the first excited 0^+ and 4^+ and second excited 2^+ levels (limit II′) of ^{76}Se in the ^{75}As(^{3}He,d)^{76}Se reaction. In the experimental data on the former two reactions,[15,19] the $\frac{3}{2}^-$ ground state, the $\frac{5}{2}^-$ level at 279.5 keV, and the $\frac{1}{2}^-$ level at 468.8 keV (associated to $\frac{1}{2}^*$) in ^{75}As are indeed favored with respect to most other $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ states. There are, however, strong transitions to the $\frac{3}{2}^-$ level at 264.7 keV (about 60% of the ground state transitions), which should be forbidden, and indications that the $\frac{7}{2}^-$ level at 821.6 keV contains part of the $1f\frac{7}{2}^-$ proton-hole orbit. In the ^{75}As(^{3}He,d)^{76}Se reaction,[13] the $l_p=1$ transition to the 0^+ ground state of ^{76}Se is strong. Moreover, the first excited 2^+ level is mainly populated by a strong $l_p=3$ transfer with some $l_p=1$ contribution, the first excited 0^+ level, by an $l_p=1$ transition with half the strength of the one to the 0^+ ground state, the second excited 2^+ level, by a mixed $l_p=1$ and 3 transfer, and the first excited 4^+ level, by a sizable $l_p=3$ transfer. The latter features are unexplained by limit II, but are in qualitative agreement with the predictions of limit II′. The strong $l_p=1$ and 3 transfers to the third 2^+ level do not agree with either limits II or II′. There are no experimental data available on the ^{75}As(d,^{3}He)^{74}Ge reaction.

V. DISCUSSION AND CONCLUSION

In the present paper, we have investigated in detail the properties of three U(5)×SU(2) boson-fermion symmetries in the IBFM and their associated supersymmetries, which involve a U(5) even-even core, and two or three single-particle orbits, with $j=\frac{3}{2}$ and $\frac{5}{2}$ or $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$. We have shown, in particular, that there are many features common to these three symmetries, concerning both the excitation energies and the level properties. As a consequence, a "simpler" symmetry with 2 s.p. orbits (limit I) may, to some extent, be considered as a particular case of a "more involved" one with 3 s.p. orbits (limit II),

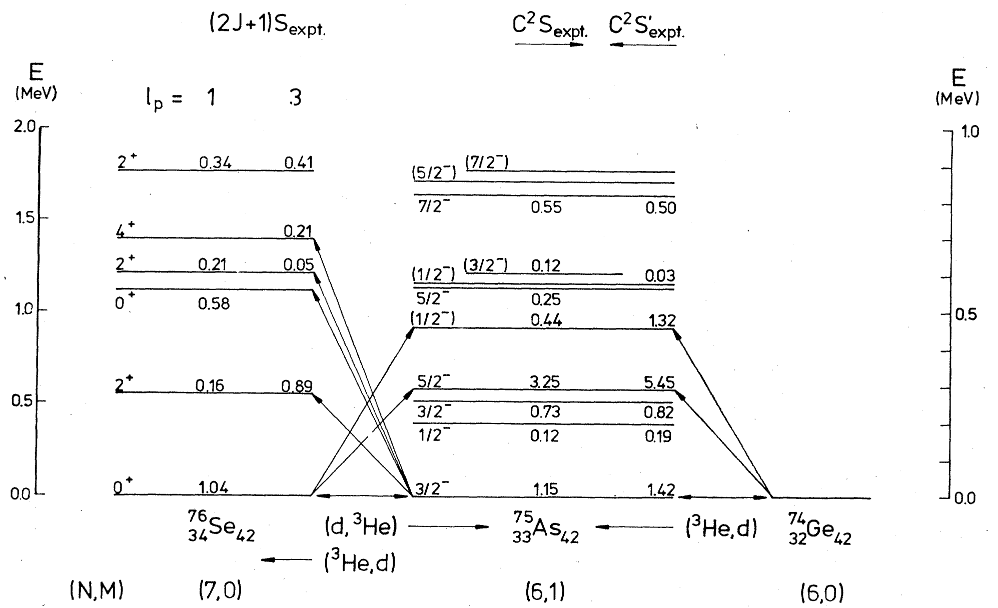

FIG. 4. Comparison between the experimental (Refs. 15, 16, and 19) and calculated [limits II and II′ of U(5)×SU(2)] one-proton transfer intensities between ^{76}Se, ^{75}As, and ^{74}Ge. The experimental data given in the ^{76}Se decay scheme deal with the ^{75}As(^{3}He,d)^{76}Se reaction, with an orbital angular momentum transfer l_p; those in the ^{75}As decay scheme deal with the ^{76}Se(d,^{3}He)^{75}As (left) and ^{74}Ge(^{3}He,d)^{75}As (right) reactions. The assignment of experimental to calculated levels is the one suggested in Fig. 2, and the assumed spins are indicated in parentheses. The energy scale on the left is valid for ^{76}Se, and on the right, for ^{75}As. The theoretically allowed transfers are marked by arrows.

TABLE III. Experimental data (Refs. 22 and 23) on the (p,t) reaction on ^{76}Se and ^{75}As. L is the orbital angular momentum of the transferred neutron pair. The energies of the final levels are given in keV. The cross sections σ are given in the papers (Refs. 22 and 33) and only indicate the qualitative behavior. The notation (NO) means not observed.

L		^{76}Se(p,t)^{74}Se (Ref. 23)	σ
0	0^+	$0^+(0)$	202.3
		$0^+(2129)$	6.1
		$0^+(2713)$	9.7
		$0^+(2920)$	11.8
2	0^+	$2^+(634)$	52.0
		$2^+(1260)$	10.5
		$2^+(3630)$	26.4
L		^{75}As(p,t)^{73}As (Ref. 22)	σ
0	$\frac{3}{2}^-$	$\frac{3}{2}^-(0)$	100
		$\frac{3}{2}^-(396)$	2.4
		$\frac{3}{2}^-(656)$	3
		$\frac{3}{2}^-(1595)$	1.5
2	$\frac{3}{2}^-$	$\frac{5}{2}^-(67.0)$	NO
		$\frac{1}{2}^-(84.5)$	NO
		$\frac{1}{2}^-(256)$	5
		$\frac{5}{2}^-(579)$	20
		$\frac{7}{2}^-(859)$	22
		$(\frac{3}{2}^-)(995)$	13.5

and the results obtained in the former have a more general validity in the framework of the latter. The same remark holds for other boson-fermion symmetries and supersymmetries.

When comparing these theoretical results with the experimental data on the nuclei ^{76}Se and ^{75}As, an approximate agreement is obtained, with, however, some discrepancies. The general structure of the negative parity levels in ^{75}As below 1 MeV resembles the one predicted by the U(5)×SU(2) symmetries, with, however, a large number of parameters in the energy formulae. The "collectivelike"properties, i.e., the quadrupole moments and $B(E2)$'s and the two-nucleon transfer reactions, display an agreement which is generally satisfactory, and which involves few parameters. The "single-particlelike" properties, i.e., the one-nucleon transfer reactions, face more problems, and some "forbidden" transfers are comparable to the "allowed" ones. The experimental data do not allow a clear-cut distinction between limits II and II′ of the U(5)×SU(2) symmetries, although the results of the ^{75}As(^{3}He,d)^{76}Se reaction slightly favor limit II′. Concerning supersymmetries, the excitation energies and electric quadrupole properties of ^{76}Se and ^{75}As can satisfactorily be described together by placing these nuclei in the same supermultiplet. These general conclusions are also valid for other boson-fermion symmetries and supersymmetries.

ACKNOWLEDGMENTS

Interesting discussions with K. Heyde are gratefully acknowledged. P.V.I. thanks the NFWO (National Fonds voor Wetenschappelijk Onderzoek) and J.J. thanks the IWONL (Instituut tot aanmoediging van het Wetenschappelijk Onderzoek in Nijverheid en Landbouw) for financial support.

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