Temperature dependence of collective states in hot nuclei

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The $T=0$ coordinate-space linear response random-phase approximation method is generalized to finite temperatures. Continuum effects are included through the temperature single-particle Green's functions. The method is utilized to investigate the temperature dependence of isovector electric dipole excitations in ^{40}Ca. The experimentally observed downward shift of the giant resonance and the spreading of the dipole strength as temperature increases are clearly reflected in the calculated cross sections. The present approach to the temperature dependent random-phase approximation method offers a remedy to some of the limitations inherent in other existing formulations of the theory.

I. INTRODUCTION

Strong experimental evidence for the existence of giant resonances in hot nuclei has been found recently in γ ray spectra associated with deep-inelastic nuclear collisions.[1–5] A broadening as well as a downward shift of the giant dipole resonance (GDR) as excitation energy increases, characterized the behavior of the measured $E1$ strength.

The rapid equilibration of the transferred energy in deep-inelastic collisions inferred from experiment[6] justifies the incorporation of statistical mechanics methods in a theoretical model for the behavior of a hot nucleus. A few theoretical investigations into the temperature dependence of collective states in a hot nucleus have been reported recently.[7–10] Common to all these investigations is the use of the temperature dependent random-phase approximation (TDRPA), which constitutes an extension of the well-known zero temperature RPA equation to finite temperatures.[7,11,12]

The finite temperature RPA is usually formulated either as an eigenvalue problem to be solved in a limited configuration space[9] or else as an integral equation satisfied by a temperature particle-hole Green's function, whose poles and residues determine the response of the nucleus to an external field. A sophisticated generalization of the linear response method to include pairing and rotational degrees of freedom as well as temperature is the one recently reported by Ring *et al.*[8]

Irrespective of the manner of formulation, in practice the corresponding TDRPA equations are never used in their full capacity. To avoid a large amount of numerical work, the usual limitations such as a truncation of the configuration space, oversimplification of the particle-hole interaction, lack of self-consistency between the RPA calculation and the underlying single-particle field, and other limitations are quite crudely imposed. The end result of such a calculation is either a discrete set of states for photonuclear strength distribution (even above the particle threshold) or else a continuous spectrum which is designed by an arbitrary choice for the width of the various collective states. This approach seems to be quite sufficient for a qualitative investigation of the temperature dependence of collective states in a hot nucleus, and indeed the general features characterizing this dependence, such as downward shift of the GDR and its broadening, have been reproduced by Vautherin and Vinh-Mau[9] and by Ring *et al.*[8] However, the study of temperature dependence implies a rapid increase of the configuration space with temperature and an arbitrary truncation may distort the final results. Moreover, the lack of any consideration for the continuum prevents any possibility for a reliable conclusion regarding the widths of the various resonances.

The purpose of the present paper is to suggest an extension of the $T=0$ linear response method of Bertsch, Tsai, and Shlomo[13] and the open-shell linear response method (OSLRM) of Bar-Touv and Moalem[14] to finite temperatures as a remedy for some of the limitations of the existing TDRPA method. The main new ingredients added to the TDRPA approach will be a coordinate-space formulation of the TDRPA integral equation and the inclusion of the whole continuum. The TDRPA equations are thus solved by inversion of a matrix, whose dimensions are determined by the mesh in coordinate space rather than by the number of configurations included in the collective state. Moreover, the inclusion of the continuum through the temperature single-particle Green's function leads in a natural way to a continuous spectrum representing a detailed temperature dependence of the nuclear response. In Sec. II an outline of the present approach to continuum TDRPA in r space is given. Application of the model to ^{40}Ca isovector GDR and comparison with other calculations are discussed in Sec. III. Conclusions are summarized in Sec. IV.

II. AN OUTLINE OF CONTINUUM TDRPA METHOD IN r SPACE REPRESENTATION

The $T=0$ RPA integral equation is given by[13]

$$G^{\mathrm{RPA}}=G^{(0)}(1-V_{\mathrm{ph}}G^{(0)})^{-1}\,, \tag{1}$$

where V_{ph} is the particle-hole (ph) interaction, G^{RPA} the perturbed ph Green's function, and $G^{(0)}$ the unperturbed ph Green's function whose coordinate space spectral representation is

$$G^{(0)}(r_1,r_2,\omega)=-\sum_{\mathrm{p}}\sum_{\mathrm{h}}\varphi_{\mathrm{p}}^{*}(r_1)\varphi_{\mathrm{h}}^{*}(r_1)\left[\frac{1}{\epsilon_{\mathrm{p}}-\epsilon_{\mathrm{h}}+\omega}+\frac{1}{\epsilon_{\mathrm{p}}-\epsilon_{\mathrm{h}}-\omega}\right]\varphi_{\mathrm{p}}(r_2)\varphi_{\mathrm{h}}(r_2)\ . \tag{2}$$

$\varphi_{\mathrm{h}}(r)$ and ϵ_{h} stand for Hartree-Fock (HF) single-hole wave function and energy and $\varphi_{\mathrm{p}}(r)$ and ϵ_{p} the HF single-particle wave function and energy.

The temperature dependence may readily be carried through the RPA equations by the occupation numbers n_j of the Fermi distribution

$$n_j(T)=\{1+\exp[(\epsilon_j-\mu)/kT]\}^{-1}\ . \tag{3}$$

ϵ_j is the single particle energy of the temperature dependent HF field and μ the chemical potential. In such a generalization of the RPA equations each of the individual states j has a dual nature of being simultaneously a hole state with fractional occupation n_j and a particle state with fractional vacancy $(1-n_j)$. Thus the thermal unperturbed ph Green's function is determined by

$$G^{(0)}(r_1,r_2,\omega,T)=-\sum_{j}\sum_{j'}n_j(1-n_{j'})\varphi_j^{*}(r_1)\varphi_{j'}^{*}(r_1)\left[\frac{1}{\epsilon_{j'}-\epsilon_j+\omega}+\frac{1}{\epsilon_{j'}-\epsilon_j-\omega}\right]\varphi_j(r_2)\varphi_{j'}(r_2)\ , \tag{4}$$

where the summations on j and j' run over all single-particle states. The double sum which goes with the product term $n_jn_{j'}$ is antisymmetric with respect to an interchange of j and j'. Since the two indices run over the same range, this double sum vanishes identically and the thermal unperturbed Green's function reduces to

$$G^{(0)}(r_1,r_2,\omega,T)=-\sum_{j}n_j\varphi_j^{*}(r_1)\left[\sum_{j'}\varphi_{j'}^{*}(r_2)\left[\frac{1}{\epsilon_{j'}-\epsilon_j+\omega}+\frac{1}{\epsilon_{j'}-\epsilon_j-\omega}\right]\varphi_{j'}(r_2)\right]\varphi_j(r_2)\ . \tag{5}$$

This expression for $G^{(0)}$ looks formally like the corresponding expression of the OSLRM.[14] However, the basic difference between the two is in the nature of the occupation numbers. The OSLRM occupations θ_j were single-particle parameters chosen to represent effectively a complex configuration mixing in the ground state wave function, while in the TDRPA they are the carriers of the temperature dependence. Expression (5) for $G^{(0)}$ may be given a form which is closer to the existing formulations of the TDRPA method[7–9] simply by changing the dummy indices of the second term to arrive at

$$G^{(0)}(r_1,r_2,\omega,T)=-\sum_{jj'}(n_j-n_{j'})\varphi_j^{*}(r_1)\varphi_{j'}^{*}(r_1)\times\frac{1}{\epsilon_{j'}-\epsilon_j+\omega}\varphi_j(r_2)\varphi_{j'}(r_2)\ . \tag{6}$$

It is important to bear in mind that the occupation numbers n_j of Eq. (5) are not the only carriers of temperature dependence. In fact, each factor in the expression for $G^{(0)}$ has its own temperature dependence originating from the temperature dependent HF equations (TDHF). The implied self-consistency between the TDHF and TDRPA equations is often neglected due to an assumed smooth and weak dependence of the HF field on temperature. As it turns out, a smooth dependence of the HF single-particle energies is in opposing directions for energies below and above the Fermi level, and with increasing temperature the effect on the occupation numbers n_j due to these variations may be significant.

The relevant approach to the thermal HF self-consistent field relies on the simplicity brought to the r space solution of the equations for zero-range Skyrme-type forces.[15] The TDHF equations are

$$h[\varphi_j]\varphi_i=\epsilon_i\varphi_i\ , \tag{7}$$

where $h[\varphi_i]$, the TDHF single-particle Hamiltonian, is a functional of its own eigenstates. The temperature dependence propagates in these self-consistency equations through the Hamiltonian density out of which the equations are derived by the variational condition. This Hamiltonian density is expressed in terms of three local temperature dependent densities: the nucleon density

$$\rho(\mathbf{r},T)\equiv\sum_{j}n_j(T)\,|\,\varphi_j(\mathbf{r},T)\,|^2\ , \tag{8}$$

the kinetic energy density

$$\tau(\mathbf{r},T)\equiv\sum_{j}n_j(T)\,|\,\nabla\varphi_j(\mathbf{r},T)\,|^2\ , \tag{9}$$

and the spin density

$$\mathbf{J}(\mathbf{r},T)=-i\sum_{j}n_j(T)\varphi_j^{*}(\mathbf{r},T)[\nabla\varphi_j(\mathbf{r},T)\times\sigma]\ . \tag{10}$$

The iterative process[15] used for solving Eq. (7) includes the constraints on the number of protons and neutrons

$$Z=\sum_{j}\tfrac{1}{2}(1-\tau_{3j})n_j \tag{11}$$

and

$$N=\sum_{j}\tfrac{1}{2}(1+\tau_{3j})n_j\ , \tag{12}$$

τ_{3j} being the third component of the isospin of particle j. These two constraints determine, as usual, the chemical potentials μ_π and μ_ν for protons and neutrons.

Having the TDHF single-particle basis and occupation numbers, one can proceed to the TDRPA equations. The unperturbed ph Green's function $G^{(0)}$ is calculated by Eq. (5) and the perturbed Green's function G^{RPA} by Eq. (1).

The ph interaction $V_{\rm ph}$ is identified according to Landau's theory of Fermi liquids with the second derivative of the energy functional with respect to the nuclear density

$$V_{\rm ph}=\delta^2 E/\delta\rho^2 \ . \tag{13}$$

In including all the continuum states in the TDRPA equations we follow Shlomo and Bertsch[13] in replacing the sum over j' in Eq. (5) with the TDHF Green's function for a single particle propagating from r_1 to r_2:

$$g_{lj}(r_1,r_2,E,T)=\frac{1}{H_0-E}=-\frac{2m}{\hbar^2}u_{lj}(r_<)v_{lj}(r_>)/W \ . \tag{14}$$

u_{lj} is the regular solution for the TDHF Hamiltonian H_0 for a wave l_j, v_{lj} is the irregular solution with the appropriate boundary conditions, $r_<$ and $r_>$ in this equation denote the lesser and the greater of r_1 and r_2, respectively, and W is the Wronskian of the two solutions. Using (14), G^0 will be determined by

$$G^{(0)}(r_1,r_2,\omega,T)=-\sum_j n_j\varphi_j^*(r_1)[g(r_1,r_2,\epsilon_j+\omega)-g(r_1,r_2,\epsilon_h-\omega)]\varphi_j(r_2) \ . \tag{15}$$

The sum over j has still to cover all single-particle states. However, due to the coefficients n_j, this sum is automatically limited to fully and partially occupied states of the TDHF single-particle field. The number of partially occupied states having a significant value for n_j obviously increases with temperature. Still this number stays quite limited if we choose to exclude states with n_j smaller than some value (e.g., $n_j<10^{-3}$) and renormalizes other n_j in order to obey the constraints on the number of protons and neutrons exactly.

Once $G^{(0)}$ and $G^{\rm RPA}$ are determined, the response of the nucleus to an external field described by the single-particle operator F is given by[13]

$$R_F(E,T)=\langle FGF\rangle = \int dr_1 dr_2 F(r_1)F(r_2)G(r_1,r_2,E,T) \tag{16}$$

and transition strength by

$$S_F(E,T)=\frac{1}{\pi}\mathrm{Im}\langle FGF\rangle \ . \tag{17}$$

III. TEMPERATURE DEPENDENCE OF GDR IN ^{40}Ca

We now turn to the application of the present approach to continuum TDRPA in r space. In order to minimize a possible interplay between pairing and rotational degrees of freedom and temperature, we choose to apply the present model to the spherical nucleus ^{40}Ca. The $T=0$ GDR in ^{40}Ca is experimentally well known; however, experimental data concerning GDR in a hot ^{40}Ca are not available. Yet ^{40}Ca may serve as a relatively simple system for comparing the present approach to TDRPA with other formulations. For that purpose we find the analysis of the temperature dependence of ^{40}Ca GDR reported recently by Vautherin and Vinh-Mau[9] most useful.

A. TDHF field in ^{40}Ca

The collective behavior of a hot nucleus is obviously intimately related to the changes in the underlying self-consistent single-particle field with temperature. Thus, to make the present discussion self-contained we describe the main features of that field which are most relevant to a GDR in ^{40}Ca. A more general discussion of the TDHF field in nuclear matter and in finite nuclei may be found in the paper of Sauer, Chandra, and Mosel[18] and in references therein. For that matter we solved ^{40}Ca TDHF equations for a Skyrme I force in a model space which included the $1s$, $1p$, $2s\,1d$, $2p\,1f$, and $3s\,2d\,1g$ shells. The temperature dependence of single-particle energies and occupations related to the dominant ph components of isovector GDR in ^{40}Ca are given in Tables I and II. The

TABLE I. ^{40}Ca TDHF solutions for neutrons: ϵ_j^ν for single-particle energy (in MeV), n_j^ν for occupation, μ_ν for chemical potential (in MeV), and $\Delta\nu$ for the number of neutrons lifted out of the $T=0$ core at temperature T (in MeV).

	$T=0$		$T=2$		$T=4$		$T=6$	
	ϵ_j^ν	n_j^ν	ϵ_j^ν	n_j^ν	ϵ_j^ν	n_j^ν	ϵ_j^ν	n_j^ν
$1s_{1/2}$	40.95	1.0	42.12	1.0	42.38	1.0	42.07	0.991
$1p_{3/2}$	31.83	1.0	32.32	1.0	32.35	0.991	31.95	0.951
$1p_{1/2}$	27.94	1.0	28.94	1.0	29.10	0.979	28.64	0.918
$1d_{5/2}$	21.54	1.0	21.64	0.986	21.54	0.878	21.05	0.759
$2s_{1/2}$	15.22	1.0	15.97	0.807	16.25	0.656	15.99	0.575
$1d_{3/2}$	14.35	1.0	14.96	0.715	15.10	0.589	14.71	0.523
$1f_{7/2}$	10.39	0	10.26	0.193	10.06	0.289	9.49	0.315
$2p_{3/2}$	4.05	0	4.46	0.013	4.56	0.093	4.42	0.165
$2p_{1/2}$	1.80	0	2.46	0.005	2.73	0.061	2.68	0.129
$1f_{5/2}$	0.66	0	1.17	0.000	1.15	0.042	0.99	0.100
μ_ν			13.11		13.65		14.16	
Δ_ν			1.608		3.146		4.581	

TABLE II. ^{40}Ca TDHF solutions for protons: ϵ_j^π for single-particle energy (in MeV), n_j^π for occupation, μ_π for chemical potential (in MeV), and $\Delta\pi$ for the number of protons lifted out of the $T=0$ core at temperature T (in MeV).

	$T=0$		$T=2$		$T=4$		$T=6$	
	ϵ_j^π	n_j^π	ϵ_j^π	n_j^π	ϵ_j^π	n_j^π	ϵ_j^π	n_j^π
$1s_{1/2}$	34.26	1.0	34.76	1.0	35.10	1.0	35.34	0.989
$1p_{3/2}$	24.77	1.0	24.82	1.0	24.99	0.989	25.14	0.944
$1p_{1/2}$	20.88	1.0	21.37	1.0	21.68	0.976	21.80	0.907
$1d_{5/2}$	14.44	1.0	14.28	0.984	14.39	0.866	14.42	0.740
$2s_{1/2}$	9.10	1.0	9.47	0.845	9.88	0.677	10.02	0.578
$1d_{3/2}$	7.38	1.0	7.72	0.695	8.06	0.571	8.19	0.502
$1f_{7/2}$	3.44	0	3.18	0.191	3.21	0.283	3.10	0.302
$2p_{3/2}$	−2.00	0	−1.93	0.018	−1.76	0.103	−1.57	0.165
$2p_{1/2}$	−4.17	0	−3.88	0.007	−3.55	0.068	−3.28	0.130
$1f_{5/2}$	−5.92	0	−5.68	0.003	−5.29	0.045	−5.03	0.100
μ_π			6.07		6.90		8.14	
Δ_π			1.628		3.257		4.882	

number of neutrons $\Delta\nu$ and protons $\Delta\pi$ lifted out of the $T=0$ ^{40}Ca core are also given in the table. The most conspicuous result regarding a GDR superimposed on a single-particle field of Tables I and II is the increasing number and weight of new ph dipole excitations between the $1p$ shell and the $(2s\,1d)$ shell and between the $(2p\,1f)$ and the $(3s\,2d\,1g)$ shells. These new ph components, together with a blocking effect for ph excitations between the $(2s\,1d)$ and $(2p\,1f)$ states due to the particles lifted out of the core, are expected to determine the spreading and downward shift of the isovector GDR found in experiment.

Since the present calculations extend over the wide range of temperatures (from $T=0$ up to $T=6$ MeV), it is necessary to examine the validity of the statistical assumptions underlying the present model for the higher temperature solutions. Some measure of confidence in the physical significance in the numerical solutions may be established if for $T=6$ MeV the nucleus is still far from the boiling point of diminishing binding energy and if the relative particle-number fluctuation stays small. For a Skyrme-type force in which a density dependent force replaces the three-body interaction the energy of the system at a temperature T is given by[15]

$$\langle\phi(T)\,|\,H\,|\,\phi(T)\rangle=\tfrac{1}{2}\sum_i n_i(t_i+\epsilon_i) - \tfrac{1}{6}t_3\int\rho_\pi(\mathbf{r})\rho_\nu(\mathbf{r})\rho(\mathbf{r})d^3r\ , \quad (18)$$

where t_i and ϵ_i are single-particle kinetic energy and HF energy, ρ_π, ρ_ν, and ρ are the proton, neutron, and nuclear densities, and t_3 is the strength parameter for the three-body force. The relative particle-number fluctuation $\Delta A/A$ at a temperature T is defined as the rms fluctuation per particle and is determined by[16]

$$\left[\frac{\Delta A}{A}\right]=\frac{1}{A}\langle(\hat{N}-A)^2\rangle^{1/2}=\frac{1}{A}\left[\sum_i n_i(1-n_i)\right]^{1/2}, \quad (19)$$

where $\hat{N}$ is the particle-number operator.

Table III shows the calculated values for rms radius, binding energy, excitation energy

$$E^*=\langle\phi(T)\,|\,\hat{H}\,|\,\phi(T)\rangle-\langle\phi(0)\,|\,H\,|\,\phi(0)\rangle\ ,$$

Fermi gas model E_F^* determined by[17]

$$E_F^*=aT^2=0.127AT^2\ , \quad (20)$$

and the relative particle number of fluctuations $\Delta A/A$ at the temperatures 0, 2, 4, and 6 MeV.

Examining the results of Table III we find that, even at the temperature of 6 MeV, the nucleus has enough binding (−5.1 MeV per nucleon) and is thus far from its boiling point. There is, however, a marked difference in the temperature dependence of the self-consistent field at temperatures up to 4 MeV compared to the behavior at 6 MeV. We find a significant change in the rate by which the nuclear volume increases. While at $T=4$ the relative increase of the volume (with respect to the $T=0$ one) is 17%, the relative change of volume at $T=6$ is already 29%. We also find a significant moderation in the rate by which the self-consistent excitation energy increases with temperature. In terms of the temperature coefficient $a(T)=E^*/T^2$, the constant value 5.08 of the Fermi gas

TABLE III. ^{40}Ca TDHF solutions: R for root-mean-square radius (in fm), $\langle\hat{H}\rangle$ for total energy (in MeV), E^* for excitation energy (in MeV), E_F^* for excitation energy for Fermi gas model, and $\Delta A/A$ for relative particle-number fluctuation for $T=0$, 2, 4, and 6 MeV.

T	0	2	4	6
R	3.298	3.300	3.380	3.556
$\langle\hat{H}\rangle$	−336.35	−312.86	−270.29	−203.88
E^*	0	23.49	66.06	132.46
E_F^*	0	20.32	81.28	182.87
$\Delta A/A$ (%)	0	7.93	10.10	12.70

model is replaced in a self-consistent field by the decreasing values 5.87, 4.13, and 3.68 at the temperatures 2, 4, and 6 MeV, respectively. The marked change of volume and excitation energy rate of change found at $T=6$ MeV reflect the weakening of the restraining role of the internal degrees of freedom of the nucleus. At $T=6$ shell effects are expected to diminish,[18] and the limit for a thermal equilibrium regime seems to be reached. The calculated values of 10% and 13% for the relative particle-number fluctuations at $T=4$ and 6 MeV, respectively, are much bigger than the corresponding values for heavier nuclei.[19] However, a relative fluctuation of 10% for ^{40}Ca still justifies the reference to this nucleus as a grand canonical ensemble with a rather well defined statistical distribution. We find further support for the above counted observations in the calculated isovector GDR to be analyzed in the next subsection.

B. Solution of continuum TDRPA equations for ^{40}Ca

Using the TDHF single-particle fields of the preceding subsection, we constructed and solved the r space TDRPA equations relevant to the response of ^{40}Ca to an external electric dipole field represented by the operator

$$F(E1,\mu)=\frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1\mu}(\Omega_i) - \frac{ez}{A}\sum_{i=Z+1}^{A} r_i Y_{1\mu}(\Omega_i) . \quad (21)$$

The calculated photonuclear cross sections for isovector $J=1^-$ excitations at $T=0$, 2, 4, and 6 MeV are displayed in Fig. 1 and Table IV. Figure 1 also includes (in dashed lines) the experimental GDR cross sections[20] for comparison and as a reference spectrum for the calculated finite temperature cross sections. The integrated cross section,

$$\sigma_{\rm int}(E)=\int_0^E \sigma(E')dE' , \quad (22)$$

derived from the calculated cross sections, is displayed in Fig. 2. Since there is not much dipole strength left at energies above 30 MeV, we may compare the integrated cross section $\sigma_{\rm int}$ (30 MeV) with the classical energy weighted sum rule (CEWS) for dipole excitations[21]

$$(\sigma_{\rm int})_{\rm class}=60\frac{NZ}{A}\text{MeV mb} . \quad (23)$$

We refer first to the $T=0$ results. We notice satisfactory agreement between the overall distribution of calculated dipole strength and experiment. In particular, good agreement is found between the energies of the main GDR peak and between the integrated cross sections

$$[\sigma_{\rm int}(30)]_{\rm exp}=580\pm60\text{ MeV mb}$$

(Ref. 20) as compared to $[\sigma_{\rm int}(30)]_{\rm th}=534$. We believe that this agreement at $T=0$ gives some measure of confidence in the calculated GDR cross sections at finite temperature, and the $T=0$ results may be used as a reference spectrum to the changes in the collective behavior of the nucleus with increasing temperature.

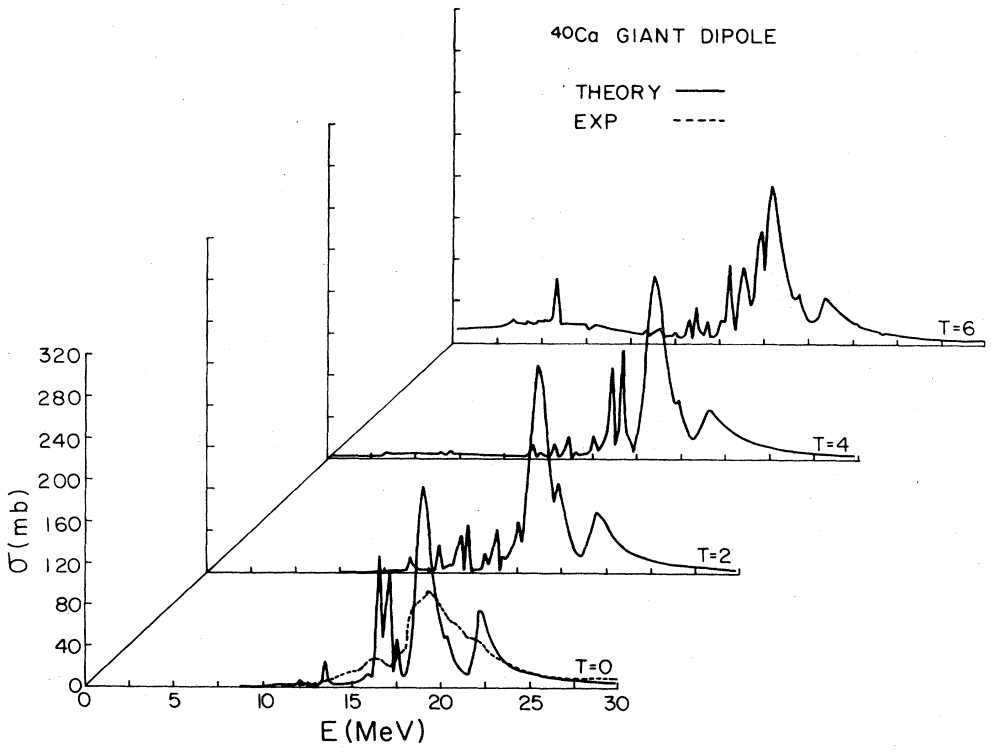

FIG. 1. ^{40}Ca isovector electric dipole cross sections for $T=0$, 2, 4, and 6 MeV. $T=0$ experimental cross sections are by Bezic *et al.* (Ref. 20).

TABLE IV. ^{40}Ca TDRPA solution for isovector electric dipole excitations: E_m for energy at the maximum of the main GRD peak, E_s for energy of the second GDR peak, $E_{\text{c.m.}}$ for GDR centroid energy, $\sigma_{\text{int}}(30\text{ MeV})$, $\sigma^L_{\text{int}}=\sigma_{\text{int}}(E)$, $\sigma^R_{\text{int}}=\sigma_{\text{int}}(30)-\sigma^L_{\text{int}}$, and α for interaction correction to dipole sum rule. Energies are given in MeV. Cross sections are given in MeV mb.

T	E_m	E_s	$E_{\text{c.m.}}$	σ_{int} (30)	σ^L_{int}	σ^R_{int}	α
0	19.0	22.2	19.0	534	234	300	0.09
2	18.6	22.0	14.8	550	207	343	0.12
4	18.6	21.6	9.5	563	272	291	0.14
6	18.0	21.0	6.9	720	446	274	0.46

The most conspicuous and nontrivial result of the calculated temperature dependence of the GDR cross sections is the sustained existence of well defined giant resonance through the whole range of temperatures used in the present calculations. In agreement with the experiment, we find a downward shift of the resonance energy as well as a large spreading of the dipole strength at the expense of the main peak when temperature increases. In complete agreement with Vautherin and Vinh-Mau,[9] we also find a weak temperature dependence of the energy of the main GDR peak. The downward shift of the peak is from 19.0 MeV at $T=0$ to 18.0 MeV at $T=6$ MeV. However, contrary to Vautherin and Vinh-Mau, we do not find a big increase in the dipole strength in the energy region above the main peak. In Table IV we give the calculated energy E_m at which the maximum of the main peak occurs, and the integrated cross sections $\sigma^L_{\text{int}}=\sigma_{\text{int}}(E_m)$ for the whole region left to E_m and $\sigma^R_{\text{int}}=\sigma_{\text{int}}(30)-\sigma^L_{\text{int}}$ right to E_m. At $T=4$ MeV we find a decrease of $\sim 3\%$ in σ^R_{int} with respect to the $T=0$ value and at $T=6$ MeV, a decrease of $\sim 10\%$. σ^L_{int}, on the other hand, increases by $\sim 10\%$ at $T=4$ and increases abruptly by $\sim 90\%$ at $T=6$. The combined effect of the downward shift of E_m and the spreading of the dipole strength may be given a quantitative measure by the values of the centeroid energy $E_{\text{c.m.}}$ This energy is defined by the ratio of the energy weighted strength to the total strength

$$E_{\text{c.m.}}=\int_0^{30\text{ MeV}} E'dE'\,\text{Im}\langle FGF\rangle \Big/ \int_0^{30\text{ MeV}} dE'\,\text{Im}\langle FGF\rangle \ . \tag{24}$$

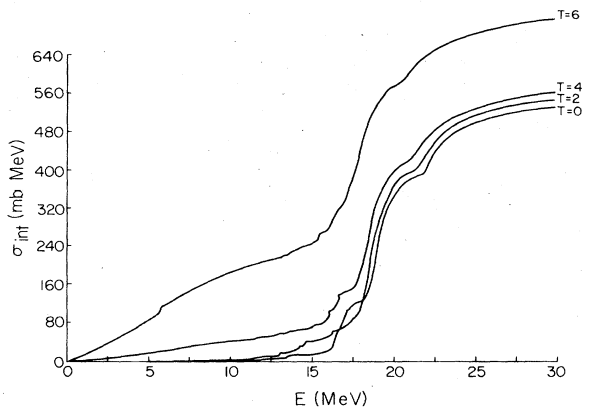

FIG. 2. TDRPA-integrated cross sections $\sigma_{\text{int}}(E)$ for isovector electric dipole excitations in ^{40}Ca.

The calculated values of $E_{\text{c.m.}}$ at each temperature are included in Table IV. These values reflect a strong temperature dependence and seem to be compatible with what the experiment indicates. Despite the significant spreading and downward shift of dipole strength there is a relatively small reduction of strength in the main peak itself. Thus, collectivity of the dipole vibration in the GDR region is sustained in a wide range of temperatures.

We now turn to analyze the temperature dependence of the energy weighted sum (EWS) associated with electric dipole excitations. In the absence of a velocity dependent component in the nuclear interaction this sum is expected to be independent of temperature.[10] In the present calculation we have used a particle-hole force which is derived from Skyrme I force. Since this force carries some velocity dependence (through the parameters t_1 and t_2 of the force), a small increase in the calculated total cross section with temperature is expected. The calculated total dipole cross sections displayed in Fig. 2 and Table IV do reflect an approximate independence of the EWS in temperatures ranging from 0 to 4 MeV. However, the small increase of 5.4% that occurs at $T=4$ MeV turns abruptly to a 35% increase at $T=6$ MeV. This drastically different behavior of the integrated cross section at 6 MeV is compatible with the different nature of the underlying self-consistent field, as stressed in Sec. III A. It may either reflect a violation of some of the basic assumptions of the TDRPA model or else a prediction for the nature of nuclear collective vibration at higher temperatures. Referring to the classical energy weighted sum rule of Eq. (23), an overestimation of this sum by the calculated cross sections due to a velocity dependent component of the force is expected regardless of temperature.[13] This interaction correction to the dipole sum rule is usually expressed in terms of a parameter α defined by

$$\sigma_{\text{int}}=(1+\alpha)(\sigma_{\text{int}})_{\text{class}} \ . \tag{25}$$

The temperature dependence of the calculated EWS may be viewed as a temperature dependence of the parameter α. The corresponding values of α at each temperature are included in Table IV.

IV. CONCLUSIONS

We have demonstrated above that the r space continuum linear response formulation of the finite temperature random-phase approximation is a most powerful method to investigate the nature of collective vibrations in

thermally equilibrated hot nuclei. Unlike other TDRPA formulations, our method is exactly solvable in the sense that one may include as many particle-hole configurations as required for an adequate description of the temperature dependence. One can concentrate on the details of a narrow segment of the energy spectrum rather than dealing with the entire spectrum. Above all, the inclusion of the continuum through the temperature single-particle Green's function enables, in a natural and transparent way, a detailed description of the broadening of particular states as a function of temperature.

Application of the present method to isovector GDR in ^{40}Ca explains in a consistent manner the spreading and downward shifting of the GDR observed in experiment. The present calculation suggests a possible violation of the thermal equilibrium regime at the temperature of 6 MeV. The rather weak temperature dependence of the GDR seems to take an abrupt turn at $T=6$ MeV.

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