Relativistic effects in the sum rules for the d(γ, p)n reaction

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(Received 16 April 1985)

The integrated and bremsstrahlung-weighted cross section for deuteron photodisintegration have been calculated taking the relativistic one-body charge density and the two-body charge density with its local and nonlocal contributions for several nucleon-nucleon potentials in pseudoscalar and pseudovector pion-nucleon couplings. The relativistic correction lowers the enhancement factor for the soft-core potentials and raises it for the hard-core potentials as well as for the Paris potential which has velocity dependence in it to simulate the effect of the hard core. This increase and decrease pattern of the enhancement factor is found to be related to the maxima of the deuteron ground-state wave functions. Though there are some changes in the enhancement factor for different potentials, they are not large enough to enable us to distinguish between the various hard-core or soft-core nucleon-nucleon potentials.

I. INTRODUCTION

It is well known that photonuclear sum rules play an important role in our understanding of the electromagnetic interactions with nuclei.[1] By knowing the ground-state wave function and nucleon-nucleon interaction one can calculate the first few moments of the deuteron photodisintegration cross section. The first sum-rule calculation for the $\gamma+\mathrm{d}\rightarrow\mathrm{n}+\mathrm{p}$ reaction was performed by Levinger[2] employing the nonrelativistic (NR) one-body charge density and the enhancement factor (k) obtained was compared with the experimental cross section integrated up to the pion threshold energy ($k=0.35\pm0.10$). Several authors followed the same procedure for a number of realistic potentials.[3–5] An integration of the experimental cross section including the Δ resonance yielded an enhancement factor of (0.80 ± 0.10) as reported in Ref. 6, which is higher than the earlier one. This difference in the enhancement factor and the suggestion of Hadjimichael[7] that the two-body charge density contributions to the integrated cross section can be used to distinguish between the various two-nucleon potentials, encouraged several workers[8–10] to reinvestigate the sum-rule calculations including the two-body charge density employing the pseudoscalar pion-nucleon (πN) coupling. It was found that contrary to Hadjimichael's suggestion, the sum-rule calculation cannot provide a guide line to distinguish among the nucleon-nucleon potentials. All these calculations were done considering NR one- and two-body (local contribution only) charge densities.

Though the contributions of the relativistic and retardation correction to the dipole sum-rule for a relativistic bound electron have been known for many years,[11–13] they have not received any attention in deuteron photodisintegration sum rules. Only recently Cambi, Mosconi, and Ricci[14] have shown that the relativistic corrections in the charge density can reduce the well-known discrepancy between the theoretical and the experimental value of the forward deuteron photodisintegration.

In this paper our aim is to include the relativistic correction to the one-body charge density and the two-body charge density considering its local and nonlocal contributions and study its effect on the enhancement factor and the bremsstrahlung-weighted cross section for the pseudoscalar (ps) and pseudovector (pv) πN coupling. In Sec. II we will describe the dipole operators for the relativistic one-body charge density and the two-body charge density including both the local and nonlocal contributions and write the expressions for the enhancement factor and the bremsstrahlung-weighted cross section. Section III is devoted to a discussion of our results and a comparison with other theoretical results.

II. CALCULATIONS

The one-body charge density electric dipole operator,

$$\mathbf{D}_1=\tfrac{1}{2}\sum_i[1+\tau_z(i)]\mathbf{r}_i\ , \tag{1}$$

will get modified for the relativistic effect as follows:[14–17]

$$\Delta\mathbf{D}_1(\mathrm{rel})=\frac{i}{8M^2}\left\{(2\mu_v-1)[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\times\boldsymbol{\nabla}_r]\frac{(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)_z}{2}+(2\mu_s-1)[(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\times\boldsymbol{\nabla}_r]\right\}, \tag{2}$$

where $\mathbf{r}_i$ are the c.m. nucleon coordinates, $\mu_s=\mu_\mathrm{p}+\mu_\mathrm{n}$, $\mu_v=\mu_\mathrm{p}-\mu_\mathrm{n}$, μ_p and μ_n are proton and neutron magnetic moments, and M is nucleon mass. The dipole operators, derived from the two-body charge density as obtained by Hyuga and Gari[18] by the unitary transformation method, with the sign correction in the pion-in-flight term pointed out by Jaus and Woolcock,[19] may be expressed as the sum of local (L) and nonlocal (NL) terms, $\mathbf{D}_2=\mathbf{D}_2(L)+\mathbf{D}_2(NL)$ and may be written as[15]

$$\mathbf{D}_2(L)=-\frac{1}{2M}\left[\frac{f^2}{4\pi}\right]\phi\left[\alpha_s(\tau_1\cdot\tau_2)\hat{\mathbf{r}}\times(\sigma_1\times\sigma_2)-\alpha_v\frac{(\tau_1-\tau_2)_z}{2}(\sigma_1\sigma_2\cdot\hat{\mathbf{r}}+\sigma_2\sigma_1\cdot\hat{\mathbf{r}})\right]$$
$$-\frac{1}{4M}\left[\frac{f^2}{4\pi}\right]\frac{(\tau_1-\tau_2)_z}{2}[(\sigma_1\sigma_2\cdot\hat{\mathbf{r}}+\sigma_2\sigma_1\cdot\hat{\mathbf{r}})\phi_1-\tfrac{1}{3}\hat{\mathbf{r}}(\phi_0\sigma_1\cdot\sigma_2+\phi_2S_{12})]\ , \tag{3}$$

$$\mathbf{D}_2(NL)=\frac{1}{4M}\left[\frac{f^2}{4\pi}\right]\frac{i}{\mu}(\tau_1\times\tau_2)_z\{\Phi_0[\sigma_2\cdot\overleftrightarrow{\nabla}\sigma_1\cdot\hat{\mathbf{r}}+\sigma_1\cdot\overleftrightarrow{\nabla}\sigma_2\cdot\hat{\mathbf{r}}]\hat{\mathbf{r}}+\tfrac{1}{3}[(3\Phi_1+\Phi_2)\sigma_1\cdot\sigma_2+\Phi_2S_{12}]\overleftrightarrow{\nabla}\}\ , \tag{4}$$

where $f^2/4\pi=0.081$, $\mu=m_\pi c/\hbar$, m_π is the pion mass, S_{12} is usual tensor operator, $\overleftrightarrow{\nabla}\equiv\overleftarrow{\nabla}_r\overrightarrow{\nabla}_r$ acts only on the initial and final wave function, and α_s and α_v are defined for pseudoscalar and pseudovector πN couplings as follows:

for pseudoscalar,

$$\alpha_s=\tfrac{1}{2}\mu_s,\ \ \alpha_v=\tfrac{1}{2}(\mu_v+1)\ , \tag{5a}$$

for pseudovector,

$$\alpha_s=\tfrac{1}{2},\ \ \alpha_v=\tfrac{1}{2}\ . \tag{5b}$$

The ϕ's are defined in Refs. 8 and 9 and the Φ's take the following forms:

$$\Phi_0(r,\mu,\Lambda)=x\phi(r,\mu,\Lambda)\frac{1+C}{2}\ , \tag{6a}$$

$$\Phi_1(r,\mu,\Lambda)=\frac{1}{x}\left\{e^{-x}+e^{-\lambda x}+\frac{4}{\lambda^2-1}[\lambda^2Y_1(\lambda x)-Y_1(x)]\right\}, \tag{6b}$$

$$\Phi_2(r,\mu,\lambda)=-x\phi_1(r,\mu,\Lambda)\ , \tag{6c}$$

where $\Lambda=1003$ MeV as obtained by Dominguez and Clark,[20] $x=\mu r$, $\lambda=\Lambda/\mu$, $Y_1(x)=(e^{-x}/x)[1+(1/x)]$, and $C=-1$ for the pseudoscalar and $C=1$ for the pseudovector πN couplings. The integrated cross section σ_0 for the deuteron is given by

$$\sigma_0=\int\sigma(W)dW$$
$$=\pi^2e^2\frac{\hbar}{Mc}(1+k)\ , \tag{7}$$

where

$$1+k=\frac{2M}{3}\sum_m\langle \mathrm{d},m\mid[D_z,[H,D_z]]\mid \mathrm{d},m\rangle\ . \tag{8}$$

Here D_z is the z component of the dipole operator

$$\mathbf{D}=\mathbf{D}_1+\Delta\mathbf{D}_1(\mathrm{rel})+\mathbf{D}_2(L)+\mathbf{D}_2(NL)\ ,$$

$|\mathrm{d},m\rangle$ is the ground state of the deuteron, and H is the Hamiltonian, the sum of the kinetic and potential energies.

The expression for the enhancement factor $(1+k)$ for the Paris potential[21] taken from Ref. 9 may be written as

$$1+k=\frac{4}{9}\sum_{JLL'ST}\int dx\left[\sum_j\langle JLST\mid U_j\Omega_j\mid JL'ST\rangle g_{JL}^{ST}g_{JL'}^{ST}+\delta_{LL'}\left\{(1+2V_s^b)\left[\frac{dg_{JL}^{ST}}{dx}\right]^2+(g_{JL}^{ST})^2\left[(1+2V_s^b)\frac{L(L+1)}{x^2}+\epsilon-\frac{d^2V_s^b}{dx^2}\right]\right\}\right]\ , \tag{9}$$

where $U_j=MV_j/m_\pi c^2$, $\epsilon=MB/m_\pi c^2$, and B is the deuteron binding energy. In our earlier paper (Ref. 9) the term proportional to $d^2V_s^b/dx^2$ was missing [Eq. (11)] though it was included in the calculation. The functions g_{JL}^{ST} corresponding to D_1 and $D_2(L)$ in the ps πN coupling have been already defined in Refs. 8 and 9 and the inclusion of the relativistic corrections and nonlocal contribution of the two-body charge density will modify g_{JL}^{ST} as follows:

$$g_{JL}^{ST}\to g_{JL}^{ST}[D_1+D_2(L)]+g_{JL}^{ST}[\Delta D_1(\mathrm{rel})]+g_{JL}^{ST}[D_2(NL)]\ , \tag{10}$$

where $g_{JL}^{ST}[\Delta D_1(\mathrm{rel})]$ for a different contributing set of J, L, S, and T are given by[15]

$$g_{01}^{11}=-(1-2\mu_v)\frac{\mu^2}{4M^2}\sqrt{2/3}(\sqrt{2}u_1+w_1)\ , \tag{11a}$$

$$g_{11}^{11}=(1-2\mu_v)\frac{\mu^2}{4M^2}(u_1-\sqrt{2}w_1)\ , \tag{11b}$$

$$g_{21}^{11}=(1-2\mu_v)\frac{\mu^2}{4M^2}\sqrt{5/3}\left[u_1-\frac{2\sqrt{2}}{5}w_1\right]\ , \tag{11c}$$

$$g_{23}^{11}=-(1-2\mu_v)\frac{\mu^2}{4M^2}\frac{3}{\sqrt{5}}w_2\ , \tag{11d}$$

$$g_{11}^{00}=\sqrt{6}\frac{1-\mu_s}{1-2\mu_v}g_{01}^{11}\ . \tag{11e}$$

Here u_1, w_1, and w_2 are defined in terms of the deuteron ground state wave functions u and w and their derivatives as

$$u_1=\frac{du}{dx}-\frac{u}{x}\ , \quad (12a)$$

$$w_1=\frac{dw}{dx}+\frac{2w}{x}\ , \quad (12b)$$

$$w_2=\frac{dw}{dx}-\frac{3w}{x}\ . \quad (12c)$$

$g_{JL}^{ST}[D_2(NL)]$ which are only nonzero for $S=T=1$ will be defined as

$$g_{JL}^{ST}[D_2(NL)]=\frac{\mu}{M}\left[\frac{f^2}{4\pi}\right]\sum_{l=0,2}\left[f_{JL}^{l}\frac{d}{dx}+\frac{1}{x}h_{JL}^{l}+\frac{1}{2}\frac{d}{dx}f_{JL}^{l}\right]u_l\ , \quad (13)$$

where $u_0=u$, $u_2=w$, and f_{JL}^{l} and h_{JL}^{l} are given below.

$$f_{01}^{0}=-\frac{1}{\sqrt{3}}(2\Phi_0-\Phi_1+\Phi_2),\quad h_{01}^{0}=\frac{1}{\sqrt{3}}(2\Phi_0-\Phi_1-\Phi_2)\ , \quad (14a)$$

$$f_{11}^{0}=-(2\Phi_0+\Phi_1+\Phi_2),\quad h_{11}^{0}=\Phi_0+\Phi_1\ , \quad (14b)$$

$$f_{21}^{0}=\frac{1}{\sqrt{15}}(2\Phi_0+5\Phi_1+\Phi_2)\ ,\quad h_{21}^{0}=\frac{1}{\sqrt{15}}(\Phi_0-5\Phi_1-2\Phi_2)\ , \quad (14c)$$

$$f_{23}^{0}=2\sqrt{2/5}(2\Phi_0+\Phi_2),\quad h_{23}^{0}=-2f_{23}^{0}\ , \quad (14d)$$

$$f_{01}^{2}=-\sqrt{2}f_{01}^{0},\quad h_{01}^{2}=\sqrt{2/3}(\Phi_0-2\Phi_1+\Phi_2)\ , \quad (14e)$$

$$f_{11}^{2}=\frac{1}{\sqrt{2}}f_{11}^{0},\quad h_{11}^{2}=-\sqrt{2}h_{11}^{0}\ , \quad (14f)$$

$$f_{21}^{2}=\frac{1}{\sqrt{30}}(14\Phi_0-\Phi_1+7\Phi_2)\ ,\quad h_{21}^{2}=\frac{2}{\sqrt{30}}(14\Phi_0-\Phi_1-7\Phi_2)\ , \quad (14g)$$

$$f_{23}^{2}=-\frac{1}{\sqrt{5}}(2\Phi_0-3\Phi_1+\Phi_2)\ ,\quad h_{23}^{2}=\frac{1}{\sqrt{5}}(\Phi_0-9\Phi_1+2\Phi_2)\ . \quad (14h)$$

The bremsstrahlung-weighted cross section for the deuteron can be written as

$$\sigma_{-1}(E1)=\int(\sigma/W)dW=\frac{4\pi^2}{9}\left[\frac{e^2}{\hbar c}\right]\frac{1}{\mu^2}\sum_{JLST}\int dx(g_{JL}^{ST})^2\ . \quad (15)$$

The bremsstrahlung-weighted cross section does not depend on the two-body potential directly but will change due to the change in g_{JL}^{ST}.

III. RESULTS AND CONCLUSIONS

We have calculated the $E1$ integrated and bremsstrahlung-weighted deuteron cross section for the Yale,[22] super-soft core,[23,24] Paris,[21] Hamada-Johnston (HJ) (Ref. 25) and modified Hamada-Johnston (HJM) potentials. The super-soft-core potential of Ref. 24 is called SSC-D here.

In Table I, the first column lists the enhancement factor k_1, when only the one-body charge density is considered. The second column displays the combined effect of the one-body charge density and the one-body relativistic correction ($k_{\rm rel}$). The effect of the relativistic correction on the one- and two-body charge density without and with the nonlocal term, respectively, are displayed in the

TABLE I. The enhancement factor for the various nucleon-nucleon potentials for the pseudoscalar and pseudovector πN coupling. The results for pseudovector πN coupling are given in the parentheses.

Potentials	k_1	$k_1+k_{\rm rel}$	k_1+k_2 (NN) $+k_2$(ret)$+k_{\rm rel}$	k_1+k_2(NN)$+k_2$(ret) $+k_2(NL)+k_{\rm rel}$
Yale	0.529	0.550	0.628	0.631
			(0.565)	(0.592)
SSC-D	0.523	0.439	0.479	0.496
			(0.451)	(0.413)
SSC-C	0.479	0.394	0.430	0.447
			(0.402)	(0.397)
SSC-B	0.441	0.374	0.410	0.426
			(0.382)	(0.363)
SSC-A	0.439	0.379	0.409	0.421
			(0.388)	(0.362)
Paris	0.497	0.486	0.525	0.600
			(0.497)	(0.543)
HJ	0.533	0.650	0.695	0.695
			(0.659)	(0.710)
HJM	0.539	0.681	0.462	0.463
			(0.451)	(0.503)

TABLE II. The position of maxima of deuteron wave functions u and w for the various nucleon-nucleon potentials.

The position of maxima	Potentials Yale	SSC-D	SSC-C	SSC-B	SSC-A	Paris	HJ	HJM
$u,x=$	1.162	1.190	1.218	1.218	1.190	1.162	1.183	1.160
$w,x=$	0.784	0.952	0.994	1.022	1.022	0.952	0.861	0.880

third and fourth columns. Our results for k_1+k_{rel} are essentially in agreement with those of Cambi *et al.*[15] for the common potentials except for the Paris. The relativistic correction has the effect of lowering the k_1 value for potentials which have no core radius, whereas k_1 is increased for the potentials with hard core. This pattern can be explained by examining the position of the maximum of the wave functions u and w because the relativistic correction involves u and w and their derivatives. Table II shows the position of the maxima of u and w for all the potentials. It is clear from Table II that the position of the maxima for u and w occur closer to the origin for the potentials having hard core than those having soft core except for the Paris potential which has velocity dependence in it and which is introduced to simulate the effect of the hard core. The hard core potentials have larger values of the wave functions between the position of the maximum and the asymptotic region and contribute more to the integrated cross section.

In the third column we have shown the effect of the relativistic correction on the one- and two-body charge densities when only the local term of the two-body charge density is considered. The relativistic effects decrease the values reported in Ref. 9 for the ps πN coupling, but the pattern of decrease and increase in k_1 still exists as was observed in the second column for k_1+k_{rel}. The inclusion of the nonlocal term to the third column increases the value for ps πN coupling but for the pv πN coupling case increases for the hard core potentials having the position of the maxima closer to the origin and decreases for the soft core potentials having the position of the maxima further away from the origin. Our results with the inclusion of nonlocal terms are in agreement with those of Cambi *et al.*[15] for the Paris and HJ potentials for ps πN coupling and with the SSC-D and SSC-B potentials for the pv πN coupling.

Table III shows the bremsstrahlung-weighted cross section for ρ_1, $\rho_1+\rho_{\text{rel}}$,

$$\rho_1+\rho_2(\text{NN})+\rho_2(\text{ret})+\rho_{\text{rel}}$$

and

$$\rho_1+\rho_2(\text{NN})+\rho_2(\text{ret})+\rho_2(NL)+\rho_{\text{rel}}$$

in the first through fourth columns, respectively. The value of $\sigma_{-1}(E1)$ for ρ_1 is higher than those of Cambi *et al.*[15] However, our results are in agreement for $\rho_1+\rho_{\text{rel}}$ with Cambi *et al.* for common cases within 0.3–2.0%. The effect of including the relativistic correction $\rho_1+\rho_2(\text{NN})+\rho_2(\text{ret})$ is to lower the value of $\sigma_{-1}(E1)$ in comparison with Ref. 9 for ps πN coupling. For pv πN

TABLE III. The bremsstrahlung-weighted cross section for the various nucleon-nucleon potentials in the pseudoscalar and pseudovector πN coupling. The numbers for pseudovector πN coupling are given in parentheses.

Potentials	$\sigma_{-1}(E1)$ ρ_1	$\sigma_{-1}(E1)$ $\rho_1+\rho_{\text{rel}}$	$\sigma_{-1}(E1)$ $\rho_1+\rho_2(\text{NN})+\rho_2(\text{ret})+\rho_{\text{rel}}$	$\sigma_{-1}(E1)$ $\rho_1+\rho_2(\text{NN})+\rho_2(\text{ret})+\rho_2(NL)+\rho_{\text{rel}}$
Yale	3.7049	3.6948	3.7134	3.7194
			(3.6980)	(3.6995)
SSC-D	3.8864	3.8416	3.8588	3.8627
			(3.8445)	(3.8447)
SSC-C	3.7912	3.7818	3.7988	3.8025
			(3.7847)	(3.7847)
SSC-B	3.8189	3.8121	3.8280	3.8313
			(3.8147)	(3.8146)
SSC-A	3.8067	3.7997	3.8158	3.8192
			(3.8023)	(3.8023)
Paris	3.7082	3.7094	3.7270	3.7310
			(3.7123)	(3.7126)
HJ	3.7610	3.7618	3.7730	3.7685
			(3.7534)	(3.7547)
HJM	3.7175	3.7093	3.7326	3.7374
			(3.7183)	(3.7196)

coupling, $\sigma_{-1}(E1)$ is less than its value for ρ_1. Inclusion of the nonlocal term leads to the same value of $\sigma_{-1}(E1)$ as reported by Cambi *et al.*[15] for common cases within a few percent.

In summary, the inclusion of the relativistic correction to one-body and one-body plus the contribution of the local two-body charge densities increases the enhancement factor for the potentials with hard core and decreases the enhancement factor for the potentials with soft core. This pattern for the enhancement factor is found to be related to the positions of the maxima of deuteron wave functions u and w for hard core and soft core potentials. The effect of relativistic correction to one-body and local and nonlocal contributions of the two-body charge densities increases the enhancement factor for ps πN coupling, whereas for pv πN coupling increase and decrease patterns remain the same. However, the changes are not large enough to enable us to distinguish between the various potentials.

This work was partially supported by the National Aeronautics and Space Administration under Grant No. NAG1442.

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