Nucleon-nucleon interaction in the quark compound bag model

V. S. Bhasin and V. K. Gupta
Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India
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A model for the N-N interaction is proposed which has a long range part to account for the low energy properties and a short range component to incorporate the quark degrees of freedom. Based on the quark compound bag model, Simonov recently derived a short range repulsive core which is nonlocal and, with a characteristic energy dependence, can be expressed in terms of the parameters relating to the six-quark compound bag. Adding to it an s-wave separable potential for the long range component which is only operative outside the bag radius, we here study n-p scattering in ${}^3S_1$ and ${}^1S_0$ states for laboratory energies up to 400 MeV and the electromagnetic form factor for the deuteron. The parameters obtained to reproduce the data are compared with those of earlier investigations. The short range parameters, and, in particular, the bag range parameter, are found to have a marked variation from the one obtained in the P-matrix analysis.

I. INTRODUCTION

While considerable success has been claimed in understanding the properties of baryons and their excited resonances in terms of the configuration of constituent quarks, at the nucleon-nucleon level, however, efforts[1] to deduce N-N interaction from the q-q force in the quantum chromodynamics (QCD) framework have had rather limited success. Besides the obvious increase in complexity due to the increasing number of quarks and the associated symmetries in the configuration, color, spin, and isospin spaces, from a dynamical standpoint, quarks are known to interact weakly at small distances, whereas at large distances they interact strongly (asymptotic freedom). In the intermediate region, absence of analytic methods hinders the efforts to deduce N-N interaction unambiguously. Recently there have been attempts[2] to derive information on the inner quark region and to see how much can be learned about the quark content in the nuclear force from our knowledge of the asymptotic behavior of N-N scattering without introducing specific models for the dynamics of the quarks. Thus, for instance, on the basis of cluster decomposition of the total wave function into two parts, viz., (i) the inner wave function of the six-quark bag and (ii) the two-nucleon bag component, Simonov[3] was able to derive a short range repulsive core for N-N interaction, which is found to be energy dependent and nonlocal in character. With an additional assumption that the interaction between quarks and nucleons is confined to the surface of the quark compound bag (QCB), one can obtain a repulsive hard core produced by a strong coupling of the nucleons to the QCB. By incorporating the information about the low energy observables (e.g., scattering length and effective range) but without assuming any specific model for the long range N-N potential, he carried out a P-matrix analysis[4] to describe ${}^3S_1$ and ${}^1S_0$ N-N scattering over a wide energy range.[5] The main finding of this analysis is that the compound bag radius turns out to be quite large (~ 1.5 fm) with a strong coupling of the nucleons to the quark bag. It would be interesting to see how far these parameters, which essentially govern the short range component of N-N dynamics in terms of the underlying quark structure, are model dependent. The object of this brief paper is to propose a model which incorporates the low energy N-N interaction ($r > b$) superimposed by the short range ($r < b$) hard repulsive force and which is simple enough to yield an explicit analytical expression for the N-N wave function and for the scattering amplitude to study the electromagnetic form factors of deuteron and ${}^3S_1$ and ${}^1S_0$ N-N scattering over a wide energy range. For the long range ($r > b$) N-N interaction we assume a separable form of the potential to account for the low energy properties, and for the short range component we adopt the form obtained by Simonov. Our analysis clearly indicates that to reproduce a fit to the N-N scattering (both ${}^3S_1$ and ${}^1S_0$ up to 400 MeV laboratory energy) and to the deuteron form factor, the short range parameters, viz., the quark bag radius and the coupling strength of the QCB to the N-N channel, appear to be model dependent and are found to have considerable deviations from the ones obtained by Simonov.[5] In the next section we briefly outline the important steps which constitute the essential ingredients of the approach and then in the subsequent section present the results of our analysis.

II. THE QUARK-COMPOSITE BAG MODEL FOR THE N-N SYSTEM

The basic ansatz[2,3] here is that the total wave function can be decomposed into two parts, viz.,

$$\Psi = \Psi_{\text{int}} + \Psi_{\text{ext}} , \tag{1}$$

where Ψ_{int} describes the internal part of the six-quark state—a quark composite bag (QCB) state, with the boundary condition that it vanishes on the surface of the bag and can be expanded in a complete set of discrete states confined to the internal region, i.e.,

$$\Psi_{\text{int}} = \sum_{\nu} \alpha_\nu \Psi_\nu , \tag{2}$$

where Ψ_ν are the energy eigenstates of the total Hamiltonian

$$H=H_0+\sum_{i<j=1,\ldots,6} V_{ij} , \quad (3)$$

$$H\Psi_\nu=E_\nu\Psi_\nu \quad (4)$$

in a confined region. The Ψ_{ext} is the wave function in the external (outer) region in which nucleons are supposed to remain undeformed. Thus, assuming cluster decomposition, Ψ_{ext} can be expressed as

$$\Psi_{exp}=A[\Psi_{NN}(\mathbf{r})\Psi_{N_1}(3q)\Psi_{N_2}(3q)] , \quad (5)$$

where A is an antisymmetrizer and $\Psi_{NN}(\mathbf{r})$ is the wave function of the relative motion of the centers of the two clusters described by the wave function $\Psi_{N_1}(3q)$ and $\Psi_{N_2}(3q)$;

$$\mathbf{r}=\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)-\tfrac{1}{3}(\mathbf{r}_4+\mathbf{r}_5+\mathbf{r}_6) . \quad (6)$$

Here it is implied that the wave function describing the relative coordinates between the two color singlet configurations can be defined even when the two-nucleon bags overlap each other at short distances.

With these essential ingredients, it is possible to derive,[3] within the framework of the cluster model, a Schrödinger equation for $\Psi_{NN}(\mathbf{r})$:

$$(H_0+V_{NN}+V_{NqN}-E)\Psi_{NN}=0 , \quad (7)$$

where V_{NN} is the long range part of the nucleon-nucleon potential and V_{NqN} is the short range component given by

$$V_{NqN}=-\sum_\nu V_{Nq}^{(\nu)}(\mathbf{r},E)V_{qN}^{(\nu)}(\mathbf{r}',E)/(E_\nu-E) . \quad (8)$$

Here $V_{Nq}^{(\nu)}$ is the nucleon-quark transition vertex given by

$$V_{Nq}^{(\nu)}(\mathbf{r},E)=\int \Psi_\nu(H-E)A[\Psi_{N_1}(3q)\Psi_{N_2}(3q)] \times\delta(\mathbf{r}-\mathbf{r}')\prod_{i=1}^{6} d\mathbf{r}_i' , \quad (9)$$

and should be nonzero only inside the bag as the quarks are not allowed outside it. Simonov, assuming a sharp transition from quarks to hadrons on the surface of the bag, suggested a form for V_{Nq} to be

$$V_{Nq}^{(\nu)}=c_\nu\delta(\mathbf{r}-\mathbf{b}) , \quad (10)$$

thus effectively giving rise to a hard core repulsion for the short range N-N interaction.

Now adopting the same form for the short range component and assuming a separable potential for the S-state N-N interaction operative only in the long range region, viz.,

$$\widetilde{V}_{NN}(r,r')=-\frac{\lambda}{m}\widetilde{g}(r)\widetilde{g}(r')\theta(r-b)\theta(r'-b) , \quad (11)$$

$$\widetilde{V}_{NqN}(r,r')=\widetilde{f}_\nu(r)\widetilde{f}_\nu(r')/(E_\nu-E) , \quad (12)$$

where, for the S wave,

$$\widetilde{f}_\nu(r)=\frac{1}{\sqrt{4\pi}}c_\nu\frac{\delta(r-b)}{r} , \quad (13)$$

and for $\widetilde{g}(r)$ we take the Yukawa shape

$$\widetilde{g}(r)=\sqrt{\pi/2}e^{-\beta r}/r . \quad (14)$$

The corresponding expressions in momentum space are

$$V_{NN}(p,p')=-\frac{\lambda}{m}g(p)g(p') \quad (15)$$

and

$$V_{NqN}(p,p')=f_\nu(p)f_\nu(p')/(E_\nu-E) , \quad (16)$$

where

$$g(p)=e^{-\beta b}[\cos(bp)+\beta\sin(bp)/p]/(p^2+\beta^2) , \quad (17)$$

$$f_\nu(p)=\frac{c_\nu}{\pi\sqrt{2}}\sin(bp)/p . \quad (18)$$

It is interesting to note here that the structure of the vertex function $g(p)$ [Eq. (17)] obtained by imposing a lower cutoff in the configuration space is strikingly different from the one most commonly employed (viz., the Yamaguchi form) in studying the 2N and 3N systems at low energies.

Now with these input forms of the separable potentials for long- as well as short-range nucleon-nucleon interaction, the Schrödinger equation (7) can easily be solved to get the wave function

$$\Psi(p)=N\{g(p)+[\lambda^{-1}-I_{11}(\alpha^2)]f_\nu(p)\}/(p^2+\alpha^2) \quad (19)$$

for the bound state case. Equivalently, in configuration space

$$\Psi(r)=N(c_1+c_2)(e^{\alpha r}-e^{-\alpha r})/r \quad \text{for } r\le b \quad (20)$$

and

$$\Psi(r)=N(d_1e^{-\alpha r}-d_2e^{-\beta r})/r \quad \text{for } r\ge b . \quad (21)$$

In Eqs. (19)—(21), N is the normalization constant of the deuteron wave function; α is the deuteron binding energy parameter (BE$=-\alpha^2/m$); and c_1,c_2 and d_1,d_2 are the simple constants in terms of the parameters α, β, b, c, etc. In addition, we have a relation

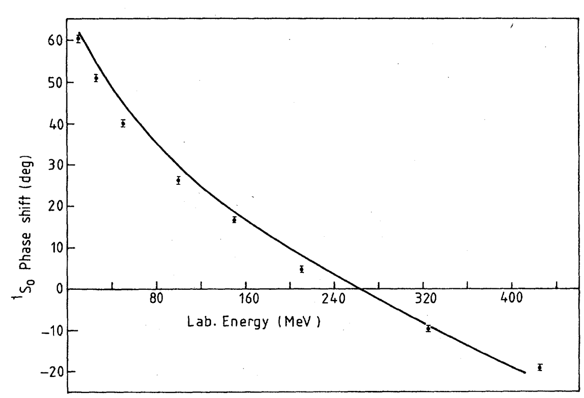

FIG. 1. $^1S_0$ neutron-proton scattering phase shifts (deg) vs laboratory energy (MeV).

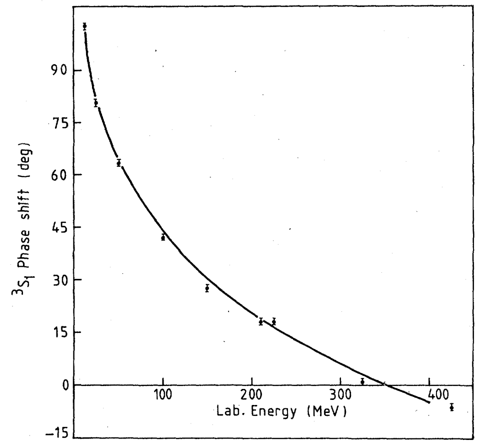

FIG. 2. $^3S_1$ neutron-proton scattering phase shifts (deg) vs laboratory energy (MeV).

$$\lambda^{-1}-I_{11}(\alpha^2)=-\left[\frac{m}{E_v-E}\right]I_{12}(\alpha^2)\times I_{21}(\alpha^2)\Big/\left[1+\frac{mI_{22}(\alpha^2)}{E_v-E}\right], \qquad (22)$$

where I_{11},I_{12},I_{22} are the simple integrals:

$$I_{11}(\alpha^2)\equiv\int d^3q\, g^2(q)/(q^2+\alpha^2)$$

$$=\pi^2e^{-2b\beta}\left[\frac{1}{\beta\alpha(\beta+\alpha)}-\frac{e^{-2b\alpha}}{\alpha(\beta+\alpha)^2}\right],$$

$$I_{12}(\alpha^2)=I_{21}(\alpha^2)\equiv\int d^3q\, g\frac{(q)f(q)}{(q^2+\alpha^2)}$$

$$=\frac{\pi c_v e^{-b\beta}}{\sqrt{2}\alpha(\beta+\alpha)}(1-e^{-2b\alpha}),$$

$$I_{22}(\alpha^2)\equiv\int d^3q\, f^2(q)/(q^2+\alpha^2)$$

$$=c_v^2(1-e^{-2b\alpha})/2\alpha. \qquad (23)$$

For the case of nucleon-nucleon scattering in the S wave, the expression for off-shell scattering amplitude is worked out as

$$a_k(p)=2\pi^2\lambda h^{-1}(g(p)g(k)-[(E_v-E)/m+I_{22}(-k^2-i\epsilon)]^{-1}\times\{g(k)f(p)I_{21}(-k^2-i\epsilon)+g(p)f(k)I_{12}(-k^2-i\epsilon)+f(p)f(k)[\lambda^{-1}-I_{11}(-k^2-i\epsilon)]\}), \qquad (24)$$

where

$$h=\{1-\lambda I_{11}(-k^2-i\epsilon)+m\lambda I_{12}^2(-k^2-i\epsilon)/[E_v-E+I_{22}(-k^2-i\epsilon)]\}.$$

The $a_k(p)$, taken on shell, is related to the phase shifts, viz.,

$$a_k(p)\,|_{p=k}=e^{i\delta}\sin\delta/k, \qquad (25)$$

III. RESULTS AND DISCUSSION

Using expressions (24) and (25) we have computed the $^1S_0$ and $^3S_1$ phase shifts for n-p scattering up to about 400 MeV laboratory energy and compared with the experimental data as shown in Figs. 1 and 2. The set of parameters used along with the corresponding values of the scattering length and effective range obtained for $^3S_1$ and $^1S_0$ cases are given in Table I. The parameter which is found to be sensitive to the position of the zero of the phase shifts is E_v, which is to be interpreted as the energy (over the two nucleon masses) of the six-quark-composite bag. For the triplet state, E_v equals 411 MeV, whereas for the singlet case it is 320 MeV. These values are to be compared with the ones obtained from the P matrix approach[5] which gives 230 MeV and 190 MeV for the triplet and singlet states, respectively. Similarly, the coupling strength of the quark-nucleon transition vertex in the present case is 6.7 MeV$^{1/2}$ compared to about 10.4 MeV$^{1/2}$ obtained in the P-matrix analysis. The marked difference is, however, observed in the value of b, the bag range parameter (which may be regarded as equivalent to the bag radius). Whereas the P-matrix analysis gives the value of the radius as large as 1.46 fm, the present model requires it to be 0.65 fm. Thus it appears that this is a rather sensitive model-dependent parameter.

To check the sensitivity of the wave function obtained in this model, we have computed, with the same set of parameters as for $^3S_1$, the body form factor

$$F(q)=\int e^{i\mathbf{q}\cdot\mathbf{r}}\,|\Psi(\mathbf{r})|^2d^3r \qquad (26)$$

for the deuteron (using the input BE$=-2.225$ MeV).

TABLE I. Nucleon-nucleon interaction parameters along with the values of the scattering length and effective range obtained.

State	E_v (MeV)	c_v (MeV$^{1/2}$)	b (fm)	λ (fm^{-3})	β (fm^{-1})	r (fm)	a (fm)
$^3S_1$	411	6.76	0.647	10.19	2.316	1.63	5.30
$^1S_0$	320	6.76	0.647	7.814	2.316	2.05	−23.28

Using this we then calculate the electromagnetic form factor $A(q^2)$ of the deuteron which is given by[7]

$$A(q^2)=F_{\rm el}^2(q^2)+\tfrac{2}{3}\eta F_{\rm mag}^2(q^2)\ , \qquad (27)$$

where

$$F_{\rm el}^2(q^2)=F_{\rm CH}^2(q^2)+\tfrac{8}{9}\eta^2 F_Q^2(q^2) \qquad (28)$$

and

$$F_{\rm mag}(q^2)=(md/m)G_{\rm mag}^S(q^2)F(q^2)\ . \qquad (29)$$

In Eq. (28), $F_Q(q^2)$ refers to the form factor due to the presence of the tensor component in the deuteron wave function, which has not been incorporated as yet in the present case. We thus have

$$F_{\rm el}(q^2)=F_{\rm CH}(q^2)=G_{\rm CH}^S(q^2)F(q^2)\ . \qquad (30)$$

In Eqs. (29) and (30) $G_{\rm CH}^S$ and $G_{\rm mag}^S$ are the isoscalar charge and magnetic form factors of the nucleon. For these we have used here the parametrizations obtained by De Vries *et al.*[8] which give the fits for these form factors over a large range of q^2. The plot of the deuteron electromagnetic form factor $A(q^2)$ vs q^2 is shown in Fig. 3 along with the experimental data.[9] Clearly the calculated values are in reasonable agreement with the experimental ones over a range of q^2 up to about 25 $\rm fm^{-2}$. Beyond this, one cannot and should not expect agreement in view of the fact that in this region the presence of a tensor component starts showing its marked effect. We have also calculated the probability of the presence of the six-quark content in the deuteron wave function which is 13.6% as compared to about 15% obtained in the earlier calculation.[3]

As observed earlier, one of the important findings of this investigation appears to be that the bag range parameter has a considerable deviation from the *P*-matrix calculations, indicating that this is presumably sensitive to the model used for the long range part of the nucleon-nucleon interaction. It would be interesting to carry out further investigations to check the sensitivity using some other specific models.

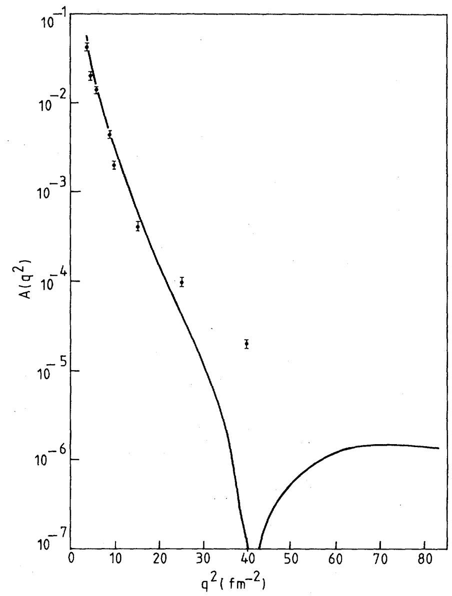

FIG. 3. Electromagnetic form factor of the deuteron vs q^2 ($\rm fm^{-2}$).

To conclude, we have here a fairly realistic and yet analytically simple model which can yield useful information about the inner quark content in the N-N system, starting from the long range scattering as well as bound state data. However, to put it on a more realistic footing, we have to include the tensor part of the N-N interaction. In addition, it will be interesting to analyze, within this framework, the contribution of isobar and three-quark-color states present within the inner quark region of the deuteron wave function. The attempts made along these directions will be the subject for future communications.

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