Microscopic calculation of the imaginary Lane isospin potential W_1

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Results of a determination of the imaginary Lane potential W_1 are presented, based on a second-order calculation of the 25 MeV ^{48}Ca(p,n)^{48}Sc 0^+ analog reaction. Inelastic (p,p^{-1}), (n,n^{-1}) and charge exchange (p,n^{-1}) particle-hole intermediate nuclear states are used, and an exact treatment of the continuum and of second-order knockout exchange are included. A complex-energy intermediate-projectile Green's function is used to account for energy averaging of the incident beam. The sign of W_1 is shown to be positive, in agreement with phenomenological results, but the calculated magnitude is about a factor of 2 weaker than the Becchetti-Greenlees phenomenological potential. The contribution from an intermediate ground-state deuteron projectile is calculated in the zero-range approximation for pickup and stripping and is found to be about $\frac{1}{3}$ of the contribution from the sum of all intermediate particle-hole states.

I. INTRODUCTION

In the past few years the calculation of the imaginary nuclear optical potential from microscopic models of nuclear matter[1,2] and of finite nuclei[3–6] has been undertaken by several groups with considerable success. In the nuclear matter approach,[2] the complex two-body t matrix in the continuum is calculated using the Bethe-Goldstone equation to take into account the Pauli principle, and a local-energy approximation is made to allow t to be applied to finite nuclei. The t matrix, which is complex, is then folded with the nuclear density to obtain both real and imaginary parts of the nuclear optical potential for finite nuclei. In the nuclear structure approach,[3–6] the optical potential is calculated in finite nuclei using a set of random-phase approximation (RPA) transition densities to describe fairly realistically the response of the nucleus to the "external field" of the projectile. Realistic nuclear forces are usually used to calculate an effective projectile-target nucleon interaction, which should, however, be calibrated against nuclear inelastic scattering cross section data. The optical potential is calculated to second-order, proceeding to the RPA particle-hole intermediate states and back to the ground state. The propagation of the projectile in the intermediate state is described by an optical-potential Green's function, which for open target channels is complex and accounts for absorption.

The transition form factor for inelastic scattering is also complex.[7] In the collective model this form factor is calculated as the derivative of the optical potential including the imaginary part.[7] The imaginary term is not only a natural result of using the collective model, but is also required to fit the data.[7,8] In the nuclear matter approach to inelastic transitions, the complex form factor results from folding a complex t matrix with transition densities,[9] although these form factors do not satisfy the criterion of Ref. 8. In the nuclear structure approach,[10] a complex form factor results from second-order excitation of the final state via some intermediate open channels. Realistic calculations of the imaginary form factor in the nuclear structure approach are difficult but technically possible.

The (p,n) reaction to the analog of the target ground state is a simple example of a nonelastic scattering process which should also have a complex transition form factor. The theoretical approaches discussed in the preceding paragraph also apply to charge exchange. Computationally, analog charge exchange in even nuclei is simpler than inelastic scattering because the requirement that $\Delta J=0$ restricts $\Delta J_1=\Delta J_2$ for the individual steps of the two-step mechanism.

Brown *et al.*[11] have undertaken a calculation of the two-nucleon isospin potential V_τ using π and ρ meson exchange potentials in first and second order. Since these particular meson potentials are purely of the spin-isospin-flip type (central plus tensor), a contribution to $\tau\cdot\tau$ comes in first order only from the exchange term, which is, however, small and varies slowly with energy.[11] The principal contribution both to magnitude and energy dependence of the $\tau\cdot\tau$ potential is due to the tensor force in second order, which gave a positive V_1, in satisfactory agreement with phenomenological interactions. Reference 11 did not treat the imaginary contribution to the second-order interaction, but claimed it to be small. By contrast, the phenomenological value[12] is large and also positive.

Because of the intrinsic interest in calculation of the imaginary part of a nonelastic process, and, because of the importance of W_1 for charge exchange and for semidirect photocapture,[13,14] we have undertaken a calculation of W_1 in second order for the closed-shell nucleus ^{48}Ca.

In calculations of the imaginary optical potential, collectivity in the intermediate inelastic transitions is impor-

tant. The residual interaction pushes isoscalar strength downward and into the open-channel region of energy, thereby enhancing the absorption by inelastic channels and therefore increasing the imaginary optical potential. The strong isoscalar collective inelastic states do not have strong matrix elements, however, for charge exchange to the analog state. Furthermore, isovector states, which would couple strongly to the analog state, are pushed up in energy by the interaction, and therefore the collectivity would tend to diminish rather than increase the absorption. We therefore choose to calculate W_1 using pure particle-hole states, knowing that we are not losing a large fraction of the charge-exchange strength. We treat the particle continuum by generating exact scattering wave functions in a Woods-Saxon potential.

Section II contains a description of the theory, Sec. III gives numerical results and compares calculated results with empirical values, and Sec. IV contains a summary and discussion.

II. THEORY

A. Derivation of the second-order Lane potential

The $V_\tau\ \tau\cdot\tau$ operator acting in second order results in a sum of isovector and isoscalar operators. By considering second-order charge exchange we are certain to pick out only the isovector part. There are four types of intermediate particle-hole excitations which will contribute in second order to the excitation of the isobaric analog state. These involve intermediate inelastic excitations and intermediate charge exchange excitations, each with either the intermediate particle or intermediate hole scattered in the second step of the excitation as in Fig. 1. The sum of the contributions from all these diagrams gives a second-order charge exchange operator $V_1^{(2)}$, which adds to the first-order real operator and contributes an imaginary part to the transition operator.

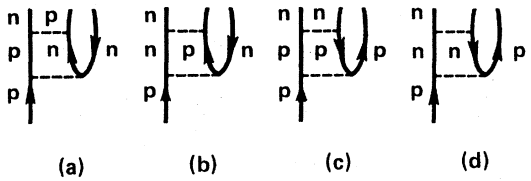

FIG. 1. Second-order particle-hole diagrams leading to the charge-exchange (p,n) reaction. These four diagrams contribute to the Lane potential V_1+iW_1 when the final nuclear state is an analog state.

Alternatively, one may obtain $V_1^{(2)}$ by considering the difference between neutron and proton elastic scattering for the nucleus. The various second-order contributions to elastic scattering are shown in Fig. 2. Subtracting the contributions of diagrams (a), (b), and (c) from those of (d), (e), and (f) and equating this to the difference between nuclear neutron and nuclear proton potentials based on the Lane model gives us an expression for the second-order contributions to the Lane potential:

$$V_n-V_p=\left[V_0+\frac{N-Z}{4A}V_1\right]-\left[V_0-\frac{N-Z}{4A}V_1\right]$$
$$=\frac{N-Z}{2A}V_1\ . \tag{1}$$

However, Eq. (1) need not be correct. The difference between V_n and V_p can be due not only to the isospin conserving Lane potential but also to isospin nonconserving Coulomb corrections (see Sec. II B). We must therefore directly calculate the four terms of the second-order charge exchange amplitude shown in Fig. 1. Details of the derivation are carried out in the Appendix with the result

$$V_1^{(2)}=\frac{4A}{N-Z}\left\{\sum_{i=f_p}^{f_n}\left[\sum_{m>f_n}^{\infty}\langle i,p\,|\,v_\tau\,|\,m,n\rangle g_{mi}^{(p)}\langle m,n\,|\,V_0-V_\tau\,|\,i,n\rangle+\sum_{m>f_p}^{\infty}\langle i,p\,|\,V_0-V_\tau\,|\,m,p\rangle g_{mi}^{(n)}\langle m,p\,|\,V_\tau\,|\,i,n\rangle\right]\right.$$
$$-\sum_{m>f_p}^{f_n}\left[\sum_{i=0}^{f_p}\langle i,p\,|\,V_\tau\,|\,m,n\rangle g_{mi}^{(p)}\langle m,p\,|\,V_0+V_\tau\,|\,i,p\rangle\right.$$
$$\left.\left.+\sum_{i=0}^{f_n}\langle i,n\,|\,V_0+V_\tau\,|\,m,n\rangle g_{mi}^{(n)}\langle m,p\,|\,V_\tau\,|\,i,n\rangle\right]\right\}+\text{like terms for } V_\sigma \text{ and } V_{\sigma\tau}\ , \tag{2}$$

where mi are various particle hole states; n refers to neutrons and p to protons; f_p and f_n are Fermi energies for protons and neutrons, respectively; V_0 and V_τ are central and tensor noncharge-exchange and charge-exchange potentials (with the $\tau\cdot\tau$ operation on target and projectile already carried out); and g_{mi} is the projectile Green's function representing the propagation of the projectile in the intermediate state. Because V_τ and $V_0\pm V_\tau$ have opposite signs, and because V_σ is small, all terms of Eq. (2) except the spin independent third and fourth have signs opposite to that of the Green's functions, which has a negative imaginary part. The third and fourth terms are the hole-scattering terms of Figs. 1(b) and (d), which were found in Ref. 10 to subtract from the particle-scattering terms. These terms tend to be relatively small and confined to the interior of the nucleus. They therefore suppress the

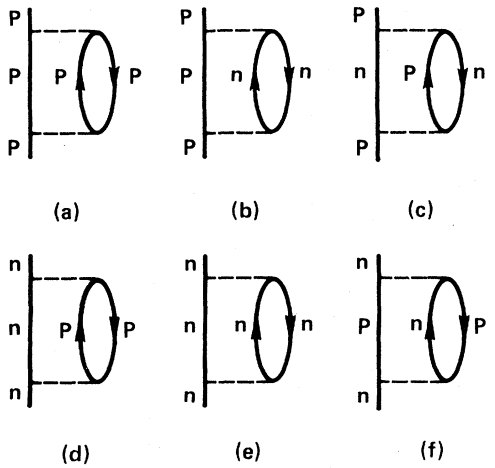

FIG. 2. Second-order particle-hole diagrams contributing to the neutron and proton optical potentials.

interior contribution to $V_1^{(2)}$ but leave the overall sign of Eq. (1) as positive in both real and imaginary terms. The sign of W_1 is therefore in agreement with the phenomenological Lane potential.

As an estimate of the dependence of $V_1^{(2)}$ on N, Z, and A, note that in each term there is a sum over a large set of single-particle states, for example, over particle states $m > f_n$ in the first term. Ignoring the energy dependence of the Green's function, which provides an upper cutoff on particle-hole energies related to the closing of channels, and ignoring the incompleteness of the sets of states, we may think of carrying out the infinite m sums using closure. There remains in each term a sum over the neutron excess (f_p to f_n) which should give a magnitude roughly proportional to $N-Z$. Furthermore, the remaining two single-hole wave functions are each normalized to the nuclear size, the product having a dependence on mass number of $1/A$. This closure argument cannot be used for the sum over hole states in the third and fourth terms of Eq. (2). For these terms we can estimate that the sums over hole states i from 0 to f_p and 0 to f_n, respectively, are proportional to Z/A and N/A, respectively, which are then roughly mass independent. The remaining particle sum, cut off by the interaction with (bound) hole states, should again depend on N,Z,A as $(N-Z)/A$. According to this argument $V_1^{(2)}$ as given by Eq. (2) should be roughly independent of A and $N-Z$.

B. Coulomb correction

Equation (2) was calculated using second-order processes including one inelastic scattering and one charge exchange. If the Lane potential is calculated from the difference in neutron and proton optical potentials as in Eq. (1), we must assume that the optical potential other than the Coulomb potential is isospin conserving. This is not the case for the real potential, for which a correction[12] for the diminished kinetic energy of protons compared to neutrons inside the nucleus is normally made. On the basis of what is known about phenomenological potentials,[15] it also need not be true for the imaginary potential. The difference between neutron and proton second-order optical potentials is then

$$V_n^{(2)} - V_p^{(2)} = \frac{N-Z}{2A} V_1^{(2)} - V_{cc}^{(2)} , \tag{3}$$

where $V_{cc}^{(2)}$ is the Coulomb correction to the second-order optical potential. The two functions on the right are not uniquely determined by the single equation Eq. (3). However, from the charge exchange we may define $V_1^{(2)}$ uniquely from the matrix element for charge exchange,

$$M_{ce}^{(2)} = \frac{\sqrt{2T_0}}{2A} V_1^{(2)} = \frac{\sqrt{N-Z}}{2A} V_1^{(2)} , \tag{4}$$

where the right-hand side is the charge-exchange matrix element of the Lane potential. Eliminating $V_1^{(2)}$ between Eqs. (3) and (4) then gives us a formula for the second-order Coulomb correction,

$$V_{cc}^{(2)} = V_p^{(2)} - V_n^{(2)} + \sqrt{N-Z}\, M_{ce}^{(2)} . \tag{5}$$

Although Eq. (2) and our numerical calculations ignore Coulomb effects, Eq. (5) gives us a way of defining and calculating the Coulomb correction to the second-order real and to the imaginary potential for a neutron excess nucleus.

C. Calculation

Following the formalism of our earlier paper,[4] the calculation of Eq. (2) was carried out including direct and exchange matrix elements in second order but with particle-hole intermediate states. Thus in the densities of Ref. 4 the spectroscopic amplitude, $x_{j_1 j_2}^{J_N J_A} + y_{j_1 j_2}^{J_N J_A}$ in Eq. (15), was replaced by the particle-hole spectroscopic amplitude, namely 1 for $j_1 j_2 = j_h j_p$.

The bound particle and hole states and the continuum particle states were calculated in a real Saxon-Woods potential. These particle-hole states are regarded as doorways to more complicated configurations, 2p-2h, 3p-3h, etc., all of which contribute to the spreading width. These configurations are approximately taken into account in leading order in the sense that we excite only the doorway part of each one. The spreading interaction just redistributes the particle-hole strength.

As our effective nucleon-nucleon interaction we have used the Eikemeyer-Hackenbroich (EH) t operator.[16] This interaction is quite close in its charge-exchange spin-flip strength to the π-plus-ρ interaction used in Ref. 11. Its strength for inelastic scattering has been tested[17] by calculating direct 3^- and 5^- inelastic cross sections in ^{40}Ca, and its V_τ strength has been tested by calculating the (p,n) analog cross section.[18] The tensor part of the EH interaction is in very close agreement with the π-plus-ρ interaction up to about $q = |k_f - k_i| = 2.5$, which would cover the relevant range in our calculation. In addition, the central and tensor spin-isospin forces are both in close agreement in their effective interactions[19] to that of Love and Franey,[19] based on the impulse approximation.

For computational convenience the actual calculations are done as though isospin is conserved, so the final state nuclear wave function is taken to be exactly

$T_+ \mid \text{g.s.}\rangle/\sqrt{N-Z}$, and the intermediate projectile is always calculated as a proton. The former approximation would seem to be very good, but one might worry about Coulomb correction effects in the intermediate propagator. In an early paper[5] we showed that for ^{40}Ca(p,p) and ^{40}Ca(n,n) propagator differences and differences in twofold charge exchange (p,n,p) and (n,p,n) were rather large. In charge exchange, however, there is no twofold charge exchange, and furthermore, the Coulomb force and the symmetry potential tend to cancel. On the average the kinetic energy is only about 0.7 MeV different in ^{48}Ca (smaller for protons), whereas in ^{40}Ca the average kinetic energy of protons in the nucleus is 8 MeV lower than that of neutrons of the same incident energy. Thus the neglect of differences in the intermediate propagator is justified for ^{48}Ca.

D. Energy averaging

The optical model represents an energy average over details of the scattering process.[20,21] Resonances due to excitation of individual compound states thus contribute only in an average way to the absorption. Calculation of the optical potential to second order using particle-hole states does not involve detailed compound states, but rather doorways as described above. Nevertheless, rather sharp potential resonances in the projectile Green's function do appear in calculation of Eq. (2), particularly for closed channels. To eliminate this undesirable feature we use here averaging over the beam energy of g_{mi} in Eq. (2), thus broadening out these resonances. This is done by using a Lorentzian distribution of beam energy E and width $2I$ and folding it with Eq. (2). Since the beam energy appears only in the Green's function g_{mi}, which is evaluated at energy $E-(\epsilon_m-\epsilon_i)$, where $\epsilon_m-\epsilon_i$ is the particle-hole energy, the averaging process involves only g_{mi}. The averaging over energy simply replaces E by $E+iI$. For this purpose and also for the treatment of closed channels, $E-(\epsilon_m-\epsilon_i)<0$, we have developed a code for calculating the Green's function at complex wave number $k=\alpha+\beta i$. The results presented below were calculated with a width of $2I=4$ MeV.

III. RESULTS

The imaginary optical potential calculated using Eq. (2) is shown in Fig. 3 along with results of a zero-range calculation of the contribution from a single intermediate ground-state deuteron channel. Since the intermediate deuteron channel also represents a particle-hole state of the nucleus, it is not orthogonal to inelastic and charge-exchange intermediate states. We have therefore not added the contributions from these two types of doorways but have displayed them both to give an estimate of the relative strength of this one strong intermediate channel. The sum of the deuteron doorway and particle-hole doorways therefore represents an upper limit of the contribution from both kinds of doorways. Also shown in Fig. 3 is the Becchetti-Greenlees[12] (BG) phenomenological potential W_1. The volume integral of the BG potential is a little

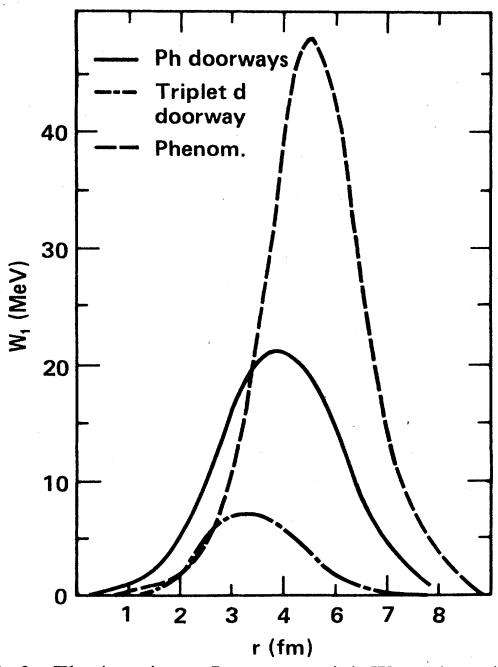

FIG. 3. The imaginary Lane potential W_1. The solid curve is the calculated result based on inelastic and charge-exchange particle-hole intermediate doorway states. The dashed-dot curve is the calculated result for the triplet ground state deuteron intermediate configuration, and the dashed curve is the Becchetti-Greenlees phenomenological W_1 potential.

greater than double that of the theoretical particle-hole potential.

In Table I the contribution from various multipolarities of the intermediate states is shown. The numbers are actually the values of the diagonal nonlocal second-order charge-exchange operator summed over all particle-hole states coupled to a given multipolarity. For computational convenience the calculation was cut off at $J=5$. All important multipoles are included. There will be small but non-negligible contributions from $J=6,7,8,9$. The contributions from multipolarities beyond 9 fall off drastically; they will involve at least an $i\frac{13}{2}$ particle (coupled to the $f\frac{7}{2}$ hole), which the angular momentum barrier effectively keeps out of the ^{48}Ca nucleus. Thus although we

TABLE I. Contributions to $W_1(r,r)$ at the nuclear surface from different multipoles.

J^π	$W_1(4.0,4.0)$
0^+	0.010 MeV fm^{-3}
0^-	0.262 MeV fm^{-3}
1^+	0.032 MeV fm^{-3}
1^-	0.064 MeV fm^{-3}
2^+	0.131 MeV fm^{-3}
2^-	1.597 MeV fm^{-3}
3^+	0.0834 MeV fm^{-3}
3^-	0.241 MeV fm^{-3}
4^+	0.103 MeV fm^{-3}
4^-	0.136 MeV fm^{-3}
5^+	0.098 MeV fm^{-3}
5^-	0.166 MeV fm^{-3}

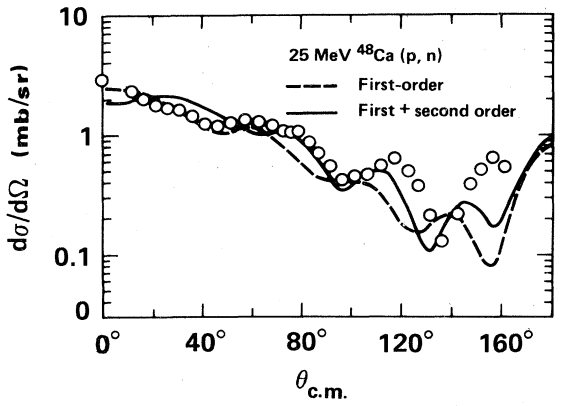

FIG. 4. A comparison of the differential cross sections calculated microscopically to first order and including the second-order imaginary terms with experimental data (Ref. 22).

have missed a small amount of strength in the higher multipoles, there is no question of a large contribution from many high multipolarities due to a slow convergence of the sum.

There is also a second-order real contribution to the Lane potential, because the Green's functions g_{mi} in Eq. (2) are complex. The real part of g_{mi} oscillates in sign with excitation energy and therefore tends to cancel itself out when many doorways are summed. For highly closed channels it has a steady sign corresponding to a positive contribution to V_1. Calculations carried out up to particle energies ϵ_m of 150 MeV show that the imaginary part of $V_1^{(2)}$ has converged but the real part has not. It can be shown, by exact integration over all plane wave particle states, that for a zero-range force the real part does not converge at all.

In Fig. 4 is shown the differential cross section for the ^{48}Ca(p,n) reaction calculated microscopically with and without the second-order imaginary contribution. It is clear that the W_1 term improves the phasing at back angles, but at the expense of a poorer fit to the data[22] at forward angles. The second-order real potential was not included because of the problems with convergence discussed above. In spite of the neglect of this correction, we obtain the correct magnitude of the (p,n) analog cross section and an angular distribution which, though not good, does bear a resemblance to the data.

IV. SUMMARY AND DISCUSSION

In summary, we have calculated the imaginary Lane charge-exchange interaction W_1 for the second order ^{48}Ca(p,n)0^+ analog reaction using all intermediate particle-hole states compatible with the Pauli principle. The second-order exchange mechanism and the particle continuum were treated exactly. The resulting W_1 potential is smaller than the phenomenological potentials by a factor of about 2.

The Eikemeyer-Hackenbroich effective interaction,[16] which we have used, is close in its charge-exchange, spin-exchange components to the π-plus-ρ exchange force used by Brown *et al.*[11] in a second-order calculation of the V_τ two-nucleon interaction. Our calculation is in principle similar to theirs except that we use all components of the EH force, whereas the π-ρ force has only the spin-isospin parts. The π-ρ interaction was used in Ref. 11 as the bare interaction to the v_τ component of the real, effective two-nucleon interaction. The authors of Ref. 11 have stated that the imaginary part was very weak. We find that the unnatural parity states, which are mainly excited by the π-ρ exchange force, contribute in second order about half of the calculated W_1 strength. The other half comes from natural parity intermediate states, which are excited through inelastic scattering or charge exchange involving $V_0 \pm V_\tau$, and V_τ.

We have used particle-hole intermediate states instead of RPA states, the collectivity of which is known to be important in the calculation of the optical potential. It is argued that collectivity should not be so important, however, for charge exchange. If anything, collectivity should lead to a reduction rather than an increase in W_1.

The contribution to W_1 from an intermediate deuteron configuration has been calculated in the zero-range approximation. This single channel gave about one-third as much contribution to W_1 as all particle-hole channels. A similar result has been found by Coulter and Satchler[23] in calculation of the imaginary optical potential in ^{40}Ca, ^{48}Ca, and ^{208}Pb. It is not appropriate to try to include all intermediate states of the deuteron, since these are redundant with the inelastic and charge-exchange particle-hole states. Thus, considering the fact that the zero-range approximation gives an overestimate of the (p,d,n) mechanism, and that even for the ground state deuteron there is redundancy between deuteron and nucleon particle-hole channels, it appears that the intermediate (p,d,n) process, though by no means negligible, cannot make a major contribution to W_1.

The contribution to the optical potential from higher-order processes, leading to more complicated configurations than one-particle—one-hole states considered here, is being considered within a soluble model. The results, which will be published in a separate paper,[24] indicate that excitation of such configurations from the doorway channels leads to a broadening of the single-particle resonances of the intermediate projectile but not to additional strength. Coupling among doorways is also considered in Ref. 24. Because of the typical weakness of coupling between doorways, and because third- and higher-order processes do not lead to a sum of individual contributions which are necessarily constructive, as they are for excitation followed by deexcitation of doorways, it seems also unlikely that coupling between doorways can make a very large contribution to imaginary optical potentials. For analog charge exchange, which we have calculated here, however, there exists a particular set of three-step processes $0^+ \rightarrow J^\pi \rightarrow J^\pi$ analog $\rightarrow 0^+$ analog, for which the nuclear matrix elements are all in phase. The analog transition $J^\pi \rightarrow J^\pi$ analog represents a coupling between inelastic doorways from the initial or final states. The phase of each such three-step amplitude then depends on the product of the propagators in the two doorway excitations. It has been shown[25] for low-lying collective doorway states,

both assumed open, that the three-step amplitude is approximately out of phase with the one-step amplitude, which means that such processes make mainly a negative contribution to the real potential. For two highly closed channels, the phase will be constructive with the one-step, therefore contributing positively to the real potential. When one channel is open and one closed, as will be the case for excitation energies between $E_{\rm proj}-\Delta E_{\rm Coul}$ and $E_{\rm proj}$, there should be a significant imaginary contribution in phase with the two-step part which we have calculated. Realistic calculation of these three-step amplitudes for many intermediate states is beyond our current capability of computation and beyond the scope of the present paper.

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APPENDIX: DERIVATION OF SECOND-ORDER LANE POTENTIAL

In this appendix we calculate the projectile isospin matrix elements and the entire target matrix elements for the second-order Lane potential with intermediate particle-hole states. Exchange may be automatically included with a nonlocal space potential when the two-body space-exchange potential is multiplied by the spin-isospin exchange operator. No details of the space-spin dependence are included since that part of our calculation is the same as in Ref. 4 applied to the special case of particle-hole states. The two second-order charge-exchange matrix elements from the ground state of the target and proton state of the projectile through intermediate particle-hole state mi to the analog-plus-projectile-neutron state are

$$\begin{aligned}M_{IS}=&\langle T_0,T_0-1,\tfrac{1}{2},\tfrac{1}{2}\mid\sum_{\kappa'}V_\tau\tau_0\cdot\tau_{\kappa'}\sum_{im\nu}a^\dagger_{m\nu}a_{i\nu}\mid T_0,T_0\tfrac{1}{2},-\tfrac{1}{2}\rangle\\&\times g_{mi}\langle T_0,T_0\tfrac{1}{2},\tfrac{1}{2}\mid a^\dagger_{i\nu}a_{m\nu}\sum_{\kappa}(V_0+V_\tau\tau_0\cdot\tau_\kappa)\mid T_0,T_0,\tfrac{1}{2},-\tfrac{1}{2}\rangle\\=&\,2\langle T_0,T_0-1\mid\sum_{\kappa'}V_\tau t_{\kappa'_-}\sum_{im\nu}a_{m\nu}a_{i\nu}\mid T_0,T_0\rangle g_{mi}\langle T_0,T_0\mid a^\dagger_{i\nu}a_{m\nu}\sum_{\kappa}(V_0-V_\tau\tau_{\kappa3})\mid T_0,T_0\rangle\\&+\text{like terms for spin forces}\end{aligned}\tag{A1}$$

for inelastic intermediate states of isospin projection ν, and

$$\begin{aligned}M_{\rm ce}=&\langle T_0,T_0-1,\tfrac{1}{2},\tfrac{1}{2}\mid\sum_{\kappa}(V_0+V_\tau\tau_0\cdot\tau_\kappa)\sum_{im}a^\dagger_{m,-1/2}a_{i,1/2}\mid T_0,T_0,\tfrac{1}{2},\tfrac{1}{2}\rangle g_{mi}\\&\times\langle T_0,T_0,\tfrac{1}{2},\tfrac{1}{2}\mid a^\dagger_{i,1/2}a_{m,-1/2}\sum_{\kappa=1}V_\kappa\tau_0\cdot\tau_\kappa\mid T_0,T_0,\tfrac{1}{2},-\tfrac{1}{2}\rangle\\=&\,2\langle T_0,T_0-1\mid\sum_{\kappa}(V_0+V_\tau\tau_{\kappa3})\sum_{mi}a^\dagger_{m,-1/2}a_{i,1/2}\mid T_0,T_0\rangle g_{mi}\langle T_0,T_0\mid a^\dagger_{i,1/2}a_{m,-1/2}\sum_{\kappa'}V_\tau(\kappa')t_{\kappa'}\mid T_0,T_0\rangle\\&+\text{like terms for spin forces}\end{aligned}\tag{A2}$$

for charge-exchange intermediate states. In Eqs. (A1) and (A2) g_{mi} is the intermediate projectile Green's function at energy $E-E_{mi}$, and V is the two-body interaction between the projectile nucleon and nuclear target nucleon. In the first matrix element of Eq. (A1) the operator $V_0-V_\tau\tau_{\kappa3}$ must produce the particle-hole pair and the operator $V_\tau t_{\kappa'_-}$ must (charge exchange) scatter either the hole or particle (see Fig. 1) to give the final analog state $\mid T_0,T_0-1\rangle$; that is,

$$\begin{aligned}\langle T_0,T_0-1\mid\sum_{\kappa'}V_\tau t_{\kappa'_-}a^\dagger_{m\nu}a_{i\nu}\mid T_0,T_0\rangle=&\langle T_0,T_0-1\mid\sum_{cd}\langle c\mid V_\tau\mid d\rangle a^\dagger_{c,-1/2}a_{d,1/2}a^\dagger_{m\nu}a_{i\nu}\mid T_0T_0\rangle\\=&\langle T_0T_0-1\mid\sum_{c}\langle c\mid V_\tau\mid m\rangle a^\dagger_{c,-1/2}a_{i,1/2}\delta_{\nu,1/2}\\&-\sum_{d}\langle i\mid V_\tau\mid d\rangle\delta_{\nu,-1/2}a^\dagger_{m,-1/2}a_{d,1/2}\mid T_0,T_0\rangle\ .\end{aligned}\tag{A3}$$

Only those particle-hole states in $J=0$ pairs with common neutron and proton single-particle quantum numbers can contribute to the analog state. We can write Eq. (A3) using $\mid T_0T_0-1\rangle=T_-\mid TT\rangle/\sqrt{2T_0}$ as

$$\frac{1}{\sqrt{2T_0}}\langle T_0T_0\mid T_+\left[\sum_c\langle c\mid V_\tau\mid m\rangle\delta_{\nu,1/2}a^\dagger_{c,-1/2}a_{i,1/2}-\sum_d\langle i\mid V_\tau\mid d\rangle\delta_{\nu,-1/2}a^\dagger_{m,-1/2}a_{d,1/2}\right]\mid T_0,T_0\rangle\tag{A4}$$

in which the T_+ operator must destroy the particle-hole operator in each term. The matrix element in Eq. (A3) is then

$$\frac{1}{\sqrt{2T_0}}(\delta_{\nu,1/2}\langle i\mid V_\tau\mid m\rangle-\delta_{\nu,-1/2}\langle i\mid V_\tau\mid m\rangle)\ .\tag{A5}$$

The other factor in Eq. (A1) is likewise

$$\langle m\nu \mid V_0+V_1\tau_3 \mid i\nu\rangle = \langle m\nu \mid V_0+2\nu V_1 \mid i\nu\rangle \ . \tag{A6}$$

Thus ignoring the difference between neutron and proton orbits, Eq. (A1) gives simply

$$M_{IS}=\frac{2}{\sqrt{2T_0}}\left[\sum_{i=f_Z}^{f_N}\sum_{m=f_N}^{\infty}\langle i \mid V_\tau \mid m\rangle g_{mi}\langle m \mid V_0-V_\tau \mid i\rangle - \sum_{i=1}^{f_Z}\sum_{m=f_Z}^{f_N}\langle i \mid V_\tau \mid m\rangle g_{mi}\langle m \mid V_0+V_\tau \mid i\rangle\right] . \tag{A7}$$

Likewise for Eq. (A2) we may write the intermediate charge-exchange matrix elements (A2) as

$$M_{\rm ce}=\langle T_0T_0-1 \mid \sum_{cd\mu}\langle c\mu \mid V_0+V_\tau\tau_3 \mid d\mu\rangle a^\dagger_{c\mu}a_{d\mu}a^\dagger_{m,-1/2}a_{i,1/2} \mid T_0T_0\rangle$$
$$\times g_{mi}\langle T_0T_0 \mid a^\dagger_{i,1/2}a_{m,-1/2}\sum_{ef}\langle e \mid V_\tau \mid f\rangle a^\dagger_{e,-1/2}a_{f,1/2} \mid T_0T_0\rangle \ . \tag{A8}$$

In the first step V creates the particle-hole mi and in the second step V scatters either the particle or the hole, leaving a different particle hole and giving

$$2\Big[\langle T_0T_0-1 \mid \sum_c\langle c-\tfrac12 \mid (V_0+V_\tau\tau_3) \mid m-\tfrac12\rangle a^\dagger_{c,-1/2}a_{i,1/2}$$
$$-\sum_d\langle i\tfrac12 \mid V_0+V_\tau\tau_3 \mid d\tfrac12\rangle a^\dagger_{m,-1/2}a_{d,1/2} \mid T_0T_0\rangle\Big] g_{mi}\langle m \mid V_\tau \mid i\rangle$$
$$=\frac{2}{\sqrt{2T_0}}\langle T_0T_0 \mid T_+\Big[\sum_c\langle c \mid V_0-V_\tau \mid m\rangle a_{c,-1/2}a_{i,1/2}$$
$$-\sum_d\langle i \mid V_0+V_\tau \mid d\rangle a^\dagger_{m,-1/2}a_{d,1/2}\Big] T_0T_0\rangle g_{mi}\langle m \mid V \mid i\rangle \ . \tag{A9}$$

With $a^\dagger_{b,1/2}a_{b,-1/2}$, Eq. (A9) gives

$$M_{\rm ce}=\frac{2}{\sqrt{2T_0}}\left[\sum_{i=f_Z}^{f_N}\sum_{m=f_Z}^{\infty}\langle i \mid V_0-V_\tau \mid m\rangle g_{mi}\langle m \mid V_\tau \mid i\rangle + \sum_{i=0}^{f_N}\sum_{m=f_Z}^{f_N}\langle i \mid V_0+V_\tau \mid m\rangle g_{mi}\langle m \mid V_\tau \mid i\rangle\right] . \tag{A10}$$

Thus combining Eqs. (A7) and (A10) gives us the second-order charge-exchange matrix element of a pure $\tau\cdot\tau$ two-body force. If we are to interpret this as the Lane potential this sum must be equal to

$$\langle T_0T_0-1,\tfrac12\tfrac12 \mid (V_1+iW_1)\frac{\mathbf{T}\cdot\mathbf{t}}{A} \mid T_0T_0,\tfrac12\tfrac12\rangle = \frac{1}{2A}\sqrt{2T_0}(V_1+iW_1) \ . \tag{A11}$$

Thus combining (A7) and (A10) and equating the sum to Eq. (A11) we have for the second-order contributions to the isospin potential

$$V_1^{(2)}=\frac{4A}{N-Z}\left[\sum_{i=f_Z}^{f_N}\sum_{m>f_N}^{\infty}\langle i \mid V_\tau \mid m\rangle g_{mi}\langle m \mid V_0-V_\tau \mid i\rangle - \sum_{i=1}^{f_Z}\sum_{m>f_Z}^{f_N}\langle i \mid V_\tau \mid m\rangle g_{mi}\langle m \mid V_0+V_\tau \mid i\rangle\right.$$
$$\left.+\sum_{i=f_Z}^{f_N}\sum_{m>f_Z}^{\infty}\langle i \mid V_0-V_\tau \mid m\rangle g_{mi}\langle m \mid V_\tau \mid i\rangle - \sum_{i=1}^{f_N}\sum_{m>f_Z}^{f_N}\langle i \mid V_0+V_\tau \mid m\rangle g_{mi}\langle m \mid V_\tau \mid i\rangle\right] . \tag{A12}$$

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