Structure of the actinides by the interacting boson model

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(Received 9 April 1985)

The interacting boson model is able to reproduce the elusive two-particle-transfer data in the actinides (uranium and thorium). The same calculation also reproduces well the energy systematics and the $B(E2)$ data.

Two-neutron-transfer reactions in the actinides have not been well understood since the first experimental data[1] became available. These data showed that the cross section for the reactions $^{A+2}X(\mathrm{p,t})^{A}X$ populating the 0_2^+ state (i.e., the β bandhead) was an appreciable fraction, typically 10—15 % of the cross section for population of the 0_1^+ ground state by the same reaction. By contrast, predictions based on the geometrical model give a very low estimate for the ratio of these two cross sections ($\approx 10^{-3}$). On the other hand, it was found that the experimental cross-section ratio for populating these same bands by the $^{A}X(\mathrm{t,p})^{A+2}X$ reaction was generally quite small ($\approx 10^{-3}$), although ratios comparable to those found for the (p,t) reaction were observed from a few product nuclei, e.g., ^{248}Cm.[2] This is well summarized in a recent publication by Janecke *et al.*[3]

Janecke *et al.*[3] have also estimated the ratio of the two-neutron-transfer (TNT) cross sections to the bandhead of the β band and the ground state in the actinides using the interacting boson model (IBM).[4] Their ratios are similar to those based on the geometrical model, i.e., two orders of magnitude lower than experimental values for (p,t) reactions. Their results have suggested that TNT reactions in the actinides cannot be described by the IBM. We show below that this is not the case. It is shown in this paper that the TNT cross sections depend critically on the choice of the IBM Hamiltonian. We have chosen a Hamiltonian different in form from that adopted by Janecke *et al.* Using this Hamiltonian we have reproduced the energy spectra and $B(E2)$ values equally well (or better) and have also been able to reproduce the TNT cross ratios for (p,t) reactions.

In the interacting boson model, the even parity collective excitations can be described by a Hamiltonian depending on six parameters.[4] In principle, these parameters can be determined for any nucleus by fitting model predictions to experimental data using least squares procedures. Typically, the fit is more sensitive to some parameters and less sensitive to others. Because of the complexity of the problem, a restricted class of Hamiltonians is generally studied and the parameters in this class are optimized. Because of the insensitivity of the spectra to some parameter variations, it may happen that quite different Hamiltonians give rise to spectra which fit experimental energy levels with roughly comparable residuals. In such cases, one must distinguish among models by probing the eigenfunctions. All such probes involve transitions. The most sensitive and most used probes are the $B(E2)$ values for transitions that depopulate the β and γ bands. Another useful probe is the TNT reactions.

The Hamiltonian used by Janecke *et al.*[3] to fit the spectra in the actinide region belongs to a restricted class of the form

$$H_\epsilon = \epsilon_d n_d + \kappa Q\cdot Q \ , \tag{1}$$

where n_d is the d-boson number operator and

$$Q = (d^\dagger s + s^\dagger d)^2 + \chi/\sqrt{5}(d^\dagger d)^2 \ . \tag{2}$$

We have recently studied[5] the properties of another restricted class of Hamiltonian of the form

$$H_\chi = -\kappa Q\cdot Q + \kappa' L\cdot L \ , \tag{3}$$

$$L = \sqrt{10}(d^\dagger d)^1 \ . \tag{4}$$

The operators $s^\dagger, d^\dagger/s, d$ in Eqs. (1)—(4) are the creation/annihilation operators for the $l=0$ (s) and 2 (d) bosons. When $\chi = -\sqrt{35}/2(0)$, the operator Q of Eq. (3) is a generator of the SU(3) [O(6)] algebra. Both Hamiltonians can reproduce spectra in reasonable agreement with the experimental spectra (with exceptions noted below), with the fits using Eq. (2) somewhat better than those obtained from Eq. (1). However, *it must be noted that when the experimental energy levels exhibit a nearly SU(3) behavior, Eq. (1) cannot adequately fit the data.*

We have tested these two Hamiltonians against the thorium and uranium isotopes for four reasons:

(i) These isotopes have well-defined β and γ bands.

(ii) There are no 0^+ intruder states below 1.5 MeV.

(iii) There are good measurements of two-neutron-transfer reactions.

(iv) The β and γ bands are rotational bands with moments of inertia slightly greater than that of the ground band.

The spectra of these nuclei have also been studied in Ref. 6 by the use of a general IBM Hamiltonian with six parameters. In this paper, we shall compare the results obtained by the use of the Hamiltonians defined by Eqs. (1) and (3). In Fig. 1, we compare the best fits obtained from the Hamiltonians of Eq. (3) to the experimental data for ^{232}U, ^{234}U, and ^{236}U [Fig. 1(a)] as well as for ^{230}Th and ^{232}Th [Fig. 1(b)].

The eigenvectors associated with experimental states are significantly different for these two Hamiltonians. This difference is easily seen in Table I. In this table, we display the overlap integrals between the three lowest 0^+ states computed with H_ϵ and H_χ. We also display the inner products for the four lowest 2^+ states. The parameters for the Hamiltonians H_ϵ and H_χ were adjusted to provide a best fit to the experimental energy levels for ^{232}U. These results show clearly that the eigenfunctions for corresponding physical states computed using the two Hamiltonians are very dissimilar. In fact, the β bandhead (0_2^+) computed using H_ϵ looks more like the ground state (0_1^+) than the 0_2^+ state computed using H_χ, and conversely. Despite this, the energy eigenvalues for corresponding states are comparable, except for the near perfect SU(3)

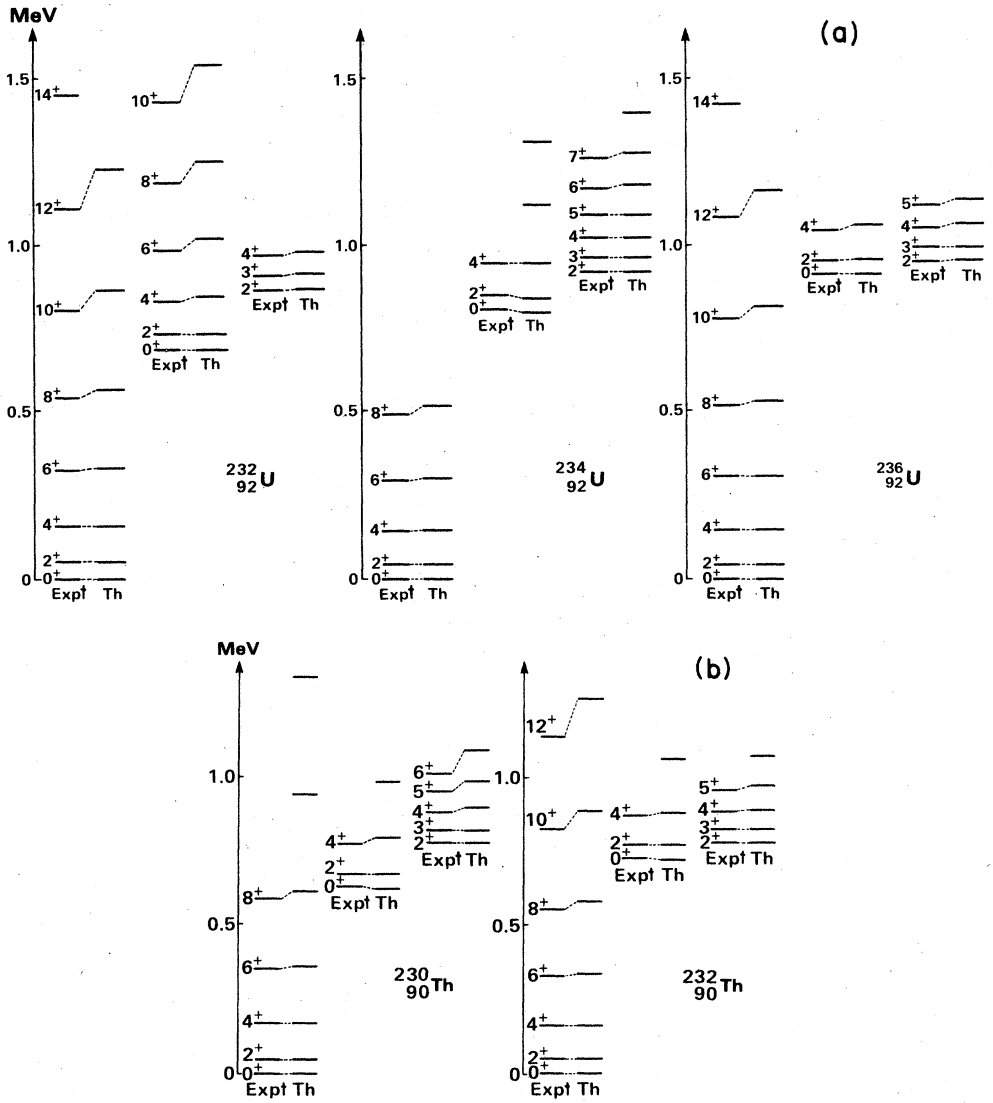

FIG. 1. Comparison between the experimental spectra and the best fit using the Hamiltonian (3). (a) Uranium isotopes. (b) Thorium isotopes. The PHINT parameter values (N,CHQ,ELL,QQ) are the following: ^{232}U (12, −5.206, 0.009 25, −0.114 33); ^{234}U (13, −3.740, 0.0070, −0.019); ^{236}U (14, −2.933, 0.000 64, −0.0228); ^{239}Th (11, −4.001, 0.01, −0.0150); ^{232}Th (12, −3.065, 0.0084, −0.0209).

TABLE I. The overlap integrals of the three lowest 0^+ states and the four lowest 2^+ states for ^{232}U computed using the Hamiltonians H_χ and H_ϵ are shown.

H_ϵ \ H_χ	0_1^+	0_2^+	0_3^+
0_1^+	0.525	0.590	−0.453
0_2^+	0.536	0.181	0.207
0_3^+	0.422	−0.119	0.338

	2_1^+	2_2^+	2_3^+	2_4^+
2_1^+	0.533	0.590	0.007	0.460
2_2^+	0.549	0.164	−0.031	−0.245
2_3^+	0.060	0.009	0.404	−0.031
2_4^+	−0.234	0.115	0.481	0.148

nuclei ^{232}Th and ^{236}U. The 2^+ states also display the same characteristics as the 0^+ states.

The $B(E2)$'s constitute another probe of the wave functions. In lowest order, the transition operator can be chosen as

$$T(E2)=e[(d^\dagger s+s^\dagger d)^2+\chi_{E2}/\sqrt{5}(d^\dagger d)^2] =eQ_{E2}\ . \quad (5)$$

Due to the necessary tensorial property, the operator $T(E2)$ in Eq. (5) has the same form as in Eq. (2). However, the parameter $\chi_{E2}\neq\chi$ stems from the different physical origin of the operators, one electromagnetic, the other nuclear.[7] We have computed the $B(E2)$ transitions using the wave functions of H_ϵ and H_χ; the results are presented in Table II for ^{234}U. The effective charge is adjusted so as to reproduce the $2_g^+\rightarrow 0_g^+$ transitions. The expectation value of the $B(E2)$ operator of Eq. (5) in the wave functions of the two Hamiltonians is a function of χ_{E2}; the $2_\gamma^+\rightarrow 0_g^+$ transition is consistent with the choice $\chi_{E2}=0$ for both Hamiltonians. On the other hand, the $2_\beta^+\rightarrow 0_g^+$ transition is not as well reproduced (see Table II).

TABLE II. The $B(E2)$ transitions for ^{234}U. The calculated values are the expectation values of the $T(E2)$ operator, with $\chi_{E2}=0$, with respect to the wave functions of H_ϵ and H_χ.

$B(E2)$	Expt.(s.p.u.)	H_ϵ	H_χ
Effective charge		1.30	1.59
$2_g^+\rightarrow 0_g^+$	51.2±0.5	51.2	51.2
$2_\beta^+\rightarrow 0_g^+$	2.3±0.3	0.033	0.69
$2_\gamma^+\rightarrow 0_g^+$	2.9±0.3	3.46	2.54

These wave functions can also be probed by TNT reactions. The simplest TNT operator describing transitions from the 0^+ ground state to 0^+ states is proportional to $s^\dagger$ (s) for TNT stripping (pickup) reactions [or s ($s^\dagger$) above half shell]. These operators must be modified by correction factors which take account of the indistinguishability of neutron and proton bosons in the IBM. We have computed the ratio for transfer into the 0_2^+ states and the 0_1^+ state as follows. The experimental energy spectra are used to determine the values of the parameters in the Hamiltonians using a least squares method. The matrix elements of the two-neutron-transfer operator s ($s^\dagger$) between the 0_1^+ ground state of A and the 0_2^+ and 0_1^+ states of $A-2$ ($A+2$) are computed, and the ratio of their absolute squares is taken. This was done for both Hamiltonians H_ϵ and H_χ. For H_ϵ, these ratios are 10^{-3}, in agreement with the results of Janecke *et al.*,[3] but in disagreement with the data. For H_χ, the ratios are consistent with experimental values. These values are presented in Table III. Note that for an arbitrary χ, the Hamiltonian H_χ does not follow a dynamical symmetry. Consequently, the TNT cross sections depend, via the wave functions, on the choice of χ.

The Hamiltonian H_χ has been shown to be applicable

TABLE III. Experimental and theoretical cross section ratios for (t,p) and (p,t) reactions.

	^{232}U	^{234}U	^{236}U	^{230}Th	^{232}Th
	(p,t) ←	(p,t) ←			(p,t) ←
Expt.	0.13±0.01[a]	0.13±0.01[a]			0.18±0.02[a]
H_ϵ	0.001	0.0006			c
H_χ	0.12	0.06			0.135
	(t,p) →	(t,p) →			(t,p) →
Expt.		<0.018[b]			0.028[b]
H_ϵ	0.15	0.17			c
H_χ	0.03	0.06			0.135

[a]The (p,t) $L=0$ cross section ratios taken from Ref. 1.
[b]The (t,p) $L=0$ cross section ratios taken from Ref. 8.
[c]The nucleus ^{232}Th is a good SU(3) nucleus and is difficult to fit with H_ϵ. On the other hand, ^{236}U is not as good an SU(3) nucleus and has been fitted as well as possible by H_ϵ.

in the rare earth region, where the β band generally occurs above the γ band (exception: ^{172}Hf).[5] In particular, it is able to describe a series of isotopes in which band inversion occurs (e.g., ^{168}Hf-^{172}Hf). Band inversion occurs as the parameter χ crosses through the SU(3) limiting value $-\sqrt{35}/2$. In the present paper, we show the applicability of this Hamiltonian in the actinide region, where the β band generally occurs below the γ band (exceptions: ^{224}Ra and heavier Cm isotopes). We have now shown that this Hamiltonian is not only useful at describing spectra throughout this region, but seems to provide reasonable wave functions and may resolve a long-standing question about the "anomalous" strength of the two-neutron-transfer cross sections into the β bandhead.

We thank Ron Brown, John Erskine, and J. L. Wood for their communications about the TNT reaction data. We thank also R. F. Casten, J. Janecke, and B. H. Wildenthal for useful discussions. All numerical computations were done with the code PHINT and FTNT by O. Scholten who kindly gave us permission to use them. This work was supported by the National Science Foundation (PHY-844-1891), Department of Energy (W-7405-ENG-48), and the Chinese Science Foundation.

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