Interacting boson model with surface delta interaction between nucleons

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Expressions for the matrix elements of the interaction between proton and neutron bosons have been obtained for the case of a surface delta interaction acting in n^2p^2 configurations. Boson-boson interaction matrix elements involving *only* bosons of angular momenta 0 and K are particularly simple, being proportional to the *particle-particle interaction energy* for orbital angular momentum K. The results obtained with the boson model are very similar to those previously obtained by diagonalization of the four-particle energy matrices for selected configurations of degenerate orbits.

I. INTRODUCTION

The development and application of the interacting boson model (IBM) (Ref. 1) is currently one of the most exciting areas of nuclear structure physics. There are, however, two important unresolved problems connected with the IBM. First, most work has been restricted to a consideration of s and d bosons, leading to the well-known SU(5), O(6), and SU(3) symmetries in limiting cases. However, there is increasing evidence that also g bosons can play a significant role, and there is some recent work dealing with the effect of g bosons in renormalizing the boson-boson interaction.[2] Second, much work remains to be done in relating the parameters of the IBM to the underlying nucleon-nucleon interaction. Until now, much of the work has been done using a pairing-plus-quadrupole model, which is meant to represent the major part of the effective nucleon-nucleon interaction. Yet it is becoming increasingly clear that we also need higher multipoles. In the absence of a microscopic theory, this will lead to more parameters in the theory.

II. THE SURFACE DELTA INTERACTION

A. General properties

In the present paper we use the surface delta interaction (SDI) as an effective nucleon-nucleon interaction.[3] The SDI has only a *single* parameter, the strength. The SDI provides a surprisingly good first approximation to some nuclear spectra.[4] The SDI has the remarkable property that the strengths of all multipole-multipole components in the NN interaction are equal, a condition which is well satisfied empirically (for $K=2$, 3, and 4).[5] In addition, for an SDI the strengths of the multipole pairing terms are equal as well, which is also found to hold.[6]

First, consider the case of a degenerate shell with an SDI,[3] which is essentially a delta interaction in angular coordinates:

$$V_{\mathrm{SDI}}=4\pi G\delta(\Omega_{ij}) ,$$

i.e., like an ordinary delta interaction, except that all *radial* integrals are equal.

This interaction has the special property of being a scalar with respect to quasispin.[7] Thus seniority, viz., the number of unpaired nucleons, is still a meaningful quantum number (no configuration mixing between states of different seniority), even when we have *several* different single particle orbits.

There are, in general, several ways for two particles to couple to a given value of the angular momentum Λ. However, for an SDI, only a *single* state of each Λ is shifted in energy. All other states have zero interaction energy. Several authors have previously shown how to map a four-nucleon problem into one involving two interacting bosons.[8,9] In this paper we use this method (generally known as the OAI mapping), and point out that things simplify considerably for an SDI nucleon-nucleon interaction.

B. Two identical nucleons

For two neutrons (or protons) in the shell, the first two excited states generally have angular momenta and parities 2^+ and 4^+. These states may be regarded as d and g bosons. For typical configurations, for example a degenerate (s,d) shell, i.e., $j_i=\frac{1}{2},\frac{3}{2},\frac{5}{2}$, the interaction energies of these states, which have seniority $v=2$, are roughly $\frac{1}{2}$ and $\frac{1}{4}$ that for the 0^+ ground state (the s boson, corresponding to seniority zero).

C. Four identical nucleons

Next consider configurations of four identical particles with surface delta interactions. For four identical particles, the $(v=0)$ ground state occurs at an energy just twice as large as for the two-particle case. Again there are $v=2$ states with $J^{\pi}=2^+$ and 4^+ at exactly the same excitation energies as for the two-particle case. These three states can be interpreted as states involving two *noninteracting* like (i.e., neutron *or* proton) bosons,[1] s^2 for 0^+, sd for 2^+, and sg for 4^+. The validity of generalized seniority for mixed configurations with general interactions was discussed by several authors.[1,10] It is well known that for single closed shell configurations, the excitation energies calculated in a generalized seniority scheme agree well with exact shell model calculations.[11]

For four identical particles, we can also have states of $v=4$. For example, the $(s,d)^4$ spectrum with SDI has three such states 0^+, 2^+, and 4^+, with excitation energies 2.33, 1.58, and 2.05 times that of the $v=2$, 2^+ state. (There are also some other $v=4$ states which we will not consider here.) In the boson picture, with no interactions between like bosons, these three states should occur at just *twice* the energy of the d boson ($v=2$, 2^+) state, which is fairly close to what happens in the exact shell model calculation. The relation between four-nucleon and two-boson spectra is discussed, for example, in Refs. 12.

III. TWO-PROTON—TWO-NEUTRON CONFIGURATIONS IN DEGENERATE ORBITS WITH SDI (INTERACTION BETWEEN PROTON BOSON AND NEUTRON BOSON)

This is the main topic considered in this paper.

A. Multipole expansion

We wish now to calculate the interaction *between* the proton and neutron bosons.[13] For this case (unlike for the case of four identical particles) the Pauli principle does not play any role, since it is possible to put all four nucleons into the same orbit. Let us then interpret the low-lying four-particle states with $S=T=0$ (which have maximum spatial symmetry) in terms of an interacting neutron and proton boson. It turns out that one can fit the energies of all these states with a particular prescription for the interaction between neutron and proton boson. The SDI can be expanded in multipoles:

$$V_{\mathrm{SDI}}=G[1+5P_2(\cos\theta_{N_1N_2})+9P_4(\cos\theta_{N_1N_2})+\cdots] .$$

Similarly we can make a multipole expansion for the boson-boson interaction.

$$V_{B_p\text{-}B_n}=F_0+5F_2P_2(\cos\theta_{B_1B_2})+9F_4P_4(\cos\theta_{B_1B_2})+\cdots .$$

We do not specify here a relation between $\theta_{N_1N_2}$ and $\theta_{B_1B_2}$, but require that the four-particle and two-boson matrix elements agree for each multipole order. We then find that

$$F_0=4G \tag{1}$$

regardless of the detailed configuration. This is expected since we have two neutrons and two protons.

B. Matrix elements involving s bosons

The multipole coefficients F_K involving transitions with $v=0\rightarrow v=2$ for both neutron and proton bosons (i.e., where we have s bosons in the initial or final state) can be obtained directly from the *two-nucleon* interaction energies. For boson-boson $00\rightarrow K^2$ and $0K\rightarrow K0$ matrix elements we find that

$$F_K=4GV_K/V_0 , \tag{2}$$

where V_K is the interaction energy for two *nucleons* coupled to angular momentum K. *This is the key relation derived here.* It appears not to have been noticed previously. We sketch here the proof for the case $0K\rightarrow K0$.

We will find it convenient to express our results in terms of V_Λ, the maximum possible interaction energy of two particles coupled to angular momentum Λ.

The two-particle matrix elements for an SDI are given by[3,7]

$$V(j_1j_2,j'_1j'_2,\Lambda)=\tfrac{1}{2}\hat{j}_1\hat{j}'_1\hat{j}_2\hat{j}'_2 \times \begin{bmatrix} j_1 & j_2 & \Lambda \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} j'_1 & j'_2 & \Lambda \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} G ,$$

where $\hat{j}=(2j+1)^{1/2}$.

For two particles in degenerate orbits interacting via an SDI, coupled to any given Λ, only a *single* state is shifted in energy. All the other states have zero interaction energy. The energy of the shifted state is

$$V_\Lambda=\sum_j\sum_{j'}\tfrac{1}{2}(2j+1)(2j'+1)\begin{bmatrix} j & j' & \Lambda \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}^2 G .$$

The sum j,j' extends over the set of degenerate orbits, which will be labeled by $[j]$ from now on. The corresponding wave function is

$$\Psi_\Lambda=\mathcal{N}\sum_j\sum_{j'}\widehat{jj'}\begin{bmatrix} j & j' & \Lambda \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}\Psi[jj']_\Lambda ,$$

where $\mathcal{N}$ is a normalization constant. This is a "Λ" boson. In particular, the wave function for the s boson is

$$\Psi_0=\left[\sum_j(2j+1)\right]^{-1/2}\left[\sum_j(2j+1)^{1/2}\Psi(j^2)_0\right] .$$

We can express the boson-boson interaction in terms of $3j$ and $6j$ coefficients:

$$V_{B_p,B_n}(\Lambda_p\Lambda_n,\Lambda'_p\Lambda'_n;J)=\hat{\Lambda}_p\hat{\Lambda}'_p\hat{\Lambda}_n\hat{\Lambda}'_n\sum_K(2K+1)\begin{bmatrix} \Lambda_p & \Lambda'_p & K \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \Lambda_n & \Lambda'_n & K \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \Lambda_p & \Lambda_n & J \\ \Lambda'_n & \Lambda'_p & K \end{Bmatrix}F_K .$$

In this section we will consider the case $0K\rightarrow K0$, the exchange interaction between an s boson and one with angular momentum K. For this case, we have $J=K$, and only a single multipole K appears in the sum. We obtain the simple result

$$V_{B_p,B_n}(0K,K0;K)=F_K .$$

For two neutrons and two protons in the same degenerate orbits j_1, j_2, etc., interacting via an SDI of strength G, it can be shown that

$$F_K(\Lambda_p\Lambda_n,\Lambda'_p\Lambda'_n;J)=4G\left[\Phi_p\Phi_n \Big/ \begin{pmatrix}\Lambda_p & \Lambda'_p & K\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\Lambda_n & \Lambda'_n & K\\ 0 & 0 & 0\end{pmatrix}\right],$$

where

$$\Phi_i=\tfrac{1}{2}\sum_{j}\sum_{j'}\sum_{\lambda=[j]}(2j+1)(2j'+1)(2\lambda+1)\begin{pmatrix}j & \lambda & \Lambda'_i\\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\begin{pmatrix}\lambda & j' & \Lambda_i\\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\begin{pmatrix}j & j' & K\\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}$$

$$\times\begin{Bmatrix}\Lambda_i & \Lambda'_i & K\\ j & j' & \lambda\end{Bmatrix}G(V_{\Lambda_i}V_{\Lambda'_i})^{-1/2}$$

($i=$p or n). This result simplifies greatly for the case $0K\rightarrow K0$:

$$F_K=4G(2K+1)\Phi^2\ ,$$

where

$$\Phi=\tfrac{1}{2}\sum_{j}\sum_{j'=[j]}(2j+1)(2j'+1)\begin{pmatrix}j & j' & K\\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}^2 G[(2K+1)V_0V_K]^{-1/2}=[V_K/(2K+1)V_0]^{1/2}G$$

from which we immediately obtain Eq. (2).

C. General expressions for boson-boson interaction multipoles

The interaction energies involving $v=2$ bosons, i.e., no s bosons, are somewhat more complicated, but can still be obtained in closed form, at least for degenerate configurations with SDI. Note that for $K>0$, all of these involve $v=2\rightarrow v=2$.

We give here the result for a single j shell, for which we have $j'=\lambda=j$.

$$F_K=4G\frac{V_K}{V_0}(2j+1)\left[\frac{\begin{Bmatrix}\Lambda_p & \Lambda'_p & K\\ j & j & j\end{Bmatrix}\begin{Bmatrix}\Lambda_n & \Lambda'_n & K\\ j & j & J\end{Bmatrix}}{\begin{pmatrix}\Lambda_p & \Lambda'_p & K\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\Lambda_n & \Lambda'_n & K\\ 0 & 0 & 0\end{pmatrix}}\right].$$

This may be obtained from the general expressions in the preceding section. Of course, if $K=0$, or if s bosons are involved, we again obtain Eqs. (1) and (2).

D. Quadrupole interaction between d bosons

The quadrupole-quadrupole term is the most important component of the boson-boson interaction not involving s bosons. The case of quadrupole bosons with general interactions has been considered by Zirnbauer and Brink.[8] They worked out the relation between the boson-boson matrix elements and the *particle-hole* nucleon-nucleon matrix elements.

We consider here the interaction between two d bosons, i.e., the case that $\Lambda_i=\Lambda'_i=K=2$ for both protons and neutrons.

We give results for two cases:

(a) single j shell and (b) degenerate shells with $j_i=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots,j$. The latter is equivalent to a degenerate harmonic oscillator shell with principal quantum number $N=j-\frac{1}{2}$ which has $l=N,N-2,\ldots,0$ or 1.

For $j=\frac{3}{2}$, we obtain $V_2/V_0=\frac{1}{5}$ and $F_2/F_0=0$. Note that our "key" relation [Eq. (2)] does not hold here, since we have only d (and no s) bosons. As j increases, so do both ratios, toward the value $\frac{1}{4}$ for $j\rightarrow\infty$.

For the case of mixed configurations, $j_i=\frac{1}{2}$ to j, we have to perform the triple summation to obtain Φ. However, again analytic expressions for the above quantities can be obtained. We found (by inspection) that

$$V_2/V_0=[(2j-1)/(2j+2)]\ ,$$

$$F_2/F_0=[(4j+1)/(4j+4)]^2\ .$$

For $j_i=\frac{1}{2},\frac{3}{2}$, we have $V_2/V_0=\frac{2}{5}$ and $F_2/F_0=\frac{49}{100}$. Note that these ratios are *closer* to each other than for the pure j case. (We recall that for the interaction involving s bosons, the ratios are *identical* regardless of the detailed configuration.) For $j\gg 1$, both ratios approach $1-(\frac{3}{2})j^{-1}$.

The author, in collaboration with Druce, has shown that analytic expressions can also be obtained for other configurations of degenerate orbitals. We are hopeful that these results will be useful for the calculation of IBM parameters in nuclei.

E. IBM and shell model results for simple four-particle $T=0$ configurations

We consider now the well-known example of four particles in the p shell. For this (single orbit) case, the SDI

TABLE I. Boson multipole coefficients and energies. All energies are given in units of the surface delta interaction strength G. $F_2(s)$ and $F_2(d)$ denote $F_2(sd\text{-}ds)$ and $F_2(dd\text{-}dd)$, respectively.

Degenerate	Boson energies			Multipole coefficient			Two-boson—(four-particle)		Energies
j_i	V_0	V_2	V_4	F_0	$F_2(s)$	$F_2(d)$	E_0	E_2	E_4
$\frac{3}{2}$	2	0.4		4	0.8	0	8.8(8.8)	7.2(7.2)	4.8(4.8)
$\frac{1}{2},\frac{3}{2}$	3	1.2		4	1.6	1.96	13.2(13.2)	11.4(11.4)	7.2(7.2)
$\frac{1}{2},\frac{3}{2},\frac{5}{2}$	6	3.43	1.43	4	2.29	2.47	23.6(23.9)	21.8(22.1)	18.9(19.1)

reduces to an ordinary delta interaction. Results are shown in Table I. For four particles coupling to $T=0$, we get an SU(3)-type spectrum with two rotational bands. In this case (unlike that of identical particles), interactions between neutron and proton bosons play a crucial role.[2,13] Only monopole and quadrupole terms enter here. For this case, the interacting boson model reproduces the four-particle energies *exactly*. It should be noted that although the boson model in the form used here is not manifestly isospin invariant (see Ref. 14 for a discussion of an isospin invariant form of the interacting boson model), the results for the energies and also the two boson wave functions are the same as the $T=0$ states obtained with an isospin conserving delta function nucleon-nucleon interaction.

It is interesting to consider also four particles in a $j=\frac{3}{2}$ orbit. Here again, the SDI is equivalent to a δ interaction. However, the energies are different from those for the p shell case.

For $(s,d)^4_{T=0}$, the exact shell model calculations[3] give a near rotational spectrum. This time the boson-boson interaction contains multipoles up to order 4. The energies of the yrast ($T=0$) states are very closely reproduced with the IBM.

IV. CONCLUSIONS

We have seen that if an SDI is used as an effective N-N interaction, the implementation of the mapping of four fermions into two bosons is greatly simplified. It is particularly encouraging that analytic expressions can be obtained for key quantities in the application of the IBM, at least for degenerate orbits. It is also interesting that, at least for an SDI, the boson interactions are closely related to their structure, especially for matrix elements involving s bosons. Indeed, for *rough* estimates, it might be adequate to use the relation $F_K/F_0=V_K/V_0$. It is a challenging task to try generalizing the simple results obtained in this paper to the more realistic case with nondegenerate single particle orbits.

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