Core polarization in inelastic scattering to ^{52}Cr(2_1^+)

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Calculations are performed for the excitation of the 2_1^+ state in ^{52}Cr by the inelastic scattering of electrons, protons, and pions. The use of probe-dependent enhancement factors of the type proposed by Brown and Madsen provides good agreement with data. The regions of large $\sigma_{\pi^+}-\sigma_{\pi^-}$ differences in the 2^+ and 4^+ spectra, predicted by f-p shell configuration, are greatly reduced with the inclusion of pion enhancement factors.

I. INTRODUCTION

High resolution inelastic scattering data, becoming available for many target nuclei, are capable of providing structure information not readily deduced from other reactions. Most of these data are from (e,e′), (p,p′), and (α,α') reactions, but they are complemented for some nuclei by data from pion scattering. Comparison of (π^+,$\pi^{+\prime}$) and (π^-,$\pi^{-\prime}$) cross sections can distinguish neutron from proton excitations, and this property should be of particular interest for nuclei in the $A=50$—60 region. The $N=28$ nuclei have low-lying states which are pure proton excitations, if $f_{7/2}^{-n}$ wave functions are a good approximation, while $N=29$ nuclei have some excited states which are predicted to arise from virtually pure neutron $(p_{3/2}f_{5/2}p_{1/2})f_{7/2}^{-1}$ excitations. Examples of the latter are the well-known $\frac{7}{2}$-hole state at 1.41 MeV in ^{55}Fe, and the 6^+ state at 1.07 MeV in ^{54}Mn.

In the present paper, theoretical and experimental cross sections are compared for ^{52}Cr, an $N=28$ nucleus for which there are (e,e′), (p,p′), and pion scattering data for excitation of the lowest 2^+ state. In particular, we investigate whether the effects of core polarization can be adequately taken into account by using probe-dependent enhancement factors of the type proposed by Brown and Madsen,[1] and the extent to which these enhancement factors modify the distribution of $\sigma(\pi^+)-\sigma(\pi^-)$ strength among higher levels. Section II describes the theoretical framework for the shell model and distorted-wave calculations, and results are compared with experimental data in Sec. III.

II. THEORY

In the distorted-wave impulse approximation (DWIA), the differential cross section for hadron inelastic scattering is given by

$$(d\sigma/d\Omega)_{\mathrm{c.m.}}=\frac{W}{2\pi^2c^2}\frac{k_f}{k_i}\frac{1}{[S][J]}\,|\,T_{fi}\,|^{2}\,, \tag{1}$$

where W is the reduced projectile energy, k_f (k_i) is the final (initial) c.m. momentum, $[S]$ is twice the projectile spin plus one, $[J]$ is twice the target spin plus one, and

$$T_{fi}=\int d^3r\,\chi^{(-)*}(r)\left\langle J_f\left|\sum_j t_{jp}\right|J_i\right\rangle\chi^{(+)}(r)\ . \tag{2}$$

For pions, the bare t matrix, t, is taken to have a simple Kisslinger form with energy and angle corrections.[2] For protons, it is taken to be the density-dependent form[3] derived from the Hamada-Johnston potential.

For the present calculations, the ground state of ^{52}Cr was assumed to be the $f_{7/2}^{-4}$ $J=0$ state. This should be a good approximation if results of a recent projected-Hartree-Fock calculation[4] in the complete fp shell are a valid guide. In the impulse approximation the only $0\hbar\omega$ states reached in inelastic scattering are then $f_{7/2}^{-4}$ states with seniority 2, and states arising from the $(p_{3/2}f_{5/2}p_{1/2})f_{7/2}^{-5}$ configuration. The latter were calculated as in Refs. 5 and 6, using the mass-independent effective interaction which successfully reproduces energy levels in ^{52}Cr and many other $A=50$—60 nuclei.

To reproduce $E2$ transition rates observed in ^{52}Cr and other $N=28$ nuclei with $f_{7/2}^{-n}$ wave functions requires an effective charge of $e_{\mathrm{p}}=1.9$, implying that core polarization (and admixtures of higher-lying $0\hbar\omega$ configurations) significantly affects the collectivity of the states. Inelastic scattering cross sections have been shown to be sensitive to collective components.[7,8] Therefore, the bare t matrix of Eq. (2) must be modified to give agreement with experiment. If t is expressed in the form

$$t_{jp}=\sum_J c_J t_J(j)\cdot t_J(p)\ , \tag{3}$$

where p and j refer to projectile and target-nucleon coordinates, one wishes to equate the reduced matrix elements of t_J between exact nuclear wave functions with those of an effective t_J between model space wave functions. Inclusion of second order corrections corresponding to the diagrams in Fig. 1 provides an effective operator

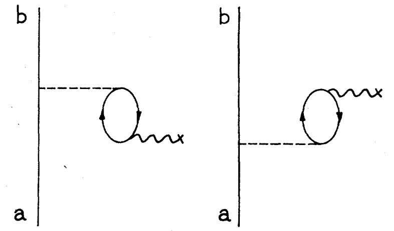

FIG. 1. Core polarization diagrams.

$$\bar{t}_J = t_J + t_J \frac{Q}{E - H_0} v + v \frac{Q}{E - H_0} t_J = t_J + t^{\text{pol}} , \qquad (4)$$

where t_J^{pol} is the contribution from core polarization and v is the nucleon-nucleon interaction. Love and Satchler[9] calculate t_J^{pol} in a collective model with a form factor proportional to $R_0 \partial U / \partial r$, where U is the elastic optical model potential. This is a useful approach for $N = Z$ nuclei. However, the isovector giant quadrupole resonance (IVGQR) will also couple to the 2^+ state and the IVGQR form factor is likely to be different from the surface oscillations implied by $R_0 \partial U / \partial r$. Petrovich *et al.*[10] have calculated t^{pol} explicitly by including two-particle—one-hole intermediate states, but a large number of states must be included to build up the collectivity and eliminate spurious center-of-mass excitation.

An alternative to explicit calculation of t_J^{pol} is the use of projectile-dependent enhancement factors. Following Brown and Madsen,[1] one may factor the spin independent t matrix

$$t_{jp} = t_0 + t_1 \boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2) \qquad (5)$$

as

$$t_{jp} = \sum_J Q_J(j) Q_J(p) [V_0 + V_1 \boldsymbol{\tau}(j) \cdot \boldsymbol{\tau}(p)] , \qquad (6)$$

where V_0 and V_1 are constants. The restriction that V_1 / V_0 be constant is reasonably accurate at small momentum transfer (q) for the NN interaction. This is illustrated in Fig. 2, where the moduli of t_0 and t_1 are plotted as functions of q for 40 MeV protons. For the pion t matrix in the (3,3) resonance region, V_1 / V_0 is approximately constant over a wider range of q, because of the p-wave dominance. This is illustrated in Fig. 3, where the moduli of t_0 and t_1 for the pion t matrix in the π-nucleus center of mass frame,

$$t = t_0 + t_1 \boldsymbol{\tau}(N) \cdot \mathbf{t}(\pi) , \qquad (7)$$

are plotted.

The target operator of Eq. (3) may now be written as

$$t_J = Q_J (a_0 + a_1 \tau_z) , \qquad (8)$$

where a_0 and a_1 are constants. The effects of core polarization can be included by setting

$$\bar{t}_J = Q_J (a_0^{\text{eff}} + a_1^{\text{eff}} \tau_z') , \qquad (9)$$

where τ_z' acts only on valence nucleons, and

$$\begin{bmatrix} a_0^{\text{eff}} \\ a_1^{\text{eff}} \end{bmatrix} = \begin{bmatrix} \epsilon_{00} & \epsilon_{01} \\ \epsilon_{10} & \epsilon_{11} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} . \qquad (10)$$

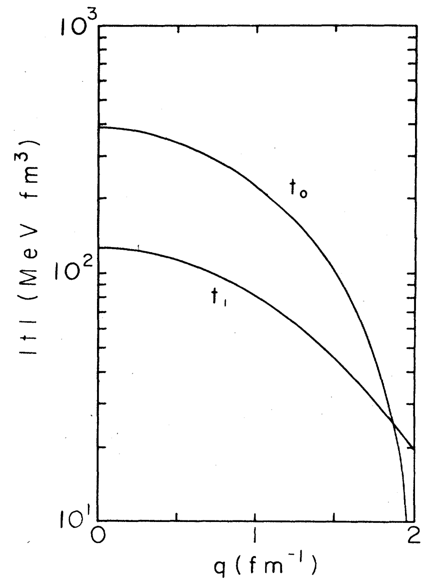

FIG. 2. The central nucleon t matrix of Ref. 3 at $E_p = 40$ MeV, nuclear density 0.126 fm^{-3}, and asymptotic energy approximation, as a function of momentum transfer.

The matrix ϵ is to be determined from a particular model.

The effective a coefficients can be expressed in terms of parameters δ^{pp}, δ^{nn}, δ^{pn}, and δ^{np}, which have more transparent physical interpretation. The parameter δ^{pp} represents the polarization of core protons caused by valence protons, δ^{np} represents the polarization of core neutrons by valence protons, and δ^{pn} and δ^{nn} represent the analogous contributions due to valence neutrons. The effective strength parameter $a_0^{\text{eff}} + a_1^{\text{eff}} \tau_z$ for (α,α'), (p,p′), (n,n′), and electromagnetic transitions is given in Table IV of Ref. 1, and for pions is given in our Table I. Here, $V_{\pi^+ p} = V_{\pi^- n} = V_0 - V_1$, and $V_{\pi^+ n} = V_{\pi^- p} = V_0 + V_1$. For

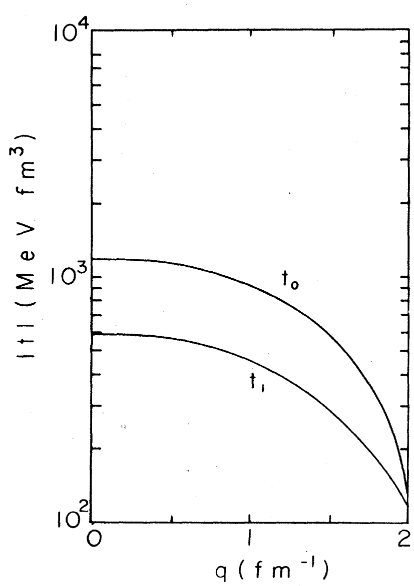

FIG. 3. The central pion t matrix in the π-nucleus c.m. system with Kisslinger potential at $E_\pi = 180$ MeV.

TABLE I. Effective strength parameter for pions.

$(\pi^+,\pi^{+\prime})$	n	$V_{\pi+\mathrm{n}}(1+\delta^{\mathrm{nn}})+V_{\pi+\mathrm{p}}\delta^{\mathrm{pn}}$
	p	$V_{\pi+\mathrm{p}}(1+\delta^{\mathrm{pp}})+V_{\pi+\mathrm{n}}\delta^{\mathrm{np}}$
$(\pi^-,\pi^{-\prime})$	n	$V_{\pi\text{-}\mathrm{n}}(1+\delta^{\mathrm{nn}})+V_{\pi\text{-}\mathrm{p}}\delta^{\mathrm{pn}}$
	p	$V_{\pi\text{-}p}(1+\delta^{\mathrm{pp}})+V_{\pi\text{-}\mathrm{n}}\delta^{\mathrm{np}}$

pions in the (3,3) resonance region, $V_{\pi+\mathrm{p}}/V_{\pi+\mathrm{n}}$ and $V_{\pi\text{-}\mathrm{n}}/V_{\pi\text{-}\mathrm{p}}\simeq 3$.

Effective charges have been calculated in a collective model by Bohr and Mottelson[11] in terms of the neutron excess $\xi=(N-Z)/A$, Lane isovector and isoscalar optical potential strengths V_1 and V_0, $\beta=\xi V_1/4V_0$, and the isovector and isoscalar polarizability coefficients χ_1 and χ_0. Their effective charge expressions can be used[1] to identify the components of the ϵ matrix and hence the a^{eff} coefficients. As pointed out in Ref. 1, the isoscalar effective charge is very sensitive to splitting of the isoscalar GQR strength, so model calculations of ϵ_{00} are unlikely to be satisfactory. We therefore follow the suggestion of Brown and Madsen in regarding ϵ_{00} as a parameter to be fitted using the observed proton effective charge e_p. The coefficients are then given by

$$\delta^{\mathrm{pp}}+1=e_\mathrm{p}\ , \tag{11a}$$

$$\delta^{\mathrm{np}}=\tfrac{1}{2}[(1-\beta+\xi)\epsilon_{00}-(1-\beta+\xi)\epsilon_{11}]\ , \tag{11b}$$

$$\delta^{\mathrm{nn}}+1=\tfrac{1}{2}[(1+\beta+\xi)\epsilon_{00}+(1-\beta-\xi)\epsilon_{11}]\ , \tag{11c}$$

$$\delta^{\mathrm{pn}}=\tfrac{1}{2}[(1+\beta-\xi)\epsilon_{00}-(1+\beta-\xi)\epsilon_{11}]\ , \tag{11d}$$

where

$$\epsilon_{00}=[2e_\mathrm{p}-(1+\beta+\xi)\epsilon_{11}]/(1-\beta-\xi)\ , \tag{11e}$$

$$\epsilon_{11}=1+\chi/(1+\beta)\ . \tag{11f}$$

A proton effective charge e_p of 1.9, together with the choices of Bohr and Mottelson of $\chi_1=-0.64$ and $V_1/4V_0=-0.65$, leads to $\delta^{\mathrm{pp}}=0.9$, $\delta^{\mathrm{np}}=1.82$, $\delta^{\mathrm{pn}}=1.41$, and $\delta^{\mathrm{nn}}=0.99$. The resulting amplitude-enhancement factors, squared, for the $f_{7/2}^{-4}\ 2^+$ state in (e,e′), $(\pi^+,\pi^{+\prime})$, and $(\pi^-,\pi^{-\prime})$ are then 3.6, 6.3, and 54, respectively, demonstrating the great importance of core polarization effects.

III. COMPARISON WITH DATA

Proton, electron, and pion data are available for inelastic scattering to the first excited state of ^{52}Cr at 1.43 MeV, which we assume to be predominantly the seniority 2, $J=2$, $f_{7/2}^{-4}$ state. The (e,e′) data of Refs. 12 and 13 are shown in Fig. 4. The dashed curve is our Born approximation calculation for the $f_{7/2}^{-4}$ configuration which includes center-of-mass and finite proton size corrections. Radial functions were those of a harmonic oscillator with size parameter $\alpha=(m\omega/\hbar)^{1/2}=0.51\ \mathrm{fm}^{-1}$, this value being required to reproduce the observed mean-square charge radius. The solid curve is the identical calculation but with the electron enhancement factor $H_\mathrm{e}^2=e_\mathrm{p}^2=3.6$. These results are similar to other shell model calculations[13] in that the fit is very good at a small momentum transfer.

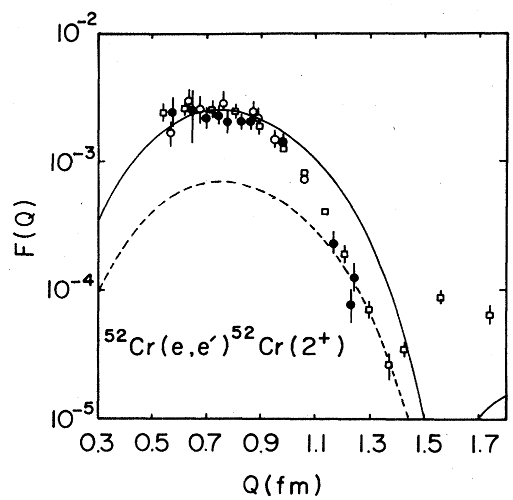

FIG. 4. The (e,e′) form factor for the 2_1^+ state of ^{52}Cr as a function of momentum transfer. The dashed curve is the calculation with bare proton change; the solid curve is with enhancement factor. The open and closed circles are data of Ref. 12. The squares are data of Ref. 13.

Also available are 39.9 MeV (p,p′) data for the 2_1^+ state. To calculate the (p,p′) enhancement factor,

$$H_\mathrm{p}=[V_{\mathrm{pp}}(1+\delta^{\mathrm{pp}})+V_{\mathrm{pn}}\delta^{\mathrm{np}}]/V_{\mathrm{pp}}\ , \tag{12}$$

one requires a value for

$$V_{\mathrm{pn}}/V_{\mathrm{pp}}=(V_0-V_1)/(V_0+V_1)\ .$$

For proton scattering the ratio V_1/V_0 can vary substantially over an energy range of 40 MeV, and may differ from the bound state interaction. In addition, the factorization performed in Eq. (6) implies a specific treatment of exchange terms. An inherent uncertainty therefore exists in V_1/V_0. Figure 5 shows the (p,p′) cross section calculated with a local energy approximation and the 39 MeV effective interaction derived from the Hamada-Johnston potential.[3] The lower curve is with no enhancement factor; it falls more than an order of magnitude below the data. The upper curve is for $H_\mathrm{p}=5.59$, calculated from $V_1/V_0=-0.34$. This value was determined by evaluating $|t_1|/|t_0|$ at $q=0.7\ \mathrm{fm}^{-1}$, where the form factor peaks, and with the nuclear density at $r=3.4$ fm, where the transition density peaks.

To compare with pion data,[15] we have used the optical potential of Stricker, McManus, and Carr (SMC),[16] with the 180 MeV parameter set of Carr.[17] The nucleon density for the optical potential was taken as a two-parameter Fermi distribution determined from electron scattering, with parameters $c=3.95$ fm and $t=2.33$ fm. It was assumed that $N\rho_\mathrm{p}=Z\rho_\mathrm{n}$. Figure 6 displays the elastic cross section for ^{52}Cr$(\pi^\pm,\pi^\pm)^{52}$Cr (2_1^+) data. The calculated curves without core polarization are one to two orders of magnitude below the data, and the $\sigma(\pi^+)/\sigma(\pi^-)$ ratio is close to 9, as expected for a pure proton excitation. Scaling by the core-polarization amplitude enhancement factors for valence protons, $H_{\pi^+}(\mathrm{p})=2.51$ and $H_{\pi^-}(\mathrm{p})=7.3$, one obtains a rather good description of the data as shown in Fig. 7. The sensitive $\sigma(\pi^+)/\sigma(\pi^-)$ ratio in the low

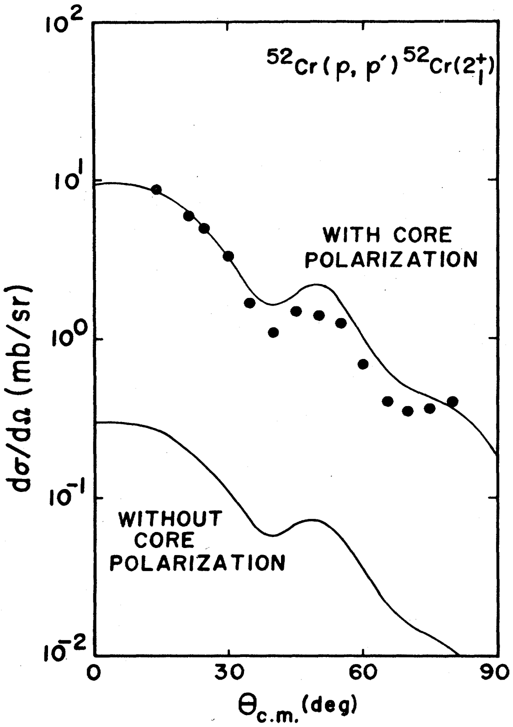

FIG. 5. The ^{52}Cr(p,p′)^{52}Cr (2_1^+) cross section at $E_p = 40$. The data are from Ref. 14. The solid curves are calculated.

momentum transfer region becomes about 1.1, similar to the data.

One sees, therefore, that use of the enhancement factors based on the Bohr and Mottelson collective model plus

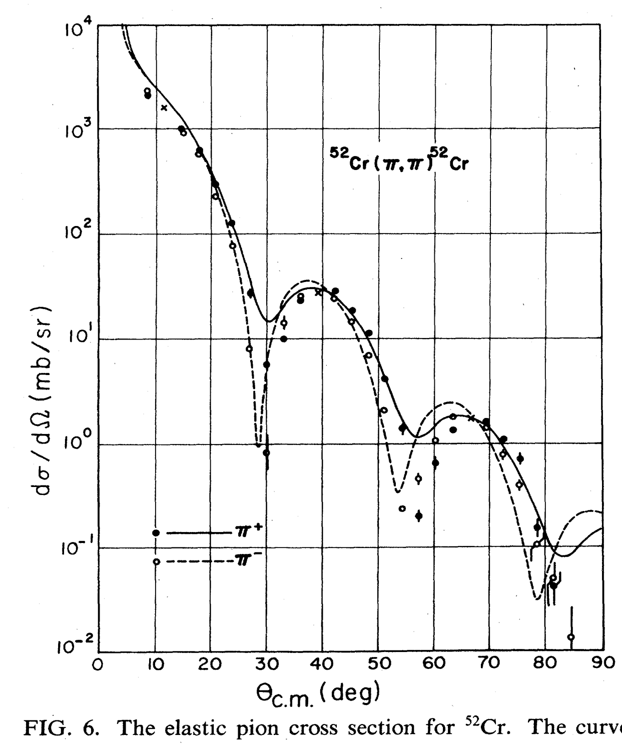

FIG. 6. The elastic pion cross section for ^{52}Cr. The curves are calculated. The data are from Ref. 13.

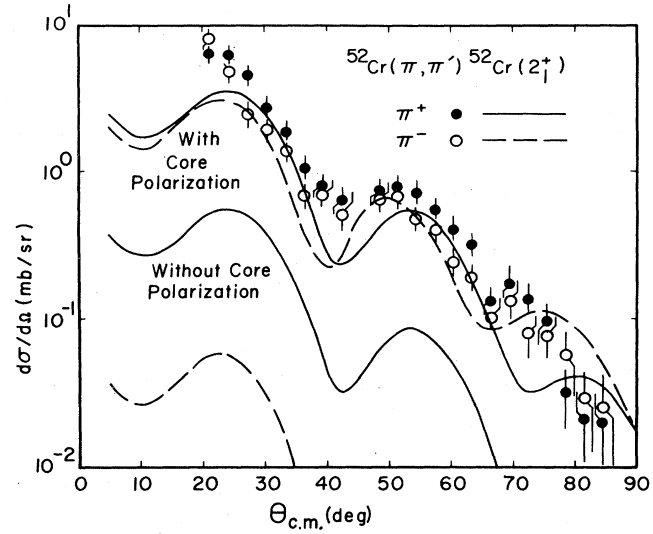

FIG. 7. The inelastic pion cross sections for the 2_1^+ state of ^{52}Cr. The data are from Ref. 15. The curves are calculated.

one free parameter, which was fit to the BE2 rate, provides very good agreement with inelastic scattering data of all three probes. The agreement is best at low momentum transfer. It cannot continue for the entire range of q because the shell model transition density is of less radial extent than the actual collective component. The most striking effect of the enhancement factors is on pion cross sections, where the $\sigma_{\pi^+}/\sigma_{\pi^-}$ ratio goes from 9 to 1.1. Therefore, the effect of enhancement factors on the entire 2^+ pion spectrum is investigated next.

Figure 8 shows the calculated ^{52}Cr($\pi^{\pm}$,$\pi^{\pm}$)^{52}Cr (2^+) excitation functions at 23°, including states arising from the $(p_{3/2}f_{5/2}p_{1/2})f_{7/2}^{-5}$ configuration. These calculations make use of the core excitation amplitude-enhancement

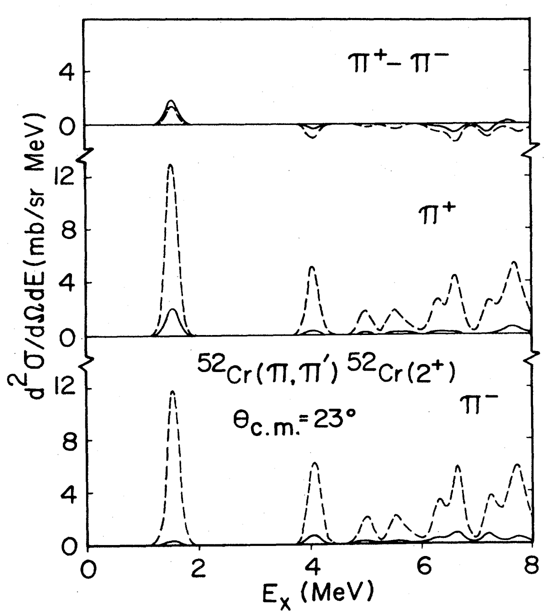

FIG. 8. The 23° spectrum for pionic excitation of the 2^+ states in ^{52}Cr. The solid curve is without enhancement factors; the dashed curve is with enhancement factors.

factors for valence neutron excitation, equal to $H_{\pi^+}(n)=6.2$ and $H_{\pi^-}(n)=2.46$, in addition to $H_{\pi^+}(p)$ and $H_{\pi^-}(p)$. Comparison of the results with and without enhancement factors shows that a primary effect of these is to greatly reduce the $\sigma(\pi^+)-\sigma(\pi^-)$ strength otherwise predicted at certain energies. A complete experimental spectrum would be of great interest. The 4^+ states are also predicted to have substantial cross sections. Therefore, in Fig. 9 is shown the calculated 4^+ excitation function at 38°. The valence proton enhancement factors appropriate to this spin were assumed to be those given by $\delta^{pp}=0.16$ and $\delta^{np}=0.70$, values determined in a microscopic calculation for ^{50}Ti by Petrovich *et al.*[10] For $J=4$ it was assumed that $H_{\pi^+}(n)=H_{\pi^-}(p)$ and $H_{\pi^-}(n)=H_{\pi^+}(p)$. Again, the regions of appreciable $\sigma(\pi^+)-\sigma(\pi^-)$ strength are reduced by the effects of core excitation.

IV. SUMMARY AND CONCLUSION

Inelastic scattering cross sections have been calculated for the 2_1^+ state of ^{53}Cr, using simple $f_{7/2}^{-4}$ wave functions for the (e,e′), (p,p′), and $(\pi^{\pm},\pi^{\pm\prime})$ reactions. Core excitation effects were approximated by the use of enhancement factors calculated using the one-parameter formulation of Brown and Madsen in conjunction with the collective model of Bohr and Mottelson, and agreement with data is good for all three probes. For pions, the enhancement factors can be calculated with reasonable confidence, since V_1/V_0 is clearly close to -2 throughout the region of the (3,3) resonance, and cross sections are in quite good agreement with experiment; in particular the ratio of $\sigma(\pi^+)$ to $\sigma(\pi^-)$ is correctly given as close to unity, rather than 9, as would be the case for a pure proton excitation. For protons, the enhancement factor is less certain, due to uncertainties in V_1/V_0, although the prescription used to calculate this ratio does provide a good fit to the low momentum-transfer data. Data at different proton energies, and 2_1^+ states in other nuclei will be useful in determining an optimum prescription for V_1/V_0. It would be ideal to have the complimentary pion data for these 2_1^+ states. Since an effective charge of $e_p=1.9$ is appropriate for other $N=28$ nuclei, it should be possible to fit electron, proton, and pion data for all 2_1^+ states with only one parameter.

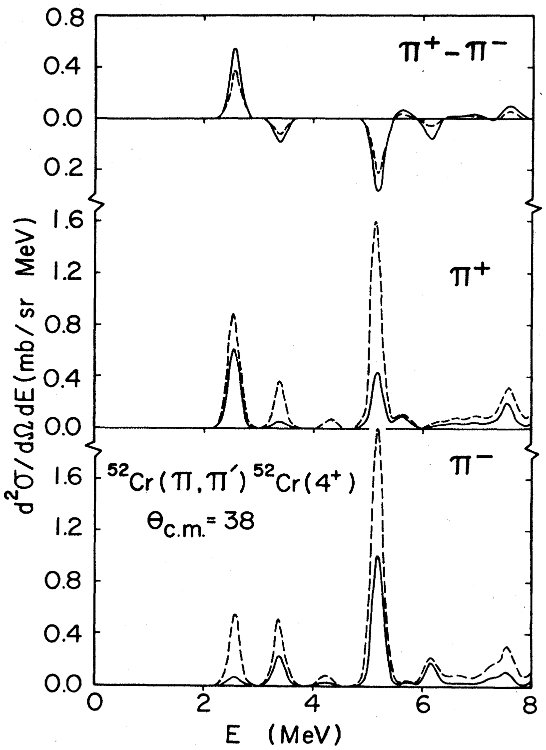

FIG. 9. The 38° spectrum for pionic excitation of the 4^+ states in ^{52}Cr. The solid curve is without enhancement factors; the dashed curve is with enhancement factors.

Calculations of pion cross sections for higher-lying 2^+ and 4^+ levels show that an effect of the enhancement factors is to significantly reduce the $\sigma(\pi^+)-\sigma(\pi^-)$ strength predicted on the basis of $(p_{3/2}f_{5/2}p_{1/2})f_{7/2}^{-5}$ wave functions. This suggests that effects of core polarization may greatly reduce the usefulness of pion scattering in testing the neutron- or proton-excitation structure of shell model wave functions.

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