Three-body unitarity and the K-matrix analyses of dibaryon resonances

I. R. Afnan and B. C. Pearce
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042, Australia
(Received 18 June 1984)

We derive a K-matrix expression starting with three-body unitarity. This is then used to parametrize the results of a Faddeev $\pi d \rightarrow \pi d$ calculation whose poles are known. We find that the K-matrix method is unable to reproduce the known pole in the system when the width of that pole is greater than the width of the Δ resonance. This is explainable in terms of the approximations used in deriving the K matrix.

I. INTRODUCTION

It is becoming increasingly obvious that a tractable method is required that enables one to establish the existence of resonance poles in systems which can only be fully described within a three-body framework. Such a system of current interest is the NN system at energies above the pion production threshold. The controversy concerning the possible existence of a (dibaryon) resonance in this system is well known.[1] The problem has been to obtain a prescription that can be used for phenomenological fits to the data that also provides an adequate description of the inelastic channels that are present. Three-body equations (such as the Faddeev equations) fulfill the second requirement well but are too cumbersome to be used for data fitting. At the other end of the scale, Argand diagrams and speed curves require negligible calculation to arrive at their predictions but are unable to provide a satisfactory description of inelastic reactions.

One approach, which has been used by Bhandari *et al.*[2] and Edwards and Thomas,[3] utilizes a coupled channel K-matrix approximation to the two-body NN T matrix. More recently Hiroshige *et al.*[4] have used the method of Edwards and Thomas to analyze the theoretical three-body πd calculation of the Lyon group.[5] In all these analyses the analytic structure of T is contained in phase space factors which describe the elastic and inelastic thresholds. For the case where the inelastic channel contains an unstable particle (such as the Δ), these phase space factors must be modified in order for the threshold cuts to move off the real axis. Edwards and Thomas utilize a prescription outlined by Basdevant and Berger[6] to do this, while the origin of Bhandari's approach is a little more obscure. By this method, both authors obtain a two-body T matrix with unitarity properties which are designed to approximate the many-body unitarity of the real problem. This approximation is fundamental to ensuing claims for the existence of dibaryon resonances. An essential ingredient missing from all such calculations is an estimate of how well the model used actually resembles reality. One way of estimating this would be to compare the unitarity of the K-matrix models with the unitarity of a many-body theory. A more direct error estimate could be made by using the K-matrix method to predict the position of a known pole. Of course, such a test would have to be made in a system that included coupling to a channel containing a two-body resonance to enable meaningful comparison to the dibaryon question.

In an attempt to establish the connection between the three-body models and the K-matrix equation, we have in Sec. II derived Bhandari's result using three-body unitarity. To achieve this we have made two approximations: (i) We have neglected the contribution to unitarity from one-particle exchange diagrams. (ii) To obtain quasi-two-body equations we need to assume that the three-body amplitudes are separable.

As a test of the K-matrix method we use it in Sec. III to parametrize the Faddeev amplitudes from a previous calculation.[7] In that paper we presented a method for determining the positions of poles in Faddeev-type equations. By fitting the K-matrix approximation to the Faddeev amplitudes we can see how well the K-matrix method reproduces the known poles of the system. To facilitate this comparison we choose to derive the K-matrix method for πd scattering with $J^{\pi}=2^{+}$ and coupling to an NΔ channel only. The motivation for our choice of $J^{\pi}=2^{+}$ is the fact that this is one of the prime candidates for a dibaryon resonance. The comparison reveals that the K-matrix method generates spurious poles.[8] Furthermore, these poles may be mistakenly identified as resonance poles if the system does not have a genuine pole with width (Γ) less than that of the Δ-resonance width (Γ_{Δ}). In particular, for $\Gamma > \Gamma_{\Delta}$ the K-matrix method generates a spurious pole very close to the NΔ branch point.

Our concluding remarks are presented in Sec. IV.

II. FORMALISM

The main difficulty in parametrizing the NN or πd data for the purpose of extracting resonance parameters is to correctly describe the inelastic thresholds. In the energy region of interest ($T_{\pi} < 300$ MeV) we need to include the breakup and NΔ thresholds (and the πd threshold in the case of NN scattering). This means that if we are to derive a suitable parametrization we must start from at least a three-body theory. Hence we use as our starting point the nonrelativistic Faddeev equations with separable two-body amplitudes. In operator form the two-body amplitude is given by

$$t_\alpha(E) = |g_\alpha\rangle \tau_\alpha(E) \langle g_\alpha| \; . \tag{1}$$

and the corresponding Faddeev equations can be expressed as[9]

$$X(E) = Z(E) + Z(E)\tau(E)X(E) \; . \tag{2}$$

In Eq. (2) $X(E)$ is the $2\to 2$ three-body amplitude, $Z(E)$ is the Born amplitude corresponding to one particle exchange, and $\tau(E)$ is the propagator for the correlated pair. In this paper we restrict our considerations to πd scattering with the NΔ and the πNN breakup as the only inelastic channels. Therefore using separable two-body input is justifiable since both two-body amplitudes present (i.e., the deuteron and the Δ) are dominated by poles.

The unitarity of the Faddeev equations is well known[9] and its general form (illustrated diagrammatically in Fig. 1 for πd scattering) is given in momentum space by

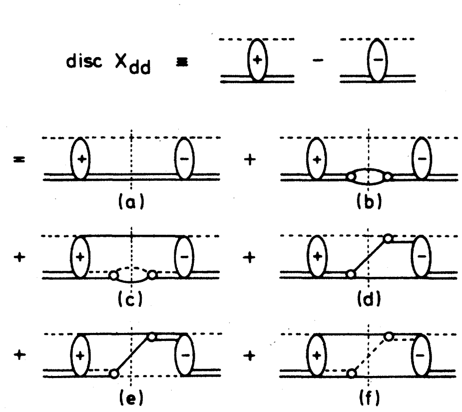

FIG. 1. Diagrammatic expression of unitarity of the Faddeev equation [Eq. (3)]. The vertical dotted line indicates that all particles are on shell.

$$\begin{aligned}\mathrm{disc}X_{\alpha\beta}(\vec{q}^{\,0}_\alpha,\vec{q}^{\,0}_\beta;E) = -2\pi i \Bigg[& \sum_\gamma \int d^3q_\gamma \langle \vec{q}^{\,0}_\alpha | X_{\alpha\gamma}(E^+) | \vec{q}_\gamma \rangle \delta\left[E - \frac{q_\gamma^2}{2\mu_\gamma} + \epsilon_\gamma\right] \langle \vec{q}_\gamma | X_{\gamma\beta}(E^-) | \vec{q}^{\,0}_\beta \rangle \\ & + \sum_{\gamma\eta} \int d^3q_\gamma d^3p_\gamma \langle \vec{q}^{\,0}_\alpha | X_{\alpha\gamma}(E^+) | \vec{q}_\gamma \rangle \tau_\gamma\left[E^+ - \frac{q_\gamma^2}{2\mu_\gamma}\right] g^+_\gamma(p_\gamma) Y^*_\gamma(\hat{p}_\gamma) \\ & \times \delta\left[E - \frac{q_\gamma^2}{2\mu_\gamma} - \frac{p_\gamma^2}{2n_\gamma}\right] Y_\eta(\hat{p}_\eta) g_\eta(p_\eta) \tau_\eta\left[E^- - \frac{q_\eta^2}{2\mu_\eta}\right] \langle \vec{q}_\eta | X_{\eta\beta}(E^-) | \vec{q}^{\,0}_\beta \rangle \Bigg] \; . \end{aligned} \tag{3}$$

α, β, γ, and η are labels that specify the spectator and the quantum numbers of the pair. In this equation the first term on the right-hand side is the contribution from elastic unitarity [Fig. 1(a)], while the second term has the contribution from breakup [Figs. 1(b)—1(f)]. This includes the breakup via the Δ, which is dominant at the energy of interest, and the breakup through the deuteron, which is small. In the three-body center of mass there are only two independent momentum variables. We use the convention that q_α is the momentum of the particle α and p_α is the relative momentum of the remaining pair. ϵ_α is the binding energy of the pair $(\beta\gamma)$. We also have our reduced masses given by

$$\mu_\gamma = \frac{m_\gamma(m_\alpha + m_\beta)}{m_\alpha + m_\beta + m_\gamma}, \quad n_\gamma = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta} \; . \tag{4}$$

We now need to approximate this to a form that is instructive when making a K-matrix approximation. The K-matrix approximation to an amplitude T consists of putting all threshold singularities into phase space factors. In matrix notation we express this as

$$T^{-1}(E) = K^{-1}(E) - i\rho(E) \; , \tag{5}$$

where the elements of K are real meromorphic functions of E (i.e., simple poles are the only singularities allowed in K) and all the branch point singularities are contained in ρ. If we now construct the discontinuity of T, we find

$$\begin{aligned}\mathrm{disc}T(E) &\equiv T(E^+) - T(E^-) \\ &= 2iT(E^+)\mathrm{Re}(\rho(E))T(E^-) \; , \end{aligned} \tag{6}$$

where all T's appearing in this equation are on shell. We observe here that unitarity determines the real part of ρ only. For the case of πd scattering with coupling to a channel containing a stable Δ we would then have

$$\begin{aligned}\mathrm{disc}T^{\mathrm{on}}_{\mathrm{dd}}(E) = 2i[\, & |T^{\mathrm{on}}_{\mathrm{dd}}(E)|^2 \mathrm{Re}\rho_{\mathrm{d}}(E) \\ & + |T^{\mathrm{on}}_{\mathrm{d}\Delta}(E)|^2 \mathrm{Re}\rho_\Delta(E)] \; . \end{aligned} \tag{7}$$

Here $T^{\mathrm{on}}_{\mathrm{d}\Delta}$ is the on-shell $\pi\mathrm{d}\to\mathrm{N}\Delta$ amplitude. The fact that the Δ is not stable means the on-shell momentum in the NΔ channel is complex. This point will be discussed later when we derive this result from three-body unitarity.

Our objective now is to derive an expression of this form by making approximations to the unitarity equations for the πNN system [Eq. (3)]. Clearly, we can only allow terms containing the form $|X_{\mathrm{d}\alpha}|^2$, $\alpha = \mathrm{d}, \Delta$. This forces us to neglect the terms arising from Figs. 1(d)—(f) which correspond to $\gamma \neq \eta$ in the second term on the right-hand side of Eq. (3). These diagrams arise fıom the contribution of $Z(E)$, the Born amplitude, to three-body unitarity. Their contribution may be small, as they are normally neglected in nucleon-nucleon scattering.[10] Neglecting these terms also allows us to perform the angle integrations over $\hat{p}_\gamma$ while the δ function takes care of the remaining p_γ integration. For the time being we shall neglect the contribution from Fig. 1(b), which is the breakup via the deuteron channel. This is only to simplify the discussion since its inclusion follows exactly the same lines as for Fig. 1(c). A standard partial-wave expansion can now be performed to reduce the remaining three-dimensional integrations over $\vec{q}_\gamma$ to one-dimensional ones, giving

$$\mathrm{disc}X_{\mathrm{dd}}(q_\pi^0,q_\pi^0;E)=-2\pi i\left[\sum_\gamma \int_0^\infty dq_\gamma q_\gamma^2\,|X_{\mathrm{d}\gamma}(q_\pi^0,q_\gamma;E)|^2\delta\left[E-\frac{q_\gamma^2}{2\mu_\gamma}+\epsilon_\gamma\right]\right.$$
$$\left.+\int_{\mathrm{Im}(p_{\mathrm N}^0)=0} dq_{\mathrm N}q_{\mathrm N}^2 n_{\mathrm N}p_{\mathrm N}^0(q_{\mathrm N})\,|X_{\mathrm{d}\Delta}(q_\pi^0,q_{\mathrm N};E)|^2\,|g_\Delta(p_{\mathrm N}^0)|^2\left|\tau_\Delta\left[E-\frac{q_{\mathrm N}^2}{2\mu_{\mathrm N}}\right]\right|^2\right], \tag{8}$$

where $p_{\mathrm N}^0$ is given by

$$p_{\mathrm N}^0(q_{\mathrm N})=\left[2n_{\mathrm N}\left[E-\frac{q_{\mathrm N}^2}{2\mu_{\mathrm N}}\right]\right]^{1/2}. \tag{9}$$

$p_{\mathrm N}^0$ arises from the δ function involved in performing the $p_{\mathrm N}$ integration and, as a consequence, we must restrict $p_{\mathrm N}^0$ to be real. This amounts to restricting $q_{\mathrm N}$ to the range $0\to(2\mu_{\mathrm N}E)^{1/2}$.

Since the sum over intermediate states in the first term on the right-hand side of Eq. (8) is restricted to the πd channel, and the integration over q_γ is easily performed, we can reduce Eq. (8) to the form

$$\mathrm{disc}X_{\mathrm{dd}}(q_\pi^0,q_\pi^0;E)=-2\pi i\left[\mu_\pi q_\pi^0\,|X_{\mathrm{dd}}(q_\pi^0,q_\pi^0;E)|^2\right.$$
$$\left.+\int_{\mathrm{Im}(p_{\mathrm N}^0)=0} dq_{\mathrm N}q_{\mathrm N}^2 n_{\mathrm N}p_{\mathrm N}^0(q_{\mathrm N})\,|X_{\mathrm{d}\Delta}(q_\pi^0,q_{\mathrm N};E)|^2\,|g_\Delta(p_{\mathrm N})^0|^2\left|\tau_\Delta\left[E-\frac{q_{\mathrm N}^2}{2\mu_{\mathrm N}}\right]\right|^2\right]. \tag{10}$$

This is very nearly of the form we require [Eq. (7)] except that $|X_{\mathrm{d}\Delta}|^2$, the amplitude for πd$\to$NΔ, appears inside the integral. If $|X_{\mathrm{d}\Delta}(q_\pi^0,q_{\mathrm N};E)|$ was a smooth function of $q_{\mathrm N}$ it could be factored out, resulting in the desired expression. One way of improving this factorization approximation is to use some of the known momentum dependences of X to define another amplitude T which is then factored out in the expectation that it is constant over the range of the integral. Consider an amplitude $X_{\alpha\beta}(q_\alpha,q_\beta;E)$ describing scattering from a state labeled by the spectator α to a state with spectator β. In the initial (final) state the spectator has angular momentum $\mathscr{L}_\alpha$ ($\mathscr{L}_\beta$) with respect to the pair. In this case it is simple to show that at threshold,

$$Z_{\alpha\beta}(q_\alpha,q_\beta;E)\underset{q_\alpha\to 0}{\to} q_\alpha^{\mathscr{L}_\alpha}. \tag{11}$$

If we now assume that $X_{\alpha\beta}(q_\alpha,q_\beta;E)$ has the same threshold behavior as $Z_{\alpha\beta}(q_\alpha,q_\beta;E)$, we can define $X_{\mathrm{dd}}(E)$ and $X_{\mathrm{d}\Delta}(E)$ in terms of new amplitudes T_{dd} and $T_{\mathrm{d}\Delta}$ by

$$X_{\mathrm{dd}}(q_\pi,q_\pi';E)=\frac{u_\pi q_\pi^{\mathscr{L}_\pi}}{(q_\pi^2+c_\pi)^{(1/2)(\mathscr{L}_\pi+\nu_\pi)}}T_{\mathrm{dd}}(E)\times\frac{u_\pi q_\pi'^{\mathscr{L}_\pi}}{(q_\pi'^2+c_\pi)^{(1/2)(\mathscr{L}_\pi+\nu_\pi)}}, \tag{12a}$$

$$X_{\mathrm{d}\Delta}(q_\pi,q_{\mathrm N};E)=\frac{u_\pi q_\pi^{\mathscr{L}_\pi}}{(q_\pi^2+c_\pi)^{(1/2)(\mathscr{L}_\pi+\nu_\pi)}}T_{\mathrm{d}\Delta}(E)\times\frac{u_{\mathrm N} q_N^{\prime\mathscr{L}_{\mathrm N}}}{(q_{\mathrm N}^2+c_{\mathrm N})^{(1/2)(\mathscr{L}_{\mathrm N}+\nu_{\mathrm N})}}. \tag{12b}$$

The arbitrary constants u_π and $u_{\mathrm N}$ can be used later to absorb various constants resulting from the integrations. Since we assume that T is independent of momentum we have eliminated the necessity to define what we mean by an on-shell πd$\to$NΔ amplitude when making a K-matrix approximation to T. We note that in the preceding analysis we have not made any approximations concerning the energy dependence of the amplitude, which would have the resonance poles.

To simplify the discussion we consider scattering with total angular momentum $J^\pi=2^+$. For the elastic channel we consider a pion in relative angular momentum 1 with a ${}^3S_1$ deuteron. We consider only one inelastic channel, namely a nucleon in relative angular momentum 0 with a delta. This enables us to specify $\mathscr{L}_\pi$ and $\mathscr{L}_{\mathrm N}$; namely $\mathscr{L}_\pi=1$ and $\mathscr{L}_{\mathrm N}=0$. With this choice and now including the contribution of Fig. 1(b), Eq. (10) becomes (performing a change of variables $M=E-q^2/2\mu$)

$$\mathrm{disc}T_{\mathrm{dd}}(E)=2i\left[\frac{(E+\epsilon_{\mathrm d})^{3/2}}{(E+c_\pi)^{\nu_\pi+1}}|T_{\mathrm{dd}}(E)|^2+|T_{\mathrm{dd}}(E)|^2\sigma\int_0^E dM\frac{(E-M)^{3/2}M^{1/2}|g_{\mathrm d}(\sqrt{2n_\pi M})|^2|\tau_{\mathrm d}(M)|^2}{(E-M+b_\pi)^{\nu_\pi+1}}\right.$$
$$\left.+|T_{\mathrm{d}\Delta}(E)|^2\int_0^E dM\frac{(E-M^{1/2})M^{1/2}|g_\Delta(\sqrt{2n_{\mathrm N}M})|^2|\tau_\Delta(M)|^2}{(E-M+b_{\mathrm N})^{\nu_{\mathrm N}}}\right], \tag{13}$$

where all constants have been absorbed into the constant σ. In Eq. (13) we have a unitarity equation identical to that given in Eq. (7). The derivation of this equation involves two approximations: (i) We have ignored all contributions to unitarity from the Born term $Z(E)$. (ii) We have assumed that the off-shell momentum dependence is determined by the threshold momentum behavior of the Born amplitude (i.e., $X_{\alpha\beta}$ is separable). In this way we were able to factorize the T matrix out of the integral, and thus reduce the unitarity equations to an algebraic relation.

If we now make a K-matrix approximation to T, then Eq. (13) gives us a prescription for choosing the real part of ρ leaving the imaginary part still undetermined. In the Appendix we show that this prescription can lead to a K matrix that is the same as that used by Bhandari *et al.*[2]

To parametrize the data we now need to choose the form of the elements of K. Since they must be meromorphic functions of the energy, we choose

$$(K^{-1})_{ij}(E)=\sum_{n=1}^{N} a_{ij}^{(n)}E^{n-1} \tag{14}$$

with $N=3$. At this point we could perform a fit to the data over the parameters $a_{ij}^{(n)}$ using the model outlined. The pole positions predicted by this model are then extracted by searching for energies where $\det(K^{-1}-i\rho)=0$.

III. NUMERICAL RESULTS

Before a K-matrix method, such as that outlined above, is used to tackle real problems, it should be tested to see that its predictions are reliable. In a previous paper[7] we presented a method for searching for poles in the solution to the Faddeev equations for complex energies. The method was then used to find the poles in the Faddeev amplitude for a simple, two-channel calculation. Since the Faddeev equations incorporate the NΔ threshold in a well-defined manner, we can use them to construct a test of the K-matrix method.

A Faddeev calculation was performed to produce phase shifts and inelasticities over a range of energies. These phase shifts were then used as data for a K-matrix analysis and the pole positions predicted were compared with the poles present in the Faddeev amplitudes.

As indicated in the preceding derivation we considered πd scattering in the $J^{\pi}=2^{+}$ partial wave with coupling to the NΔ channel. The two-body input to the Faddeev calculation was identical to that used in the previous investigation of poles in Faddeev equations.

The K-matrix model for T used consists of Eqs. (5), (14), and (A5). The parameters b_{π}, $b_{\rm N}$, α_{π}, $\alpha_{\rm N}$, σ, and $a_{ij}^{(n)}$ were adjusted to obtain satisfactory fits to the phase shifts obtained from the Faddeev calculation.

We considered two cases:

(a) All three particles are treated nonrelativistically when solving the Faddeev equation. Although the Faddeev amplitudes in this case do not produce cross sections that are in agreement with experiment, they provide a useful test case. As we reported in a previous paper,[7] the Faddeev amplitudes in this case have a pole on the second energy sheet (i.e., the second sheet of the πd and πNN, the

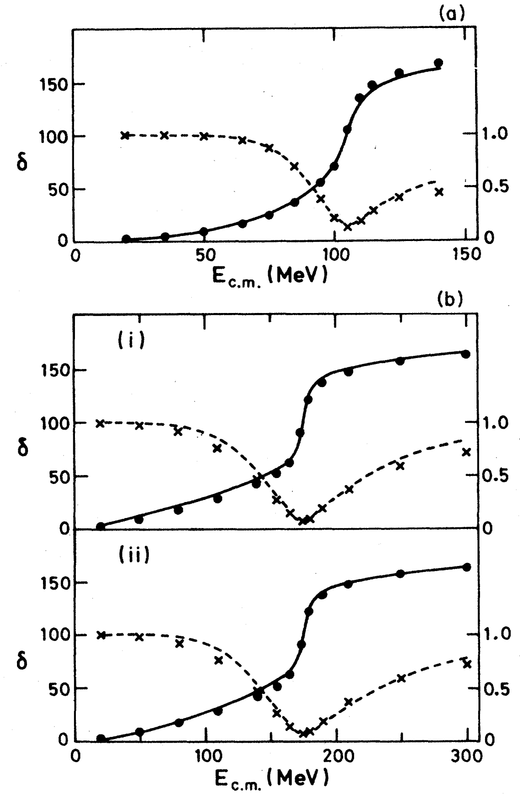

FIG. 2. The K-matrix fit to phase shifts and inelasticities from the Faddeev amplitudes. The smooth curves are the K-matrix fits. (a) Nonrelativistic case. (b) Semirelativistic case (i.e., pion only treated relativistically).

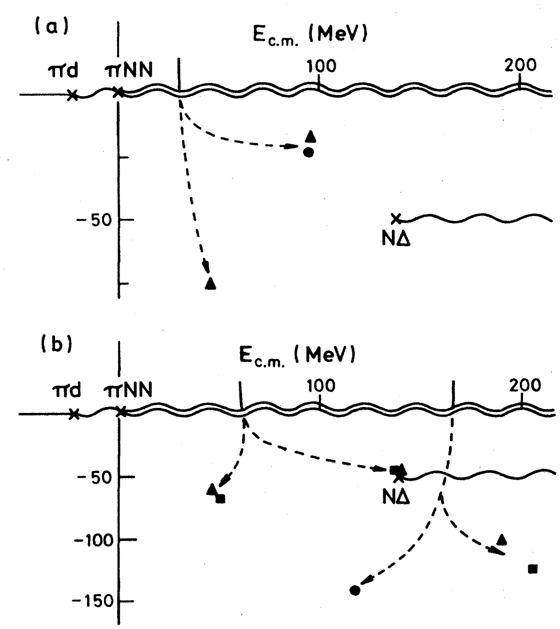

FIG. 3. A comparison of the poles found using the K-matrix analysis and the poles present in the Faddeev amplitudes (represented by circles). (a) Nonrelativistic case [see Fig. 2(a)]. (b) Semirelativistic case. The poles resulting from the fit illustrated in Fig. 2(b) (i) are indicated by triangles and those from Fig. 2(b) (ii) by squares.

first sheet of the NΔ threshold cuts) quite close to the real axis at a center of mass kinetic energy of (94.5, −23) MeV. A K-matrix fit was performed resulting in the fit illustrated in Fig. 2(a). A search for poles revealed two poles on the second energy sheet, one at (47, −75.5) MeV and the other at (97, −16) MeV, close to the Faddeev pole. These results are summarized in Fig. 3(a).

(b) The pion only is treated relativistically, which considerably improves the agreement between theory and experiment. In this case the Faddeev amplitudes have a pole on the third sheet (i.e., the second sheet of the πd, πNN, and NΔ threshold cuts) at (118, −141) MeV. This time two fits to the Faddeev amplitudes were obtained, the second one yielding a slightly lower χ^2, as can be seen from Fig. 2(b). Despite the different parameters needed for the two fits, the positions of the poles predicted are in qualitative agreement with each other, but are in strong disagreement with the pole present in the Faddeev amplitude, as can be seen in Fig. 3(b). The first fit predicted poles at (47, −60) MeV and (141, −45) MeV on the second sheet and a third pole at (190, −103) MeV on the third sheet, while the second fit had poles at (50.5, −67) MeV and (137, −47) MeV on the second sheet and at (205, −125) MeV on the third.

From these results it is clear that the method is quite capable of producing more than one pole in order to describe the combined effect of a single real pole and a complex inelastic threshold. It appears that this is more likely to occur if the real pole is further from the real axis than the complex threshold. Although it would appear that the method reproduces the real pole reasonably well when it is close to the real axis (i.e., $\Gamma < \Gamma_\Delta$), it fails badly when $\Gamma > \Gamma_\Delta$.

In the semirelativistic case both fits predicted a spurious pole in the vicinity of the NΔ threshold. Several K-matrix analyses of the NN data[2,3] and one πd analysis[4] all predict resonance poles in close proximity to the NΔ threshold cut and, in the πd analysis, the existence of other spurious poles was noted. This pole, which appears to be a feature of the K-matrix method, probably appears as a means of recovering from the approximate treatment of the threshold within the model, and should not be interpreted as a resonance pole.

Having established that the K-matrix method fails to correctly and unambiguously predict pole positions, we now examine the approximations that were made to unitarity in order to derive it.

The first approximation was to neglect the terms that arise from the discontinuity in the Born terms Z. Although we are unable to estimate the effects of this, we note that coupled channel calculations of NN scattering[10] also neglect this contribution.

The second approximation was to assume that $T(E)$ [see Eq. (12)] is independent of momentum. This was necessary in order to factorize T from the integral. In order to verify this assumption we used the amplitudes $X_{d\Delta}(q_\pi^0, q_N; E)$ (which are calculated for a range of momenta q_N whilst solving the Faddeev equation) and Eq. (12) to examine the dependence of $T_{d\Delta}(E)$ on momentum. For the semirelativistic case we find that $T_{d\Delta}$ is not constant but varies by as much as a factor of 2 over the momentum range required for the integration.

To estimate the combined effect of these two approximations we calculated the right-hand side of Eq. (13) (using the Faddeev amplitudes to estimate a value for $T_{d\Delta}$) and compared it with the left-hand side. Since we have established that $T_{d\Delta}$ is not momentum independent as required, the results of this test depend on the value of momentum q_N at which $|T_{d\Delta}|^2$ is evaluated. It is possible to choose a value for q_N that satisfies unitarity, but if the wrong choice is made unitarity can be violated by as much as a factor of 10.

These results can perhaps be understood in the context that Eq. (12) amounts to assuming that the three-body $\pi d \to \pi d$ and $\pi d \to N\Delta$ amplitudes are separable. The circumstances under which this approximation is most valid is in the vicinity of a pole. If a pole does not exist or is a long way from the real axis, this approximation breaks down and the method is no longer reliable. The fact that in the nonrelativistic case, where the known pole lies close to the real axis, the method works better supports this idea. This implies that the presence of a pole is implicit to the K-matrix approach, and hence the method should not be used to establish the existence or otherwise of a pole.

IV. CONCLUSION

Our results show that the K-matrix method is not suitable for establishing the existence of dibaryon resonances. More specifically, if an S-matrix pole exists with a width greater than the width of the Δ resonance, the K-matrix results are ambiguous. As a consequence, K-matrix methods are only useful in estimating the position of a pole that is known to lie close to the real axis. These results suggest that previous investigations of dibaryon resonances that rely mainly on K-matrix techniques should be regarded with a large degree of caution.

We would like to thank the Australian Research Grants Scheme for their financial support.

APPENDIX

In this appendix we show that the discussion of Sec. II leads to a K-matrix approximation that is very similar to that used by Bhandari *et al.*[2] in searching for dibaryon resonances.

To further approximate the remaining integrals in Eq. (14) we use the fact that $\tau_\gamma(E)$ has a pole at $E = -\epsilon_\gamma$ (ϵ_γ complex for a resonance). This means we can write

$$\tau_\gamma(E) = (E+\epsilon_\gamma)^{-1} R_\gamma(E) , \tag{A1}$$

where $R_\gamma(E)$ is the residue of the pole. We also assume a Yamaguchi form for the form factor g_γ,

$$g_\alpha(p) = \langle p_\alpha | g_\alpha \rangle = \frac{p_\alpha^{l_\alpha}}{(p_\alpha^2+\beta_\alpha^2)^{(1/2)(l_\alpha+\kappa_\alpha)}} . \tag{A2}$$

For the $^3S_1$ deuteron $l_d = 0$ and we choose $\kappa_d = 2$, while for the p_{33} we have $l_\Delta = 1$ and we take $\kappa_\Delta = 1$. Applying these to Eq. (13) and comparing the resulting equation with Eq. (7) gives

$$\mathrm{Re}\rho_{\mathrm{d}}(E)=\frac{(E+\epsilon_{\mathrm{d}})^{3/2}}{(E+a_\pi)^{\nu_\pi+1}}+\sigma\int_0^E dM\frac{(E-M)^{3/2}M^{1/2}\,|\,R_{\mathrm{d}}(M)\,|^2}{(E-M+b_\pi)^{\nu_\pi+1}(M+\alpha_\pi)^2(M+\epsilon_{\mathrm{d}})^2}\,, \qquad \text{(A3a)}$$

$$\mathrm{Re}\rho_\Delta(E)=\int_0^E dM\frac{(E-M)^{1/2}M^{3/2}\,|\,R_\Delta(M)\,|^2}{(E-M+b_{\mathrm{N}})^{\nu_{\mathrm{N}}}(M+\alpha_{\mathrm{N}})^2[(M-m_0)^2+\frac{1}{4}\Gamma_\Delta^2]}\,, \qquad \text{(A3b)}$$

where m_0 and $-\frac{1}{2}\Gamma_\Delta$ are the real and imaginary parts, respectively, of the Δ mass, and $b_\gamma=c_\gamma/2\mu_\gamma$, $a_\pi=b_\pi+\epsilon_{\mathrm{d}}$. If the free parameters b_π and b_{N} are chosen sufficiently large, $(E-M+b_\gamma)^\nu\approx(E+b_\gamma)^\nu$ for M anywhere in the interval $0\rightarrow E$. If we also assume that the residues $R_\gamma(M)$ are independent of M throughout the integration interval, they can be factored out of the integral and absorbed into the constants u_π, u_{N} [see Eq. (12)], and σ. Choosing $\nu_\pi=\nu_{\mathrm{N}}=\frac{1}{2}$ and setting $b_\pi=a_\pi$ (since $b_\pi\gg\epsilon_{\mathrm{d}}$) results in

$$\mathrm{Re}\rho_{\mathrm{d}}(E)=\frac{1}{(E+b_\pi)^{3/2}}\left[(E+\epsilon_{\mathrm{d}})^{3/2}+\sigma\int_0^E dM\frac{(E-M)^{3/2}M^{1/2}}{(M+\alpha_\pi)^2(M+\epsilon_{\mathrm{d}})^2}\right], \qquad \text{(A4a)}$$

$$\mathrm{Re}\rho_\Delta(E)=\frac{1}{(E+b_{\mathrm{N}})^{1/2}}\int_0^E dM\frac{(E-M)^{1/2}M^{3/2}}{(M+\alpha_{\mathrm{N}})^2[(M-m_0)^2+\frac{1}{4}\Gamma_\Delta^2]}\,. \qquad \text{(A4b)}$$

We now have only to choose the imaginary part for ρ_{d} and ρ_Δ. Since $(E-M)^{1/2}$ is purely imaginary for $M>E$, extending the integration limit up to ∞ will not affect the real part of the integral. Hence we make the (nonunique) choice

$$\rho_{\mathrm{d}}(E)=\frac{1}{(E+b_\pi)^{3/2}}\left[(E+\epsilon_{\mathrm{d}})^{3/2}+\sigma\int_0^\infty dM\frac{(E-M)^{3/2}M^{1/2}}{(M+\alpha_\pi)^2(M+\epsilon_{\mathrm{d}})^2}\right], \qquad \text{(A5a)}$$

$$\rho_\Delta(E)=\frac{1}{(E+b_{\mathrm{N}})^{1/2}}\int_0^\infty dM\frac{(E-M)^{1/2}M^{3/2}}{(M+\alpha_{\mathrm{N}})^2[(M-m_0)^2+\frac{1}{4}\Gamma_\Delta^2]}\,. \qquad \text{(A5b)}$$

If we had ignored the contribution to three-body unitarity of Fig. 1(b) (i.e., $\sigma=0$), Eq. (A5) would be nearly identical to the prescription given by Bhandari.[2]

[1]A. Yokosawa, Phys. Rep. **64**, 47 (1980).

[2]R. Bhandari, R. A. Arndt, L. D. Roper, and B. J. VerWest, Phys. Rev. Lett. **46**, 1111 (1981).

[3]B. J. Edwards and G. H. Thomas, Phys. Rev. D **22**, 2772 (1980).

[4]N. Hiroshige, W. Watari, K. Takabayashi, and M. Yonenzawa, Osaka University report, 1984.

[5]T. Mizutani, C. Fayard, G. H. Lamot, and R. S. Nahabetian, Phys. Lett. **107A**, 177 (1981).

[6]J. L. Basdevant and E. L. Berger, Phys. Rev. D **19**, 239 (1979).

[7]B. C. Pearce and I. R. Afnan, Phys. Rev. C **30**, 2022 (1984).

[8]I. J. R. Aitchison and R. J. A. Golding, Phys. Lett. **59B**, 288 (1975).

[9]I. R. Afnan and A. W. Thomas, in *Modern Three-Hadron Physics,* edited by A. W. Thomas (Springer, Heidelberg, 1977).

[10]T.-S. H. Lee, Phys. Rev. C **29**, 194 (1984).