# Four-nucleon potential due to exchange of pions

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A four-body force due to the exchange of pions has been derived by means of an effective Lagrangian which is approximately invariant under chiral and gauge transformations. It includes effects corresponding to pion-pion scattering, pion production, and pion-nucleon rescattering. The strength parameters of this four-body potential are typically one order of magnitude smaller than those of the two-pion-exchange three-body force.

## I. INTRODUCTION

It is well known nowadays that the nucleon-nucleon interaction does not suffice for a precise description of many important effects in nuclear physics. For instance, the study of the nuclei of $^3$H and $^3$He by means of different techniques has shown that realistic two-body forces underbind these trinucleon systems by about 1.5 MeV.[1] This situation has led researchers in the field to look elsewhere for the explanation of this and other discrepancies. In this context, three-body forces have deserved much attention, particularly that which is due to pion exchange, whose effects have been shown to be important.[2]

The study of the four-body system is in a much less advanced stage. Nevertheless, the present possibility of tackling the problem by means of various techniques and different nucleon-nucleon potentials allows one to foresee that the inclusion of many body forces will be performed soon.[3] This makes opportune a discussion of the role of four-body forces.

The many-body forces of longer range are those due to the exchange of pions. In the case of the alpha particle, these forces are the result of proper interactions among either three or four nucleons. By proper interactions one means processes in which there are no nucleons propagating forward in time. The two-pion-exchange three-body force corresponds to diagrams in which a virtual pion, emitted by one of the nucleons, is scattered by another and absorbed by a third one. In the most accurate theoretical treatments of this force the intermediate pion-nucleon scattering amplitude is described by means of chiral symmetry,[4,5] since the interactions of pions with other hadrons are approximately invariant under transformations of the group SU(2)×SU(2). The symmetry is a crucial ingredient in the calculation of the force because it produces a pion-nucleon amplitude which is consistent with on shell data and is suitable for off-shell extrapolation.

The dynamical content of the pion-exchange four-body force, on the other hand, is related to three different types of intermediate processes, namely, pion-pion scattering, pion production, and pion-nucleon rescattering, as depicted in Fig. 1. The first of them corresponds to the interaction of the virtual pions exchanged between different pairs of nucleons. The amplitude for pion production contributes in the case where a virtual pion, emitted by a nucleon, interacts with another nucleon, producing two pions, which are absorbed by the remaining nucleons. Finally, in the third type of process, a nucleon emits one pion, which is scattered in succession by two other nucleons and absorbed by a fourth one. The four-body potential associated with Figs. 1(a) and (b) has already been considered by McManus and Riska,[6] who did not include the contribution of the delta resonance to the intermediate pion production amplitude. The double scattering diagram, given by Fig. 1(c), has also been studied by Blatt and McKellar,[7] who employed a rather simplified description of the "elementary" pion nucleon scattering amplitude. These results correspond to special cases of those derived here.

In this work the four-body force due to exchange of pions is obtained by means of an effective Lagrangian that is approximately invariant under chiral and gauge transformations. In Sec. II this Lagrangian is employed in the calculation of the amplitudes for the intermediate processes. One derives the four-body potential in Sec. III by evaluating the contributions of proper interaction to

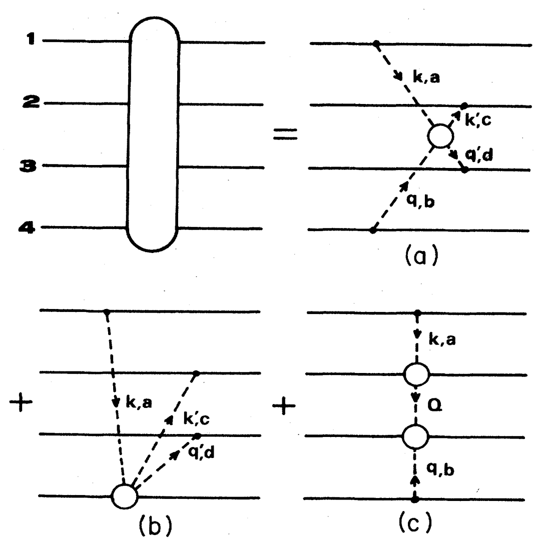


FIG. 1. Contributions to the pion-exchange four-body force: pion-pion scattering (a), pion production (b), and pion-nucleon rescattering (c).

 

the scattering of four nonrelativistic nucleons. Finally, conclusions are presented in Sec. IV.

## II. INTERMEDIATE AMPLITUDES

The relationship between the four-body potential and the scattering amplitude of free nucleons is totally analogous to that of the three-body case. As discussed in the Introduction, the potential is based on proper diagrams describing the propagation of pions in tree approximation and containing the amplitudes for pion-pion scattering, pion production, and pion-nucleon rescattering. The pions exchanged in the various processes are off shell, and hence the evaluation of these subamplitudes must be performed with the help of some theory.

The most successful theory describing pionic processes is based on the assumption that their interactions are approximately invariant under transformations of the group $SU(2)\times SU(2)$. There are two main approaches for applying this symmetry, known as chiral symmetry, to the interactions of low-energy pions with other hadrons. One of them uses the so-called current algebra, whereas the other is based upon effective Lagrangians. They are physically equivalent, but correspond to rather different calculational techniques. The former approach has the disadvantages of requiring much algebraic effort when the number of pions is not small, and of hiding the dynamical implications of the soft pion limit.[8]

These problems are not present in the alternative approach, which is based on effective or phenomenological Lagrangians, built in such a way as to reproduce the results of current algebra when used in lowest order perturbation theory.[8] It is important to stress that the use of these effective Lagrangians should not be understood as an attempt to apply ordinary perturbation theory in calculations of strong processes. The main advantage of the Lagrangian approach is that it allows for a clear understanding of the dynamical content of the intermediate amplitudes and hence is well suited for guiding one's intuition.

The elastic $\pi$N scattering has been extensively studied by means of chiral symmetry, and agreement with experiment is good both below threshold and for pion energies up to 350 MeV.[9,10] In the Lagrangian approach the amplitude for the process $\pi N\rightarrow\pi N$ is assumed to be given by the diagrams of Fig. 2. The vertices for the interactions $\pi$NN, $\pi N\Delta$, $\pi\pi\rho$, and $\rho$NN are extracted from the nonlinear effective Lagrangian, whose terms are displayed in Eqs. (1)–(12). The last diagram of Fig. 2 represents the contribution of the pion-nucleon $\sigma$ term and is related to a controversial aspect of the use of Lagrangians. In the case of current algebra this contribution is due to the equal-time commutator of the axial current and its divergence. In the effective Lagrangian approach, on the other hand, it cannot be ascribed to the exchange of realistic particles, since no serious candidate for the sigma field seems to exist. Hence the usual procedure consists of considering this contribution by means of a parametrized form.[9]

The experimental knowledge of the reaction $\pi N\rightarrow\pi\pi N$ is in a much less advanced state than that of elastic $\pi$N

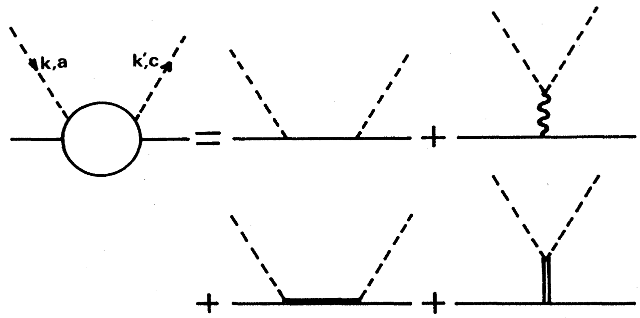


FIG. 2. Low-energy pion-nucleon amplitude. Pions, rhos, and sigma are denoted by broken, wavy, and double lines, respectively. Full and thick lines represent nucleons and deltas.

scattering. Nevertheless, a recent detailed analysis of a particular process[11] has shown that it is well represented near threshold by an isobar model satisfying the current algebra constraints.[12] This model is essentially equivalent to the diagrams given in Fig. 3. The meaning of the various contributions is discussed in subsection B.

In this work one assumes that the amplitude for the process $\pi\pi\rightarrow\pi\pi$ is given by the diagrams displayed in Fig. 4. A comparison of the predictions of this model with experiment is produced in Ref. 13, where further references can be found. There it is shown that the theoretical results for the isoscalar ($a_0$) and isotensor ($a_2$) scattering lengths agree well with experiment as far as the combination $2a_0-5a_2$ is concerned. On the other hand, the experimental values of $a_0$ are about twice that provided by the model. This situation could be improved by considering the contribution of isoscalar resonances.[13,14] However, this procedure is not adopted here, since the introduction into the problem of rather uncertain coupling constants and masses would be fully justified only if the effects of the four-body potential prove to be important in realistic calculations.

The intermediate amplitudes are calculated by means of an effective Lagrangian which is approximately invariant under chiral and gauge symmetries. It describes the interactions among nucleons, deltas, pions, rhos, and the axial-vector mesons $A_1$. The relevant terms to the present discussion are the following:

$$L_{\pi\pi\pi\pi}=\frac{1}{8f_\pi^2}[2(1-\xi)\phi^2\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi}+(\tfrac{1}{2}-\xi)\partial_\mu\phi^2\partial^\mu\phi^2-\mu^2(1-\tfrac{3}{2}\xi)\phi^4]\,, \tag{1}$$

$$L_{\pi\pi\rho}=\gamma_0\vec{\rho}_\mu\cdot(\vec{\phi}\times\partial^\mu\vec{\phi})+\frac{\gamma_0}{4m_\rho^2}(\delta-1)(\delta_\mu\vec{\rho}_\nu-\delta_\nu\vec{\rho}_\mu)\cdot(\delta^\mu\vec{\phi}\times\delta^\nu\vec{\phi})\,, \tag{2}$$

$$L_{\pi\pi\pi A_1}=\frac{\gamma_0}{f_\pi}[\phi^2\partial^\mu\vec{\phi}\cdot\vec{A}_\mu-\tfrac{1}{2}\partial^\mu\phi^2\vec{\phi}\cdot\vec{A}_\mu]\,, \tag{3}$$

$$L_{\pi\rho A_1}=\frac{1}{2f_\pi}[\vec{\phi}\cdot\partial^\mu\vec{\rho}^{\,\nu}\times(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu)-\partial_\nu\vec{\phi}\cdot(\partial^\mu\vec{\rho}^{\,\nu}-\partial^\nu\vec{\rho}^{\,\mu})\times\vec{A}_\mu]\,, \tag{4}$$

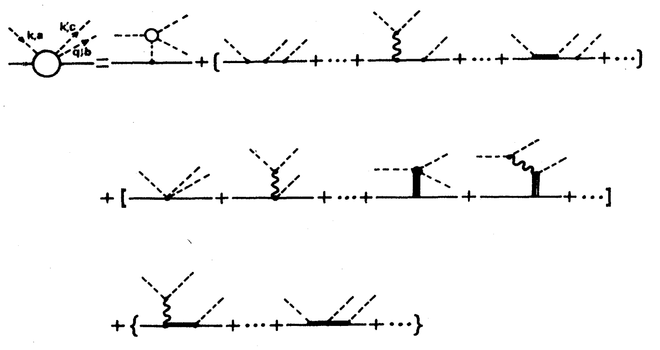

FIG. 3. Low-energy pion production amplitude. Pions, rhos, and axial-vector mesons are represented by broken, wavy, and triple lines, respectively. Full and thick lines denote nucleons and deltas. The symbol ( $\cdots$ ) indicates permutations of the pions.

$$L_{\pi NN}=\frac{g}{2m}\bar{N}\vec{\tau}\gamma^{\mu}\gamma_5 N\cdot\partial_{\mu}\vec{\phi}\ , \tag{5}$$

$$L_{\pi\pi\pi NN}=\frac{g}{2m}\bar{N}\vec{\tau}\gamma^{\mu}\gamma_5 N\cdot\frac{1}{4f_{\pi}^2}[(1-\xi)\partial_{\mu}\phi^2\vec{\phi}-\xi\phi^2\partial_{\mu}\vec{\phi}]\ , \tag{6}$$

$$L_{\rho NN}=\frac{\gamma_0}{2}\bar{N}\vec{\tau}\cdot\left[\gamma^{\mu}\vec{\rho}_{\mu}+\frac{\mu_p-\mu_n}{2m}\sigma^{\mu\nu}(\partial_{\mu}\vec{\rho}_{\nu}-\partial_{\nu}\vec{\rho}_{\mu})\right]N\ , \tag{7}$$

$$L_{\pi\rho NN}=\frac{g}{2m}\gamma_0\bar{N}\vec{\tau}\gamma^{\mu}\gamma_5 N\cdot\vec{\rho}_{\mu}\times\vec{\phi}\ , \tag{8}$$

$$L_{A_1 NN}=\frac{g}{m}f_{\pi}\gamma_0\bar{N}\vec{\tau}\gamma^{\mu}\gamma_5 N\cdot\vec{A}_{\mu}\ , \tag{9}$$

$$L_{\pi N\Delta}=g_{\Delta}\bar{\Delta}_{\mu}\vec{M}[g^{\mu\nu}-(Z+\tfrac{1}{2})\gamma^{\mu}\gamma^{\nu}]N\cdot\partial_{\nu}\vec{\phi}+\text{H.c.}\ , \tag{10}$$

$$L_{\rho N\Delta}=i\gamma_{\Delta}\bar{\Delta}_{\mu}\vec{M}\left[g^{\mu\nu}-\frac{\lambda}{4}\gamma^{\mu}\gamma^{\nu}\right]\gamma^{\theta}\gamma_5 N\cdot(\partial_{\theta}\vec{\rho}_{\nu}-\partial_{\nu}\vec{\rho}_{\theta}) +\text{H.c.}\ , \tag{11}$$

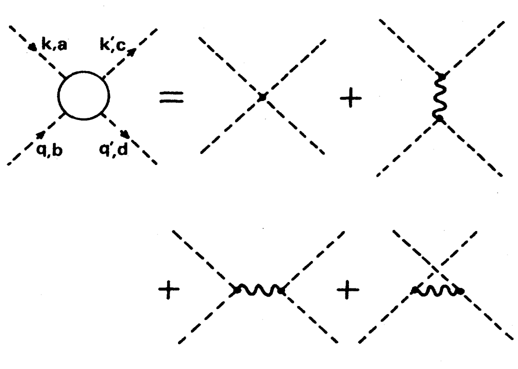

FIG. 4. Low-energy pion-pion scattering amplitude. Pions and rhos are represented by broken and wavy lines, respectively.

$$L_{\pi\Delta\Delta}=C_{\pi\Delta\Delta}\bar{\Delta}^{\sigma}\vec{T}[\gamma_{\lambda}g_{\sigma\theta}-(1-\tfrac{1}{2}\zeta)g_{\sigma\lambda}\gamma_{\theta} -(1-\tfrac{1}{2}\zeta)\gamma_{\sigma}g_{\lambda\theta} +(1-\zeta+\tfrac{3}{8}\zeta^2)\gamma_{\sigma}\gamma_{\lambda}\gamma_{\theta}] \times\gamma_5\Delta^{\theta}\cdot\partial^{\lambda}\vec{\phi}\ . \tag{12}$$

The $\pi$N coupling in the nonlinear Lagrangian approach is of the pseudovector type. In the above expressions the symbols N, $\Delta_{\mu}$, $\vec{\phi}$, $\vec{\rho}_{\mu}$, and $\vec{A}_{\mu}$ denote, respectively, the nucleon, delta, pion, rho, and $A_1$ fields, whose masses are $m$, $M_{\Delta}$, $\mu$, $m_{\rho}$, and $m_A$. The matrices $\vec{\tau}$, $\vec{M}$, and $\vec{T}$ combine two nucleons, one nucleon and one delta, and two deltas into isospin 1 states. The parameter $\xi$ was introduced by Olsson and Turner[15] and is determined by the group transformation properties of the chiral symmetry breaking term in the Lagrangian. The universal vector coupling constant is $\gamma_0$, $\delta$ is a parameter measurable in the decay $\rho\rightarrow\pi\pi$, and $\mu_p$ and $\mu_n$ are the proton and neutron anomalous magnetic moments. The parameters $Z$, $\lambda$, and $\zeta$ represent the possibility of spin $\frac{1}{2}$ components in the off-pole delta wave function. The delta couplings are associated with the following form for its propagator:

$$G_{\mu\nu}(p)=\frac{(\not{p}+M_{\Delta})}{p^2-M_{\Delta}^2}\left[g_{\mu\nu}-\tfrac{1}{3}\gamma_{\mu}\gamma_{\nu}-\frac{\gamma_{\mu}p_{\nu}}{3M_{\Delta}}+\frac{p_{\mu}\gamma_{\nu}}{3M_{\Delta}}-\frac{2p_{\mu}p_{\nu}}{3M_{\Delta}^2}\right]. \tag{13}$$

This expression corresponds to setting $A=-1$ into Fronsdal's[16] $\Delta$ Lagrangian, which has also been used to generate $L_{\pi\Delta\Delta}$ by means of an axial gauge transformation.

In the derivation of the four-body potential the momenta of the nucleons are consistently assumed to be of the order of the pion mass. The momentum $p$ of a nucleon is written as

$$p\equiv(E,\vec{p})\cong\left[m+\frac{\vec{p}^{\,2}}{2m},\vec{p}\right], \tag{14}$$

whereas the momentum $k$ of a pion emitted by this nucleon is given by

$$k\equiv(\omega,\vec{k})\cong\left[\frac{\vec{p}^{\,2}-\vec{p}\,'^{2}}{2m},\vec{p}-\vec{p}\,'\right]. \tag{15}$$

Therefore, the orders of magnitude of these kinematical variables are the following: $E\sim m$, $|\vec{p}|\sim|\vec{k}|\sim\mu$, $\omega\sim\mu^2/m$.

### A. Intermediate pion-pion scattering

The process $\pi^a(k)\pi^b(q)\rightarrow\pi^c(k')\pi^d(q')$ is described by the diagrams of Fig. 4, representing a contact term and three exchanges of a rho meson. The corresponding amplitude, denoted by $T^{(a)}$, is obtained from the Lagrangian elements already displayed and has the following form:[15]

$$T^{(a)}=\frac{1}{f_\pi^2}\{\delta_{ac}\delta_{bd}[-(1-\xi)(k-k')\cdot(q-q')-\xi(k\cdot k'+q\cdot q')-\mu^2(1-\tfrac{3}{2}\xi)]$$
$$+\delta_{ad}\delta_{bc}[-(1-\xi)(k-q')\cdot(q-k')-\xi(k\cdot q'+q\cdot k')-\mu^2(1-\tfrac{3}{2}\xi)]$$
$$+\delta_{ab}\delta_{cd}[(1-\xi)(k+q)\cdot(k'+q')+\xi(k\cdot q+k'\cdot q')-\mu^2(1-\tfrac{3}{2}\xi)]\}\ . \tag{16}$$

In this derivation one has used the relation $\gamma_0^2/m_\rho^2=\frac{1}{2}f_\pi^2$.[17] The corresponding expression for nonrelativistic nucleons is

$$t^{(a)}=\frac{1}{f_\pi^2}\{\delta_{ac}\delta_{bd}[(1-\xi)(\vec{k}-\vec{k}')\cdot(\vec{q}-\vec{q}')+\xi(\vec{k}\cdot\vec{k}'+\vec{q}\cdot\vec{q}')-\mu^2(1-\tfrac{3}{2}\xi)]$$
$$+\delta_{ad}\delta_{bc}[(1-\xi)(\vec{k}-\vec{q}')\cdot(\vec{q}-\vec{k}')+\xi(\vec{k}\cdot\vec{q}'+\vec{q}\cdot\vec{k}')-\mu^2(1-\tfrac{3}{2}\xi)]$$
$$+\delta_{ab}\delta_{cd}[-(1-\xi)(\vec{k}+\vec{q})\cdot(\vec{k}'+\vec{q}')-\xi(\vec{k}\cdot\vec{q}+\vec{k}'\cdot\vec{q}')-\mu^2(1-\tfrac{3}{2}\xi)]\}\ . \tag{17}$$

### B. Intermediate pion production

The dynamical content of the reaction

$$\pi^a(k)N(p)\rightarrow\pi^c(k')\pi^d(q')N(p')$$

for free particles at low energies is shown in Fig. 3. The first diagram represents the pion-pole amplitude and cannot be included in the four-body potential, since this would mean the double counting of the pion-pion process. One also must not include the diagrams within round brackets, because they contain nucleon propagators and hence correspond to iterations of two and three body potentials.

The square bracket contains a seagull term besides others describing the propagation of vector mesons. When the effective Langrangian adopted in this work is used, the contribution of the diagrams including the $\pi\rho$NN vertex is canceled to leading order in $\mu^2/m_\rho^2$ by that describing the propagation of the $A_1$. Moreover, processes containing both $\rho$ and $A_1$ propagators produce only corrections to the leading term. Thus, the most important contribution from the diagrams within square brackets comes from the seagull term. The corresponding amplitude, represented by $T^{(b,S)}$, is

$$T^{(b,S)}=i\frac{g}{2m}\frac{1}{2f_\pi^2}\bar{u}\gamma^\mu\gamma_5 u\{\delta_{ac}\tau_d[-\xi q'_\mu+(\xi-1)(k_\mu-k'_\mu)]+\delta_{ad}\tau_c[-\xi k'_\mu+(\xi-1)(k_\mu-q'_\mu)]$$
$$+\delta_{cd}\tau_a[\xi k_\mu-(\xi-1)(k'_\mu+q'_\mu)]\}\ . \tag{18}$$

The nonrelativistic limit of this expression is

$$t^{(b,S)}=-i2m\frac{g}{2m}\frac{1}{2f_\pi^2}\{\delta_{ac}\tau_d[-\xi\vec{\sigma}\cdot\vec{q}'+(\xi-1)\vec{\sigma}\cdot(\vec{k}-\vec{k}')]+\delta_{ad}\tau_c[-\xi\vec{\sigma}\cdot\vec{k}'+(\xi-1)\vec{\sigma}\cdot(\vec{k}-\vec{q}')]$$
$$+\delta_{cd}\tau_a[\xi\vec{\sigma}\cdot\vec{k}-(\xi-1)\vec{\sigma}\cdot(\vec{k}'+\vec{q}')]\}\ . \tag{19}$$

The diagrams within curly brackets represent two types of processes, namely those containing one and two delta propagators. They are referred to as single and double delta diagrams and correspond to the amplitudes $T^{(b,\Delta)}$ and $T^{(b,\Delta\Delta)}$. The single delta processes depicted in Fig. 5(a) yield the following amplitude:

$$(T^{(b,\Delta)})=\gamma_0[i\epsilon_{acd}(T_\mu^+)_\Delta+(\delta_{ac}\tau_d-\delta_{ad}\tau_c)(T_\mu^-)_\Delta]\frac{g^{\mu\nu}-Q^\mu Q^\nu/m_\rho^2}{Q^2-m_\rho^2}(q'_\nu-k'_\mu)\ , \tag{20}$$

where $(T_\mu^\pm)_\Delta$ are the same subamplitudes that contribute to the pion-rho-exchange three-body force.[18] Their most important parts are those proportional to the poles of the delta, and are given by

$$(T_\mu^+)_\Delta=-i\frac{\gamma_\Delta g_\Delta}{9M_\Delta^2}\bar{u}\gamma_5\left\{\left[\frac{1}{s-M_\Delta^2}+\frac{1}{u-M_\Delta^2}\right][m\alpha+(Q^2-2m^2)\beta+3M_\Delta^2(k^2-2Q\cdot k-2m^2-2mM_\Delta)]\frac{\gamma^\lambda\gamma^\nu}{2}\right.$$
$$+\left[\frac{1}{s-M_\Delta^2}-\frac{1}{u-M_\Delta^2}\right]\alpha Q\frac{\gamma^\lambda\gamma^\nu}{2}+\beta(m\gamma^\lambda-Q^\lambda)\left[\frac{(P+k)^\nu}{s-M_\Delta^2}+\frac{(P-k)^\nu}{u-M_\Delta^2}\right]$$

$$+\left[\frac{1}{s-M_\Delta^2}+\frac{1}{u-M_\Delta^2}\right]6M_\Delta^2k^\lambda P^\nu+3M_\Delta^2(m+M_\Delta)\gamma^\lambda\left[\frac{(P-k)^\nu}{s-M_\Delta^2}+\frac{(P+k)^\nu}{u-M_\Delta^2}\right]\Bigg\}$$

$$\times(g_{\mu\lambda}Q_\nu-g_{\mu\nu}Q_\lambda)u\ , \qquad (21)$$

$$(T_\mu^-)_\Delta=-i\frac{\gamma_\Delta g_\Delta}{18M_\Delta^2}\bar u\gamma_5\Bigg\{\left[\frac{1}{s-M_\Delta^2}-\frac{1}{u-M_\Delta^2}\right][m\alpha+(Q^2-2m^2)\beta+3M_\Delta^2(k^2-2Q\cdot k-2m^2-2mM_\Delta)]\frac{\gamma^\lambda\gamma^\nu}{2}$$

$$+\left[\frac{1}{s-M_\Delta^2}+\frac{1}{u-M_\Delta^2}\right]\alpha Q\frac{\gamma^\lambda\gamma^\nu}{2}+\beta(m\gamma^\lambda-Q^\lambda)\left[\frac{(P+k)^\nu}{s-M_\Delta^2}-\frac{(P-k)^\nu}{u-M_\Delta^2}\right]$$

$$+\left[\frac{1}{s-M_\Delta^2}-\frac{1}{u-M_\Delta^2}\right]6M_\Delta^2k^\lambda P^\nu+3M_\Delta^2(m+M_\Delta)\gamma^\lambda\left[\frac{(P-k)^\nu}{s-M_\Delta^2}-\frac{(P+k)^\nu}{u-M_\Delta^2}\right]\Bigg\}$$

$$\times(g_{\mu\lambda}Q_\nu-g_{\mu\nu}Q_\lambda)u\ , \qquad (22)$$

where

$$\alpha\equiv(m+M_\Delta)(M_\Delta^2-m^2)+k^2(m+2M_\Delta)\ , \qquad (23)$$

$$\beta\equiv(2M_\Delta^2+mM_\Delta-m^2+k^2)\ , \qquad (24)$$

$$P=(p+p')\ . \qquad (25)$$

When the nucleons are assumed to be nonrelativistic and the diagrams corresponding to the permutations of the pions are added, one obtains the following form for the single delta amplitude:

$$t^{(b,\Delta)}=i\,2m\frac{4\gamma_0\gamma_\Delta g_\Delta}{9m_p^2(M_\Delta-m)}\Big[12\epsilon_{acd}\vec{k}\cdot\vec{k}'\times\vec{q}'+\delta_{ad}\tau_c(\vec{k}'\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}-2\vec{k}\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}'+\vec{k}\cdot\vec{k}'\vec{\sigma}\cdot\vec{q}')$$

$$+\delta_{ac}\tau_d(\vec{k}'\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}+\vec{k}\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}'-2\vec{k}\cdot\vec{k}'\vec{\sigma}\cdot\vec{q}')$$

$$+\delta_{cd}\tau_a(-2\vec{k}'\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}+\vec{k}\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}'+\vec{k}\cdot\vec{k}'\vec{\sigma}\cdot\vec{q}')\Big]\ . \qquad (26)$$

In the derivation of this expression one has neglected the difference between the nucleon and delta masses.

The double delta amplitude, shown in Fig. 5(b), receives its dominant contribution from the double pole term, corresponding to the following partial amplitude:

$$(T^{(b,\Delta\Delta)})=g_\Delta^2C_{\pi\Delta\Delta}(\tfrac{5}{6}i\epsilon_{acd}+\tfrac{2}{3}\delta_{ac}\tau_d-\tfrac{1}{6}\delta_{cd}\tau_a-\tfrac{1}{6}\delta_{ad}\tau_c)\bar u\gamma_5\frac{\not Q'-M_\Delta}{Q'^2-M_\Delta^2}$$

$$\times\Bigg\{\left[2M_\Delta k\cdot k'-\frac{4}{3M_\Delta}(Q\cdot kQ\cdot k'+Q'\cdot kQ'\cdot k')+\frac{8}{9M_\Delta^3}(M_\Delta^2+Q\cdot Q')Q\cdot kQ'\cdot k'\right]$$

$$+\left[-\tfrac{2}{3}Q\cdot k'-\frac{4}{9M_\Delta^2}(2M_\Delta^2-Q\cdot Q')Q'\cdot k'\right]\not k+\left[\tfrac{2}{3}Q'\cdot k+\frac{4}{9M_\Delta^2}(2M_\Delta^2-Q\cdot Q')Q\cdot k\right]\not k'$$

$$-\frac{2}{9M_\Delta}(4M_\Delta^2+Q\cdot Q')\not k'\not k\Bigg\}\frac{\not Q+M_\Delta}{Q^2-M_\Delta^2}u\ . \qquad (27)$$

Taking the nonrelativistic limit of this expression and including the contributions of all the other permutations of the pion quantum numbers, one gets

$$t^{(b,\Delta\Delta)}=i2m\frac{g_\Delta^2 C_{\pi\Delta\Delta}}{3(M_\Delta-m)^2}[\tfrac{25}{3}\epsilon_{acd}\vec{k}\cdot\vec{k}'\times\vec{q}'+\delta_{ad}\tau_c(\tfrac{7}{9}\vec{k}'\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}-2\vec{k}\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}'+\tfrac{7}{9}\vec{k}\cdot\vec{k}'\vec{\sigma}\cdot\vec{q}')$$
$$+\delta_{ac}\tau_d(\tfrac{7}{9}\vec{k}'\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}+\tfrac{7}{9}\vec{k}\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}'-2\vec{k}\cdot\vec{k}'\vec{\sigma}\cdot\vec{q}')$$
$$+\delta_{cd}\tau_a(-2\vec{k}'\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}+\tfrac{7}{9}\vec{k}\cdot\vec{q}'\vec{\sigma}\cdot\vec{k}'+\tfrac{7}{9}\vec{k}\cdot\vec{k}'\vec{\sigma}\cdot\vec{q}')] . \quad (28)$$

It is worth pointing out that Eqs. (26) and (28) do not depend on the parameters $\lambda$ and $\zeta$ associated with the off-shell $\rho N\Delta$ and $\pi\Delta\Delta$ couplings, respectively. This occurs because the single and double delta amplitudes are dominated by their poles, since they yield denominators proportional to $(M_\Delta-m)\sim\mu$.

### C. Intermediate pion-nucleon scattering

The amplitude for the process

$$\pi^a(k)N(p)\to\pi^c(k')N(p') ,$$

for nucleons on shell, can be parametrized as

$$T^{(c)}=\bar{u}\left[\left[A^+ +\frac{\not{k}'+\not{k}}{2}B^+\right]\delta_{ac} + \left[A^- +\frac{\not{k}'+\not{k}}{2}B^-\right]i\epsilon_{cae}\tau_e\right]u . \quad (29)$$

When the nucleons are nonrelativistic, this expression can be written as

$$t^{(c)}=2m\left[\left[f^+ +i\frac{b^+}{2m}\vec{\sigma}\cdot\vec{k}'\times\vec{k}\right]\delta_{ac} + \left[f^- +i\frac{b^-}{2m}\vec{\sigma}\cdot\vec{k}'\times\vec{k}\right]i\epsilon_{cae}\tau_e\right] , \quad (30)$$

where

$$f^\pm=a^\pm+\nu b^\pm , \quad (31)$$

$$\nu\equiv(p+p')\cdot(k+k')/(4m) , \quad (32)$$

and $a^\pm$ and $b^\pm$ are the nonrelativistic limits of $A^\pm$ and $B^\pm$. The dynamical content of the pion-nucleon scattering amplitude is shown in Fig. 2. The diagram describing the propagation of a nucleon represents an iteration of the two-body potential and must not be considered. The relativistic expression for the contribution of the delta pole and rho exchange to $A^\pm$ and $B^\pm$ can be found in Ref. 5 and will not be reproduced here. Their nonrelativistic limits produce the following nonvanishing terms:

$$f_\Delta^+=\frac{8g_\Delta^2}{9(M_\Delta-m)}\vec{k}\cdot\vec{k}' , \quad (33)$$

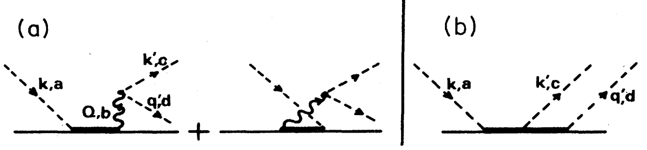


FIG. 5. Single (a) and double (b) delta diagrams.

$$b_\Delta^-=\frac{4g_\Delta^2 m}{9(M_\Delta-m)} , \quad (34)$$

$$f_\rho^-=\frac{1}{2f_\pi^2}\nu , \quad (35)$$

$$b_\rho^-=\frac{1}{2f_\pi^2}(1+\mu_p-\mu_n) . \quad (36)$$

The delta contribution is dominated by its pole term and hence does not depend on the parameter $Z$. The function $f_\rho^-$ is proportional to the velocity and produces nonlocal factors in the potential; therefore it is neglected in this work.

The $\sigma$ term contributes only to the function $A^+$ of the relativistic $\pi$N amplitude. In Ref. 5 this contribution has been parametrized as

$$A_\sigma^+=\alpha_\sigma+\beta_\sigma k\cdot k' , \quad (37)$$

where $\alpha_\sigma$ and $\beta_\sigma$ are constants extracted from experiment. This form has been taken from Ref. 9 and is adequate for on-shell pions. When the pions are not asymptotic, one has to include off-shell effects, and the above form has to be modified, as has been correctly pointed out in Ref. 2. The parametrization used in Ref. 4, on the other hand, does not suffer from these difficulties and is consistent with the theoretical single and double soft-pion limits of the $\pi$N amplitude. It is proportional to the "measurable" parameter $\sigma$, which is the $\pi$N sigma term, and is equivalent to the following form for $A_\sigma^+$:

$$A_\sigma^+=\frac{\sigma}{f_\pi^2}\left[(1-\beta)\left[\frac{k^2+k'^2}{\mu^2}-1\right]+\beta\left[\frac{t}{\mu^2}-1\right]\right]$$
$$=\frac{\sigma}{f_\pi^2}\left[1-2\beta\frac{k\cdot k'}{\mu^2}+\frac{1}{\mu^2}(k^2-\mu^2)+\frac{1}{\mu^2}(k'^2-\mu^2)\right]$$
$$\equiv\alpha_\sigma+\beta_\sigma k\cdot k'+\gamma_\sigma[(k^2-\mu^2)+(k'^2-\mu^2)] , \quad (38)$$

where one has made the identifications $\alpha_\sigma\equiv(\sigma/f_\pi^2)$, $\beta_\sigma\equiv-(2\beta/\mu^2)(\sigma/f_\pi^2)$, and $\gamma_\sigma\equiv(1/\mu^2)(\sigma/f_\pi^2)$.

Comparing Eqs. (37) and (38) it is possible to see that they differ by the terms proportional to $\gamma_\sigma$, which describe off-shell effects. In the evaluation of the four-body amplitude, they cancel pion propagators, yielding potentials in coordinate space containing parts that are proportional to δ functions. In the absence of form factors, the short range repulsion between nucleons allows one to expect that these contact interactions would influence numerical results very little. However, realistic calculations do require the use of form factors in order to regularize the behavior of the potential for vanishing internucleon distances. In a recent study of the role of form factors in the *s*-wave component of the two-pion-exchange three-nucleon potential[19] it was argued that the regularization of the results of chiral symmetry must be made by eliminating all possible δ functions *before* the inclusion of form factors. In doing the opposite one would be using form factors to regularize δ functions, producing huge distortions in the form of the potential.[19]

With the purpose of avoiding this sort of undesired behavior in the case of the four nucleon potential, one eliminates from the amplitude terms that correspond to δ functions in coordinate space. In the present situation this amounts to disregarding the factor proportional to $\gamma_\sigma$ in Eq. (38). The amplitude $A_\sigma^+$ modified in this way produces the following contribution to Eq. (30), the nonrelativistic πN amplitude

$$\hat{f}_\sigma^+ = \alpha_\sigma - \beta_\sigma \vec{k}\cdot\vec{k}' \ . \tag{39}$$

The caret indicates the modification mentioned above.

## III. THE FOUR-BODY POTENTIAL

The four-body potential in momentum space is defined as

$$\langle \vec{p}\,'_1\vec{p}\,'_2\vec{p}\,'_3\vec{p}\,'_4 \,|\, W^{1234} \,|\, \vec{p}_1\vec{p}_2\vec{p}_3\vec{p}_4\rangle \equiv -(2\pi)^3\delta^3(\vec{p}_f-\vec{p}_i)\frac{1}{(2m)^4}t_{4N} \ , \tag{40}$$

where $t_{4N}$ is the amplitude for the elastic scattering of four nonrelativistic nucleons, excluding the contribution of intermediate nucleons propagating forward in time.

In the evaluation of the potential one uses the following kinematical variables:

$$k=p_1-p'_1,\quad k'=p'_2-p_2\ ,$$
$$q'=p'_3-p_3,\quad q=p_4-p'_4,\quad Q=k-k'=q'-q \ . \tag{41}$$

Energy-momentum conservation means that

$$k+q=k'+q' \ . \tag{42}$$

The amplitude for the process 4N→4N due to the intermediate scattering of pions, as represented in Fig. 1(a), is given by

$$\begin{aligned} t_{4N}^{(a)} = (2m)^4 \left[\frac{g}{2m}\right]^4 & \frac{1}{2f_\pi^2}\frac{1}{\vec{k}^2+\mu^2}\frac{1}{\vec{k}'^2+\mu^2}\frac{1}{\vec{q}\,'^2+\mu^2}\frac{1}{\vec{q}^{\,2}+\mu^2}\vec{\sigma}^{(1)}\cdot\vec{k}\,\vec{\sigma}^{(2)}\cdot\vec{k}'\,\vec{\sigma}^{(3)}\cdot\vec{q}\,'\,\vec{\sigma}^{(4)}\cdot\vec{q} \\ & \times\{\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)}[-(1-\xi)(\vec{k}^2+\vec{k}'^2+\vec{q}\,'^2+\vec{q}^{\,2}+4\mu^2)+2(\vec{k}\cdot\vec{k}'+\vec{q}\cdot\vec{q}\,')+\mu^2(2-\xi)] \\ & +\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}[-(1-\xi)(\vec{k}^2+\vec{k}'^2+\vec{q}\,'^2+\vec{q}^{\,2}+4\mu^2)+2(\vec{k}\cdot\vec{q}\,'+\vec{q}\cdot\vec{k}')+\mu^2(2-\xi)] \\ & +\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}[-(1-\xi)(\vec{k}^2+\vec{k}'^2+\vec{q}\,'^2+\vec{q}^{\,2}+4\mu^2)-2(\vec{k}\cdot\vec{q}+\vec{k}'\cdot\vec{q}\,')+\mu^2(2-\xi)]\} \ . \end{aligned} \tag{43}$$

In this expression $\sigma^{(i)}$ and $\tau^{(i)}$ indicate expectation values.

The contribution of the pion production process to $t_{4N}$, indicated in Fig. 1(b), is composed of three terms, namely, the seagull, single delta, and double delta. The first of them results in the following value for the amplitude:

$$\begin{aligned} t_{4N}^{(b,S)} = (2m)^4 \left[\frac{g}{2m}\right]^4 & \frac{1}{2f_\pi^2}\frac{1}{\vec{k}^2+\mu^2}\frac{1}{\vec{k}'^2+\mu^2}\frac{1}{\vec{q}\,'^2+\mu^2}\vec{\sigma}^{(1)}\cdot\vec{k}\,\vec{\sigma}^{(2)}\cdot\vec{k}'\,\vec{\sigma}^{(3)}\cdot\vec{q}\,' \\ & \times\{\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)}[(1-\xi)\vec{\sigma}^{(4)}\cdot\vec{q}-\vec{\sigma}^{(4)}\cdot\vec{q}\,']+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}[(1-\xi)\vec{\sigma}^{(4)}\cdot\vec{q}-\vec{\sigma}^{(4)}\cdot\vec{k}'] \\ & +\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\cdot\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}[(1-\xi)\vec{\sigma}^{(4)}\cdot\vec{q}+\vec{\sigma}^{(4)}\cdot\vec{k}]\}+(1\leftrightarrow 4)+(2\leftrightarrow 4)+(3\leftrightarrow 4) \ . \end{aligned} \tag{44}$$

The single delta diagram leads to

$$\begin{aligned} t_{4N}^{(b,\Delta)} = (2m)^4 \left[\frac{g}{2m}\right]^3 & \frac{4\gamma_0\gamma_\Delta g_\Delta}{9m_\rho^2(M_\Delta-m)}\frac{1}{\vec{k}^2+\mu^2}\frac{1}{\vec{k}'^2+\mu^2}\frac{1}{\vec{q}\,'^2+\mu^2}\vec{\sigma}^{(1)}\cdot\vec{k}\,\vec{\sigma}^{(2)}\cdot\vec{k}'\,\vec{\sigma}^{(3)}\cdot\vec{q}\,' \\ & \times\{-12\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\times\vec{\tau}^{(3)}\vec{k}\cdot\vec{k}'\times\vec{q}\,'+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)}(-\vec{k}'\cdot\vec{q}\,'\vec{\sigma}^{(4)}\cdot\vec{k}-\vec{k}\cdot\vec{q}\,'\sigma^{(4)}\cdot\vec{k}'+2\vec{k}\cdot\vec{k}'\vec{\sigma}^{(4)}\cdot\vec{q}\,') \end{aligned}$$

$$+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}(-\vec{k}'\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}+2\vec{k}\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}'-\vec{k}\cdot\vec{k}'\vec{\sigma}^{(4)}\cdot\vec{q}')$$

$$+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}(2\vec{k}'\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}-\vec{k}\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}'-\vec{k}\cdot\vec{k}'\vec{\sigma}^{(4)}\cdot\vec{q}')\}$$

$$+(1\leftrightarrow 4)+(2\leftrightarrow 4)+(3\leftrightarrow 4)\,. \tag{45}$$

The double delta contribution is

$$t_{4\mathrm{N}}^{(b,\Delta\Delta)}=(2m)^4\left[\frac{g}{2m}\right]^3\frac{g_\Delta^2 C_{\pi\Delta\Delta}}{3(M_\Delta-m)^2}\frac{1}{\vec{k}^2+\mu^2}\frac{1}{\vec{k}'^2+\mu^2}\frac{1}{\vec{q}'^2+\mu^2}\vec{\sigma}^{(1)}\cdot\vec{k}\vec{\sigma}^{(2)}\cdot\vec{k}'\vec{\sigma}^{(3)}\cdot\vec{q}'$$

$$\times[-\tfrac{25}{3}\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\times\vec{\tau}^{(3)}\vec{k}\cdot\vec{k}'\times\vec{q}'$$

$$+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)}(-\tfrac{7}{9}\vec{k}'\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}-\tfrac{7}{9}\vec{k}\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}'+2\vec{k}\cdot\vec{k}'\vec{\sigma}^{(4)}\cdot\vec{q}')$$

$$+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}(-\tfrac{7}{9}\vec{k}'\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}+2\vec{k}\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}'-\tfrac{7}{9}\vec{k}\cdot\vec{k}'\vec{\sigma}^{(4)}\cdot\vec{q}')$$

$$+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}(2\vec{k}'\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}-\tfrac{7}{9}\vec{k}\cdot\vec{q}'\vec{\sigma}^{(4)}\cdot\vec{k}'-\tfrac{7}{9}\vec{k}\cdot\vec{k}'\vec{\sigma}^{(4)}\cdot\vec{q}')]$$

$$+(1\leftrightarrow 4)+(2\leftrightarrow 4)+(3\leftrightarrow 4)\,. \tag{46}$$

In the evaluation of the four-nucleon amplitude corresponding to the pion rescattering diagram of Fig. 1(c) one neglects both nonlocal terms and those corresponding to δ functions in coordinate space. As before, the latter modification is indicated by a caret; one obtains

$$\hat{t}_{4\mathrm{N}}^{(c)}=-\tfrac{1}{2}(2m)^4\left[\frac{g}{2m}\right]^2\frac{1}{\vec{k}^2+\mu^2}\frac{1}{\vec{Q}^2+\mu^2}\frac{1}{\vec{q}^2+\mu^2}\vec{\sigma}^{(1)}\cdot\vec{k}\vec{\sigma}^{(4)}\cdot\vec{q}$$

$$\times\Bigg\{\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\left[\alpha_\sigma+\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\vec{k}\cdot\vec{Q}\right]\left[\alpha_\sigma-\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\vec{Q}\cdot\vec{q}\right]$$

$$-\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}\times\vec{\tau}^{(1)}\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_\mathrm{p}-\mu_\mathrm{n})}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]\vec{\sigma}^{(2)}\cdot\vec{Q}\times\vec{k}\left[\alpha_\sigma-\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\vec{Q}\cdot\vec{q}\right]$$

$$+\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)}\times\vec{\tau}^{(1)}\left[\alpha_\sigma+\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\vec{k}\cdot\vec{Q}\right]\vec{\sigma}^{(3)}\cdot\vec{q}\times\vec{Q}\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_\mathrm{p}-\mu_\mathrm{n})}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]$$

$$+\left[\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}-\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}\right]\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_\mathrm{p}-\mu_\mathrm{n})}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]^2$$

$$\times\vec{\sigma}^{(2)}\cdot\vec{Q}\times\vec{k}\,\vec{\sigma}^{(3)}\cdot\vec{q}\times\vec{Q}\Bigg\}+(\text{all nucleon permutations})\,. \tag{47}$$

The factor $\frac{1}{2}$ which precedes the amplitude has been introduced because every independent diagram is double counted when one performs *all* possible permutations of the nucleon indices.

The potential in configuration space is given by

$$\langle\vec{r}'_1\vec{r}'_2\vec{r}'_3\vec{r}'_4\,|\,W^{1234}\,|\,\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4\rangle=-\frac{(2\pi)^3}{(2m)^4}\int\frac{d\vec{p}_1}{(2\pi)^3}\cdots\frac{d\vec{p}'_4}{(2\pi)^3}\delta^3(\vec{p}_f-\vec{p}_i)e^{i\vec{p}_1\cdot\vec{r}_1}\cdots e^{-i\vec{p}'_4\cdot\vec{r}'_4}t_{4\mathrm{N}}\,. \tag{48}$$

The expressions for $t_{4\mathrm{N}}$ already obtained allow one to write

$$\langle\vec{r}'_1\vec{r}'_2\vec{r}'_3\vec{r}'_4\,|\,W^{1234}\,|\,\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4\rangle\equiv\delta^3(\vec{r}'_1-\vec{r}_1)\cdots\delta^3(\vec{r}'_4-\vec{r}_4)W_{4\mathrm{B}}\,, \tag{49}$$

where

$$W_{4\mathrm{B}}=-\frac{(2\pi)^3}{(2m)^4}\int\frac{d\vec{k}}{(2\pi)^3}\frac{d\vec{k}'}{(2\pi)^3}\frac{d\vec{q}'}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\delta(\vec{k}-\vec{k}'-\vec{q}'+\vec{q})e^{i(\vec{k}\cdot\vec{r}_1-\vec{k}'\cdot\vec{r}_2-\vec{q}'\cdot\vec{r}_3+\vec{q}\cdot\vec{r}_4)}t_{4\mathrm{B}}\,. \tag{50}$$

The function $W_{4B}$ is the four-body potential. It is made up of various terms, representing the partial contributions from pion-pion scattering, seagull, single delta, double delta, and pion-nucleon rescattering. It is given by

$$W_{4B}=W_{4B}^{(a)}+W_{4B}^{(b,S)}+W_{4B}^{(b,\Delta)}+W_{4B}^{(b,\Delta\Delta)}+\hat{W}_{4B}^{(c)} . \qquad (51)$$

The explicit form of the potential contains the Yukawa function $U$, defined as

$$U(\mu r)\equiv\frac{4\pi}{\mu}\int\frac{d\vec{k}}{(2\pi)^3}\frac{e^{-i\vec{k}\cdot\vec{r}}}{\vec{k}^2+\mu^2}=\frac{e^{-\mu r}}{\mu r} . \qquad (52)$$

This function is not regular at the origin. Its regularization can be achieved by means of form factors and hard cores.[5] The former correspond to cutoffs in momentum space, whereas the latter are cutoffs in configuration space. These procedures are not mutually exclusive, since they are motivated by different physical causes. In this work the Yukawa function is assumed to be somehow regularized. However, one does not choose a specific method of regularization because applications of the potential are not being considered here.

The form of the functions $W_{4B}$ becomes simpler when one uses the dimensionless variables $\vec{x}_i\equiv\mu\vec{r}_i$ and $\vec{x}_{ij}\equiv\vec{x}_i-\vec{x}_j$. These definitions result in the following expressions for the partial contributions to the four-body potential.

### A. Intermediate pion-pion scattering

$$\begin{aligned}W_{4B}^{(a)}=&-\left[\frac{1}{4\pi}\right]^3\left[\frac{g}{2m}\right]^4\frac{1}{2f_\pi^2}\mu^7(\vec{\sigma}^{(1)}\cdot\vec{\nabla}_1\vec{\sigma}^{(2)}\cdot\vec{\nabla}_2\vec{\sigma}^{(3)}\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_4)\\&\times\{-(1-\xi)(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)}+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\tau^{(4)}+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})\\&\times[U(x_{14})U(x_{24})U(x_{34})+(1\leftrightarrow4)+(2\leftrightarrow4)+(3\leftrightarrow4)]\\&+2[(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)})(\vec{\nabla}_1\cdot\vec{\nabla}_2+\vec{\nabla}_3\cdot\vec{\nabla}_4+1-\tfrac{1}{2}\xi)+(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)})\\&\quad\times(\vec{\nabla}_1\cdot\vec{\nabla}_3+\vec{\nabla}_2\cdot\vec{\nabla}_4+1-\tfrac{1}{2}\xi)+(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})(\vec{\nabla}_1\cdot\vec{\nabla}_4+\vec{\nabla}_2\cdot\vec{\nabla}_3+1-\tfrac{1}{2}\xi)]\\&\times\frac{1}{4\pi}\int d\vec{x}\,U(|\vec{x}_1-\vec{x}|)U(|\vec{x}_2-\vec{x}|)U(|\vec{x}_3-\vec{x}|)U(\vec{x}_4-\vec{x}|)\} ,\end{aligned} \qquad (53)$$

where $\vec{\nabla}_i$ acts on $\vec{x}_i$.

### B. Intermediate pion production

The seagull term is

$$\begin{aligned}W_{4B}^{(b,S)}=&-\left[\frac{1}{4\pi}\right]^3\left[\frac{g}{2m}\right]^4\frac{1}{2f_\pi^2}\mu^7\{[(1-\xi)(\vec{\sigma}^{(1)}\cdot\vec{\nabla}_1\vec{\sigma}^{(2)}\cdot\vec{\nabla}_2\vec{\sigma}^{(3)}\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_4)\\&\times(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)}+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})]\\&+(\vec{\sigma}^{(1)}\cdot\vec{\nabla}_1\vec{\sigma}^{(2)}\cdot\vec{\nabla}_2\vec{\sigma}^{(3)}\cdot\vec{\nabla}_3)[(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)})\vec{\sigma}^{(4)}\cdot\vec{\nabla}_3\\&+(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)})\vec{\sigma}^{(4)}\cdot\vec{\nabla}_2\\&+(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})\vec{\sigma}^{(4)}\cdot\vec{\nabla}_1]\}\\&\times U(x_{14})U(x_{24})U(x_{34})+(1\leftrightarrow4)+(2\leftrightarrow4)+(3\leftrightarrow4) .\end{aligned} \qquad (54)$$

It is worth noting that the terms proportional to $(1-\xi)$ in Eqs. (53) and (54) have opposite signs and cancel when both contributions are added together. This cancellation has already been obtained in Ref. 6 and is due to chiral symmetry, as discussed in Sec. IV.

The single delta term is

$$W_{4B}^{(b,\Delta)}=-\left[\frac{1}{4\pi}\right]^3\left[\frac{g}{2m}\right]^3\frac{4\gamma_0\gamma_\Delta g_\Delta}{9m_\rho^2(M_\Delta-m)}\mu^9(\vec\sigma^{(1)}\cdot\vec\nabla_1\vec\sigma^{(2)}\cdot\vec\nabla_2\vec\sigma^{(3)}\cdot\vec\nabla_3)$$
$$\times\{12(\vec\tau^{(1)}\cdot\vec\tau^{(2)}\times\vec\tau^{(3)})(\vec\nabla_1\cdot\vec\nabla_2\times\vec\nabla_3)$$
$$+(\vec\tau^{(1)}\cdot\vec\tau^{(2)}\vec\tau^{(3)}\cdot\vec\tau^{(4)})[\vec\nabla_2\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_1+\vec\nabla_1\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_2-2\vec\nabla_1\cdot\vec\nabla_2\vec\sigma^{(4)}\cdot\vec\nabla_3]$$
$$+(\vec\tau^{(1)}\cdot\vec\tau^{(3)}\vec\tau^{(2)}\cdot\vec\tau^{(4)})[\vec\nabla_2\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_1-2\vec\nabla_1\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_2+\vec\nabla_1\cdot\vec\nabla_2\vec\sigma^{(4)}\cdot\vec\nabla_3]$$
$$+(\vec\tau^{(1)}\cdot\vec\tau^{(4)}\vec\tau^{(2)}\cdot\vec\tau^{(3)})[-2\vec\nabla_2\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_1+\vec\nabla_1\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_2+\vec\nabla_1\cdot\vec\nabla_2\vec\sigma^{(4)}\cdot\vec\nabla_3]\}$$
$$\times U(x_{14})U(x_{24})U(x_{34})+(1\leftrightarrow4)+(2\leftrightarrow4)+(3\leftrightarrow4)\,. \quad (55)$$

The double delta term is

$$W_{4B}^{(b,\Delta\Delta)}=-\left[\frac{1}{4\pi}\right]^3\left[\frac{g}{2m}\right]^3\frac{g_\Delta^2C_{\pi\Delta\Delta}}{3(M_\Delta-m)^2}\mu^9(\vec\sigma^{(1)}\cdot\vec\nabla_1\vec\sigma^{(2)}\cdot\vec\nabla_2\vec\sigma^{(3)}\cdot\vec\nabla_3)$$
$$\times\{\tfrac{25}{3}(\vec\tau^{(1)}\cdot\vec\tau^{(2)}\times\vec\tau^{(3)})(\vec\nabla_1\cdot\vec\nabla_2\times\vec\nabla_3)$$
$$+(\vec\tau^{(1)}\cdot\vec\tau^{(2)}\vec\tau^{(3)}\cdot\vec\tau^{(4)})[\tfrac{7}{9}\vec\nabla_2\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_1+\tfrac{7}{9}\vec\nabla_1\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_2-2\vec\nabla_1\cdot\vec\nabla_2\vec\sigma^{(4)}\cdot\vec\nabla_3]$$
$$+(\vec\tau^{(1)}\cdot\vec\tau^{(3)}\vec\tau^{(2)}\cdot\vec\tau^{(4)})[\tfrac{7}{9}\vec\nabla_2\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_1-2\vec\nabla_1\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_2+\tfrac{7}{9}\vec\nabla_1\cdot\vec\nabla_2\vec\sigma^{(4)}\cdot\vec\nabla_3]$$
$$+(\vec\tau^{(1)}\cdot\vec\tau^{(4)}\vec\tau^{(2)}\cdot\vec\tau^{(3)})[-2\vec\nabla_2\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_1+\tfrac{7}{9}\vec\nabla_1\cdot\vec\nabla_3\vec\sigma^{(4)}\cdot\vec\nabla_2+\tfrac{7}{9}\vec\nabla_1\cdot\vec\nabla_2\vec\sigma^{(4)}\cdot\vec\nabla_3]\}$$
$$\times U(x_{14})U(x_{24})U(x_{34})+(1\leftrightarrow4)+(2\leftrightarrow4)+(3\leftrightarrow4)\,. \quad (56)$$

### C. Intermediate pion-nucleon scattering

$$\hat W_{4B}^{(c)}=-\frac{1}{2}\left[\frac{1}{4\pi}\right]^3\left[\frac{g}{2m}\right]^2\mu^9(\vec\sigma^{(1)}\cdot\vec\nabla_{12}\vec\sigma^{(4)}\cdot\vec\nabla_{34})$$
$$\times\left\{\vec\tau^{(1)}\cdot\vec\tau^{(4)}\left[-\left[\frac{\alpha_\sigma}{\mu^2}\right]^2+\frac{\alpha_\sigma}{\mu^2}\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right](\vec\nabla_{23}\cdot\vec\nabla_{34}+\vec\nabla_{12}\cdot\vec\nabla_{23})\right.\right.$$
$$\left.-\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]^2(\vec\nabla_{12}\cdot\vec\nabla_{23}\vec\nabla_{23}\cdot\vec\nabla_{34})\right]$$
$$-\frac{\alpha_\sigma}{\mu^2}\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right][(\vec\tau^{(2)}\cdot\vec\tau^{(1)}\times\vec\tau^{(4)})(\vec\nabla_{12}\times\vec\nabla_{23})+(\vec\tau^{(3)}\cdot\vec\tau^{(1)}\times\vec\tau^{(4)})(\vec\nabla_{23}\times\vec\nabla_{34})]$$
$$+\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right][(\vec\tau^{(2)}\cdot\vec\tau^{(1)}\times\vec\tau^{(4)})(\vec\nabla_{23}\cdot\vec\nabla_{34}\,\vec\sigma^{(2)}\cdot\vec\nabla_{12}\times\vec\nabla_{23})$$
$$+(\vec\tau^{(3)}\cdot\vec\tau^{(1)}\times\vec\tau^{(4)})(\vec\nabla_{12}\cdot\vec\nabla_{23}\,\vec\sigma^{(3)}\cdot\vec\nabla_{23}\times\vec\nabla_{34})]$$
$$+\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]^2(\vec\tau^{(1)}\cdot\vec\tau^{(4)}\vec\tau^{(2)}\cdot\vec\tau^{(3)}-\vec\tau^{(1)}\cdot\vec\tau^{(3)}\vec\tau^{(2)}\cdot\vec\tau^{(4)})$$
$$\left.\times(\sigma^{(2)}\cdot\vec\nabla_{12}\times\vec\nabla_{23}\sigma^{(3)}\cdot\vec\nabla_{23}\times\vec\nabla_{34})\right\}U(x_{12})U(x_{23})U(x_{34})+(\text{all nucleon permutations})\,. \quad (57)$$

The final form for the potential derived in this work is that shown in Eq. (51), where the partial contributions are those given by Eqs. (53)–(57). Here, as in the case of three-body forces, each term of the potential is written as the product of four kinds of terms, namely a strength parameter with dimension of energy, an isospin operator, and a spin operator coupled to derivatives, acting on Yukawa functions. There would be, of course, other ways of writing the potential. For instance, the explicit evaluation of the derivatives of the functions $U$ would produce a result in terms of the functions $U_0$, $U_1$, and $U_2$ used in Ref. 5. The main advantage of the form adopted above is that the expressions tend to be more compact than the alternative ones.

Actual calculations require the knowledge of the various parameters entering Eqs. (53)–(57). In their numerical evaluation one adopts the following values for the "experimental" masses and coupling constants: $\mu=139.57$ MeV, $m_\rho=770$ MeV, $m=938.28$ MeV, $M_\Delta=1220$ MeV (Ref. 9), $g=13.39$, $g_\Delta=1.84\ \mu^{-1}$ (Ref. 9), $f_\pi=93$ MeV (Ref. 20), $\mu_p-\mu_n=3.7$, $\gamma_0=6.0$, and $\gamma_\Delta=2.0\ \mu^{-1}$. The value of $\gamma_0$ has been derived from the relation $\gamma_0=m_\rho/\sqrt{2}f_\pi$, whereas $\gamma_\Delta$ is linked to the $\gamma N\Delta$ form factor $C$ by $\gamma_\Delta=C\gamma_0$. The value of $C$ can be extracted from electroproduction, and here one adopts $C=0.34\ \mu^{-1}$.[21] The sigma parameters are $\alpha_\sigma=1.05\ \mu^{-1}$ (Ref. 20) and $\beta_\sigma=-0.80\ \mu^{-3}$ (Ref. 9). The symmetry SU(4) yields $C_{\pi\Delta\Delta}=\frac{6}{5}g/2m$.[23] The value of $\xi$ is compatible with zero.[11,22] Finally, it is worth pointing out that the potential does not depend on $Z$, $\lambda$, and $\zeta$, as the contributions of the $\Delta$ are dominated by the pole terms.

These "experimental" parameters produce the following values for the strength constants of the various partial contributions,

$$C_{(a)}=C_{(b,S)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^4\frac{1}{2f_\pi^2}\mu^3=0.0779\ \text{MeV}\,,$$

$$C_{(b,\Delta)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^3\frac{4\gamma_0\gamma_\Delta g_\Delta}{9m_\rho^2(M_\Delta-m)}\mu^6=0.0111\ \text{MeV}\,,$$

$$C_{(b,\Delta\Delta)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^3\frac{g_\Delta^2C_{\pi\Delta\Delta}}{3(M_\Delta-m)^2}\mu^6=0.0230\ \text{MeV}\,,$$

$$C_{(c,\sigma-\sigma)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^2\left[\frac{\alpha_\sigma}{\mu^2}\right]^2\mu^7=0.0769\ \text{MeV}\,,$$

$$C_{(c,\sigma-\Delta\sigma)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^2\left[\frac{\alpha_\sigma}{\mu^2}\right]\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\mu^7$$
$$=0.1677\ \text{MeV}\,,$$

$$C_{(c,\Delta\sigma-\Delta\sigma)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^2\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]^2\mu^7$$
$$=0.3661\ \text{MeV}\,,$$

$$C_{(c,\sigma-\rho\Delta)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^2\left[\frac{\alpha_\sigma}{\mu^2}\right]\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]\mu^7$$
$$=0.0561\ \text{MeV}\,,$$

$$C_{(c,\Delta\sigma-\Delta\sigma)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^2\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\times\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]\mu^7$$
$$=0.1225\ \text{MeV}\,,$$

$$C_{(c,\rho\Delta-\rho\Delta)}\equiv\left[\frac{1}{4\pi}\right]^3\left[\frac{g\mu}{2m}\right]^2\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]^2\mu^7$$
$$=0.0410\ \text{MeV}\,.$$

The meaning of these results is discussed in the next section. Before doing this, however, it is useful to compare them with those of other works. The results of McManus and Riska[6] correspond to keeping only $C_{(a)}$ and $C_{(b,S)}$ and neglecting all the other strength parameters. The results of Blatt and McKellar,[7] on the other hand, are formally obtained by making

$$(1/f_\pi^2)=\gamma_\Delta=C_{\pi\Delta\Delta}=\alpha_\sigma=\beta_\sigma=0$$

in the preceding expressions.

## IV. CONCLUSIONS

The derivation of the pion-exchange four-body potential presented in this work is based on the assumption that the nucleon momenta are comparable to the pion mass. The spin and isospin structures of the potential are somewhat complex and a precise assessment of the relative importance of its various contributions can only be done in specific applications. Nevertheless, several semiquantitative conclusions can be drawn by inspecting the strength parameters already displayed.

First, one notes that the seagull term $(b,S)$ dominates the contribution of the intermediate pion-production amplitude, relative to the single delta $(b,\Delta)$ and double delta $(b,\Delta\Delta)$ terms. This result is the direct consequence of chiral and gauge symmetries and hence is similar to the case of the pion-rho-exchange three-body force, where it has been argued that a seagull diagram could be ten times more important than that of the delta.[8]

The contributions from the intermediate pion-pion scattering $(a)$ and seagull in pion production $(b,S)$ have the same strength and cancel partially when they are added together, as has been shown in the work of McManus and Riska.[7] This behavior can be ascribed to the symmetries and is analogous to that observed on the exchange current contribution to the elastic pion-deuteron scattering.[24]

The strength parameters associated with the intermediate pion-nucleon rescattering are larger than those arising from pion-pion scattering and pion production. The largest term is due to $p$ waves in the isospin even amplitude and comes from the diagrams describing the delta pole and sigma "exchange" in Fig. 2.

The actual relevance of four-nucleon potentials for physical processes can only be assessed in realistic calculations. However, the present unavailability of this kind of results suggests that the relative importance of four, three, and two nucleon potentials could be roughly estimated by comparing their strengths. This comparison can be performed provided one bears in mind that it is supposed to yield only very crude indications about the roles of the various potentials, since their space, spin, and isospin structures are not considered.

The three-body force derived in Ref. 5 is characterized by the following parameters:

$$C_s=\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\left[\frac{\alpha_\sigma}{\mu^2}\right]\mu^4=0.92\text{ MeV},$$

$$C_p=-\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right]\mu^4$$

$$=-2.01\text{ MeV},$$

$$C_p'=-\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_{\rm p}-\mu_{\rm n})}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]\mu^4$$

$$=-0.67\text{ MeV}.$$

These values are typically one order of magnitude greater than those of the four body force. The three-body parameters are, in turn, one order of magnitude smaller than that of the one pion exchange nucleon-nucleon potential, which is given by

$$C_{\rm OPEP}=\left[\frac{1}{4\pi}\right]\left[\frac{g\mu}{2m}\right]^2\mu=11.02\text{ MeV}.$$

The comparison among these various strength parameters shed some light on the hierarchy of many body forces due to pion exchange. These forces correspond, in general, to a succession of vertices, describing interactions, and pion propagators. The latter are represented, in configuration space, by a factor

$$\left[\frac{1}{4\pi}U(x)\right]$$

for each pion, where $U(x)$ is a Yukawa function. The vertices, on the other hand, produce the remaining factors of the strength parameters.

In general, the strength of a many body potential should depend on the number of its vertices and propagators. However, inspection of the strength parameters of two-, three-, and four-body potentials allows one to conclude that the contributions of the vertices are roughly independent of their number, provided that the pion mass is adopted as a unit for the momenta. This means that the propagation of pions is the dominant factor in determining the strength of the potential. This influence is felt both through the radial variation of the Yukawa function and the factors $(1/4\pi)$. The various powers of the latter determine the different orders of magnitude of the strength parameters of two-, three-, and four-body potentials.

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