Alpha stripping and pickup to even Ge nuclei

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Coexistence wave functions for even Ge nuclei, previously derived from fits to two-neutron transfer data, are used to calculate cross section ratios for $^{A-4}$Zn(^{6}Li,d)^{A}Ge and $^{A+4}$Se(d,^{6}Li)^{A}Ge.

I. INTRODUCTION

Two-neutron transfer cross section ratios[1–9] for an excited 0^+ level and the ground state of the even-even Ge nuclei have been used[10] to determine wave-function amplitudes in a standard two-state model:

$$\Psi^A_{\rm g.s.}({\rm Ge})=\alpha_A\varphi^A_g({\rm Ge})+\beta_A\varphi^A_e({\rm Ge}) ,$$
$$\Psi^A_{0_2^+}({\rm Ge})=\beta_A\varphi^A_g({\rm Ge})-\alpha_A\varphi^A_e({\rm Ge}) . \quad (1)$$

The coefficients α_A and β_A were deduced in Ref. 10 from the expressions

$$x_A(R;r)=\frac{\alpha_A}{\beta_A}=\frac{R+K_A(1+Z_A)(R+1)}{-r+K_A(T_A+P_AZ_A)(R+1)}=\frac{-r-K_A(T_A+P_AZ_A)(R+1)}{1+K_A(1+Z_A)(R+1)} , \quad (2)$$

where K_A, T_A, P_A, Z_A are all obtainable from experimental quantities, as described in Ref. 10, and R is restricted to being between $R=0.75$ and $R=1.33$. We use a semicolon in the expression $x_A(R;r)$ to indicate that the parameters R and r are not independent—they are related by the expression $r^2=R+K_A(R+1)^2$.

Figure 1 gives a plot of α_A^2 deduced in Ref. 10. These squares of wave-function amplitudes are given in terms of one parameter, R, and fits to two-neutron transfer data are identical for all R values in the range indicated in Fig. 1. As a further test of the wave functions, the proton occupation numbers[11] in germanium were fitted[12] for any value of R between $R=0.88$ and 1.26. We therefore believe that these wave functions give an accurate quantitative account of the first 0^+ state and ground state in even isotopes of germanium. In this paper we use these wave functions to calculate the $^{A+4}$Se(d,^{6}Li)^{A}Ge and $^{A-4}$Zn(^{6}Li,d)^{A}Ge 0_2^+/g.s. cross-section ratios.

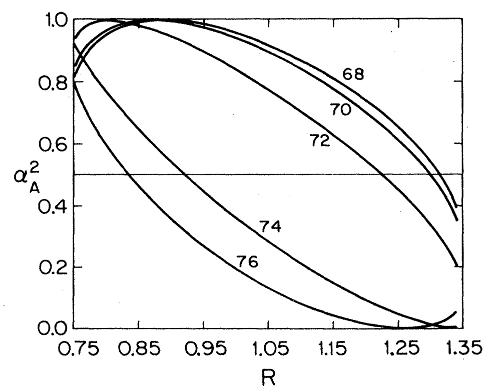

FIG. 1. α_A^2 vs R from Ref. 10.

II. MODEL AND ANALYSIS

Cross sections for the reactions $^{A+4}$Se(d,^{6}Li)^{A}Ge were measured by Van den Berg *et al.*[13] We define the ratios

$$\text{dLi}(A)=\frac{\sigma(^{A+4}\text{Se(d},^6\text{Li})^A\text{Ge}(0_2^+))/\sigma_{\rm DWBA}(0_2^+)}{\sigma(^{A+4}\text{Se(d},^6\text{Li})^A\text{Ge(g.s.)})/\sigma_{\rm DWBA}(\text{g.s.})} .$$

The distorted-wave Born approximation (DWBA) cross sections, $\sigma_{\rm DWBA}$, are identical except for Q-value and binding-energy effects. The experimental cross sections are divided by $\sigma_{\rm DWBA}$ simply to remove these kinematic effects from the data.

These ratios for $^{A+4}$Se$\rightarrow$^{A}Ge are plotted in Fig. 2, and given in Table I, in which we note a rapid dependence on A. Clearly our ratio is just the ratio of alpha-particle

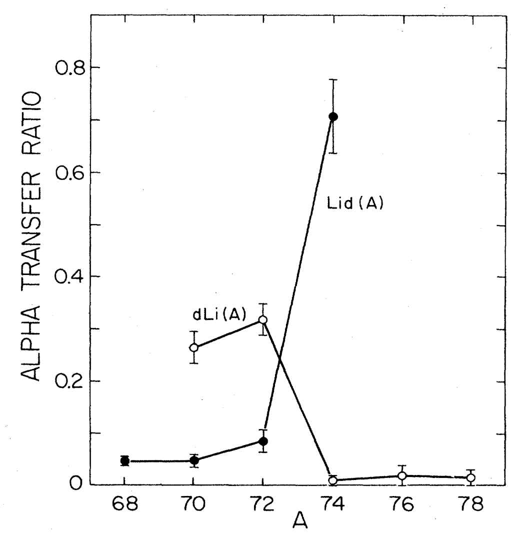

FIG. 2. Experimental 0_2^+/g.s. cross-section ratios in (^{6}Li,d) and (d,^{6}Li) leading to ^{A}Ge.

TABLE I. Values of alpha-transfer cross section ratios between the excited state listed and the ground state of even Ge isotopes.

A	dLi(A)$\pm\Delta_A$[a]	Lid(A)$\pm\Delta'_A$[b]	E_x (0^+) (MeV)
68		0.047±0.008	3.47
70	0.265±0.03	0.046±0.012	1.216
72	0.318±0.03	0.085±0.021	0.6915
74	(0.0095±0.0095)[c]	0.707±0.07	1.486
76	(0.0189±0.0189)		1.911
78	(0.015±0.015)		1.547

[a]Reference 13.
[b]Reference 14.
[c]Results in parentheses are given only as limits in Ref. 13.

spectroscopic factors from the physical Se ground states to the physical 0^+ excited and ground states in Ge.

Similar results exist[14] for ${}^{A-4}$Zn(^{6}Li,d)^{A}Ge and are also plotted in Fig. 2. Again, the A dependence is rapid. In a manner similar to that for pickup, we have defined a ratio

$$\text{Lid}(A)=\frac{\sigma({}^{A-4}\text{Zn}({}^6\text{Li,d})^A\text{Ge}(0_2^+))/\sigma_{\text{DWBA}}(0_2^+)}{\sigma({}^{A-4}\text{Zn}({}^6\text{Li,d})^A\text{Ge(g.s.)})/\sigma_{\text{DWBA}}(\text{g.s.})} . \quad (3)$$

To calculate these ratios with our Ge wave functions, we require the alpha-particle overlaps (actually only their ratio) between the Se physical ground states and our Ge basis states, as depicted in Fig. 3. From the figure, it is obvious that

$$\text{dLi}(A)=\left[\frac{1-x_A(R;r)u_A}{x_A(R;r)+u_A}\right]^2 , \quad (4)$$

where $x_A(R;r)$ are given in Eq. (2) and

$$u_A=\frac{\langle\varphi_{\text{g.s.}}^{A+4}(\text{Se})\,|\,\varphi_e^A(\text{Ge})+\alpha\rangle}{\langle\varphi_{\text{g.s.}}^{A+4}(\text{Se})\,|\,\varphi_g^A(\text{Ge})+\alpha\rangle} .$$

If we assume u_A are functions of A, then Eq. (4) can be inverted, and the experimental ratios determine u_A as functions of R. But first we investigate the possibility that u_A is independent of A, in which case

$$\text{dLi}(A)=\left[\frac{1-x_A(R;r)u}{x_A(R;r)+u}\right]^2 . \quad (5)$$

We therefore have two parameters, R and u, for which we try to fit four experimental results, [dLi(A),A =70,72,74,76]. To find the best fit we attempt to minimize χ^2/N denoted by $f(u,R;r)$ which is given by

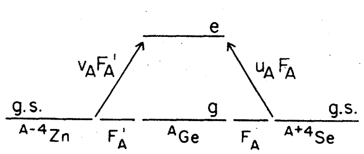

FIG. 3. Definition of the overlaps connecting the physical Zn and Se ground states with the Ge basis states.

$$f(u,R;r)=\frac{1}{n}\sum_{A=70}^{76}\frac{1}{\Delta_A^2}\left\{\left[\frac{1-x_A(R;r)u}{x_A(R;r)+u}\right]^2-\text{dLi}(A)\right\}^2 , \quad (6a)$$

where we have divided by the number of data points fitted minus the number of degrees of freedom, and Δ_A are measures of uncertainties in the ${}^{A+4}$Se(d,^{6}Li)^{A}Ge 0_2^+/g.s. cross-section ratios. For any "reasonable" fit we seek $f(u,R;r)$ to be less than or of order unity. For ${}^{A-4}$Zn(^{6}Li,d)^{A}Ge, Fig. 3 shows that with

$$v_A=\frac{\langle\varphi_{\text{g.s.}}^{A-4}(\text{Zn})+\alpha\,|\,\varphi_e^A(\text{Ge})\rangle}{\langle\varphi_{\text{g.s.}}^{A-4}(\text{Zn})+\alpha\,|\,\varphi_g^A(\text{Ge})\rangle}$$

and independent of A, we have

$$\text{Lid}(A)=\left[\frac{1-x_A(R;r)v}{x_A(R;r)+v}\right]^2 ,$$

which then leads to a χ^2/N given by

$$f'(v,R;r)=\frac{1}{n}\sum_{n=68}^{74}\frac{1}{\Delta_A'^2}\left\{\left[\frac{1-x_A(R;r)v}{x_A(R;r)+v}\right]^2-\text{Lid}(A)\right\}^2 , \quad (6b)$$

where Δ'_A is the uncertainty in Lid(A). Hence, both reactions yield the same form for χ^2/N and so, mathematically, both can be investigated simultaneously.

The method used in minimizing $f(u,R;r)$ is to calculate f for a range of values of R [by using $x_A(R;r)$ and search on f for the location of its minimum and thereby construct the function $u_{\min}=u_{\min}(R)$]. As we begin with no restriction on u, it might appear that this search would need to cover the entire range $(-\infty,\infty)$ to make sure that the absolute minimum value of f is found. This, of course, is not desirable, but in fact it turns out not to be necessary. From Eq. (2) and the K_A equation, it is clear that

TABLE II. Results of the ${}^{A+4}$Se(d,^{6}Li)^{A}Ge calculations. An (I) indicates the number was included in the minimization of χ^2/N, an (O) indicates the number was not included.

dLi (A) 70	72	74	76	χ^2/N
I	I	I	I	19.7
I	I	I	O	20.3
I	I	O	I	29.6
I	O	I	I	18.0
O	I	I	I	0.84
I	I	O	O	15.1
I	O	I	O	6.8
O	I	I	O	1.0
I	O	O	I	35.2
O	I	O	I	0.69
O	O	I	I	0.25

$$x_A(R;r)=\frac{-1}{x_A\left[\frac{1}{R};-r\right]},$$

and using this in Eq. (6) gives the relation

$$f(u,R;r)=f\left[-\frac{1}{u},\frac{1}{R};-r\right].$$

So to search over $f(u,R;r)$ when $u\leq -1$, we need only search on $f(u,1/R;-r)$ for $0\leq u\leq 1$, and to search over $f(u,R;r)$ when $u\geq 1$, we need only search on $f(u,1/R;-r)$ for $-1\leq u\leq 0$. Therefore to find the absolute minimum value of $f(u,R;r)$ over the entire interval $(-\infty,\infty)$ we need only search over $f(u,R;r)$ for $-1\leq u\leq 1$ and $f(u,1/R;-r)$ for $-1\leq u\leq 1$.

When this search is done for various values of R, a remarkable observation emerges. The absolute minimum value of $f(u,R;r)$ appears to be independent of R. The proof of the result is as follows: From Eq. (6a) we calculate $\partial f/\partial u$ and $\partial f/\partial R$ and obtain

$$\frac{\partial f}{\partial u}=\frac{-4}{n}\sum_A\left\{\left[\frac{1-x_A(R;r)u}{x_A(R;r)+u}\right]^2-\mathrm{dLi}(A)\right\}\frac{[1+x_A^2(R;r)]}{[x_A(R;r)+u]^3}\frac{[1-x_A(R;r)u]}{\Delta_A^2} \tag{7}$$

and

$$\frac{\partial f}{\partial R}=\frac{-4}{n}(u^2+1)\sum_A\left\{\left[\frac{1-x_A(R;r)u}{x_A(R;r)+u}\right]^2-\mathrm{dLi}(A)\right\}\frac{[1-x_A(R;r)u]}{\Delta_A^2[x_A(R;r)+u]^3}\left[\frac{\partial x_A(R;r)}{\partial R}\right]. \tag{8}$$

But using Eq. (2) and other results in Ref. 10 one can easily show that

$$-\frac{\partial x_A}{\partial R}=\frac{1+x_A^2(R;r)}{2r(R+1)}. \tag{9}$$

Putting Eq. (9) into Eq. (8) and then comparing it to Eq. (7) yields the result

$$2r(R+1)\frac{\partial f}{\partial R}+(1+u^2)\frac{\partial f}{\partial u}=0. \tag{10}$$

This equation clearly demonstrates that at a minimum point of f with respect to u, at which

$$\frac{\partial f}{\partial u}(u_{\min},R;r)=0$$

by definition, we must also have

$$\frac{\partial f}{\partial R}(u_{\min},R;r)=0;$$

and therefore

$$f_{\min}=f(u_{\min},R;r)$$

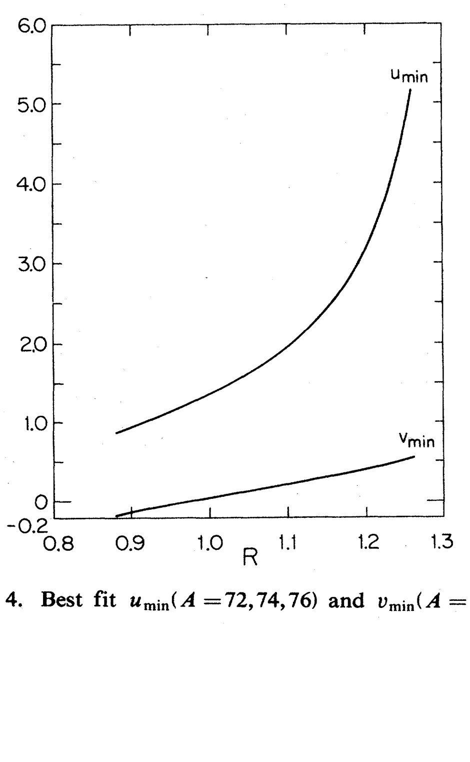

FIG. 4. Best fit $u_{\min}(A=72,74,76)$ and $v_{\min}(A=68,70,72)$ vs R.

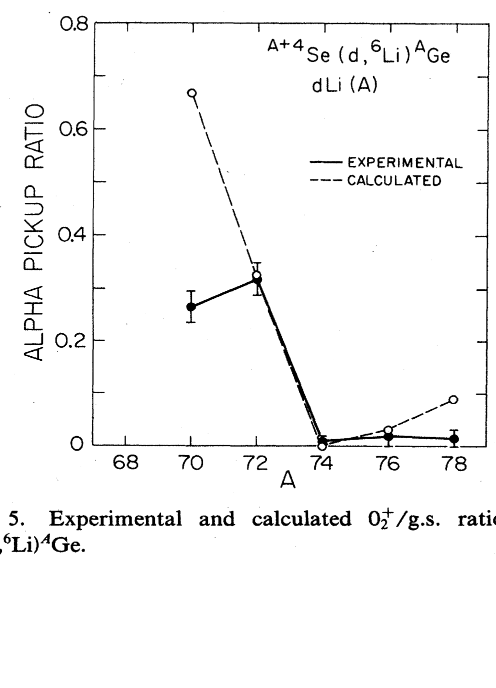

FIG. 5. Experimental and calculated 0_2^+/g.s. ratios for ${}^{A+4}$Se(d,^{6}Li)^{A}Ge.

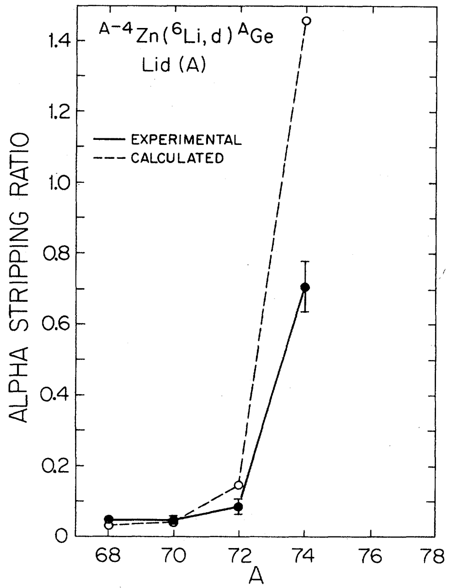

FIG. 6. Experimental and calculated 0_2^+/g.s. ratios for $^{A-4}$Zn(^{6}Li,d)^{A}Ge.

is independent of R.[15]

If we couple the preceding result with the previous observation about $f[-(1/u),1/R;-r]$ then the search for $u_{\min}$ is quite straightforward. Of course, even though the value of $f_{\min}$ is independent of R, the value of u at $f_{\min}$ does depend on R. When this search is done using all four data points for pickup (note that ^{78}Ge is excluded, as its wave function is not clearly determined in Ref. 10) we find $f_{\min}=19.7$—a result which clearly demonstrates that fitting all four points is not possible for any values of R and u. (The wave functions for ^{70,72,74,76}Ge were calculated using the techniques of Ref. 10 and the 0^+ excited states at $E_{70}=1.216$, $E_{72}=0.6915$, $E_{74}=1.486$, and $E_{76}=1.911$ MeV.) We next attempt to determine how many of these four points can be fitted. For each possible combination of two, three, and four data points, we have calculated $f_{\min}$, and these are summarized in Table II. This table reveals an interesting result. No A-independent value of u fits the data if ^{74}Se is included. However, if we exclude it, the remaining three points can be fitted quite well, giving a minimum χ^2/N of 0.84. This would imply that the overlap of the ^{74}Se physical ground state with our ^{70}Ge basis states is different from that for the other even Se nuclei. But, for ^{76,78,80}Se, the wave functions derived[10] for germanium g.s. and 0^+ fit the ^{76,78,80}Se(d,^{6}Li) 0_2^+/g.s. cross-section ratios, quite well, assuming α overlap ratios independent of A. The value of $u_{\min}$ (using the ^{72,74,76}Ge data) is plotted as a function of R in Fig. 4, and the calculated values of dLi(A) are compared with the experimental results in Fig. 5.

The agreement is excellent for ^{72,74,76}Ge, it fits the trend for ^{78}Ge, but the calculated ratio for ^{70}Ge is too large by more than a factor of 2.

We repeat the analysis of the previous section but for $^{A-4}$Zn(^{6}Li,d)^{A}Ge. Here the excited state used in ^{68}Ge is at 3.47 MeV, as it is the state for which Zn(^{6}Li,d) data exist. We must leave open the possibility that the calculation involving ^{68}Ge is in question. We present it here until data for the ^{64}Zn(^{6}Li,d)^{68}Ge reaction to the 1.754-MeV state become available. For ^{70,72,74}Ge, we used the same wave functions as in the Se(d,^{6}Li) analysis. As in the $^{A+4}$Se(d,^{6}Li)^{A}Ge case, all four data points cannot be fitted. For various combinations of three points, the best fit occurs if ^{70}Zn is removed from the analysis. The $v_{\min}(R)$ that minimized χ^2/N here is plotted in Fig. 4, and the calculated values of Lid(A) along with experimental results are shown in Fig. 6.

Agreement is best for ^{68,70,72}Ge, but the calculated value for ^{74}Ge is almost twice as large as experiment. The fact that the reaction ^{70}Zn(^{6}Li,d)^{74}Ge cannot be fitted along with any other data point suggests that the α overlap amplitudes connecting ^{70}Zn and our ^{74}Ge basis states are different from those for the other even Zn nuclei.

III. CONCLUSIONS

Using Ge two-state wave functions derived from (t,p) and (p,t) data on germanium nuclei and making simple assumptions about the physical Se and Zn ground states, we can satisfactorily fit the 0_2^+/g.s. cross-section data for the reactions $^{A+4}$Se(d,^{6}Li)^{A}Ge ($A=72,74,76$) and the trend in $^{A-4}$Zn(^{6}Li,d)^{A}Ge ($A=68,70,72$). Any attempts to include ^{74}Se(d,^{6}Li)^{70}Ge or ^{70}Zn(^{6}Li,d)^{74}Ge in these fits prove unsatisfactory—implying that the ground states of ^{74}Se and ^{70}Zn (both having $N=40$) are more complicated. We emphasize that these fits are completely independent of R, and so any wave function in Fig. 1 for germanium will do. The next step in the analysis would be to use (p,t) and (t,p) data on the selenium and zinc isotopes to derive two-state wave functions for them and see if they can be made consistent with the Zn(^{6}Li,d) and Se(d,^{6}Li) data, including the reactions ^{74}Se(d,^{6}Li) and ^{70}Zn(^{6}Li,d). The data necessary for such a calculation do not exist at present.

Finally, we note that even though our calculated values for pickup are small for all three of ^{74,76,78}Ge, they reach a minimum at ^{74}Ge. (The amplitude passes through zero near there.) So we believe that a careful α pickup measurement should reveal a larger cross-section ratio in ^{76}Ge and ^{78}Ge than in ^{74}Ge.

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[1]F. Guilbault *et al.*, Phys. Rev. C **16**, 1840 (1977).
[2]G. C. Ball *et al.*, Nucl. Phys. **A231**, 334 (1974).
[3]A. C. Rester, J. B. Ball, and R. L. Auble, Nucl. Phys. **A346**, 371 (1980).
[4]C. Lebrun *et al.*, Phys. Rev. C **19**, 1224 (1979).
[5]S. Mordechai, H. T. Fortune, R. Middleton, and G. Stephans, Phys. Rev. C **19**, 1733 (1979).
[6]S. LaFrance, S. Mordechai, H. T. Fortune, and R. Middleton, Nucl. Phys. **A307**, 52 (1978).
[7]S. Mordechai, H. T. Fortune, R. Middleton, and G. Stephans, Phys. Rev. C **18**, 2498 (1979).
[8]J. F. Mateja *et al.*, Phys. Rev. C **17**, 2047 (1978).
[9]F. Guilbault *et al.*, Phys. Rev. C **15**, 894 (1977).
[10]M. Carchidi, H. T. Fortune, G. S. F. Stephans, and L. C. Bland, Phys. Rev. C **30**, 1293 (1984).
[11]G. Rotbard *et al.*, Phys. Rev. C **18**, 86 (1978).
[12]H. T. Fortune, M. Carchidi, and S. Mordechai, Phys. Lett. **145B**, 4 (1984).
[13]A. M. Van den Berg *et al.*, Nucl. Phys. **A379**, 239 (1982).
[14]D. Ardouin, D. L. Hanson, and Nelson Stein, Phys. Rev. C **22**, 2253 (1980).
[15]Note that in the allowed range of R, r is never zero.