

Evidence of a shape transition in even- A Ge isotopes

S. Sen,* S. E. Darden, R. C. Luhn, and N. O. Gaiser

Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556

G. Murillo and J. Ramirez

Centro Nuclear, Instituto Nacional de Investigaciones Nucleares, Mexico D.F., Mexico

(Received 16 July 1984)

Cross-section and vector-analyzing-power measurements have been carried out for $^{70,72,74,76}\text{Ge}(\vec{d},d)$ at $E_d=8$ and 16 MeV, (\vec{d},d') 2^+ at $E_d=16$ MeV, and (\vec{p},p) and (\vec{p},p') 2^+ at $E_p=11.5$ MeV. Exploratory measurements of $^{70,72,74}\text{Ge}(\vec{d},d)$ at $E_d=6$ MeV have also been performed. The elastic analyzing-power data for $A=70,72$ are nearly identical, as are those for $A=74,76$, and a sudden decrease in the amplitude of A_y occurs in going from $A=72$ to 74 . The inelastic-scattering data also show near identity for the $A=70,72$ target nuclei as well as for $A=74,76$, with characteristic changes occurring between $A=72$ and 74 . Coupled-channels calculations have been performed for all four isotopes using a macroscopic form factor employing both vibrational and rotational models. It is found that the anomalous behavior of the elastic-scattering data at $E_d=16$ MeV and at $E_p=11.5$ MeV is by itself not indicative of ground-state shape, but only shows the effects of target excitation on elastic scattering. The 2^+ inelastic-scattering data, however, can be explained only if a vibrational model is assumed for $^{70,72}\text{Ge}$ and a prolate symmetric rotational model is assumed for $^{74,76}\text{Ge}$. These results are indicative of a shape transition between ^{72}Ge and ^{74}Ge , consistent with the conclusions drawn from transfer-reaction measurements.

I. INTRODUCTION

The structure of nuclei in the mass region $A=68-82$ has been a subject of extensive experimental and theoretical investigations. In particular, the even-even isotopes of Ge are complex systems and show evidence of shape instability. Ardouin *et al.*¹ have reported a systematic investigation of one-nucleon transfer reactions in the region and have suggested the possibility of a shape transition between nuclei with $N=40$ and 42 . In addition, spectroscopic measurements indicate that some of these nuclei exhibit both collective and noncollective features. In particular, the 0^+ excited state in ^{72}Ge at $E_x=0.685$ MeV presumably arises from coupling two quasiparticles with collective excitations.² Anomalies in two-nucleon transfer reactions strongly suggest both the existence of a shape transition from spherical to weakly deformed between $N=40$ (^{72}Ge) and $N=42$ (^{74}Ge) and a coexistence of different types of deformation.³⁻⁵ Recent Coulomb-excitation measurements⁶ have shown, in addition, a possible structural change between $N=38$ (^{70}Ge) and $N=40$ (^{72}Ge). These ideas have been integrated in a recent report by LeComte *et al.*⁷

In a number of investigations, it has been suggested that a coupled-channels (CC) analysis of elastic and inelastic scattering of polarized protons or deuterons is sensitive to the nature of the collectivity of the states involved.⁸⁻¹⁴ In the case of a 0^+-2^+ rotational excitation, for example, there are two matrix elements: one between the 0^+ and 2^+ states, which is related to the $E2$ transition amplitude, and the other involving two 2^+ wave functions, which is essentially the quadrupole moment of the excited state. For scattering to the 2^+ state, the cross section is dom-

inated by the transition strength. The analyzing power, on the other hand, is a relative measurement and is nearly independent of the transition matrix element, but is rather sensitive to the diagonal element,^{15,16} which enters in the interference term of the scattering process.

Szaloky *et al.*¹⁷ have measured angular distributions of cross section and vector analyzing power (VAP) for $^{70,72,74}\text{Ge}(\vec{d},d)$ and (\vec{d},d') at $E_d=16.0$ MeV. Although remarkable differences in the VAP data in going from ^{72}Ge to ^{74}Ge were observed and CC calculations were performed, the above-mentioned ideas concerning the nuclear-shape sensitivity of the analyzing powers were not fully exploited. In fact, the emphasis in the analysis was on spectroscopy of the inelastic states and no firm conclusions concerning the ground-state shapes of these nuclei could be made. Analyzing powers for the (p,p') reaction for these isotopes have not been published.

VAP data, particularly for deuteron elastic scattering, are interesting in another respect. Recent measurements^{19,20} of $^{76,78,80,82}\text{Se}(\vec{d},d)$ at $E_d=12$ MeV show a linear increase of the amplitude of the vector analyzing power $iT_{11}(\theta)$ with mass number, changing by nearly a factor of 2 as one goes from ^{76}Se to ^{82}Se . A similar situation in the $A=90$ mass region has been reported by Bieszk and Ulbricht.²¹ There are other interesting data such as those on even-even Ni isotopes for $E_d=6-12$ MeV,²² where one observes that as the bombarding energy is decreased, the VAP changes from a symmetric oscillation about zero to a predominantly "one-signed" value in an isotopically systematic way. The one-signed behavior at low deuteron bombarding energies (close to the Coulomb barrier) has also been observed for many other medium mass nuclei.²² These drastic variations in the

behavior of the VAP are unexpected from a simple optical-model description of the elastic scattering.

Our purpose is to investigate the collective properties of the even-even isotopes of Ge(70,72,74,76) as manifested in the elastic and inelastic scattering to the first 2^+ state of vector polarized deuterons and protons from these nuclei. We seek a comprehensive understanding of the behavior of the VAP data simultaneously with the differential cross-section angular distributions in the framework of a CC analysis of the data using a macroscopic form factor.²³

Part of the data for $^{70,72,74}\text{Ge}(\vec{d},d')$ at $E_d=16$ MeV has been taken from Ref. 17. We have remeasured all the cross-section data and some of the iT_{11} data at this energy. In addition, measurements have been carried out for ^{76}Ge . Elastic-scattering data for the four isotopes at $E_d=8$ MeV have been measured. Exploratory measurements of $^{70,72,74}\text{Ge}(\vec{d},d_0)$ at $E_d=6$ MeV have been carried out. Analyzing-power and cross-section data for elastic and inelastic scattering of protons at $E_p=11.5$ MeV for all the four isotopes have also been acquired. The measurements, analysis of the data, and the results, are discussed in the ensuing sections.

II. EXPERIMENTAL PROCEDURE

All of the data in this experiment were obtained at the Notre Dame tandem accelerator laboratory using beams mostly from the polarized ion source. The targets consisted of self-supporting Ge foils between 300 and 350 $\mu\text{g}/\text{cm}^2$ thick and were prepared by Szaloky *et al.*¹⁷ The isotopic enrichments of $^{70,72,74,76}\text{Ge}$ were 84.6%, 96.2%, 94.5%, and 73.9%, respectively. In the case of deuteron-induced reactions, the reaction products were detected in a 43-cm-diam scattering chamber using four ΔE - E solid-state detector telescopes in conjunction with a two-parameter data-handling program. VAP data were obtained for 24 laboratory angles between 30° and 145° in 5° steps. Detector telescopes were located at the same angle on both left and right sides of the incident beam and measurements were performed with the beam polarized alternately in directions parallel and antiparallel to the normal to the scattering plane. The beam polarization was determined by a vector polarimeter²⁴ located downstream from the scattering chamber. The polarimeter also served as a Faraday cup for current integration. The magnitude of beam polarization it_{11} was typically -0.47 ± 0.01 ($p_y=0.54$). Polarized beam intensity on the target was approximately 12 nA. Narrow entrance slits (1.5 mm) to the scattering chamber and the positioning of detectors on both left and right sides were used in order to avoid the problem of false asymmetries that might result if the beam spot on the target moved systematically as one goes from the up mode to the down mode, or if the target were nonuniform. This is particularly important at $E_d=8$ and 6 MeV where the measured elastic analyzing powers are relatively small [$iT_{11}(\theta) \leq 0.05$].

Measurements with polarized protons were carried out at a proton energy $E_p=11.5$ MeV. No telescopes were used in this case. The scattered protons were detected by 15 solid-state detectors fixed in 10° intervals from 25° to

165° . Pulses from the detectors were routed into the PDP-9 computer through two analog-to-digital converters (ADC's) and the spectra were written on magnetic tape. Data were obtained for 29 laboratory angles between 25° and 165° in 5° steps. The beam polarization was measured using a downstream polarimeter employing $^4\text{He}(\vec{p},p)^4\text{He}$ scattering. The magnitude of the beam polarization was typically $p_y=0.60$. Polarized proton beam intensity on the target was approximately 7 nA.

For deuteron elastic scattering, the relative cross sections were measured using an unpolarized beam and one detector system of fixed geometry over the entire angular range. The yield/charge for a few angles at $E_d=3.2$ MeV were also measured. Absolute cross sections were obtained assuming that the scattering at $E_d=3.2$ MeV is all Rutherford scattering. An optical-model calculation indicates that this is true within 1% even at back angles. Proton elastic-scattering data at $E_p=11.5$ MeV were normalized by comparing the yield/charge with those measured at $E_p=3.0$ MeV.

A separate normalization was determined for each isotope studied. Both the VAP and cross-section data were corrected for the isotopic abundances.

Inelastic scattering cross-section data for (d,d') at $E_d=16$ MeV and for (p,p') at $E_p=11.5$ MeV were obtained using the normalization described above (isotopic abundance corrected). These cross sections were also calculated from VAP data,²⁵ assuming the incident beam to be purely vector polarized (which applies only to deuterons) and the beam polarization to have the same value in the two modes of polarization. Errors introduced by these assumptions lie within the uncertainty of the measurements and were considered unimportant. If the cross-section values obtained by the two methods were inconsistent, they were excluded from consideration. In some cases, the peak of interest in a spectrum is partially masked by a contaminant peak. In such situations, it has sometimes been possible to calculate the VAP using a part of the peak, but the corresponding cross-section data could not be extracted.

In order to determine the extent of agreement between our measurements and those of other workers, we measured elastic cross sections for $^{70,72}\text{Ge}(p,p)$ at $E_p=14.5$ MeV for five laboratory angles between 60° and 105° . Curtis *et al.*²⁶ have reported similar measurements, and assign an uncertainty of 10%. These authors normalized their data for 20° , 30° , 40° , and 50° with cross sections calculated for $^{70}\text{Ge}(p,p)$ using an average optical model potential.²⁷ Our measurements yield values approximately 16% lower than those of Ref. 26. The estimated overall uncertainty in the measured cross sections reported in this paper is of the order of $\pm 10\%$.

The elastic and inelastic cross-section and VAP data are displayed in Figs. 1–6. Errors shown are statistical and, where error bars are not used, the size of the data point indicates the approximate statistical error.

III. DATA ANALYSIS

We first discuss the characteristic features of the deuteron elastic-scattering data. The measurements at $E_d=16$

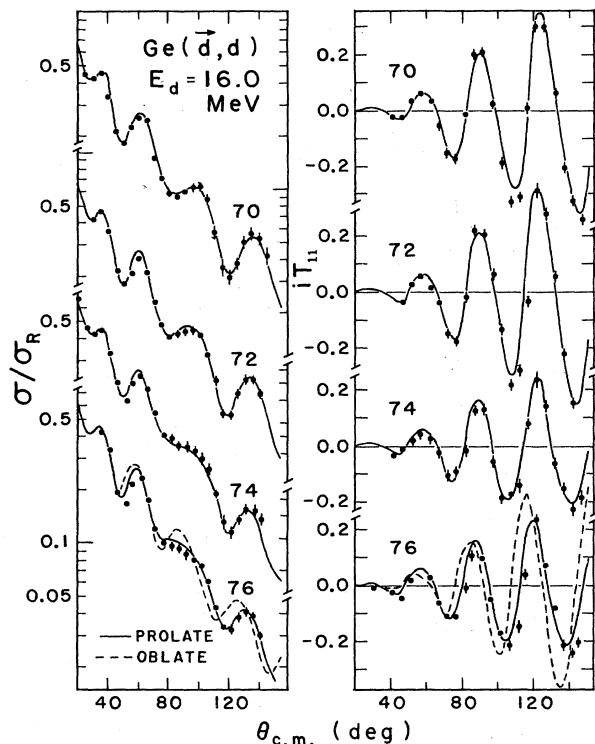


FIG. 1. Data for $\text{Ge}(\vec{d},d)$ at $E_d=16.0$ MeV. Solid curves represent vibrational model CC calculations for $^{70,72}\text{Ge}$ and prolate rotational model CC calculations for $^{74,76}\text{Ge}$. The dashed curves for ^{76}Ge correspond to an oblate rotational model calculation. The coupling scheme considered is 0^+-2^+ .

MeV are shown in Fig. 1. One observes that the cross-section data for all four isotopes are quite similar, although the maximum at 100° is somewhat suppressed for $^{74,76}\text{Ge}$. While the iT_{11} data for ^{70}Ge and ^{72}Ge are almost identical, the amplitudes of oscillation at larger angles are drastically smaller for ^{74}Ge and ^{76}Ge , the data for these two isotopes again being almost identical. In fact, the amplitudes at $\theta_{\text{lab}}=90^\circ, 105^\circ,$ and 120° are reduced by approximately 40%.

The elastic-scattering data at $E_d=8$ MeV are shown in Fig. 2. Although the absolute magnitudes of the analyzing powers are rather small at the lower bombarding energy, the basic behavior that the iT_{11} for $^{70,72}\text{Ge}$ is drastically different in amplitude of oscillations from the $^{74,76}\text{Ge}$ data, persists. Our measurements of $^{70,72,74}\text{Ge}(\vec{d},d_0)$ at $E_d=6$ MeV (Fig. 3) and unpublished data of Yoh²⁸ at $E_d=12$ MeV show a similar behavior.

It is interesting to note that the characteristic behavior of deuteron elastic-scattering from the even-even Ge isotopes in mass and energy dependence is quite different from the anomalous behavior of (d,d_0) VAP data given in Refs. 18–21. There is a sudden change in going from ^{72}Ge to ^{74}Ge , in contrast to isotopically systematic changes observed in the cases discussed in Refs. 18–21.

It is obvious that a conventional optical-model analysis

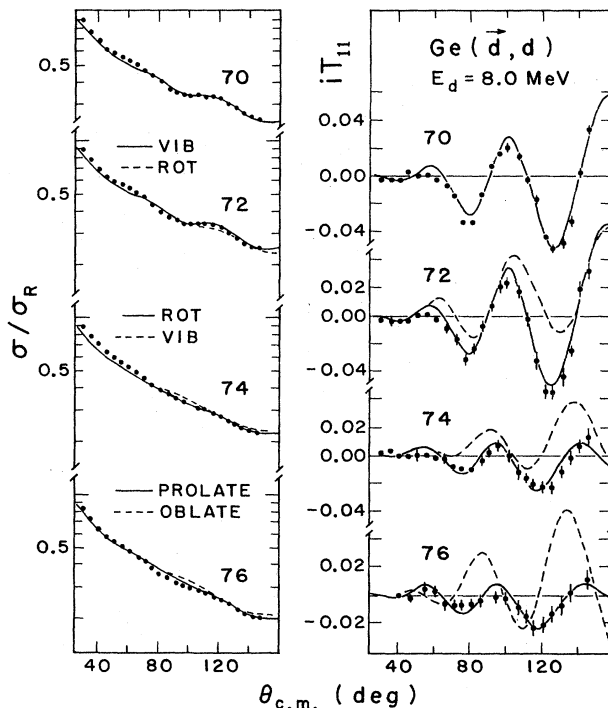


FIG. 2. Data for $\text{Ge}(\vec{d},d)$ at $E_d=8.0$ MeV. Solid curves represent vibrational model CC calculations for $^{70,72}\text{Ge}$ and prolate rotational model CC calculations for $^{74,76}\text{Ge}$. The dashed curves for ^{72}Ge and ^{74}Ge represent CC calculations assuming a prolate rotational model for ^{72}Ge and a vibrational model for ^{74}Ge . The dashed curves for ^{76}Ge correspond to an oblate rotational model calculation. The coupling scheme considered is 0^+-2^+ .

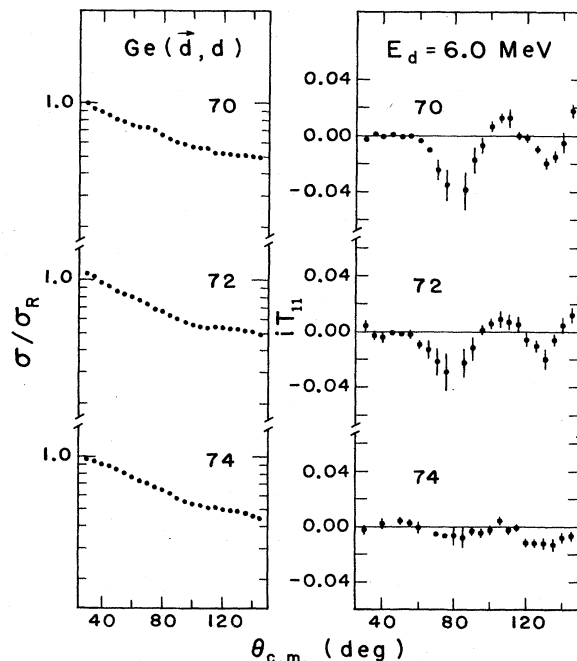


FIG. 3. Data for $^{70,72,74}\text{Ge}(\vec{d},d)$ at $E_d=6.0$ MeV.

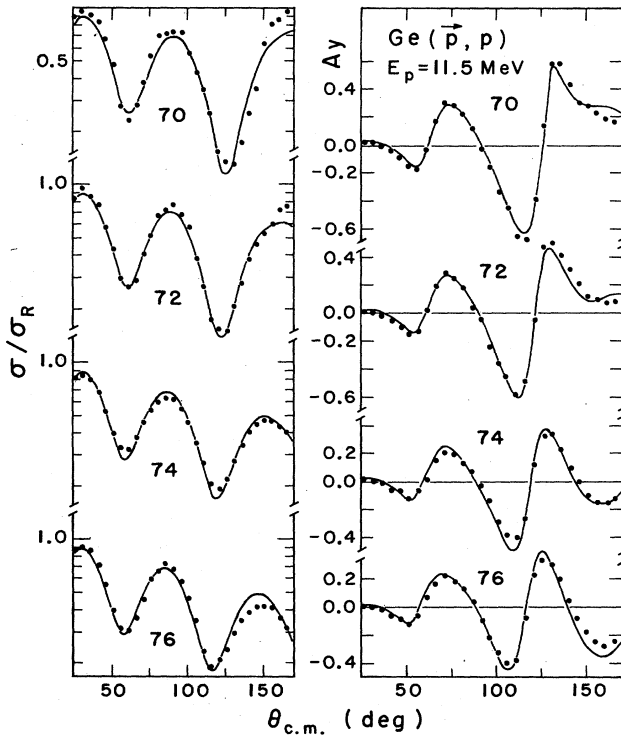


FIG. 4. Data for $\text{Ge}(\bar{p}, p)$ at $E_p = 11.5$ MeV. The curves are the results of CC calculations assuming a vibrational model for $^{70,72}\text{Ge}$ and a prolate rotational model for $^{74,76}\text{Ge}$. The coupling scheme considered is $0^+ - 2^+$.

will not reproduce the characteristics of the data. In other words, starting with a global set of deuteron optical-model potential parameters,^{29,30} if we search and fit the $^{72,74}\text{Ge}(d, d_0)$ data at any one energy, one obtains two very different sets of parameters, which cannot be related by any realistically acceptable isotopic dependence.³⁰

In order to investigate whether the behavior of the elastic-scattering data is a reflection of nuclear shape (structure) effects, we have performed proton elastic and inelastic-scattering measurements from the Ge isotopes at $E_p = 11.5$ MeV. The elastic scattering cross-section and VAP (A_y) data are shown in Fig. 4. Once again we observe that while the A_y data for ^{70}Ge and ^{72}Ge are almost identical, there is a decrease in amplitude of A_y for ^{74}Ge and ^{76}Ge . In addition, both the cross-section and the VAP data show a qualitative change in shape at extreme back angles between $^{70,72}\text{Ge}$ and $^{74,76}\text{Ge}$. Optical-model analyses using the average potentials of Becchetti and Greenlees³¹ show a systematic variation of the analyzing power because of a large symmetry dependence of the well depths, but the near-identity of the 70,72 data and 74,76 data and the sudden change between ^{72}Ge and ^{74}Ge cannot be explained.

In an attempt to understand the anomalous behavior of the data, we recall that the optical model was introduced to reduce an infinite-channel problem to a one-channel problem (the usual optical model for elastic scattering) or

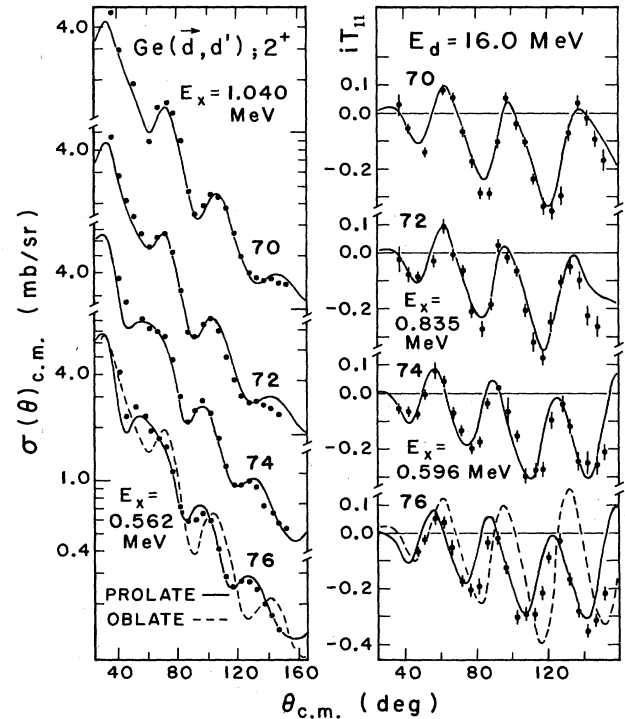


FIG. 5. Data and CC fits (solid curves) to the first 2^+ excited states in the reactions $\text{Ge}(\bar{d}, d')$, 2^+ at $E_d = 16.0$ MeV using a $0^+ - 2^+$ coupling scheme and assuming a vibrational model for $^{70,72}\text{Ge}$ and a prolate rotational model for $^{74,76}\text{Ge}$. The dashed curves for ^{76}Ge correspond to an oblate rotational model calculation.

a few-channel problem (coupling between a few low-lying states).^{32,33} The Hamiltonian for the entire system (projectile a + target A) including relative motion and projectile-target interaction is

$$H = H_a + H_A + T + V.$$

A straight-forward derivation of the coupled equations leads to an infinite set. Following Feshbach,³² if one introduces the projection operators P , which refer to the elastic and a few inelastic channels of interest, and Q , which refer to all other channels, one can formally replace the infinite-channel problem governed by H by a modified problem governed by

$$\mathcal{H} = H_a + H_A + T + \mathcal{V}, \quad (1)$$

where

$$\mathcal{V} = V + V \frac{Q}{E - H + i\eta} V. \quad (2)$$

This modified problem is to be solved only within the finite subspace of the P channels. The matrix elements of the effective interaction \mathcal{V} that are diagonal in the intrinsic structure of a and A are conveniently parameterized by the optical potential,

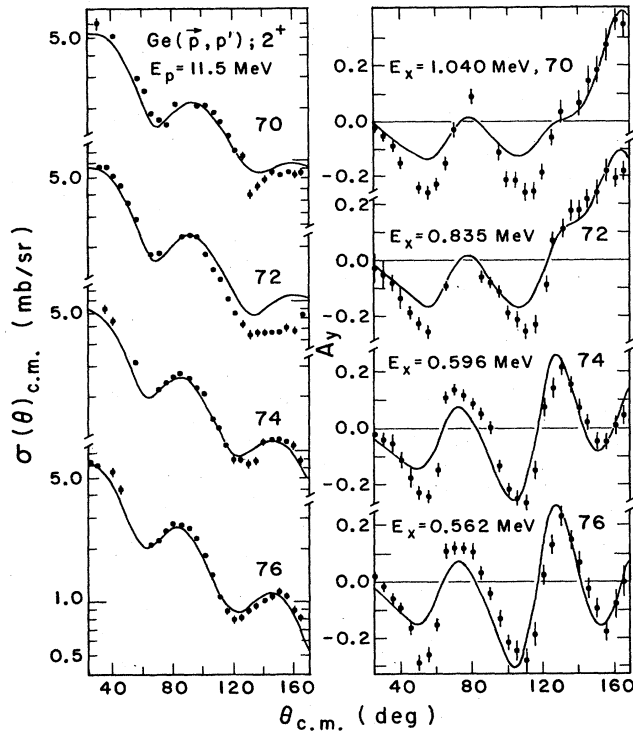


FIG. 6. Data and CC fits (solid curves) to the first 2^+ excited states in the reactions $\text{Ge}(\bar{p}, p') 2^+$ at $E_p = 11.5$ MeV using a $0^+ - 2^+$ coupling scheme and assuming a vibrational model for $^{70,72}\text{Ge}$ and a prolate rotational model for $^{74,76}\text{Ge}$.

$$U_{\text{opt}} = \langle 0 | \mathcal{V} | 0 \rangle. \quad (3)$$

Using (2), one can write the optical potential

$$U_{\text{opt}} = \langle 0 | V | 0 \rangle + \sum_{Q \neq 0} \langle 0 | V | Q \rangle \frac{1}{E - E_Q + i\eta} \langle Q | V | 0 \rangle, \quad (4)$$

where Q is summed over all channels except the elastic channel. For spherical nuclei these include all intrinsic excitations. For deformed nuclei, they include rotations as well as intrinsic excitations.

We introduce the phenomenological model via the parametrization of the nuclear surface,

$$R = R_0 \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right] = R_0 + \delta R.$$

For rotational nuclei, the $\alpha_{\lambda\mu}$ (or α_λ for the axially symmetric case) are c numbers. For even-even deformed nuclei, the lowest lying states are the members of the ground-state rotational band and thus correspond to the same intrinsic state of motion. The excitation of rotational states of the g.s. band of a deformed nucleus is exceptional in the sense that only the diagonal intrinsic matrix elements of the effective interaction are involved.

For vibrational nuclei, the $\alpha_{\lambda\mu}$ are dynamical operators (combinations of phonon creation and annihilation opera-

tors) and we need both $\langle c | \mathcal{V} | c \rangle$ and $\langle c | \mathcal{V} | c' \rangle$ ($c \neq c'$), i.e., matrix elements which are diagonal and non-diagonal, respectively, with respect to the intrinsic states.

In expression (4), we split the \sum_Q into two parts: a sum \sum_R referring to the rotational levels of the ground state band, which are present if the nucleus is deformed, and a sum over all other channels,

$$U_{\text{opt}} = \langle 0 | V | 0 \rangle + \sum_{Q \neq 0, R} \langle 0 | V | Q \rangle \frac{1}{E - E_Q + i\eta} \langle Q | V | 0 \rangle + \sum_R \langle 0 | V | R \rangle \frac{1}{E - E_R + i\eta} \langle R | V | 0 \rangle = U_s + \sum_R \langle 0 | V | R \rangle \frac{1}{E - E_R + i\eta} \langle R | V | 0 \rangle, \quad (5)$$

where

$$U_s = \langle 0 | V | 0 \rangle + \sum_{Q \neq 0, R} \langle 0 | V | Q \rangle \frac{1}{E - E_Q + i\eta} \langle Q | V | 0 \rangle. \quad (6)$$

If the coupling to the states of the ground-state rotational band is explicitly included, as in a coupled-channels calculation, then the rotational terms R in (5) ought not to appear in the corresponding potential U_s used in the coupled equation. This U_s should apply to both spherical and deformed nuclei since it takes into account only intrinsic excitation in both cases. However, there may exist vibrational nuclei whose intrinsic excitations are such that some vibrational states belonging to Q are strongly coupled to the ground state. In such cases, these collective vibrational levels are included explicitly in CC calculations and their effect thereby eliminated from \mathcal{V} . The resulting optical potential should be essentially the same as U_s , since the \sum_Q is dominated by the region of the spectrum where the level density is high, i.e., the high-excitation energy region. To the extent that the level density at high excitation does not depend on the deformed nature of the ground state, U_s should be a smooth function of mass number, and should therefore vary only slightly across the Ge isotopes.

It is not easy to explicitly calculate U_{opt} and U_s , but we can parametrize them. In the standard procedure, U_{opt} is the phenomenological optical-model potential that reproduces the elastic-scattering data, while the parameters characterizing U_s are to be chosen such that when the coupling to the collective states is explicitly included in the calculations via the \sum_R terms, the predicted elastic observables are essentially identical to those predicted by U_{opt} .

The CC calculations were performed using the code ECIS74,³⁴ assuming both the central and spin-orbit parts of the potential to be deformed. Coulomb deformation was also included. The methods adopted for numerical solution of the coupled differential equations are discussed in Ref. 35. In these calculations we include coupling to the first 2^+ excited state and assume a vibrational model for $^{70,72}\text{Ge}$ and a rotational model in the case of $^{74,76}\text{Ge}$. The effects of including other states in the coupling scheme will be discussed in the next section. Grid searches on the

deformation parameters and the optical parameters were performed, with the requirement that we should be able to fit the 0^+ g.s. and 2^+ state data for all the isotopes using basically one set of potential parameters.

The code includes the full-Thomas deformation of the spin-orbit potential which could be complex, and has provisions for using imaginary and/or spin-orbit deformation parameters different from those of the real central potential. Although the deformation parameters quoted in the next section refer to the deformation of the real central potential, in the actual analysis the deformation parameters of the individual parts of the optical potential are correlated to each other by the constraint of a constant deformation length.

IV. RESULTS AND DISCUSSION

A. $E_d = 16.0$ MeV data

Results of the analysis for (d,d) and (d,d') at $E_d = 16$ MeV for the g.s. and the first 2^+ excited state considering a $0^+ - 2^+$ coupling scheme are shown in Figs. 1 and 5, respectively. The choice of this particular coupling scheme and the effect of including additional inelastic excitations in the calculations will be discussed later in this subsection. We have assumed a vibrational model for $^{70,72}\text{Ge}$ and a symmetric prolate rotational model for $^{74,76}\text{Ge}$. Parameters characterizing the U_s part of the optical model potential as discussed in the preceding section and the deformation parameters are listed in Table I. Both the elastic and inelastic cross-section and VAP data are well reproduced by the calculations.

It can be seen that the U_s parameters (Table I) for the four isotopes, although very similar, are not quite identical. We recall that one expects only U_s to be a smooth function of A , and there will be some residual mass dependence. There exists no prescription for these dependences when CC calculations are performed. However, it is reasonable to assume that the A dependence of the parameters of the conventional optical-model potential U_{opt} , as found in global searches,^{29,30} provide an upper limit on the corresponding variations in U_s . The differences in the U_s parameter values of Table I are small and well within these limits.

It is to be noted that a rather small depth imaginary spin-orbit component has been used in the calculations. There is no *a priori* reason why the absorption should not be spin dependent, and such potential components have been found to improve the phase relationships between the oscillations in the deuteron elastic-scattering data and phenomenological optical-model predictions.³⁶ The effects are, however, small and the imaginary spin-orbit depth for medium mass nuclei at $E_d \approx 15$ MeV has been found to be between $\frac{1}{3}$ and $\frac{1}{2}$ the real spin-orbit depth.^{36,37} These results are based on analyses of elastic-scattering data only. In the present analysis it was found that while the elastic predictions were only marginally affected, the inelastic scattering VAP fits were very sensitive to the parameters characterizing the imaginary spin-orbit potential. A small depth and a geometry identical to that of the real spin-orbit part best reproduce the data.

As stated earlier, we have used a vibrational model for $^{70,72}\text{Ge}$ and a rotational (prolate) model for $^{74,76}\text{Ge}$. However, an interesting feature that emerges from the analysis is that if one interchanges the models, i.e., if one uses a rotational model for $^{70,72}\text{Ge}$ and a vibrational model for $^{74,76}\text{Ge}$, retaining the U_s and β parameters listed in Table I, the elastic scattering data are equally well reproduced. The fits are indistinguishable from those shown in Fig. 1. On the other hand, neither the cross-section nor the VAP data for the 2^+ inelastic scattering can be reproduced when the models are interchanged. Two examples, one for $^{70}\text{Ge}(d,d')$; 2^+ (solid curve: vibrational and dashed curve: rotational) and the other for $^{76}\text{Ge}(d,d')$; 2^+ (solid curve: rotational and dashed curve: vibrational) are shown in the upper half of Fig. 7. The predictions are for the most part out of phase with the data. The characteristic changes in the elastic-scattering data between ^{72}Ge and ^{74}Ge are, therefore, not indications of a shape transition, but simply show the effect of target excitation on the elastic scattering as described by the second term in Eq. (4) of Sec. III. The reproduction of the first 2^+ -state data, however, is sensitive to the model assumed and shows evidence of a shape transition from spherical to deformed in the region ^{72}Ge to ^{74}Ge .

Calculations were also performed assuming that the deformation is oblate instead of being prolate. Neither the elastic- nor the inelastic-scattering data are reproduced by these calculations. An example is shown for ^{76}Ge in Figs. 1 and 5 (dashed curves).

At this point, some discussion of the characteristics of the data for inelastic scattering to other states in the Ge nuclei is appropriate. Considering the published data (Ref. 17) on the first excited 4^+ states in $^{70,72,74}\text{Ge}$ ($E_x = 2.160, 1.727, \text{ and } 1.473$ MeV, respectively) one observes that the data for the three isotopes are almost identical. The iT_{11} are negative at all angles of measurement and neither the magnitudes nor the phases of the oscillations show any characteristic or systematic isotopic dependence. Considering the data for the second-excited 2^+ states in $^{70,72,74}\text{Ge}$ ($E_x = 1.710, 1.466, \text{ and } 1.210$ MeV, respectively), although a small gradual increase in the amplitudes of the oscillations occurs for both $\sigma(\theta)$ and iT_{11} with increasing A , there is no abrupt change in the data between $A=72$ and 74 , and in any case, such changes as exist are within the uncertainties in the data. The scatter in the data points and large error bars prevent any meaningful comparison of the data for the first excited 0^+ states among the different nuclei. Data for the 3^- states ($E_x = 2.570, 2.522, 2.546$ MeV for $^{70,72,74}\text{Ge}$, respectively) are also nearly identical in magnitude and phase for the three isotopes. A small decrease in the amplitude of oscillations of iT_{11} at back angles in ^{74}Ge is within the error limits of the data points. Data for these additional excited states in ^{76}Ge are not available. We thus conclude that, as far as the cross section and vector analyzing powers for the states considered are concerned, the sudden changes between ^{72}Ge and ^{74}Ge are exhibited only by the data for the ground and first-excited states. It is also necessary to determine the effect of including explicitly more channels in the CC calculations. The use of a $0^+ - 2^+ - 3^-$ vibrational model coupling scheme for ^{70}Ge or ^{72}Ge and a $0^+ - 2^+ -$

TABLE I. The U_s part of the optical potential as discussed in Sec. III of the text and used in the CC calculations. Standard notations are used. The deformation parameters for the real part of the central potential are listed.

Projectile	Incident energy (MeV)	Target <i>A</i>	<i>V</i> (MeV)	<i>r</i> ₀ (fm)	<i>a</i> ₀ (fm)	<i>W</i> _{<i>D</i>} (MeV)	<i>r</i> _{<i>i</i>} (fm)	<i>a</i> _{<i>i</i>} (fm)	<i>V</i> _{<i>LS</i>} ^{<i>R</i>} (MeV)	<i>r</i> _{<i>so</i>} ^{<i>R</i>} (fm)	<i>a</i> _{<i>so</i>} ^{<i>R</i>} (fm)	<i>V</i> _{<i>LS</i>} ^{<i>I</i>} (MeV)	<i>r</i> _{<i>so</i>} ^{<i>I</i>} (fm)	<i>a</i> _{<i>so</i>} ^{<i>I</i>} (fm)	<i>r</i> _{<i>c</i>} (fm)	<i>E</i> _{<i>x</i>} (2 ⁺) (MeV)	β_2 (real central)
\bar{d}	16.0	70	97.5	1.151	0.750	11.85	1.330	0.800	5.0	0.72	0.40	0.300	0.72	0.40	1.30	1.040	0.17± 0.015
		72	97.5	1.151	0.755	11.65	1.330	0.790	5.0	0.72	0.40	0.300	0.72	0.40	1.30	0.835	0.17± 0.015
		74	97.5	1.151	0.750	11.80	1.330	0.800	5.0	0.72	0.40	0.300	0.72	0.40	1.30	0.596	0.197± 0.010
		76	97.5	1.151	0.760	11.80	1.330	0.800	5.0	0.72	0.40	0.200	0.72	0.40	1.30	0.562	0.197± 0.010
\bar{p}	8.0	70	94.0	1.154	0.780	8.3	1.65	0.84	6.2	0.55	0.40	2.9	0.55	0.40	1.30	1.040	0.17± 0.015
		72	93.0	1.154	0.795	8.2	1.65	0.82	6.2	0.55	0.40	2.9	0.55	0.40	1.30	0.835	0.17± 0.015
		74	98.0	1.154	0.800	8.6	1.65	0.85	6.2	0.55	0.40	2.9	0.55	0.40	1.30	0.596	0.197± 0.010
		76	96.0	1.154	0.820	8.7	1.65	0.86	6.2	0.55	0.40	2.9	0.55	0.40	1.30	0.562	0.197± 0.010
\bar{p}	11.5	70	57.5	1.156	0.795	8.7	1.334	0.575	6.0	0.92	0.74	0.0	0.92	0.74	1.25	1.040	0.202± 0.010
		72	57.6	1.156	0.795	8.85	1.334	0.580	6.0	0.92	0.74	0.0	0.92	0.74	1.25	0.835	0.203± 0.010
		74	57.75	1.156	0.785	9.10	1.334	0.590	6.0	0.92	0.74	-0.30	0.92	0.74	1.25	0.596	0.208± 0.010
		76	57.8	1.156	0.785	9.20	1.334	0.595	6.0	0.92	0.74	-0.30	0.92	0.74	1.25	0.562	0.211± 0.010

4^+ rotational model coupling scheme for ^{74}Ge or ^{76}Ge does not affect the results of the CC calculations in any fundamental way. When these more extended coupling schemes are used, an increase in β_2 between 5% and 10% provides a quality of reproduction of the g.s. and 2^+ -state data equal to that obtained using the more restricted coupling.

The fits to the 3^- state data (^{70}Ge or ^{72}Ge) are of the same quality as given in Ref. 17 (cross-section data are well-reproduced but the VAP are not well fit). The quality of the fit to the 4^+ data (^{74}Ge) using $0^+-2^+-4^+$ rotational model coupling was also quite satisfactory.

Nurzynski *et al.*³⁸ have investigated the influence of various second-order processes on the elastic scattering of 12 MeV deuterons from $^{76,78,80,82}\text{Se}$. They found that two-step processes involving transfer channels contribute little to the elastic scattering and almost the entire effect arises from (d,d') 2^+ (d',d). We have assumed this to be true in the case of the Ge isotopes. Some calculations involving transfer channels were performed for a lower bombarding energy and will be discussed in the following subsection.

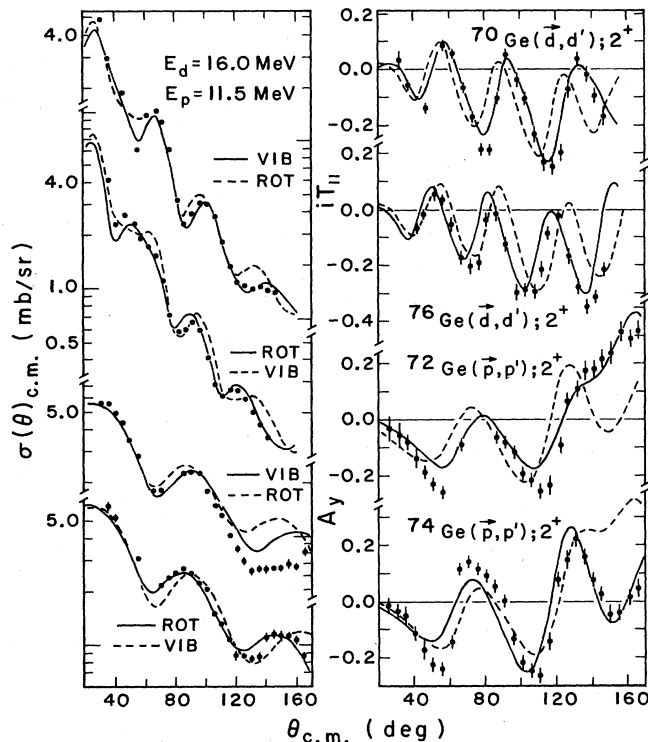


FIG. 7. Data and CC calculations for the first 2^+ excited state. $^{70}\text{Ge}(\vec{d},d')$ 2^+ , $E_d=16$ MeV; solid curves correspond to vibrational-model and dashed curves represent prolate rotational-model calculations. $^{76}\text{Ge}(\vec{d},d')$ 2^+ , $E_d=16$ MeV: solid curves represent prolate rotational and dashed curves correspond to vibrational-model calculations. $^{72}\text{Ge}(\vec{p},p')$ 2^+ , $E_p=11.5$ MeV: solid curves correspond to vibrational-model and dashed curves represent prolate rotational-model calculations. $^{74}\text{Ge}(\vec{p},p')$ 2^+ $E_p=11.5$ MeV: solid curves represent prolate rotational model and dashed curves correspond to vibrational-model calculations.

On the basis of the preceding discussions it is reasonable to conclude that as far as the observed anomalies in the elastic and the 2^+ inelastic scattering data are concerned, it is appropriate to consider the elastic channel and the first 2^+ inelastic channel to be sufficient to constitute the P subspace (Sec. III) of the coupled-channel calculations. Consequently a 0^+-2^+ coupling scheme has been adopted throughout.

B. $E_d=8.0$ and 6.0 MeV data

At $E_d=8.0$ MeV, because of the weakness of the inelastic scattering, only elastic-scattering data could be extracted with reasonable statistical accuracy. The analyzing powers are an order of magnitude smaller than those at $E_d=16.0$ MeV.

The solid curves in Fig. 2 show the results of CC calculations assuming a 0^+-2^+ coupling scheme. We assume the same g.s. shapes as in Sec. IV A and the U_s part of the potential parameters are listed in Table I. We note that even the cross-section data show some distinguishability in going from ^{72}Ge to ^{74}Ge for $\theta_{c.m.}=100^\circ-120^\circ$ and all the data are well reproduced by the calculations.

The U_s parameters (Table I) for the four isotopes are quite similar, but not identical. In particular, the real well depths show considerable fluctuations, especially for ^{74}Ge . Many attempts were made to reduce this fluctuation in a consistent way, but it always resulted in a deterioration of the fits. It is to be noted that most of the global sets of conventional optical model parameters^{29,39} are prescribed for higher energies, and little in the way of systematic studies exist for energies near the Coulomb barrier.

Other characteristics of the U_s parameters are the relatively large values of r_i and a_i compared to those at 16 MeV. Even a conventional optical model analysis of the $^{70}\text{Ge}(d,d)$, $E_d=8$ MeV data (i.e., U_{opt}) requires $r_i \approx 1.70$ fm and $a_i \approx 0.88$ fm, although such a large r_i value is not ruled out in a global prescription by Hodgson.³⁹ In Sec. I, we have mentioned the work of Wong and Quin²² on deuteron scattering from even-even Ni isotopes and from other nuclei in the f - p shell at energies near the Coulomb barrier. Analyses of these data⁴⁰ using an optical model (U_{opt}) show that the data and, in particular, the one-signed behavior can be reproduced using relatively large r_i and a_i values ($r_i \approx 1.7$ fm, $a_i \approx 0.9$ fm).

In the study of heavy-ion elastic scattering near or below the Coulomb barrier, the effect of strongly-coupled inelastic channels upon elastic scattering has been studied in detail.^{41,42} Analyses in terms of coupling to the Coulomb-excited 2^+ state provide a satisfactory explanation of the data. An alternative description would be to use additional terms in the optical potential which arise from two-step contributions to elastic scattering. A long-range imaginary potential approximating the effect of Coulomb excitation has been derived.^{41,42}

One expects that when the appropriate couplings are included in the analysis, the parameters characterizing U_s will become smooth and will probably be of conventionally accepted magnitude. At $E_d \leq 8$ MeV, inelastic scattering to the 2^+ state is weak and a large fraction of the total

reaction cross section may be attributed to transfer reactions. In a recent paper, Tostevin and Johnson⁴³ show that coupling to weakly-bound neutron transfer channels can produce large changes in the elastic VAP at sub-Coulomb energies. For the ⁷⁰⁻⁷⁶Ge nuclei, no information on weakly-bound-neutron states exists in the literature. In the present experiment, no such states with large cross section were observed in the proton spectrum. In the absence of any information, model calculations were done using the coupling scheme

$$(d,d) + (d,d') 2^+(d',d) \\ + \text{Coulomb excitation} + (d,p) \frac{5}{2}^+(p,d).$$

A neutron binding energy of 0.200 MeV and values of the spectroscopic factors as large as 0.25 were assumed. Calculations were performed using the code CHUCK2,⁴⁴ using a vibrational model for ⁷⁰Ge and a rotational model for ⁷⁴Ge. The inclusion of the transfer channel in the coupling scheme had a small effect on the predictions and does not affect our conclusions. Model calculations using a 0⁺-2⁺-3⁻ vibrational coupling scheme for ⁷⁰Ge and a 0⁺-2⁺-4⁺ rotational scheme for ⁷⁴Ge showed no significant differences from those employing the 0⁺-2⁺ coupling scheme.

A complex spin-orbit potential was necessary to reproduce the VAP data, particularly the dips around 80° and 120° c.m. The imaginary depths are approximately one-half of the real depths. This is in agreement with the results of Refs. 36 and 37 but, in the absence of inelastic scattering data, the significance of this agreement is not clear.

An interesting feature of the analysis is that, unlike the $E_d = 16$ MeV case, the elastic scattering appears to be sensitive to the model used. Calculations were performed by exchanging the models, i.e., assuming a rotational model for ^{70,72}Ge and a vibrational model for ^{74,76}Ge, without altering the U_s and β parameters. Two examples (for ⁷²Ge and ⁷⁴Ge) are shown by the dashed curves in Fig. 2. Calculations were also carried out using, for example, the ⁷⁴Ge potential and a rotational model for ⁷⁰Ge, and the ⁷⁰Ge potential and a vibrational model for ⁷⁴Ge. In the ^{74,76}Ge cases, calculations were also performed assuming an oblate deformation. This is shown for the case of ⁷⁶Ge in Fig. 2. It is seen that only the solid curves, where ^{70,72}Ge are assumed vibrational and ^{74,76}Ge are assumed rotational, fit the data.

The sensitivity of the fits to the model used, which is observed at this energy, is not completely understood. The calculations are also quite sensitive to the potential parameters. As noted above, the parameters show some fluctuations which could not be avoided. There may also be some cancellation between the amplitudes corresponding to coupling to reaction channels⁴³ not included in the calculations. These effects may be small, but the observed analyzing powers are also small in magnitude, and the cumulative effect may be substantial. In the absence of a more detailed investigation, the conclusion that at 8 MeV the elastic scattering analyzing powers show a nuclear shape dependence is to be considered tentative.

Measurements at $E_d = 6.0$ MeV shown in Fig. 3 are of

an exploratory nature and are incomplete. The main purpose in making these measurements was to obtain a qualitative idea of the energy dependence of the VAP. The characteristics of the data are similar to those at 8 MeV. Calculations employing the same potential parameters used in the analysis of the 8-MeV data are in qualitative agreement with the data of Fig. 3. No attempt has been made to further analyze these data.

C. $E_p = 11.5$ MeV data

Figs. 4 and 6, respectively, show the results of CC analyses of the (p,p) and (p,p') data at $E_p = 11.5$ MeV. The solid curves correspond to vibrational-model calculations for ^{70,72}Ge and to rotational-model calculations for ^{74,76}Ge, using the 0⁺-2⁺ coupling scheme. Table I contains the U_s and β parameters used. The elastic-scattering data are quite well reproduced by the calculations and the inelastic-scattering data are fairly well fit. The phases of oscillation in the VAP are in good agreement with the measured values, but the magnitudes at forward angles are underpredicted by the calculations.

The U_s parameters for the four isotopes are again quite similar (Table I). Small systematic variations in the central potential depths are well within the ($N-Z$) dependence of a conventional U_{opt} .³¹ In the case of ^{74,76}Ge, the use of a small imaginary spin-orbit component improves the overall reproduction of the data. Both the magnitude (0.3 MeV) and the negative sign of this component are in good agreement with those predicted by Brieda and Rook⁴⁵ from a microscopic optical model.

As for the $E_d = 16$ MeV data, if we interchange the models, i.e., use a rotational model for ^{70,72}Ge and a vibrational model for ^{74,76}Ge, retaining the U_s and β parameters, the proton elastic data including the qualitative differences at extreme back angles are equally well reproduced. The calculated results are nearly identical to the solid lines in Fig. 4. Here again, the observed anomalies in the elastic-scattering data can be understood on the basis of the discussion in Sec. III and seem to be relatively insensitive to the g.s. shapes. However, the reproduction of the 2⁺ inelastic-scattering data, especially the VAP, is sensitive to the model adopted. In particular, the sharp rise of the VAP beyond 120° c.m. and the shoulder at ~140° c.m. for ^{70,72}Ge and the oscillatory dip between 130° and 165° c.m. for ^{74,76}Ge cannot be reproduced if the models are interchanged. Two examples of this, one for ⁷²Ge(p,p'), 2⁺ (solid curve: vibrational and dashed curve: rotational) and the other for ⁷⁴Ge(p,p'), 2⁺ (solid curve: rotational and dashed curve: vibrational), are shown in the lower half of Fig. 7. The inelastic proton scattering data thus also provide evidence of a shape transition in the region ⁷²Ge to ⁷⁴Ge.

Although proton inelastic scattering data for other excited states were not extracted, sample calculations were performed including the 3⁻-state coupling in ⁷⁰Ge and 4⁺-state coupling in ⁷⁴Ge. By increasing the value of β between 3% and 5% in these calculations, equally good fits as those obtained using the 0⁺-2⁺ coupling were achieved. For ^{74,76}Ge, the use of an oblate deformation does not reproduce either the elastic- or inelastic-

scattering data.

Tamir *et al.*⁴⁶ and Ramstein *et al.*⁴⁶ have reported cross-section angular-distribution measurements of $^{74}\text{Ge}(p,p')$ and $^{76}\text{Ge}(p,p')$, respectively, at $E_p=22$ MeV. These authors performed CC analyses of the data using a vibrational-model description. While the data for the 2_1^+ and 3^- states are quite well described, those for the first 4^+ , second 2^+ , and excited 0^+ states are not well fit. Our measurements and analysis have shown that most of the structure-dependent characteristic features are contained in the analyzing powers, but unfortunately such data are not available at $E_p=22$ MeV. In the case of $^{76}\text{Ge}(p,p')$, CC calculations⁴⁶ were also performed employing the asymmetric rotor model (ARM). Calculations were performed for the first 2^+ and 4^+ and second 2^+ states. The calculations show that the main improvement provided by the ARM is in the reproduction of data for the 4^+ and the second-excited 2^+ states. A γ parameter somewhat different from those obtained from Coulomb-excitation measurements was required to fit the second 2^+ state data. For the first 2^+ state, which is of main interest in our analysis, the improvement in reproduction of the data provided by the ARM is marginal. The predictions of the elastic scattering cross sections do not depend in any significant way on the detailed description of the collective states assumed in the CC calculations.

In a recent paper Delaroche *et al.*⁴⁷ have reported cross-section and VAP angular-distribution measurements for $^{76,78,80,82}\text{Se}(\bar{p},p')$ at $E_p=16$ MeV. It is important to point out that while both the Se and Ge nuclei occur in the same mass region, both proton and deuteron elastic and inelastic-scattering data for these nuclei show quite different characteristics. The measurements of Ref. 47 show the following: (1) At $E_p=16$ MeV, the (p,p) analyzing-power data for the four isotopes are almost identical; (2) an earlier measurement⁴⁸ at $E_p=12$ MeV showed a systematic increase in amplitude of the oscillations of A_y with mass number, but in agreement with that expected from the A dependence of global optical-model potential parameters;³¹ (3) the A_y data for inelastic scattering to the 2^+ state at $E_p=16$ MeV are nearly identical for the four isotopes, except in the angular range $90^\circ-120^\circ$. This is in contrast to the Ge data, where one observes sudden changes in both the elastic and inelastic (2_1^+) analyzing-power data at $E_p=11.5$ MeV. In addition, as pointed out in Sec. III, the deuteron elastic- and inelastic-scattering data for the Ge isotopes are completely different from those of the Se isotopes at comparable incident deuteron energies. The data of Ref. 47 have been analyzed in the CC framework assuming the vibrational model (VM), the rotation-vibration model (RVM), the asymmetric rotor model (ARM), and its extension to β -vibration states (EARM). It is seen (Figs. 3–6, 11, 12, 13, of Ref. 47) that (1) the data for the 4^+ state are *not* well reproduced by any of the calculations; (2) the data for the only resolved excited 0^+ state in ^{76}Se are *not* reproduced by the EARM calculations; (3) the data for the second excited 2^+ states are fairly well reproduced by the ARM model assuming a γ value smaller than that obtained from Coulomb excitation measurements; (4) the elastic and the first 2^+ state data are almost equally well repro-

duced by all of the calculations, although none of the calculations reproduce the 2^+ analyzing-power data in $^{80,82}\text{Se}$ in the angular range $90^\circ-120^\circ$. It is interesting to note that Szaloky *et al.*¹⁷ were able to fit the $\text{Ge}(\bar{d},d')$ data for the second 2^+ state fairly well using the vibrational model. The $^{70}\text{Ge}(2_2^+, 1.710 \text{ MeV})$ data were fairly well reproduced assuming it to be a pure two-phonon state. In order to reproduce the $^{74}\text{Ge}(2^+, 1.210 \text{ MeV})$ data in the VM, a one-phonon component in the transition amplitude was needed in addition to the two-phonon part. The necessity of including a one-phonon component in this case is consistent with our conclusion that the vibrational model is inappropriate for ^{74}Ge . For $^{72}\text{Ge}(2_2^+, 1.466 \text{ MeV})$, the analyzing-power data are well reproduced assuming the state to be a pure two-phonon state. The cross-section data, however, are not well reproduced.

The calculations of Refs. 46 and 47 thus show the ARM predictions to be nearly identical with VM predictions for the Ge and Se isotopes as far as the g.s. and first 2^+ states are concerned. Our analysis clearly shows that VM calculations cannot reproduce the inelastic deuteron analyzing-power measurements for the first 2^+ state in ^{74}Ge and ^{76}Ge . The assumption of a change in shape between ^{72}Ge and ^{74}Ge explains the characteristics of the data. An analysis employing more extended models such as the ARM or the rotation-vibration model would require high-quality analyzing-power data for inelastic scattering to the 4_1^+ , 2_2^+ , and 0_2^+ states, and these data are as yet unavailable.

D. General comments and discussion

Sherif and Blair⁴⁹ have demonstrated the importance of including a spin-orbit deformation of the full-Thomas form in the interaction potential. Their investigation and several others^{50,51} propose the use of a spin-orbit deformation substantially greater than the deformation of the real potential, in order to improve the quality of reproduction of the proton inelastic-scattering data, for example. Calculations performed with $\beta_{so}=2\beta_{real}$ did not yield any significant improvement in the fits either for the deuteron or the proton scattering cases. This observation is in agreement with the results of Refs. 9 and 10. In fact, the predictions are quite insensitive to whether the spin-orbit part of the potential is considered to be deformed or not. All calculations reported in this paper, however, have been performed using a full Thomas form of the spin-orbit potential. In addition, since the deformation lengths for each component of the potential were held constant and the spin-orbit radius parameter is of smaller magnitude than its real central counterpart, in effect β_{so} is larger than β_{real} .

The β values obtained from the deuteron scattering data are approximately 10% smaller than those of Ref. 17. However, as discussed in Sec. IV A, the β values increase when more channels are included in the coupling scheme and come to reasonable agreement with those of Ref. 17. The deformation parameters obtained from analysis of the proton scattering data are approximately 10% lower than those of Ref. 26. These disagreements are within the normalization differences between the present measurements

and those of Ref. 26, as discussed in Sec. II. We also note that the β values obtained from the 16-MeV deuteron data are 10–15% lower than those obtained from the 11.5 MeV proton scattering data. A remeasurement of the cross-section data shows that the difference is not connected with any discrepancy in the normalization constants. A number of investigations^{52–54} have yielded a deformation parameter for nuclear neutrons in general different from that of protons. This difference results in a dependence of the measured deformation parameter on the external field which produces the transition. The observed differences between β_2 (protons) and β_2 (deuterons) may be indicative of the probe dependence of nuclear excitation.

The abrupt change in the analyzing-power data in the region ^{72}Ge – ^{74}Ge that we have observed is not the only indication of a shape transition in these nuclei. Two-nucleon stripping and pickup reactions on even-even Ge nuclei have been studied extensively.^{3–5,55–58} It is observed that the cross section for populating the first-excited 0^+ state varies dramatically³ from isotope to isotope. This state is populated^{4,5} in the (p,t) reaction with about 30% of the ground state strength, and the strength is strongly peaked at $n=40$. In the case of the (t,p) reaction,³ the maximum cross section for this state is about 25% of that of the ground state transition, and the ratio is strongly peaked at $N=42$ for the final nucleus. Similar discontinuities at $N=40$ have also been observed in other nuclear reactions.^{1,59} In the study of the (d,³He) reaction on the even isotopes of Ge, for example, Rotbard *et al.*⁵⁹ found that the ground-state proton occupation number exhibits anomalous behavior between ^{72}Ge and ^{74}Ge .

Attempts have been made to understand the nature of these abrupt changes from both microscopic^{5,58} and macroscopic^{3,7,59} points of view. The microscopic approaches are based on orthogonality of the 0^+ ground and first-excited states in either the neutron^{55,58} or proton configurations.⁵ Becker *et al.*⁵⁸ have analyzed the angular distributions for (p,t) reactions to 0^+ states in even Ge isotopes at sub-Coulomb triton energies in the framework of CC calculations. The form factors were determined from simple shell-model wave functions with ^{70}Ge considered as an inert core, and the remaining neutrons occupying only $2p_{1/2}$ and $1g_{9/2}$ subshells. Their calculations provide a fairly good description of the $0^+ \rightarrow 0^+$ transitions in $^{72,74,76}\text{Ge}$ (p,t), but other experimental facts, particularly the results of single-proton transfer reactions, which led to the introduction of orthogonal proton configurations,⁵ are not easily explained. A recent analysis⁶⁰ using the interacting boson model (IBM) reproduces quite well the general trend in the (t,p) and (p,t) data as a function of mass number.

From a macroscopic point of view, the characteristics of the transfer-reaction data indicate a shape transition between $N=40$ and 42. Since the maxima in the cross sections leading to the first-excited 0^+ state observed in both the (p,t) and (t,p) reactions do not occur for the same final nuclei, but for nuclei differing by two mass units, it is reasonable to eliminate the possibility of a subshell closure at $N=40$. Comparison of the results in this mass region with other transitional regions suggests a shape tran-

sition.³ A summary of the experimental data obtained in Coulomb excitation and two-nucleon transfer-reaction measurements has been presented by Lecompte *et al.*,⁷ and a consistent picture emerges from the data if the existence of a shape transition is assumed. The measurements and analysis reported in the present paper support the concept of a shape transition between ^{72}Ge and ^{74}Ge .

V. SUMMARY AND CONCLUSIONS

Cross-section and vector-analyzing-power measurements have been carried out for $^{70,72,74,76}\text{Ge}(\vec{d},d)$ at $E_d=16$ and 8 MeV, (\vec{d},d') 2^+ at $E_d=16$ MeV, (p,p) and (p,p') 2^+ at $E_p=11.5$ MeV, and measurements of $^{70,72,74}\text{Ge}(\vec{d},d)$ at $E_d=6$ MeV are reported. The data have been analyzed using the coupled-channels formalism assuming a collective-model form factor. Most of the calculations were performed using a $0^+ \rightarrow 2^+$ coupling scheme. The elastic-scattering data for $^{70,72}\text{Ge}$ are nearly identical, as are those of $^{74,76}\text{Ge}$. However, there is a sudden transition in the cross-section shapes and the amplitudes of oscillations of the VAP as one goes from ^{72}Ge to ^{74}Ge . The inelastic-scattering data also show near identity for ^{70}Ge and ^{72}Ge , but differ from the $^{74,76}\text{Ge}$ data, which are nearly identical. In the case of deuteron inelastic scattering, the main difference between $^{70,72}\text{Ge}$ and $^{74,76}\text{Ge}$ data occurs in the phase of the VAP oscillations. For proton inelastic scattering, considerable difference also exists, particularly at the back angles. Calculations were performed using a vibrational model for $^{70,72}\text{Ge}$ and a prolate rotational model for $^{74,76}\text{Ge}$. Essentially all of the data are well reproduced by the calculations. The analysis shows that the characteristic features of the elastic-scattering data of $E_d=16$ MeV and $E_p=11.5$ MeV are not by themselves particularly sensitive to the ground state shape of the nuclei. The inelastic-scattering data, however, are sensitive to the model assumed, and, within the framework of the CC calculations, can be reproduced only by assuming that the nuclei $^{70,72}\text{Ge}$ are vibrational while $^{74,76}\text{Ge}$ are rotational, suggesting a shape transition between ^{72}Ge and ^{74}Ge . At the lower bombarding energies investigated, the deuteron elastic-scattering analyzing powers are smaller by an order of magnitude than those observed at 16 MeV, but show characteristics similar to the higher-energy data.

We note that ^{74}Ge shows a $0^+ \rightarrow 2^+ \rightarrow 4^+$ triad at approximately twice the excitation energy of the first 2^+ state. The 0^+ , first 2^+ , and first 4^+ excitation energies do not precisely satisfy the $I(1+1)$ rule characteristic of well-developed rotational bands. These nuclei are complex and no simple model or, for that matter, no one model is expected to unravel all the structure information of the states. There are features in these nuclei which imply evidence of shape coexistence and the existence of noncollective states. Our analysis is not powerful enough to discern all these aspects. However, the present analysis is inherently simple and provides a satisfactory overall description of a fairly large data set.

ACKNOWLEDGMENTS

The authors wish to thank Dr. G. Szaloky for preparing the targets used in the present investigation. This

work was supported in part by the National Science Foundation under contracts Nos. PHY82-00426 and INT81-14699 and by the Consejo Nacional de Ciencia y Tecnología (CONACyT), Mexico.

- *Present address: Department of Physics, Emory and Henry College, Emory, VA 24327.
- ¹D. Ardouin, R. Tamisier, M. Vergnes, G. Rotbard, J. Kalifa, G. Berrier, and B. Grammaticos, *Phys. Rev. C* **12**, 1745 (1975).
 - ²M. Didong, H. Muther, K. Goeke, and A. Faessler, *Phys. Rev. C* **14**, 1189 (1976).
 - ³M. N. Vergnes, G. Rotbard, F. Guibault, D. Ardouin, C. Lebrun, E. R. Flynn, D. L. Hanson, and S. D. Orbesen, *Phys. Lett.* **72B**, 447 (1978).
 - ⁴C. Lebrun, F. Guibault, D. Ardouin, E. R. Flynn, D. L. Hanson, S. D. Orbesen, G. Rotbard, and M. Vergnes, *Phys. Rev. C* **19**, 1224 (1979).
 - ⁵M. Vergnes, in *Proceedings of the International Conference on the Structure of Medium-Heavy Nuclei, Rhodes, 1979*, edited by the "Demokritos" Tandem Accelerator Group, Athens (The Institute of Physics, Bristol, 1980), p. 25.
 - ⁶R. Lecomte, M. Irshad, S. Landsberger, P. Paradis, and S. Monaro, *Phys. Rev. C* **22**, 1530 (1980); R. Lecomte, M. Irshad, S. Landsberger, G. Kajrys, P. Paradis, and S. Monaro, *ibid.* **22**, 2420 (1980).
 - ⁷R. Lecomte, G. Kajrys, S. Landsberger, P. Paradis, and S. Monaro, *Phys. Rev. C* **25**, 2812 (1982).
 - ⁸R. C. Brown, A. A. Debenham, J. A. R. Griffith, O. Karban, D. C. Cocher, and S. Roman, *Nucl. Phys.* **A208**, 589 (1973).
 - ⁹F. T. Baker, S. Davis, C. Glashauser, and A. B. Robbins, *Nucl. Phys.* **A233**, 409 (1974); **A250**, 79 (1975).
 - ¹⁰S. Sen and S. E. Darden, *Nucl. Phys.* **A266**, 173 (1976).
 - ¹¹M. Nakamura, H. Sakaguchi, K. Imai, K. Hatanaka, A. Goto, T. Noro, F. Ohtani, S. Kobayashi, K. Hosono, M. Kato, K. Ogino, and Y. Kadota, *J. Phys. Soc. Jpn. Suppl.* **44**, 557 (1978).
 - ¹²R. DeLeo, B. D'Erasmus, A. Pantaleo, G. Pasquariello, G. Viesti, M. Pignanelli, and H. V. Geramb, *Phys. Rev. C* **19**, 646 (1979).
 - ¹³H. Clement, R. Frick, G. Graw, F. Merz, P. Schiemenz, N. Seichert, and Sun Tsu Hsun, *Phys. Rev. Lett.* **45**, 599 (1980).
 - ¹⁴K. Hatanaka, M. Nakamura, K. Imai, T. Noro, H. Shimizu, H. Sakamoto, J. Shirai, T. Matsusue, and K. Nisimura, *Phys. Rev. Lett.* **46**, 15 (1981).
 - ¹⁵S. Matsuki, T. Higo, T. Ohsawa, T. Shiba, T. Yanabu, K. Ogino, Y. Kadota, K. Haga, N. Salamoto, K. Kume, and M. Matoba, *Phys. Rev. Lett.* **51**, 1741 (1983).
 - ¹⁶J. Raynal, in *Proceedings of the Fourth International Symposium on Polarization Phenomena in Nuclear Reactions, Zürich, 1975*, edited by W. Grüebler and V. König (Birkhäuser, Basel, 1976), p. 271.
 - ¹⁷G. Szaloky, L. A. Montestruque, M. C. Cobian-Rozak, and S. E. Darden, *Phys. Rev. C* **18**, 750 (1978).
 - ¹⁸W. H. L. Moonen, P. J. Van Hall, S. S. Klein, G. J. Nijgh, C. W. A. M. van Overveld, R. H. A. L. Petit, and O. J. Poppe, in *Proceedings of the International Conference on Nuclear Physics, Florence, Italy, 1983*, edited by P. Blasi and R. A. Ricci (Tipografia Compositori, Bologna, Italy, 1983), p. 260.
 - ¹⁹J. Nurzynski, W. Grüebler, V. König, R. Risler, R. A. Hardekopf, H. R. Bürgi, and B. Jenny, *Phys. Lett.* **67B**, 23 (1977).
 - ²⁰Y. Tagishi, B. L. Burks, T. B. Clegg, E. J. Ludwig, R. L. Varner, J. F. Wilkerson, and W. J. Thompson, in *Polarization Phenomena in Nuclear Physics—1980 (Fifth International Symposium, Santa Fe)*, Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, AIP Conf. Proc. No. 69, edited by G. G. Ohlson, R. E. Brown, N. Jarmie, M. W. McNaughton, and G. M. Hale (AIP, New York, 1981), p. 467.
 - ²¹J. A. Bieszk and J. Ulbricht, *Phys. Rev. C* **24**, 56 (1981).
 - ²²W. H. Wong and P. A. Quin, in *Polarization Phenomena in Nuclear Physics—1980 (Fifth International Symposium, Santa Fe)*, Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, AIP Conf. Proc. No. 69, edited by G. G. Ohlson, R. E. Brown, N. Jarmie, M. W. McNaughton, and G. M. Hale (AIP, New York, 1981), p. 464; W. H. Wong, Ph.D. thesis, University of Wisconsin, 1979 (unpublished).
 - ²³T. Tamura, *Rev. Mod. Phys.* **37**, 679 (1965).
 - ²⁴R. R. Cadmus, Jr. and W. Haerberli, *Nucl. Instrum. Methods* **129**, 403 (1975).
 - ²⁵S. Sen, W. A. Yoh, and M. T. McEllistrem, *Phys. Rev. C* **10**, 1050 (1974).
 - ²⁶T. H. Curtis, H. F. Lutz, and W. Bartolini, *Phys. Rev. C* **1**, 1418 (1970).
 - ²⁷F. G. Perey, *Phys. Rev.* **131**, 745 (1963).
 - ²⁸W. A. Yoh, Ph.D. dissertation, University of Notre Dame, 1975 (unpublished).
 - ²⁹W. W. Daehnick, J. D. Childs, and Z. Vrcelj, *Phys. Rev. C* **21**, 2253 (1980).
 - ³⁰J. M. Lohr and W. Haerberli, *Nucl. Phys.* **A232**, 381 (1974).
 - ³¹F. D. Becchetti and G. W. Greenlees, *Phys. Rev.* **182**, 1190 (1969).
 - ³²H. Feshbach, *Ann. Phys. (N.Y.)* **19**, 287 (1962); R. H. Lemmer, *Rep. Prog. Phys.* **29**, 131 (1966).
 - ³³N. K. Glendenning, in *Proceedings of the International School of Physics, "Enrico Fermi"*, edited by M. Jean, Course XL 1967 (Academic, New York, 1969), p. 332; N. K. Glendenning, D. L. Hendrie, and O. J. Jarvis, *Phys. Lett.* **26B**, 131 (1968); N. K. Glendenning, *Direct Nuclear Reactions* (Academic, New York, 1983).
 - ³⁴J. Raynal, Coupled-channels code ECIS 74, Centre d'Etudes Nucléaires de Saclay (Saclay), private communication (unpublished).
 - ³⁵J. Raynal, in *Computing as a Language in Physics* (IAEA, Vienna, 1972).
 - ³⁶R. P. Goddard and W. Haerberli, *Phys. Rev. Lett.* **40**, 701 (1978).
 - ³⁷W. W. Daehnick, in *Polarization Phenomena in Nuclear Physics—1980 (Fifth International Symposium, Santa Fe)*, Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, AIP Conf. Proc. No. 69, edited by G. G. Ohlson, R. E. Brown, N. Jarmie, M. W. McNaughton, and G. M. Hale (AIP, New York, 1981), p. 487.
 - ³⁸J. Nurzynski, W. Grüebler, H. R. Bürgi, V. König, R. Risler, and B. Jenny, *Nucl. Phys.* **A359**, 61 (1981).

- ³⁹P. E. Hodgson, *Nuclear Reactions and Nuclear Structure* (Clarendon, Oxford, 1971), p. 249.
- ⁴⁰S. Sen (unpublished).
- ⁴¹W. G. Love, T. Terasawa, and G. R. Satchler, *Phys. Rev. Lett.* **39**, 6 (1977).
- ⁴²A. J. Baltz, S. K. Kaufmann, N. K. Glendenning, and K. Pruess, *Phys. Rev. Lett.* **40**, 20 (1978).
- ⁴³J. A. Tostevin and R. C. Johnson, *Phys. Lett.* **124B**, 135 (1983).
- ⁴⁴P. D. Kunz, code CHUCK, University of Colorado, private communication (unpublished).
- ⁴⁵F. A. Brieva and J. R. Rook, *Nucl. Phys.* **A297**, 206 (1978).
- ⁴⁶R. Tamisier, B. Ramstein, P. Avignon, L. H. Rosier, G. LaRana, F. Guilbault, C. Lebrun, and C. Jeanperrin, *Nucl. Phys.* **A385**, 430 (1982); B. Ramstein, R. Tamisier, L. H. Rosier, P. Avignon, and J. P. Delaroche, *Nucl. Phys.* **A411**, 231 (1983), and references therein.
- ⁴⁷J. P. Delaroche, R. L. Varner, T. B. Clegg, R. E. Anderson, B. L. Burks, E. J. Ludwig, and J. F. Wilkerson, *Nucl. Phys.* **A414**, 113 (1984).
- ⁴⁸R. L. Varner, J. F. Wilkerson, W. J. Thompson, Y. Tagishi, E. J. Ludwig, T. B. Clegg, and B. L. Burks, in *Polarization Phenomena in Nuclear Physics—1980 (Fifth International Symposium, Santa Fe)*, Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, AIP Conf. Proc. No. 69, edited by G. G. Ohlson, R. E. Brown, N. Jarmie, M. W. McNaughton, and G. M. Hale (AIP, New York, 1981), p. 430.
- ⁴⁹H. Sherif and J. S. Blair, *Phys. Lett.* **26B**, 489 (1968); *Nucl. Phys.* **140**, 33 (1970).
- ⁵⁰C. Glashausser, R. De Swiniarski, T. Goudergues, R. M. Lombard, N. Mayer, and J. Thirion, *Phys. Rev.* **184**, 1217 (1969).
- ⁵¹B. J. Verhaar, *Phys. Rev. Lett.* **32**, 307 (1974).
- ⁵²V. A. Madsen, V. R. Brown, and J. D. Anderson, *Phys. Rev. Lett.* **34**, 1398 (1975).
- ⁵³H. Clement, R. Frick, G. Graw, F. Merz, H. J. Scheerer, P. Schiemenz, N. Seichert, and Sun Tsu Hsun, *Phys. Rev. Lett.* **48**, 1082 (1982).
- ⁵⁴A. M. Bernstein, V. R. Brown, and V. A. Madsen, in *Comments in Nuclear and Particle Physics XI* (Gordon and Breach, London, 1983), p. 203.
- ⁵⁵G. C. Ball, R. Fournier, J. Kroon, T. H. Hsu, and B. Hird, *Nucl. Phys.* **A231**, 334 (1974).
- ⁵⁶D. Ardouin, B. Remaud, K. Kumar, F. Guilbault, P. Avignon, R. Seitz, M. Vergnes, and G. Rotbard, *Phys. Rev. C* **18**, 2739 (1978).
- ⁵⁷S. Mordechai, H. T. Fortune, R. Middleton, and G. Stevens, *Phys. Rev. C* **18**, 2498 (1978); **19**, 1733 (1979).
- ⁵⁸A. Becker, E. A. Bakkum, and K. Kamermans, *Phys. Lett.* **110B**, 199 (1982).
- ⁵⁹G. Rotbard, G. LaRana, M. Vergnes, G. Berrier, J. Kalifa, F. Guilbault, and R. Tamisier, *Phys. Rev. C* **18**, 86 (1978).
- ⁶⁰P. D. Duval, D. Goutte, and M. Vergnes, *Phys. Lett.* **124B**, 297 (1983).