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^{4}He D-state effects in the ^{2}H($\vec{d}$, γ)^{4}He reaction

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A discussion of tensor analyzing powers of the ^{2}H($\vec{d}$, γ)^{4}He reaction at low energy ($E_d < 20$ MeV), where the process is predominantly $E2$, is presented. The inclusion of the ^{4}He D state generates transitions from the $^5S_2$, $^5D_2$, and $^5G_2$ initial states. It is shown that the tensor analyzing powers depend linearly on the D/S ratio ρ of ^{4}He for angles near $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$. Using scattering wave functions with phase shifts obtained from resonating group calculations good agreement with recent T_{20} data at $E_d = 9.7$ MeV is obtained for $-0.5 < \rho < -0.4$.

The ^{2}H($\vec{d}$, γ)^{4}He reaction is a particularly interesting radiative capture process because of the simplicity resulting from symmetry considerations.[1] For deuteron energies below 20 MeV the differential cross section data exhibit a distinct $\sin^2 2\theta$ shaped angular distribution in good agreement with theoretical expectations for an $E2$, $^1D_2 \rightarrow {}^1S_0$, transition.[2-4] Cross section data also exist around $E_d = 400$ MeV (Ref. 5) but at these higher energies the reaction mechanism is not well understood.

More recently, Weller *et al.* have measured the tensor analyzing power T_{20} at $E_d = 9.7$ MeV (Ref. 6) and showed that the ^{4}He D state has a large effect upon this observable. An example of an analogous situation is found in the ^{1}H($\vec{d}$, γ)^{3}He reaction where the tensor analyzing powers T_{2q} are strongly dependent on the D-state component of the relative motion between the deuteron cluster and the spectator proton in ^{3}He.[7] Here we present a discussion of the analyzing powers in the ^{2}H($\vec{d}$, γ)^{4}He reaction at low energy ($E_d < 20$ MeV) and consider specifically the effect of the ^{4}He D state.

Using the notation of Rose and Brink[8] the interaction Hamiltonian for the emission of a photon, with momentum $\mathbf{p}$ and polarization $\boldsymbol{\epsilon}_n$, is given in first order perturbation theory by the expression

$$H_e(\mathbf{p}, \boldsymbol{\epsilon}_n) = -\sum_{LM\pi} n^\pi T_{LM}(\pi)^\dagger D^{L}_{Mn}(R)^* \quad , \tag{1}$$

where $T_{LM}(\pi)$ is a multipole operator of rank L and $\pi = 0$ and 1 correspond to electric and magnetic operators, respectively. R is a rotation taking the z axis into the direction $\mathbf{p}$. We use the Madison Convention coordinate system where the z axis is along the momentum $\mathbf{k}$ of the incident deuteron and the y axis is along $\mathbf{k} \times \mathbf{p}$. The transition amplitude is a sum of terms involving matrix elements that can be written, using the Wigner-Eckart theorem as

$$\langle 0 | T_{LM}(\pi)^\dagger | {}^{2s+1}l_J; JM' \rangle = (-1)^L (2L+1)^{-1/2} \delta_{LJ} \times \delta_{MM'} (0 \| T_L(\pi) \| {}^{2s+1}l_J) \quad . \tag{2}$$

$|0\rangle$ is the $J=0$ ^{4}He ground state, $|{}^{2s+1}l_J; JM\rangle$ is a two-deuteron initial state with channel spin s, orbital angular momentum l, and total angular momentum J. The identity of the two deuterons in the entrance channel restricts l and s to be of the same parity. With multipoles of order $L \leq 2$ the allowed transitions are $(E1;{}^3P_1)$, $(M1;{}^3D_1)$, $(E2;{}^1D_2)$, $(E2;{}^5S_2)$, $(E2;{}^5D_2)$, $(E2;{}^5G_2)$, $(M2;{}^3P_2)$, and $(M2;{}^3F_2)$. Conservation of isospin in a self-conjugate nucleus implies that, in the long wavelength approximation, the $E1$ transition is forbidden and that the $M1$ transition is strongly suppressed between states of equal isospin.[9] The $E2$ transition is therefore expected to be dominant and, for a pure S state ^{4}He, it is of the form $({}^1S_0 | E2 | {}^1D_2)$. This result is in good agreement with measured cross section angular distributions for $E_d < 20$ MeV. The inclusion of the tensor component of the nucleon-nucleon interaction generates D states in the deuteron and ^{4}He ground states. In the case of ^{4}He we consider a $^5D_0$ state given, as in Ref. 10, by

$$|\phi_D\rangle = G_0 \sum_{i<j} V_T^{ij} |\phi_S\rangle \quad , \tag{3}$$

where ϕ_S is the $^1S_0$ state, G_0 a Green's operator and V_T^{ij} the tensor interaction between nucleons i and j. The ^{4}He D state ϕ_D generates amplitudes $({}^5D_0 | E2 | {}^5S_2)$, $({}^5D_0 | E2 | {}^5D_2)$, and $({}^5D_0 | E2 | {}^5G_2)$. In addition the deuteron internal D state gives a nonvanishing $({}^1S_0 | E2 | {}^5S_2)$ amplitude. Thus, the $E2$ transition strength from the $^5S_2$ initial state receives contributions from both the ^{4}He and deuteron D states. The latter will be neglected in the present work.

At low energy we can expect that the most important contributions to the polarization observables arise from terms linear in the $({}^1S_0 | E2 | {}^1D_2)$ amplitude. This, however, does not apply to scattering angles near to $\theta = \frac{1}{2}\pi$ because $({}^1S_0 | E2 | {}^1D_2)$ is proportional to $\sin 2\theta$. We find that the vector analyzing power iT_{11} has terms of the form $\mathrm{Im}[(E2;{}^1D_2)(E1;{}^3P_1)^*]$, $\mathrm{Im}[(E2;{}^1D_2)(M2;{}^3P_2)^*]$, $\mathrm{Im}[(E2;{}^1D_2)(M2;{}^3F_2)^*]$ and is therefore expected to be small since the $E1$ and $M2$ transitions are strongly suppressed. The tensor analyzing powers have terms of the form $\mathrm{Re}[(E2;{}^1D_2)(E2;{}^5S_2)^*]$, $\mathrm{Re}[(E2;{}^1D_2)(E2;{}^5D_2)^*]$, $\mathrm{Re}[(E2;{}^1D_2)(E2;{}^5G_2)^*]$, $\mathrm{Re}[(E2;{}^1D_2)(M1;{}^5D_1)^*]$; those with the $E2,E2$ interference being expected to be larger. Here we consider only the $E2$ amplitudes. The contributions from meson exchange currents were not included in the present calculations. These are expected, however, to be strongly reduced because of the isoscalar nature of the $^2\mathrm{H}(\vec{d},\gamma)^4\mathrm{He}$ process.

In the calculation of the $E2$ matrix elements we assume that the position vectors of the protons, within Siegert's form of the $E2$ operator, are proportional to the displacement $\mathbf{r}$ between the centers of mass of the two deuterons. With this approximation the $E2$ matrix elements depend on the internal structure of the ^{4}He through the overlap[10]

$$\langle \phi_d^{\sigma_1}\phi_d^{\sigma_2}|\phi_\alpha\rangle = \frac{1}{2}\sum_{L'=0,2}(-1)^{\sigma_1}(L'M'1\sigma_2|1-\sigma_1) \times u_{L'}(r)\,Y_{L'}^{M'}(\hat{r}) \ . \quad (4)$$

Asymptotically

$$u_{L'}(r) \underset{r\to\infty}{\longrightarrow} -N_{L'} i^{L'} h_{L'}(i\alpha r) \ , \quad (5)$$

where $\alpha = 1.072\ \mathrm{fm}^{-1}$ is the wave number corresponding to the separation energy of two deuterons from ^{4}He (23.85 MeV). The asymptotic D/S state ratio is denoted $\rho = N_2/N_0$.

For the description of the initial state we follow closely the works of Thompson[11] and Chwieroth, Tang, and Thompson[12] which consider elastic d-d scattering, for $E_d < 20$ MeV, within a one-channel resonating group method (RGM) calculation. This analysis gives a good account of the experimental d-d elastic scattering differential cross section data and also total cross section measurements for the $^4\mathrm{He}(\gamma,d)^2\mathrm{H}$ photodisintegration process.[13] In Ref. 12 the d-d continuum radial wave functions $\chi_{ls}(r)$ are not explicitly J dependent but are l and s dependent. With the above approximations the $E2$ matrix elements are given, up to a common multiplicative factor, by

$$A = (0||E2||^1D_2) = \int_0^\infty u_0(r)\chi_{20}(r)j_2(pr)r^2\,dr \ , \quad (6a)$$

$$B = (0||E2||^5S_2) = -\frac{1}{5}\int_0^\infty u_2(r)\chi_{02}(r)j_2(pr)r^2\,dr \ , \quad (6b)$$

$$C = (0||E2||^5D_2) = -\sqrt{2/7}\int_0^\infty u_2(r)\chi_{22}(r)j_2(pr)r^2\,dr \ , \quad (6c)$$

$$D = (0||E2||^5G_2) = -(9/5)\sqrt{2/7}\int_0^\infty u_2(r)\chi_{42}(r) \times j_2(pr)r^2\,dr \ . \quad (6d)$$

An important aspect of the relations above, for low deuteron energies, is that the integrand contains a factor r^4, since $j_2(pr)$ can be approximated by $(pr)^2/15$ (long wavelength approximation). This means that the radial integrals and the transition amplitudes probe the asymptotic region of large r in the ^{4}He radial wave functions $u_L(r)$. This long wavelength approximation is not made in the numerical calculations presented.

Keeping only terms linear in A we obtain

$$T_{20} \cong 2\mathrm{Re}[B/A - \sqrt{2/7}(C/A - D/A)] \ , \quad (7a)$$

$$T_{21} \cong 2\sqrt{2/3}\mathrm{Re}[B/A - C/(\sqrt{14}A) - 2D/(\sqrt{21}A)]\cot 2\theta \ , \quad (7b)$$

$$T_{22} = -\sqrt{2/3}\mathrm{Re}\{B/A + \sqrt{2/7}[C/A + D/(6A)]\} \ . \quad (7c)$$

These approximate expressions are only valid in the angular regions where $\sin^2 2\theta$ is large, that is at $\theta = \frac{1}{4}\pi$ and $\frac{3}{4}\pi$. At $\theta = \frac{1}{2}\pi$ T_{21} is zero and T_{20} and T_{22} become ratios of bilinear functions of B, C, and D. In particular,

$$T_{20}(\tfrac{1}{2}\pi) = \frac{5|B + C/(2\sqrt{14})|^2 + |B + D/\sqrt{14}|^2 + (9|C|^2/8 - 8|D|^2/9)/7 - 11|B|^2/2}{\sqrt{2}[2|B + (C-D)/(2\sqrt{14})|^2 + 9|C + 5D/9|^2/28]} \ . \quad (8)$$

The analyzing power A_{yy} has the remarkable property that it has no linear dependence on B. In fact, from Eqs. (7) we obtain

$$A_{yy} \cong 4/\sqrt{7}Re[C/A - 5D/(12A)] \ . \quad (9)$$

To calculate the $E2$ amplitudes we use d-d scattering wave functions χ_{ls} with the phase shifts δ_{ls} obtained in Ref. 12. The χ_{02} wave, strongly distorted at low energy, is generated in a rank two separable potential, comprising Yamaguchi type S-wave form factors[14] adjusted to reproduce the RGM phase shifts for 0 MeV $< E_d < 20$ MeV. The use, in Ref. 6, of the same local potential in the $^1D_2$ scattering state as binds the $^1S_0$ state and of the same local potential in the $^5S_2$, $^5D_2$, $^5G_2$ scattering states as binds the $^5D_0$ state produces phase shifts at considerable variance with those obtained from RGM calculations. In the present work the phase shifts for the $^1D_2$ and $^5D_2$ channels, which are relatively small for $E_d < 10$ MeV, were fitted for $E_d < 20$ MeV to rank one separable interactions of D-wave Yamaguchi form[14] and the corresponding χ_{ls} generated. For the $^5G_2$ state we write $\chi_{42}(r) = j_4(kr)$ since this state is only very weakly distorted. The radial overlap functions $u_L(r)$ were generated in a Wood-Saxon well ($r_0 = 1.5$ fm, $a = 0.5$ fm) adjusted so that the S-state finite range parameter β has the value 1.5 fm^{-1} (Ref. 10).

Figure 1 shows the result of calculations for T_{20} at $E_d = 9.7$ MeV obtained with the phase shifts $\delta_{02} = -69.1°$, $\delta_{20} = 14.6°$, and $\delta_{22} = 4.8°$ for different values of the D/S ratio ρ. Good agreement with the T_{20} measurements[3] close to $\theta = \frac{1}{4}\pi$ and $\theta = \frac{3}{4}\pi$ is obtained for $-0.5 < \rho < -0.4$. In

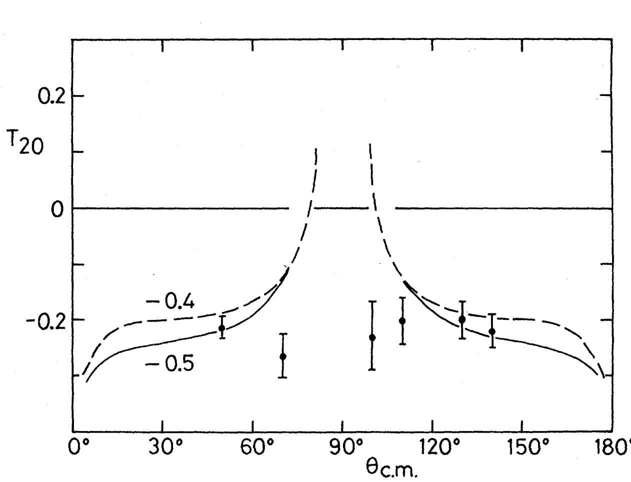

FIG. 1. Calculated tensor analyzing power T_{20} for the $^2\mathrm{H}(\vec{d},\gamma)^4\mathrm{He}$ reaction at $E_d = 9.7$ MeV for the values of the D/S state ratio indicated. The data are from Ref. 6.

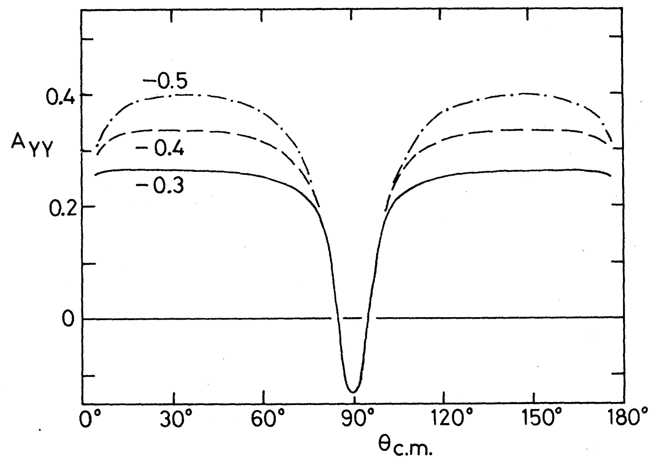

FIG. 2. Calculated tensor analyzing power A_{yy} for ^{2}H($\vec{d}$,γ)^{4}He reaction at $E_d = 9.7$ MeV for the values of the D/S state ratio indicated.

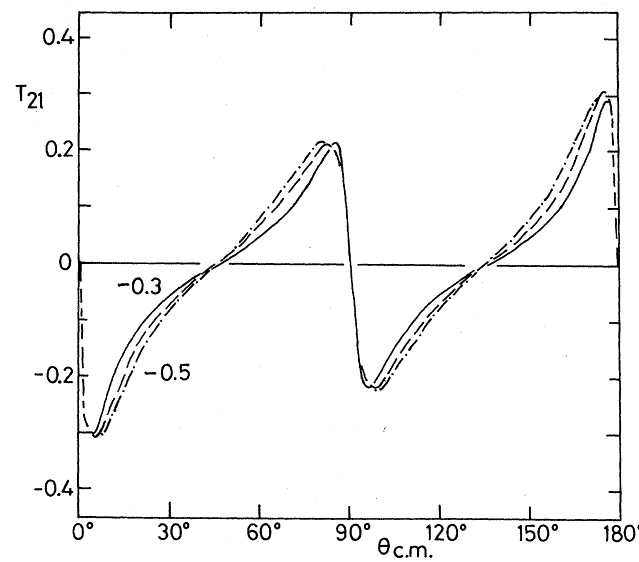

FIG. 3. As for Fig. 2 for the tensor analyzing power T_{21}.

this angular region T_{20} and T_{22} show a distinct linear dependence upon ρ, as suggested by Eqs. (7a) and (7b). For angles near to $\frac{1}{2}\pi$ it was not possible to reproduce the T_{20} data with reasonable variations of the $E2$ amplitudes. This discrepancy does not however affect the value estimated for ρ since theoretically T_{20} becomes independent of ρ as θ approaches $\frac{1}{2}\pi$.

At $E_d = 9.7$ MeV, a result of the strong distortion in the $^5S_2$ channel is that Re(B/A) is small compared with Re(C/A). Since Re(D/A) is also considerably smaller than Re(C/A), the latter term dominates in Eqs. (7). The relatively weak distortion in the $^1D_2$ and $^5D_2$ channels implies therefore that $Re(C/A)$ has the opposite sign to ρ. Thus, for $\theta = \frac{1}{4}\pi$ and $\frac{3}{4}\pi$ T_{20} has the same sign as ρ while A_{yy} has the opposite sign and is larger than $|T_{20}|$ by approximately $\sqrt{2}$. We conclude that A_{yy}, shown in Fig. 2 for different values of ρ, is more favorable than either T_{20} or T_{22} to empirically determine ρ. A_{yy} is more sensitive to variations in ρ, and more importantly has a weaker dependence on the initial state interactions. Calculations for T_{21} (Fig. 3) show this observable to have a stronger angular dependence, less sensitivity to variations in ρ, and from the experimental point of view, the disadvantage of being small where the cross section is largest.

From the present analysis of the T_{20} data of Weller, Colby, Roberson, and Tilley[6] we deduce the value $-0.5 < \rho < -0.4$, which is in reasonable agreement with recent determinations of $D_2 = -0.3 \pm 0.1$ fm^2 obtained from the analysis of tensor analyzing power data for the ($\vec{d}$,α) reaction.[15] To a good approximation $\rho = \alpha^2 D_2 = (1.072)^2 D_2$. It should be noted that the relation between ρ and the probabilities P_S and P_D associated with the wave functions $u_0(r)$ and $u_2(r)$, respectively, depends crucially on the detailed nature of the wave functions at short distances. The presently available T_{20} data are clearly rather insensitive to the details of this short range behavior.

It is interesting and encouraging that the analysis of two quite different reaction processes yields approximately the same value for ρ. The question, therefore, which now arises is to what extent it is possible to reproduce this value of ρ using realistic ^{4}He wave functions derived from four-body calculations which incorporate the effects of the nucleon-nucleon tensor force. Calculations in which the point-deuteron approximation, made in the $E2$ operator, is relaxed and in which the effect of the deuteron D state, through the $(^1S_0|E2|^5S_2)$ amplitude, is taken into account are in progress.

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