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$^{12}C+^{12}C$ resonances within the nuclear vibron model

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The barrier region resonances of the $^{12}C+^{12}C$ system have been analyzed by Erb and Bromley in terms of the O(4) dynamical symmetry of the nuclear vibron model. Using the general Hamiltonian of the model based on σ and π bosons, we have reanalyzed these data. With parameters characterizing a situation close to the U(3) dynamical symmetry, good agreement was also achieved.

The analysis of the barrier region resonances of the $^{12}C+^{12}C$ system by Erb and Bromley[1] was the first application of the vibron model[2] in nuclear physics and indicated its usefulness in the description of nuclear molecular spectra. Their analysis has been performed in terms of the O(4) dynamical symmetry which is one of the two limiting cases of the model. By generating the molecule spectra with scalar $\sigma(J^\pi=0^+)$ and vector $\pi_\mu(\mu=0,\pm1;J^\pi=1^-)$ bosons the vibron model has a Hamiltonian of U(4) group structure in its general form, and it has two dynamical symmetries characterized by the (I) $U(4)\supset O(4)\supset O(3)$, and (II) $U(4)\supset U(3)\supset O(3)$ group chains. So far other applications to the quasimolecular resonances[3] have also concentrated on the possibility of parametrizing the experimental data in terms of the dynamical symmetries. Here we report an application based on the general form of the Hamiltonian. The $^{12}C+^{12}C$ resonances of the barrier region from Ref. 1 have been reanalyzed to find out (i) how good description of the experimental data is available with the general case of the spectrum generating algebra based on the σ and π bosons, and (ii) how close is this system to the dynamical symmetry.

The experimental spectrum consists of 38 resonances, and they are arranged into multiplets according to the relative energy differences in Ref. 1. We followed the same procedure with a little bit more emphasis on the classification scheme of the model,[4] i.e., we preferred the completeness of the collective bands to the agreement between the experimental and theoretical energies. In this way we got the experimental spectrum of Fig. 1, which is slightly different from that of Ref. 1. As for the number of bosons, we took the smallest value required by the relations between the quantum numbers[4] for this band structure, and this value turned out to be $N=20$.

To diagonalize the Hamiltonian we have used the ROTVIB code[5] and the parameters have been searched by a least square fitting procedure. The nonvanishing elements of the energy matrix in this calculation were the following:

$$\begin{aligned} H^L_{n_\pi,n_\pi} &= h_0+h_1n_\pi+h_2n_\pi^2+h_3L(L+1) \\ &\quad +\tfrac{1}{4}h_4[\,n_\pi(n_\pi+1)-L(L+1) \\ &\quad +(N-n_\pi)(N-n_\pi-1)]\ , \\ H^L_{n_\pi,n_\pi+2} &= -\tfrac{1}{4}h_4[\,(N-n_\pi+2)(N-n_\pi+1) \\ &\quad \times(n_\pi+L+1)(n_\pi-L)]^{1/2}\ . \end{aligned} \tag{1}$$

Here L is the spin of the state, n_π is the number of π bosons, N is the total number of bosons, while h_0, h_1, h_2, h_3, and h_4 are parameters to fit. The h_0 term is an extra one in comparison with the original version;[5] it has been added to the Hamiltonian in order to account for the molecular band head.[4]

We have performed the calculations with several different initial sets of parameters. First of all, the spectra corre-

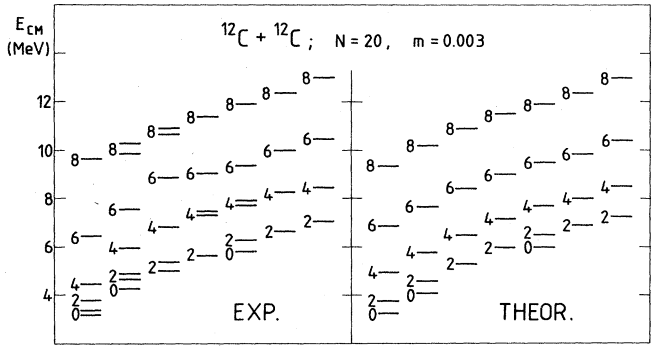

FIG. 1. Energies of experimental and model states.

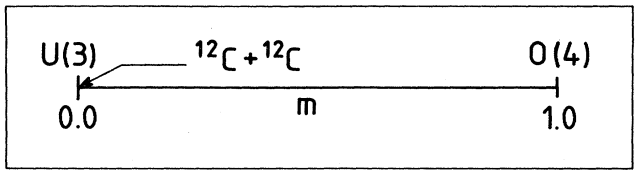

FIG. 2. Position of the $^{12}C+^{12}C$ system on the phase diagram according to our best fit.

TABLE I. Results of different calculations for the $^{12}C+^{12}C$, $N=20$ system with band structure of Fig. 1.

Dynamical symmetry	Input data	h_0 (MeV)	h_1 (MeV)	h_2 (MeV)	h_3 (MeV)	h_4 (MeV)	m	Σ (MeV^2)
O(4)	O(4)	3.31	0.000	0.0000	0.084	0.039	1.00	2.02
U(3)	U(3)	3.11	0.296	−0.0006	0.057	0.000	0.00	3.30
· · ·	O(4)	3.20	0.012	0.0000	0.084	0.039	0.38	2.01
· · ·	U(3)	3.10	0.291	−0.0006	0.058	0.0002	0.000	3.10
· · ·	1st	5.56	−0.222	0.0001	0.091	0.046	0.003	1.98
· · ·	2nd	3.06	0.037	−0.0010	0.084	0.039	0.06	2.01
· · ·	3rd	2.78	0.190	−0.0138	0.088	0.049	0.003	1.84

sponding to the dynamical symmetries have been fitted to the experimental one using the energy formulas[3]

$$E^{\mathrm{I}}=\alpha\omega(\omega+2)+\beta L(L+1)+\epsilon\ ,$$
$$E^{\mathrm{II}}=\gamma n_\pi+\delta n_\pi(n_\pi+2)+\beta L(L+1)+\epsilon\ , \tag{2}$$

which correspond to the dynamical symmetries (I) and (II), respectively. Here ω is related to the irreducible representations of the O(4) group in the same way as L and n_π are related to those of the O(3) and U(3) groups.[4] The α, β, γ, δ, and ϵ parameters could be uniquely determined, i.e., their final values were independent of the initial parameter set. These results, as well as other values, have been used as input data for the calculation with the Hamiltonian of Eq. (1). It turned out that in this case one can find several local minima of the sum of the squared differences.

When we deal with a situation intermediate between the two dynamical symmetries then it is straightforward to measure the distances from these limiting cases. To this end we have chosen the quantity m defined with the coefficients of the Casimir operators in the Hamiltonian

$$H=\alpha C_{2O4}+\beta C_{2O3}+\gamma C_{1U3}+\delta C_{2U3}+\epsilon\ , \tag{3}$$

$$m=\frac{\alpha^2}{\alpha^2+\gamma^2+\delta^2}\ . \tag{4}$$

Here C_{2O4} is the quadratic Casimir operator of the O(4) group, C_{1U3} is the linear Casimir operator of the U(3) group, etc. Obviously, $m=0$ means that the U(3) symmetry is valid, while $m=1$ means that the O(4) symmetry is valid.

Table I contains some illustrative results of our calculations. In the last column we listed Σ which is defined as $\Sigma=\Sigma_i(E_i^{\mathrm{exp}}-E_i^{\mathrm{th}})^2$. The h_3 rotational parameter is close, in each case, to that of Ref. 1, 0.076 MeV, and so, to that of a dumbbell configuration consisting of two, touching ^{12}C nuclei.

It was an interesting finding that the O(4) symmetry fits better to the experimental spectrum, yet we found the best agreement with a parameter set that gives a rather low m value: $m=0.003$, i.e., a point of the one dimensional phase diagram quite close to the U(3) limit (Fig. 2). Calculations for the experimental spectrum having a slightly different band structure in which the positions of the 0^+ and 2^+ states are those of Ref. 1 gave the best fit also at $m=0.003$ with parameters very close to those of the last line of Table I. We have made several attempts, without success, to obtain a better agreement between the experimental and the model spectra by allowing different classification for the two dynamical symmetries.[6]

To sum up, as for the possibility of parametrizing the quasimolecular spectra in terms of the nuclear vibron model, our calculations with the general case of the Hamiltonian gave some further support. As for the presence of the dynamical symmetries, however, it is not easy to make a definite choice. Our results prefer a position very close to the U(3) limit, but this preference is not very strong. To improve the situation, more complete experimental data and the use of more elaborate versions of the model would be needed.

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