Inelastic electron scattering form factors involving the second excited 2^+ levels in the nuclei ^{48}Ti and ^{50}Cr

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A microscopic description of the recent data on the Coulomb form factors for the $0^+_{g.s.} \rightarrow 2^+_2$ transitions in the nuclei ^{48}Ti and ^{50}Cr is attempted in terms of the prolate and oblate intrinsic states resulting from realistic effective interactions operating in the $2p$-$1f$ shell. The results for the higher momentum-transfer region show significant improvements compared to the form factor estimates obtained in some recent shell model calculations involving the $f_{7/2} + f_{7/2}^{-1} p_{3/2}$ configurations.

Inelastic electron scattering provides significant information concerning nuclear structure.[1,2] Measurement of the cross section for the scattered electron permits one to determine the Fourier transform of the charge and nuclear-current densities for the nuclear states involved in the transition. In contrast with the experimental studies with ordinary γ transitions, the spatial structure of the nuclear matrix elements is expected to provide a sensitive test of the nuclear wave functions as well as the transition operators involved in the theoretical description.

Some recent inelastic electron scattering experiments[3–5] have provided valuable data on the $0^+_{g.s.} \rightarrow 2^+_1$ as well as $0^+_{g.s.} \rightarrow 2^+_2$ transitions in the $N=26$ nuclei ^{48}Ti and ^{50}Cr. Within a measured momentum-transfer range up to $q_{\text{eff}} \sim 2$ fm^{-1}, the $C2$ form factors involving $0^+_{g.s.} \rightarrow 2^+_1$ transitions are characterized by two peaks appearing at $q_{\text{eff}} \sim 0.7$ and 1.7 fm^{-1}. Although the qualitative features of the form factors associated with $0^+_{g.s.} \rightarrow 2^+_2$ transitions are quite similar to those of the observed form factors for the $0^+_{g.s.} \rightarrow 2^+_1$ transitions, the magnitudes of $|F|^2$ for the former transitions are smaller by an order of magnitude than those for the latter.

Recently Iwamoto *et al.*[6] studied the $C2$ form factors involving the first and the second excited 2^+ states in some Ti and Cr isotopes in terms of the wave functions resulting from empirical effective interactions operating in the restricted valence space consisting of the $f_{7/2} + f_{7/2}^{-1} p_{3/2}$ configurations. In their work, Iwamoto *et al.*[6] employed the empirical effective interactions given by Oda *et al.*[7] as well as by Yokoyama *et al.*[8] The results for the low momentum-transfer ($q_{\text{eff}} < 1.5$ fm^{-1}) region of $|F|^2(0^+_{g.s.} \rightarrow 2^+_1)$ were consistent with experiments; the results were also quite stable towards the choice of different effective interactions. However, the choice of the $f_{7/2} + f_{7/2}^{-1} p_{3/2}$ space proved too restrictive to permit an unambiguous interpretation of the available data for the high momentum-transfer part of the $0^+_{g.s.} \rightarrow 2^+_1$ transition, as well as the $0^+_{g.s.} \rightarrow 2^+_2$ transition involving 0.5 fm$^{-1} < q_{\text{eff}} < 2.0$ fm^{-1}; the calculated results displayed large variation with respect to the choice of the effective interactions. Furthermore, none of the two effective interactions yielded acceptable simultaneous agreement for the $0^+_{g.s.} \rightarrow 2^+_1$ as well as the $0^+_{g.s.} \rightarrow 2^+_2$ transitions, although relatively large values of the neutron effective charges $-e_n = 1.0e$ and $e_n = 1.2e$ for the nuclei ^{48}Ti and ^{50}Cr, respectively—were employed.

We have recently shown[9] that the yrast wave functions projected from the Hartree-Fock-Bogolubov (HFB) intrinsic states of prolate shapes, resulting from a slightly modified version of the Kuo-Brown (KB) effective interaction[10,11] for the full $2p$-$1f$ space, provide a good description of the observed form factors for the $0^+_{g.s.} \rightarrow 2^+_1$ transitions in several $2p$-$1f$ shell nuclei. The purpose of this paper is to show that a fairly satisfactory interpretation of the available $|F|^2$ data for the $0^+_{g.s.} \rightarrow 2^+_2$ transitions in the nuclei ^{48}Ti and ^{50}Cr can also be attempted in terms of reasonable values of the proton and neutron effective charges by describing the yrare states in terms of the oblate variational solutions resulting from the modified KB interaction.

The details concerning the calculation of the inelastic scattering form factors in terms of wave functions projected from single HFB intrinsic state have been discussed in our earlier work.[9] In the framework of the plane wave Born approximation (PWBA), the squared form factors $|F|^2$ ($0^+_{g.s.} \rightarrow 2^+_2$), however, involve angular momentum projection from two different intrinsic states:

$$|F(q)|^2 = \frac{4\pi}{Z^2} \frac{1}{2J_i+1} |\langle J_f(\text{oblate}) || f^{(2)} || J_i(\text{prolate}) \rangle|^2 , \tag{1}$$

where the one-body operator $f^{(2)}$ is given by

$$f^{(2)} = \sum_k e_k j_2(qr_k) Y^{(2)}_M(\Omega_k) . \tag{2}$$

Here, e_k is the effective charge on the kth nucleon. The matrix element appearing in Eq. (1) can be computed from Eq. (8) given in Ref. 9 by carrying out appropriate changes in the density matrix $\rho(\theta)$.

Table I summarizes our results concerning the self-consistent intrinsic states associated with the yrast as well as the yrare bands in the nuclei ^{48}Ti and ^{50}Cr. It can easily be seen that the calculated energy separation between the prolate and oblate variational solutions in these nuclei are consistent with the observed excitation energies of the 2^+_2 states relative to the $O^+_{g.s.}$ states. The difference in the intrinsic energies of the prolate and oblate solutions represents roughly the difference in the energies of the $J^\pi = 0^+$ states projected from these solutions. Further, the excitation energies of the 2^+_2 states relative to the 0^+_2 states are usually about 1 MeV in the nuclei with $N=26$ in the $2p$-$1f$ shell; this assumption is supported by earlier calculation[12] involv-

TABLE I. Details of the variational intrinsic states associated with the yrast as well as the yrare levels in the nuclei ^{48}Ti and ^{50}Cr. The intrinsic quadrupole moments have been given in units of b^2, where $b=(\hbar/M\omega)^{1/2}$. Here $\langle Q_0^2\rangle_\pi(\langle Q_0^2\rangle_\nu)$ gives the contribution of protons (neutrons) towards the total quadrupole moment.

Nucleus	Variational solution	E (MeV)	$\langle Q_0^2\rangle(\langle Q_0^2\rangle_\pi, \langle Q_0^2\rangle_\nu)$
^{48}Ti	Prolate	−17.5	16.7(6.8, 9.9)
	Oblate	−16.2	−13.7(−6.0, −7.7)
^{50}Cr	Prolate	−29.9	25.5(12.7, 12.8)
	Oblate	−28.1	−12.7(−6.5, −6.2)

ing explicit angular momentum projection on the intrinsic states with quadrupole deformations quite similar to the ones calculated here, as well as by the observed $[E(2_2^+)-E(0_2^+)]$ differences in the nuclei ^{48}Ti and ^{50}Cr. Thus, the results given in Table I imply that the $J^\pi=2_2^+$ states projected from the oblate intrinsic wave function in the nuclei ^{48}Ti and ^{50}Cr are likely to occur at 2.3 and 2.8 MeV, respectively, relative to the ground states. These estimates are remarkably close to the observed excitation energies of the 2_2^+ states—2.42 and 2.92 MeV in the nuclei ^{48}Ti and ^{50}Cr, respectively. It is thus reasonable to expect that an adequate description of the observed $0_{g.s.}^+$ and 2_2^+ states can be obtained by projecting them from the prolate and oblate intrinsic wave functions, respectively.

Figure 1 compares the calculated and experimental $|F|^2$ for the $0_{g.s.}^+\rightarrow 2_1^+$ as well as $0_{g.s.}^+\rightarrow 2_2^+$ transitions in the nucleus ^{48}Ti. We have also given here the results obtained by Iwamoto *et al.*[6] in the framework of the shell model involving the $f_{7/2}+f_{7/2}^{-1}p_{3/2}$ configurations. In the earlier work, two sets of empirical effective interactions were employed. One was the set of matrix element obtained by Oda *et al.*[7] through a chi-square fit to the 38 levels chosen from Ca and Sc isotopes and the other was the set obtained by Yokoyama *et al.*[8] through a fit to the 63 data selected from the $N=27$ and 28 isotones ($A=47$–55). Hereafter we shall refer to these sets of matrix elements as the V(Ca–Sc) and the V(27–28) interactions, respectively.

The results presented in Fig. 1 bring out the sensitivity of the restricted shell model calculations for the high momentum-transfer part of the form factors associated with the $0_{g.s.}^+\rightarrow 2_{1,2}^+$ transitions towards the choice of the effective interactions. Furthermore, none of the effective interactions employed earlier in conjunction with the $f_{7/2}+f_{7/2}^{-1}p_{3/2}$ space provides an adequate *simultaneous* interpretation of the available data involving *both* the $0_{g.s.}^+\rightarrow 2_1^+$ as well as the $0_{g.s.}^+\rightarrow 2_2^+$ transitions. Whereas the interaction V(Ca–Sc) leads to reasonable agreement with the experiments for $q_{\rm eff}>1.5$ fm^{-1} in the case of the $0_{g.s.}^+\rightarrow 2_1^+$ transition, the calculated values (not shown in the figure) for high momentum transfer involving the $0_{g.s.}^+\rightarrow 2_2^+$ transition are smaller by more than an order of magnitude. The shell model estimates for $|F|^2$ around its second maximum obtained with the interaction V(27–28), on the other hand, are smaller than the observed ones by roughly a factor of two for the $0_{g.s.}^+\rightarrow 2_1^+$ as well as the $0_{g.s.}^+\rightarrow 2_2^+$ transitions.

It is seen that the present calculation yields an adequate overall quantitative description of the available data for both the $0_{g.s.}^+\rightarrow 2_1^+$ as well as the $0_{g.s.}^+\rightarrow 2_2^+$ transitions. In particular, the PHFB values for the squared form factor around $q_{\rm eff}=1.7$ fm^{-1} show substantial improvements over the earlier results. Our choice of the neutron effective charge also contrasts keenly with the values employed in the earlier work. The present calculation involves the effective charges $(e_p,e_n)=(1.4e,0.4e)$, whereas the restricted shell model calculation employed $(e_p,e_n)=(1.4e,1.0e)$.

A significant discrepancy between the calculated and the observed results occurs around the first maximum associated with the $0_{g.s.}^+\rightarrow 2_2^+$ transition where the PHFB results overestimate the observed values by about a factor of 2. In view of the sensitivity of the form factors for the $0_{g.s.}^+\rightarrow 2_2^+$ transition towards small configurational admixtures, an inclusion of the quasiparticle excitations in the oblate HFB states is likely to result in further improvements.

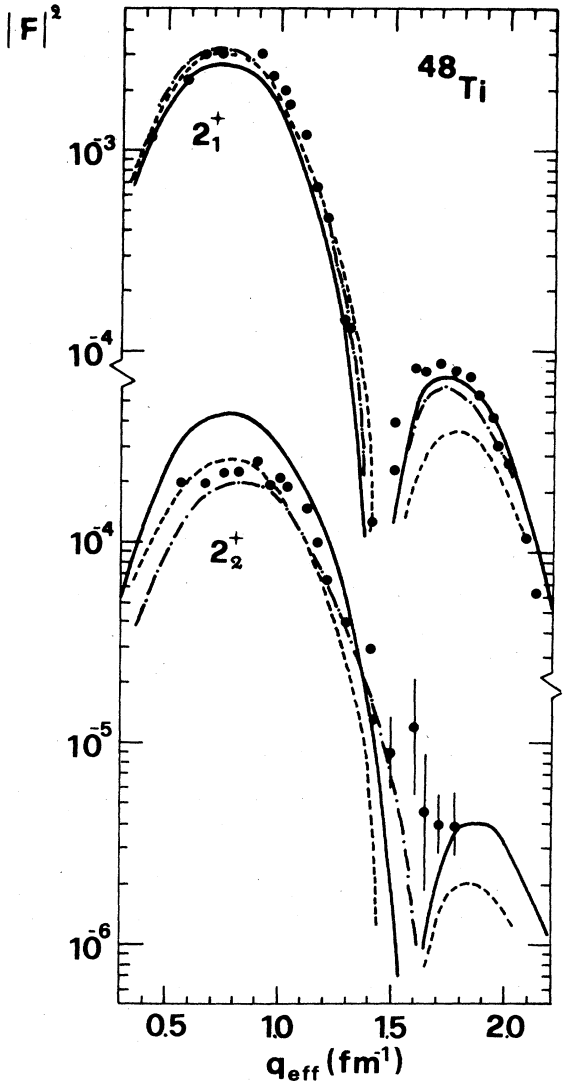

FIG. 1. Experimental[3,4] and calculated squared form factors $|F|^2$ for the $0_{g.s.}^+\rightarrow 2_{1,2}^+$ transitions in the nucleus ^{48}Ti. The broken and the dotted-dashed curves represent the results obtained by Iwamoto *et al.* (Ref. 6) with the empirical effective interactions V(27–28) and V(Ca–Sc), respectively, in conjunction with the $f_{7/2}+f_{7/2}^{-1}p_{3/2}$ configurations. The solid curves give the results obtained in the present calculation.

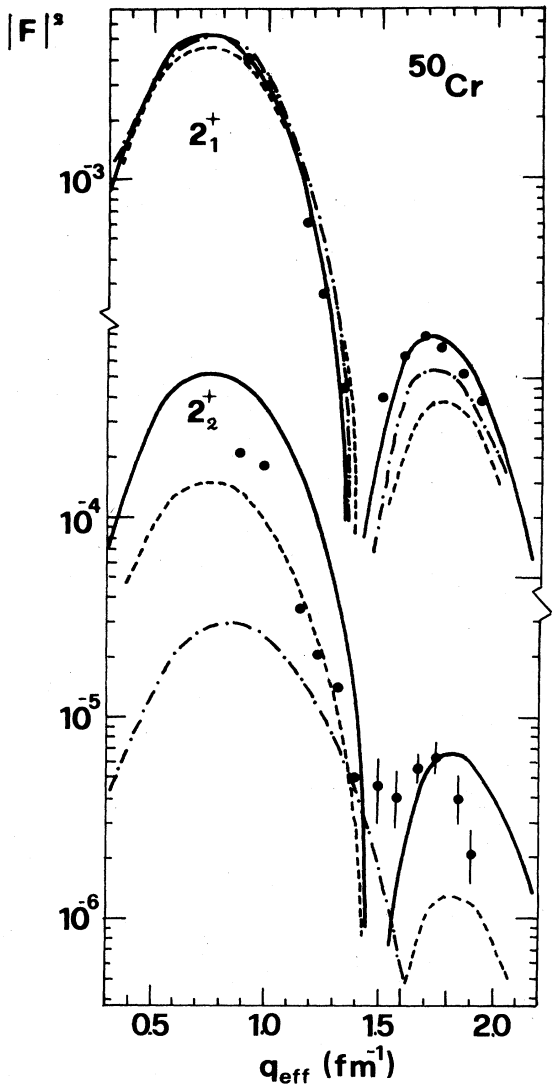

FIG. 2. Experimental (Ref. 5) and calculated squared form factors $|F|^2$ for the $0^+_{g.s.} \rightarrow 2^+_{1,2}$ transitions in the nucleus ^{50}Cr.

We next present, in Fig. 2, the results for the nucleus ^{50}Cr. We have employed the same set of effective charges—$e_p = 1.4e$ and $e_n = 0.4e$—as the ones employed in the nucleus ^{48}Ti. The effective charges employed in the earlier shell model calculation, however, are $(e_p, e_n) = (1.6e, 1.2e)$. In view of the relatively large values of the calculated intrinsic quadrupole moments for the yrast states (see column 4, Table I), one expects an enhanced involvement of the configurations outside the $(f_{7/2} + f_{7/2}^{-1} p_{3/2})$ space in the nucleus ^{50}Cr. The increased values of the effective charges employed in the earlier work[6] are intended to simulate this effect to some extent.

In the nucleus ^{48}Ti only the part of the calculated form factor for the $0^+_{g.s.} \rightarrow 2^+_2$ transition involving $q_{eff} > 1.5$ fm^{-1} is found to be sensitive towards the choice of the effective interaction employed in the restricted shell model work. In contrast to this, even the low momentum-transfer part of the restricted shell model estimates for $|F|^2(0^+_{g.s.} \rightarrow 2^+_2)$ displays significant interaction dependence in the nucleus ^{50}Cr; the $|F|^2$ values resulting from V(Ca–Sc) and V(27–28) differ by more than a factor of 5 throughout the range $0.5 \leq q_{eff} \leq 2.0$ fm^{-1}. Furthermore, both the interaction V(Ca–Sc) as well as V(27–28) fail to reproduce even qualitatively the observed form factors for the $0^+_{g.s.} \rightarrow 2^+_2$ transition; the magnitude of $|F|^2$ around its second minimum are much too low compared with the experiments.

The use of the projected HFB wave functions for the 2^+_2 state leads to the required enhancement around the second maximum. One also observes a shift of the dip in the $|F|^2$ values towards the lower momentum-transfer region. Although the PHFB results for the $0^+_{g.s.} \rightarrow 2^+_2$ form factors for $q_{eff} \sim 0.8$ fm^{-1} are still somewhat larger compared to the experiments, the present calculation is seen to yield significant improvement vis-á-vis simultaneous description of the form factors for the $0^+_{g.s.} \rightarrow 2^+_1$ as well as the $0^+_{g.s.} \rightarrow 2^+_2$ transitions in the nucleus ^{50}Cr.

As mentioned earlier, an important feature that characterizes the observed form factors for the $0^+_{g.s.} \rightarrow 2^+_2$ transitions in the nuclei ^{48}Ti and ^{50}Cr is their reduced magnitude—by an order of magnitude—compared to those for the $0^+_{g.s.} \rightarrow 2^+_1$ transitions. Present microscopic description offers a qualitative understanding of this feature of the $0^+_{g.s.} \rightarrow 2^+_2$ transitions in terms of the reduced overlap between the initial $(0^+_{g.s.})$ and the final (2^+_2) states due to their different intrinsic parentage.

Summarizing, we have shown that the use of the yrast and yrare states with $J^\pi = 0^+, 2^+$ projected from self-consistent prolate and oblate intrinsic states resulting from the modified KB interaction for the $2p$-$1f$ shell permits a reasonably adequate simultaneous description of the observed data involving the $0^+_{g.s.} \rightarrow 2^+_1$ as well as $0^+_{g.s.} \rightarrow 2^+_2$ transitions in the nuclei ^{48}Ti and ^{50}Cr.

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