Insensitivity of weak interaction amplitudes to relativistic nuclear dynamics

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The influence of strong relativistic nuclear dynamics on weak nuclear processes is examined. The constraint of partial conservation of the axial current is shown to render the weak nuclear amplitudes insensitive to the relativistic nuclear dynamics.

Relativistic Dirac based models of the nucleon-nucleus interaction have been developed over the past several years with reasonable success.[1] The dominant feature of these models is the existence of strong attractive (Lorentz) scalar and repulsive (Lorentz) vector interactions with typical strengths of −450 MeV and +350 MeV, respectively. Such strengths make the nuclear system intrinsically relativistic. Recently, a fully constrained approach to nucleon-nucleus scattering, based on a relativistic generalization of the impulse approximation, has successfully described a large body of intermediate energy elastic scattering data.[2] The relativistic optical potentials emerging from that effort are consistent with the large strengths determined in the earlier relativistic models, and open the possibility for an understanding of both the nuclear structure properties and nuclear scattering phenomena from a single theoretical perspective.

As part of this larger effort, we recently studied the impact of the strong relativistic dynamics on nuclear single particle wave functions with the intention of seeking a relatively unambiguous signature of these dynamics in electron scattering [Ref. 3, and hereafter referred to as SRSM (Shepard, Rost, Siciliano, and McNeil)]. We found that in most cases the relativistic dynamics had a negligible effect. However, for the special case of transverse isoscalar transitions, a substantial effect was found, the relativistic form factor being roughly twice its nonrelativistic counterpart. This increase is a direct consequence of the strong relativistic dynamics of the bound nuclear system. Experiments to test this conclusion have been proposed.

Given this dramatic result in the electromagnetic sector, one must ask whether a further signature of the relativistic dynamics can be seen in weak nuclear processes as well. In this work we examine this question and find that weak processes will be largely insensitive to the relativistic dynamics. The vector part of the weak nuclear current is insensitive due to its isovector character (see SRSM). A naive examination of the axial part of the weak nuclear current, however, suggests potential sensitivity, but imposing the constraint of partial conservation of the axial current (PCAC) in fact renders the axial current insensitive as well.

In any discussion of weak nuclear processes the object of central importance is the weak nuclear current,

$$\tilde{\mathcal{J}}^{\mu}=\tilde{\mathcal{J}}^{\mu}(V)+\tilde{\mathcal{J}}^{\mu}(A) \ , \tag{1}$$

where V and A refer to the vector and axial vector pieces of the weak current, respectively. In impulse approximation, the nuclear current matrix elements are defined by

$$\mathcal{J}^{\mu}_{fi}=\langle f|\gamma^{0}\tilde{\mathcal{J}}^{\mu}|i\rangle \ , \tag{2}$$

where $i(f)$ refers to the initial (final) nuclear state. Consider the vector current first. In accordance with the conserved vector current (CVC) hypothesis, we have

$$\tilde{\mathcal{J}}^{\mu}(V)=(J^{\mu}_{\rm EM})_{T=1}=\left[F_1\gamma^{\mu}+\frac{iK}{2m}F_2\sigma^{\mu\nu}q_{\nu}\right]_{T=1} \ , \tag{3}$$

where $J^{\mu}_{\rm EM}$ is the free nuclear electromagnetic current operator. As discussed in SRSM, the $\vec{\gamma}$ piece of the first term in $J^{\mu}_{\rm EM}$ is the only sizeable term which is sensitive to relativistic nuclear dynamics, and it is dominant only for isoscalar transverse transitions for which (1) the Coulomb amplitude is absolutely forbidden and (2) the second term is small because the isoscalar anomalous magnetic moment is small ($k_{T=0}=-0.12$, $k_{T=1}=3.70$). Since only the isovector portion of $J^{\mu}_{\rm EM}$ is relevant for weak interactions, relativistic dynamics can be expected to have no discernable signature in matrix elements of the vector piece of the weak current. (Except perhaps through coincidence measurements where the role of small amplitudes can be emphasized through interference effects.) To see this, we introduce the following matrices which act only in Dirac upper/lower component space (see SRSM):

$$\Gamma_1=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ \Gamma_2=\begin{bmatrix}1&0\\0&-1\end{bmatrix},\ \Gamma_3=\begin{bmatrix}0&1\\1&0\end{bmatrix},\ \Gamma_4=\begin{bmatrix}0&1\\-1&0\end{bmatrix} \ . \tag{4}$$

Using the Γ's, we can separate the spin dependence and the upper/lower component dependence of the Dirac matrices. The essential point to remember is that the strong relativistic dynamics significantly affect the lower components only, so only matrix elements of operators off diagonal in component space will be substantially affected by these dynamics (see SRSM). Consider the vector current first. The timelike (Coulomb) piece of the vector current, for example, has the component structure,

$$\gamma^{0}\gamma^{0}=\Gamma_1 \ , \tag{5}$$

which is diagonal in component space and hence will be insensitive to relativistic dynamics. In contrast, we have

$$\gamma^{0}\vec{\gamma}=\vec{\sigma}\Gamma_3 \ , \tag{6}$$

implying that matrix elements of the spacelike piece of the vector current will be sensitive to relativistic dynamics.

However, the competing spacelike piece from the anomalous magnetic moment has the structure,

$$\frac{i\kappa}{2m}\gamma^0\sigma^{i\nu}q_\nu=\frac{i\kappa}{2m}(\vec{\sigma}\times\vec{q})^i\Gamma_2 \ , \tag{7}$$

which is diagonal in component space and therefore insensitive to the relativistic dynamics. Usually the diagonal terms dominate, but for transverse isoscalar transitions the anomalous magnetic moment is small allowing one to see the relativistically sensitive term, Eq. (6). These results for the vector current were demonstrated with explicit calculations in SRSM for electron scattering.

Turning to the axial vector current, we note that the point axial vector current operator, $\gamma^\mu\gamma^5$, is significantly corrected by hadronic processes, the principal one being that due to virtual emission of a pion. Including the pion induced contribution, the free axial vector weak current operator can have one of two possible forms depending on whether we choose pseudoscalar or pseudovector coupling for the πNN vertex,

$$\mathcal{J}^\mu(A)=\alpha\left[F_A\gamma^\mu\gamma^5+F_P\frac{2m}{q^2-\mu^2}\gamma^5q^\mu\right], \text{ pseudoscalar} \tag{8a}$$

$$=\alpha\left[F_A\gamma^\mu\gamma^5-F_P\frac{q_\nu\gamma^\nu\gamma^5}{q^2-\mu^2}q^\mu\right], \text{ pseudovector} \ , \tag{8b}$$

where $\alpha=g_A/g_V=-1.23\pm0.01$, μ is the pion mass, and

$$1=F_A(0)=F_P(\mu^2)\approx F_A(0) \ . \tag{8c}$$

The coefficient of the second term in each form of $\mathcal{J}^\mu(A)$ has been chosen to be consistent with partial conservation of the axial current (PCAC), i.e.,

$$\lim_{\mu\to0}\partial_\nu\bar{u}_f\mathcal{J}^\nu(A)u_i=-i\lim_{\mu\to0}q_\nu\bar{u}_f\mathcal{J}^\nu(A)u_i=0 \ , \tag{9}$$

where $q=p_i-p_f$.

We note that the two forms, (8a) and (8b), are equivalent only for free nucleons because the proof of equivalence requires that the nucleon spinors, u, satisfy the free Dirac equation,

$$(\gamma_\mu p^\mu-m)u=\bar{u}(\gamma_\mu p^\mu-m)=0 \ . \tag{10}$$

The two forms, in general, will not be equivalent to the nuclear environment.

The timelike piece of the first term of the axial vector current operator is of particular interest because its Dirac component structure,

$$\gamma^0\gamma^0\gamma^5=\Gamma_3 \ , \tag{11}$$

is such that its matrix elements should be sensitive to the relativistic dynamics of nuclear wave functions. On the other hand, the spacelike piece of the first term in the axial vector current is

$$\gamma^0\vec{\gamma}\gamma^5=\vec{\sigma}\Gamma_1 \ , \tag{12}$$

which will be insensitive to the relativistic dynamics. Since it is this operator which gives rise to the usual allowed Gamow-Teller transitions $(\Delta L=0, \Delta J=\Delta S=1, \Delta\pi=0)$, we do not expect to observe relativistic effects in the corresponding β decays.

We turn now to the pionic contribution to the axial current which may alter these conclusions. In deciding which form (pseudoscalar or pseudovector) to employ, it is useful to invoke the PCAC hypothesis as a constraint. We see that the PCAC condition, Eq. (9), is automatically met by the axial vector current operator based on the pseudovector πNN coupling $(F_A=F_P)$. We have

$$\lim_{\mu\to0}q_\nu\mathcal{J}^\nu_{fi}(A)=\lim_{\mu\to0}\alpha\bar{\Psi}_f\left[F_Aq_\nu\gamma^\nu\gamma^5-F_P\frac{q_\nu\gamma^\nu q^2}{q^2-\mu^2}\gamma^5\right]e^{i\vec{q}\cdot\vec{r}}\Psi_i$$
$$=0 \ , \tag{13}$$

independent of the initial and final nuclear states, i.e., it is true as an operator statement. In particular, PCAC is obeyed when the free spinors of Eq. (10) are replaced by the bound state spinors, Ψ, which satisfy a Dirac equation containing potentials of arbitrary Lorentz character. For example, here we consider the scalar and timelike vector potentials giving the following Dirac equation:[1]

$$[\gamma_\mu p^\mu-m-S(r)-\gamma^0V(r)]\Psi_{i,f}=0 \ . \tag{14}$$

In contrast, we find that using the form of $\tilde{g}^\mu(A)$ based on pseudoscalar coupling, Eq. (8a), we have

$$\partial_\nu\mathcal{J}^\nu_{fi}(A)=-i\bar{\Psi}_f\left[2\mathcal{M}(r)F_A-\frac{2mF_Pq^2}{q^2-\mu^2}\gamma^5\right]e^{i\vec{q}\cdot\vec{r}}\Psi_i \ , \tag{15}$$

where use has been made of the bound state Dirac equation, Eq. (14), and where we have defined $\mathcal{M}(r)=m+S(r)$. Using the conditions of Eq. (8c), we have

$$\lim_{\mu\to0}\partial_\nu\mathcal{J}^\nu_{fi}(A)=-2i\alpha\lim_{\mu\to0}\bar{\Psi}_f\left[\frac{[\mathcal{M}(r)-m]q^2-\mu^2\mathcal{M}(r)}{q^2-\mu^2}\right]$$
$$\times\gamma^5e^{i\vec{q}\cdot\vec{r}}\Psi_i\neq0 \ ; \tag{16}$$

because $\mathcal{M}(r)\neq m$. As it stands, the pseudoscalar based form of, Eq. (8a), must be discarded because it is not consistent with the PCAC hypothesis when used with Dirac bound state wave functions. One can recover the PCAC condition for pseudoscalar coupling by considering the nuclear mass term in Eq. (8a) to be the operator $\mathcal{M}(r)=m+S(r)$. This remedy of the pseudoscalar form leads to a constraint condition on the timelike part of the axial vector current (the potentially relativistically sensitive part) equivalent to that obtained using the pseudovector form. However, we find the model dependence of the remedy distasteful. Furthermore, the pseudovector choice is consistent with chiral invariant models of the πN system and has other attractive features as well. On the other hand, the pseudoscalar choice has a history of problems in Dirac calculations.[4] For these reasons from this point on we will confine our attention to the pseudovector based form, Eq. (8b).

We now observe that the four-divergence of the nuclear axial vector current is given by

$$q_\mu\mathcal{J}^\mu_{fi}(A)=\alpha\bar{\Psi}_f\left[F_Aq_\mu\gamma^\mu\gamma^5-F_P\frac{q_\mu\gamma^\mu q^2}{q^2-\mu^2}\gamma^5\right]e^{i\vec{q}\cdot\vec{r}}\Psi_i$$
$$=\frac{2\alpha\mu^2F_A}{q^2-\mu^2}e^{i\vec{q}\cdot\vec{r}}\bar{\Psi}_f\mathcal{M}(r)\gamma^5e^{i\vec{q}\cdot\vec{r}}\Psi_i \ , \tag{17}$$

where we have again used Eq. (14). We can rewrite Eq. (17) as

$$\mathcal{J}^0_{fi}(A)=q_0^{-1}\vec{q}\cdot\vec{\mathcal{J}}_{fi}(A)+2q_0^{-1}\frac{\alpha\mu^2F_A}{q^2-\mu^2}\bar{\Psi}_f\mathcal{M}(r)\gamma^5e^{i\vec{q}\cdot\vec{r}}\Psi_i \ . \tag{18}$$

This expression tells us a great deal about $0^\pm\to0^\mp$ weak transition amplitudes which can only proceed via the timelike piece of the axial vector operator which we have previously speculated to be sensitive to relativistic dynamics. For such transitions, $\vec{\mathscr{J}}_{fi}(A)=0$, leaving only the second term on the right-hand side of Eq. (18).

The significance of this result is that the amplitude is proportional to the matrix element of the operator $\gamma^5\mathscr{M}(r)/m$. Letting $\mathscr{E}=E-V(r)$, the relativistic dynamics increase lower components by a factor of $(E+m)/(\mathscr{E}+\mathscr{M})$ (≈ 1.7 in the nuclear interior) over the values obtained using the free Dirac relation. By itself, this would increase the matrix element by a similar amount, resulting in the anticipated sensitivity to relativistic dynamics. However, the factor $\mathscr{M}(r)/m$ almost exactly cancels the $(E+m)/(\mathscr{E}+\mathscr{M})$ factor, implying that is, in fact, insensitive to relativistic dynamics. We conclude that relativistic models of nuclear structure cannot be discriminated from nonrelativistic models on the basis of their predictions of nuclear weak interaction observables such as β-decay rates.

Remembering that the PCAC hypothesis relates the four-divergence of the axial vector current to the strength of the pion field,[5] we can use Eq. (17) to write

$$\Phi_\pi(q)=i\frac{g_{\pi NN}\sqrt{2}\mu^2F_A}{q^2-\mu^2}\overline{\Psi}_f\frac{\mathscr{M}(r)}{m}\gamma^5 e^{\vec{q}\cdot\vec{r}}\Psi_i \ , \tag{19}$$

where Φ_π is the pion field strength and $g_{\pi NN}$ is the πNN coupling constant. This result tells us that the pion field strength inside the nucleus is also insensitive to relativistic dynamics and that related effects such as π-meson-exchange currents[6] will be similarly insensitive. This is a consequence of the pseudovector πNN coupling to which we were led by PCAC.

In this work we have shown that matrix elements of the weak nuclear current operator are essentially unaffected by the strong relativistic dynamics of current interest. The isovector character of the vector current insures the dominance of matrix elements diagonal in component space and, therefore, insensitive to strong relativistic dynamics. For the axial part of the weak current, the constraint of PCAC introduces a factor of $\mathscr{M}(r)$ which compensates the expected enhancement of matrix elements of the axial timelike current. This insensitivity extends to pionic exchange effects as well.

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