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Isospin composition of giant resonances and asymmetries in π^+ compared to π^- inelastic scattering

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A fully microscopic calculation for pion inelastic excitation of giant resonances in which random phase approximation transition densities are used is presented for the first time. The mixing of isoscalar and isovector modes is incorporated in the random phase approximation calculations and its effect on $\sigma(\pi^-)$ vs $\sigma(\pi^+)$ is discussed. Comparison with existing data is made and reasonable agreement is achieved.

Nuclear giant resonances are considered as fundamental modes of nuclear motion and are usually classified as either isoscalar or isovector modes of vibration (labeled $\tau=0$ or $\tau=1$, respectively). In terms of the constituent nucleons, $\tau=0$ and $\tau=1$ modes consist of particle-hole (p-h) pairs whose isospins indicate a charge-symmetric state and a charge-antisymmetric state. For a p-h pair in $N>Z$ nuclei, however, charge symmetry is not an exact symmetry, and the classification of giant resonances as either a $\tau=0$ or a $\tau=1$ mode is not precise. In such nuclei, mixing between these modes is expected from symmetry-breaking p-h excitations involving the excess neutrons only or involving the protons whose neutron analogs are excluded by the excess neutrons due to the Pauli principle. This mixing of modes should not be confused with isospin mixing between the *total* isospins obtained by coupling the p-h isospins to the ground-state isospin $T=(N-Z)/2$.

In addition to symmetry-breaking p-h configurations, a small amount of mode mixing is expected from charge-symmetry-breaking parts of the Hamiltonian itself, such as the Coulomb interaction. (This is the only source of mode mixing for an $N=Z$ nucleus and in this case is also isospin mixing.) Still, the classification of modes into $\tau=0$ and $\tau=1$ is meaningful for the case of collective states in medium and heavy nuclei since, as implied by the properties of the nucleon-nucleon interaction,[1] the mixing should be quite small.

To understand processes that may be sensitive to small amounts of mixing, it is necessary to include this effect when calculating these processes. For example, a meaningful comparison among various electromagnetic and hadronic probes that inelastically excite the same giant resonance relies upon the detailed isospin composition of the giant resonance. One process in particular is pion inelastic scattering. The comparison of π^+ to π^- inelastic scattering around the (3,3) resonance to the same nuclear state has proven to be very useful in determining isospin mixing and the proton-neutron content of states in light nuclei.[2] Recently, π^+ and π^- inelastic scattering for ^{118}Sn and ^{208}Pb was performed[3,4] and in the case of the giant isoscalar quadrupole the ratio of cross sections $R=\sigma(\pi^-)/\sigma(\pi^+)$ in the region of the first maxima was observed to be $R\simeq 1.9$ for ^{118}Sn and $R\simeq 2.8$ for ^{208}Pb.

We present results of a fully self-consistent, unrestricted p-h continuum random phase approximation (RPA) calculation[5] of isoscalar-isovector mode mixing. We use the results to calculate π^+ and π^- inelastic scattering to giant electric quadrupole resonances and compare these cross sections to those recently measured.[4,5]

Giant resonances are related to strength distributions corresponding to one-body operators of either isoscalar or isovector type,

$$Q^{(0)}=\sum_{k=1}^{A} f(\mathbf{r}_k,\boldsymbol{\sigma}_k) \quad\text{or}\quad Q^{(1)}=\sum_{k=1}^{A} f(\mathbf{r}_k,\boldsymbol{\sigma}_k)\tau_\mu(k) \ . \tag{1}$$

Here $f(\mathbf{r}_k,\boldsymbol{\sigma}_k)$ denotes a function of space and spin, and $\tau_\mu(\mu=0,\ \pm 1)$ are the nucleon isospin operators. In this work we consider only the $\mu=0$ component. If, for a given nuclear state $|i\rangle$, the transition strengths

$$S_i(\tau)=|\langle i|Q^{(\tau)}|0\rangle|^2, \quad \text{for } \tau=0,1 \tag{2}$$

are such that $S_i(0)>>S_i(1)$ $[S_i(1)>>S_i(0)]$, then this state is predominantly a $\tau=0(\tau=1)$ mode. The nuclear response can also be discussed in terms of the neutron and proton transition strengths. In this case the two relevant operators are

$$Q^{(\mathrm{n,p})}=\sum_{k=1}^{A} f(\mathbf{r}_k,\boldsymbol{\sigma}_k)\tfrac{1}{2}[1\pm\tau_0(k)] \ . \tag{3}$$

Our calculations of the $J=2^+$ resonance in a series of nuclei are performed using the Green's function method[5] with a Skyrme III interaction,[6] taking into account all relevant isoscalar and isovector one-body operators simultaneously.

TABLE I. Total strengths and average energies of the giant electric quadrupole resonances. The energies in parentheses are experimental values.

Nucleus	E (MeV)	$S(0)$ (fm^4)	$S(1)$ (fm^4)	$S(\mathrm{n})$ (fm^4)	$S(\mathrm{p})$ (fm^4)	$\frac{S(\mathrm{n})}{S(\mathrm{p})}$	$\left(\frac{N}{Z}\right)^2$
^{40}Ca	17.2 (17.1)	390.8	40.3	97.8	118.0	0.83	1.0
^{48}Ca	4.7 (3.83)	97.3	22.7	53.5	6.5		
	17.5	5.14.8	28.5	179.8	91.8	1.96	1.96
^{90}Zr	5.9	319.2	36.0	142.4	35.2		
	14.8 (14.1)	1635.4	6.0	448.2	372.5	1.2	1.56
^{120}Sn	6.1	184.2	3.4	34.4	59.4		
	13.6 (13.3)	2414.6	115.3	857.9	407.1	2.10	1.96
	25.9	146.8	1705.5	513.5	412.7	1.24	
^{208}Pb	6.4 (4.1)	2822.6	116.9	1022.0	447.7		
	12.0 (10.9)	6434.7	388.0	2299.6	1111.5	2.07	2.36

Therefore, our calculations yield, among other quantities, the amount of mode mixing. The distribution of strength for the operators in Eq. (1) is computed, taking $f(\mathbf{r}) = r^2 Y_{2\lambda}(\Omega_r)$. The mixing of the $\tau=0$ and $\tau=1$ modes has a very small effect on the energy positions. We also calculate separately the neutron and proton strengths $S(\mathrm{n})$ and $S(\mathrm{p})$, using the operators $Q^{(\mathrm{n,p})}$ in Eq. (3).

We find the isoscalar strength $S(0)$ to be concentrated in two peaks in all calculated nuclei except ^{40}Ca, in agreement with other calculations.[5] The lower 2^+ state is located below the particle-emission threshold, and the higher state (the *giant isoscalar quadrupole)* is narrow and carries about 70% of the isoscalar energy-weighted sum rule (92% in ^{40}Ca). In Table I we present the $S(0)$, $S(1)$, $S(\mathrm{n})$, and $S(\mathrm{p})$ for the 2^+, "$\tau=0$" states. The results concerning the low-frequency 2^+ states in ^{90}Zr and ^{120}Sn should not be taken too seriously, since these states are known to be sensitive to the degree of shell fillings.[1] For ^{120}Sn, as a representative case, the results for the region of the isovector quadrupole are also presented. Here the strength is very fragmented.

We note, in general, that mode mixing in the region of the *giant isoscalar quadrupole* is small and the ratio of $S(1)/S(0)$ in this region is only a few percent. The largest mixing occurs in ^{120}Sn where this ratio is about 0.05, and the purest $\tau=0$ state in ^{90}Zr where the ratio is less than 0.01. In ^{40}Ca the $\tau=0$, $\tau=1$ mixing is equivalent, of course, to the isospin mixing in the giant resonance and is due to the Coulomb force. The mixing in the region of the isovector quadrupole is also small. For example, in ^{120}Sn the ratio of $S(0)/S(1)$ is 0.09. In spite of the usually very small $\tau=0$, $\tau=1$ mixing, the $S(\mathrm{n})$ and $S(\mathrm{p})$ strengths sometimes differ substantially in the region of the giant resonance (see Table I). The ratios $S(\mathrm{n})/S(\mathrm{p})$ for the giant isoscalar quadrupole fluctuate around the value $(N/Z)^2$. The closeness of $S(\mathrm{n})/S(\mathrm{p})$ to $(N/Z)^2$ for the giant resonances signifies the degree of collectivity (coherence) of the excitation.[7]

Within the present framework we can determine the transition densities separately for protons and neutrons. As an example, we show in Fig. 1 the radial parts $F^{(\mathrm{n})}(r)$ and $F^{(\mathrm{p})}(r)$ of the transition densities

$$\rho^{(\mathrm{n,p})}(\mathbf{r}) = F^{(\mathrm{n,p})}(r)\, Y_{2\lambda}(\Omega_r)$$

for the *peak* energy of the giant isoscalar quadrupole in ^{120}Sn. The curves are normalized, respectively, to the *total* $S(\mathrm{n})$ and $S(\mathrm{p})$ strengths integrated over the area of the peaks.

The transition densities obtained from the above structure calculations can be used directly in distorted-wave impulse approximation (DWIA) calculations of pion inelastic scattering. Within this reaction framework, the amplitude for a $\pi^{\pm}$-induced transition is given in terms of $\pi^{\pm}$ distorted waves and a nuclear matrix element of the pion-nucleon transition operator. For the calculations presented here we adopt the DWIA formulation of Ref. 8.

In Table II we list our calculated π^- and π^+ differential cross sections and the ratios $R = \sigma(\pi^-)/\sigma(\pi^+)$ for the quadrupole excitations given in Table I. Some experimental

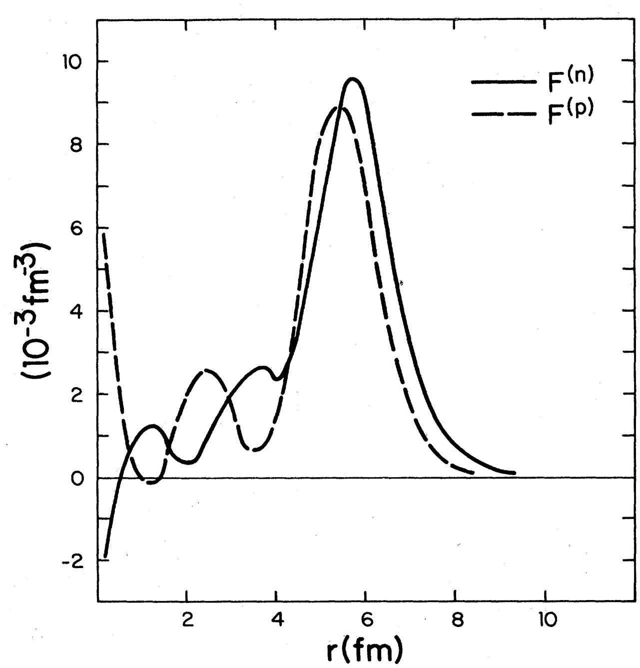

FIG. 1. Radial parts of the microscopic transition densities for the isoscalar giant quadrupole in ^{120}Sn.

TABLE II. Differential cross sections and ratios at angles for which $\sigma(\pi^-)$ maxima occur. The values of σ_- and σ_+ are in units of mb/sr.

Nucleus	E (MeV)	T_π (MeV)	Model 1 σ_-	Model 1 σ_+	Model 1 R	Model 2 σ_-	Model 2 σ_+	Model 2 R	Model 3 σ_-	Model 3 σ_+	Model 3 R	Experiment σ_-	Experiment σ_+	Experiment R
^{40}Ca	17.2	130	2.31	1.78	1.30	2.31	1.78	1.30	2.44	2.05	1.19	1.12 ± 0.23	1.07 ± 0.28	1.05 ± 0.35[a]
^{48}Ca	4.7	180							1.40	0.59	2.38	3.51 ± 0.46	1.52 ± 0.37	2.31 ± 0.4[b]
^{48}Ca	17.5	180	3.60	3.73	0.97	4.17	3.19	1.31	4.64	3.60	1.29			
^{90}Zr	5.9	165							1.31	0.60	2.17			
^{90}Zr	14.8	165	4.90	4.32	1.13	5.40	3.91	1.38	4.07	3.33	1.22			
^{120}Sn	6.1	130							0.17	0.19	0.90			
^{120}Sn	13.6	130	3.19	2.59	1.23	3.63	2.29	1.59	2.68	1.67	1.61	2.06 ± 0.33	1.09 ± 0.16	1.9 ± 0.4[c]
^{120}Sn	25.9[d]	130							1.18	0.88	1.34			
^{208}Pb	6.4	165	...	...	...	...	...	...	2.01	1.22	1.65			
^{208}Pb	12.0	165	5.07	4.36	1.16	6.09	3.60	1.69	3.80	2.24	1.70	3.30 ± 0.65	1.18 ± 0.25	2.80 ± 0.8[e]

[a]Reference 9.
[b]Reference 10.
[c]These results were obtained for the ^{118}Sn target (Ref. 3).
[d]The average energy of the $\tau = 1$, 2^+ distribution of strength.
[e]Reference 4.

cross sections are also included in Table II. The incident kinetic energies (T_π) were chosen to match the available experiments, and the values of the calculated cross sections were taken at the angles at which the $\sigma(\pi^-)$'s reach their first maxima. The columns labeled models 1–3 correspond to the following choices of ground state and transition densities.

For the ground states in models 1 and 2 we used the proton distribution (ρ_p) determined from electron scattering[11] and we assumed the neutron distributions had the same shape as ρ_p; that is, $\rho_n = N\rho_p/Z$. In model 3 we used the Hartree-Fock densities calculated with the Skyrme III interaction. For the transition densities, in models 1 and 2 we used the same Tassie[12] shape for the neutrons and protons, $F^{(n,p)}(r) = \beta^{(n,p)} r \rho'_{gs}$, but with different normalizations. In model 1 we set $\beta^{(n)} = \beta^{(p)} = \frac{1}{2}\beta_{\rm CIS}$ (where $\beta_{\rm CIS}$ is the classical isoscalar sum-rule result[1]); and in model 2 we set $\beta^{(n)} = N\beta_{\rm CIS}/A$ and $\beta^{(p)} = Z\beta_{\rm CIS}/A$. The transition densities used in model 3 were the *microscopic* RPA transition densities normalized to the strengths in Table I.

Many interesting points can be seen from Table II. Under model 1 we see that even for a pure isoscalar response the ratio $R \neq 1$. This primarily results from the difference between the π^- and π^+ distortions caused by the Coulomb field of the nucleus. Models 2 and 3 incorporate the effects of mode mixing in the nuclear response, and when compared with model 1 show the extent to which this affects the cross sections. Although model 3 includes all of the microscopic ingredients as compared with model 2, there is very little difference between these two models for the ratio R. This is due to the fact that the microscopic response is indeed a collective response. Note the $N = Z$ case of ^{40}Ca, where models 1 and 2 give identical results for R, but in model 3 R is reduced to 1.2. This reflects the fact that $\rho^{(p)} > \rho^{(n)}$ in the surface. For the other ($N > Z$) cases the R's of model 3 (and 2) are larger than those of model 1 because $\rho^{(p)} < \rho^{(n)}$ in the surface.

Although the ratios R do not differ significantly between models 2 and 3, the σ's do show a sensitivity to the detailed shapes of the microscopic ρ's. For the giant resonances the σ's given in model 3 are smaller than those of model 2 and are in better agreement with the existing data. In ^{48}Ca the only data are for the low-lying 2^+ state and, as compared with our calculations, the large values may indicate that it has more strength than given by our RPA results. For ^{120}Sn we have also calculated σ's associated with the isovector giant quadrupole by integrating (at fixed θ) the double differential cross sections over the region where most of this strength is located (24 to 30 MeV).

Our ratios in Table II tend to be somewhat smaller than the experimental results, particularly for ^{208}Pb. A source of discrepancy is that our $\sigma(\pi^+)$'s are too large. It is possible that the Skyrme III force may overestimate the coherence in the proton p-h contributions in this case. Also, corrections to the reaction mechanism arising from medium modifications of the pion-nucleon interaction may influence the ratios of the π^- to π^+ cross sections. For energies around the (3,3) resonance, such corrections can be treated in the framework of the Δ-hole theory.[13] Recently Karapiperis and Moniz[14] discussed such corrections to π^+/π^- inelastic scattering. Using their parameters for the renormalized π-N amplitudes, we find that the ratio R for the giant quadrupole in model 3 now becomes 1.72 in ^{120}Sn and 1.81 in ^{208}Pb. Thus medium corrections suggested in Ref. 14 increase the ratio R by about 6%, making the agreement with experiment better. We feel that the comparison of π^- to π^+ inelastic scattering to giant resonances can help constrain the theoretical models of collective motion and experiments to other giant resonances, such as the isoscalar monopole, would be of considerable interest.

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