PHYSICAL REVIEW C VOLUME 31, NUMBER 2 FEBRUARY 1985

Rotational structure of highly deformed ^{99}Y: Particle-rotor model calculations

F. K. Wohn and John C. Hill
Ames Laboratory and Iowa State University, Ames, Iowa 50011

R. F. Petry
University of Oklahoma, Norman, Oklahoma 73019
(Received 16 July 1984)

In the preceding paper, experimental evidence for five rotational bands in the deformed nucleus $^{99}_{39}Y_{60}$ was presented and discussed qualitatively. In the present paper, a quantitative analysis of these bands using the particle-rotor model is presented. One-quasiparticle calculations of level energies, single-particle $E1$ and $M1$ transitions, and collective $E2$ transitions provide an excellent description of the level structures and transition rates for the $\frac{5}{2}[422]$, $\frac{5}{2}[303]$, $\frac{3}{2}[301]$, and $\frac{1}{2}[431]$ bands in ^{99}Y and the $\frac{5}{2}[422]$, $\frac{5}{2}[303]$, and $\frac{3}{2}[301]$ bands in ^{103}Nb. The Coriolis interaction is shown to be crucial in obtaining the level energy and transition patterns of these bands. The only feature which cannot be explained by the one-quasiparticle calculations is the $E1$ strength from the $\frac{3}{2}[431]$ band in ^{99}Y.

I. INTRODUCTION

In the preceding paper,[1] the level scheme of the highly deformed nucleus ^{99}Y was presented and discussed qualitatively. A brief review was given of the current knowledge of both even-even and odd-A nuclei in the neutron-rich deformed region around mass $A \sim 100$. Deformations in the range 0.3—0.4 have been deduced for the even-even isotopes of Sr ($Z=38$) and Zr ($Z=40$). The energy level systematics for Sr and Zr, although still quite incomplete, appear to be consistent with the assumption of highly prolate, axially symmetric deformations. The evidence behind these statements is reviewed in paper I.

For odd-A nuclei in the $A \sim 100$ deformed region, evidence for rotational structure and Nilsson band assignments has only recently become available. The first evidence came from the report[2] of the decay of an 8.6 μs isomer of ^{99}Y that populates eight members of a $K=\frac{5}{2}$ ground-state band. Recently we observed[3] six similar rotational bands in the odd-A nuclei ^{99}Sr, ^{99}Y, ^{101}Y, and ^{101}Zr and presented evidence that both ^{99}Y and ^{101}Y have $\pi\frac{5}{2}[422]$ Nilsson assignments for their ground-state rotational bands. The $N=61$ isotones ^{99}Sr and ^{101}Zr have similar $K=\frac{3}{2}$ ground-state bands that are likely to be based upon either the $\nu\frac{3}{2}[411]$ or the $\nu\frac{3}{2}$ [541] Nilsson orbitals.[3] Recent reports on levels in the odd-neutron nuclei ^{103}Mo and ^{105}Mo (Ref. 4) and the odd-proton nucleus ^{103}Nb (Ref. 5) show that their rotational structure also indicate ground-state Nilsson assignments of $\nu\frac{3}{2}[411]$ and $\pi\frac{5}{2}[422]$, respectively, for the odd nucleon. In addition, $\pi\frac{5}{2}[303]$ and $\pi\frac{3}{2}[301]$ bands have been reported[5] for ^{103}Nb and $\nu\frac{9}{2}[404]$ bands have been reported[4] for ^{103}Mo and ^{105}Mo. Finally, in paper I we presented evidence for five rotational bands in ^{99}Y: $\pi\frac{5}{2}[422]$, $\pi\frac{5}{2}[303]$, $\pi\frac{3}{2}[301]$, $\pi\frac{1}{2}[431]$, and $\pi\frac{3}{2}[431]$. The allowed beta decay to the $\pi\frac{5}{2}[422]$ ground state of ^{99}Y supports the $\nu\frac{3}{2}[411]$ assignment[1] for the ^{99}Sr ground state.

Prior to this work, only "first-order" analyses of bands in the neutron-rich $A \sim 100$ region had been published. The term first order is used here to refer to analyses that treat each band as a "pure-K" band, ignoring higher-order terms such as the rotation particle (Coriolis) and rotation-vibration couplings that admix pure-K bands. In these first-order analyses, the rotational parameters that determine both energy levels and γ-ray deexcitation patterns were treated as effective parameters for each band, independent of the other bands. Such first-order analyses were given in Refs. 2—5.

The purpose of the present paper is to present the results of a higher-order calculation of the energy levels and deexcitation patterns in ^{99}Y. By explicitly including, via matrix diagonalization, the Coriolis term in the particle-rotor model, we provide a consistent analysis of the five rotational bands observed in ^{99}Y, including both intraband $M1+E2$ as well as intraband $E1$ and $M1+E2$ transitions. The rotation-vibration term is shown to have a minor effect on the one-quasiparticle bands. The calculated transition probabilities are extrapolated to describe the three known bands in ^{103}Nb.

Since there is not an overabundance of data available on the deformed $A \sim 100$ region, we chose to follow a "traditional" or "textbook" version of the particle-rotor model, as is outlined in the review paper by Bunker and Reich.[6] This version, which does not incorporate recent modifications to the particle-rotor model, was chosen in order to allow direct comparison of the parameters for ^{99}Y with values of the same parameters that have been systematically determined for the well-studied deformed rare-earth nuclei. In so doing we have followed a natural extension of the calculations beyond the first-order analyses of Refs.

2—5. Section II describes the methods used to determine the model parameters needed to calculate the properties of one-quasiparticle states in ^{99}Y. Section III gives a critique of the calculation and discusses possible modifications or improvements which incorporate recent advances in the particle-rotor model. Expressions for $E1$, $M1$, and $E2$ transition probabilities are given in the Appendix, which we include as a useful compilation.

II. PARTICLE-ROTOR MODEL CALCULATIONS

As was shown in paper I, to lowest order the ground-state band in ^{99}Y can be considered as a simple one-quasiparticle $\frac{5}{2}[422]$ rotational band. In the level energy expression for a rotor (see paper I), the terms b and a_{2K} determined by the fit to the eight members of the band, although small, indicate the presence of higher-order corrections due to the rotation-vibration and rotation-particle (Coriolis) interactions. The latter effect, in particular, must be included in order to account for the observed "signature" term a_5 in the $K=\frac{5}{2}$ ground band and, as the following analysis shows, is crucial in understanding the γ transition patterns among the various bands observed in ^{99}Y. Before considering the Coriolis interaction, however, a brief examination of deformed even-even Sr and Zr nuclei is presented to show that the rotation-vibration coupling appears to be weak enough to be analyzed via a simple perturbation treatment.

A. Effects of the rotation-vibration interaction

The rotation-vibration coupling in a deformed even-even nucleus mixes states with the same J^π of the $K^\pi=0^+$ ground band, the $K^\pi=0^+$ β band, and the $K^\pi=2^+$ γ band. If the bandheads lie at energies 0, E_β, and E_γ, respectively, and a_0 is the inertial parameter of the unmixed rotational bands, then the level energies of the ground band (obtained by a perturbation treatment of the band mixing) can be written as[7]

$$E_{I,0}=(a_0+a_{\rm vib})I(I+1)+b_{\rm vib}[I(I+1)]^2 .$$

Expressions are given in Ref. 7 for $a_{\rm vib}$ and $b_{\rm vib}$ in terms of the parameters a_0, E_β, and E_γ. For the rare earth ($A\sim160$) and actinide ($A\sim240$) deformed regions, values of $b_{\rm vib}$ calculated using experimental E_β and E_γ agree well with experimental values of b.[7]

For deformed nuclei in the $A\sim100$ region, experimental values of E_β and E_γ are only poorly known at present. However, they can be estimated from the systematic variation with A of E_β and E_γ for known nuclei with prolate deformations of ~0.3. These systematics can then be used to obtain values for $A\sim100$. Using even-even nuclei with $\delta\sim0.3$ for the $A\sim24$, 160, and 240 regions, we find that $E_\beta\sim E_\gamma$. Letting $E_{\rm vib}$ be the average of E_β and E_γ for a deformed nucleus of mass A, we find systematic variation with A of $E_{\rm vib}\sim51A^{-3/4}$ MeV. For $A=24$, 100, 160, and 240, this gives $E_{\rm vib}\sim4.7$, 1.6, 1.1, and 0.84 MeV, respectively. Note that the 1.6-MeV estimate of $E_{\rm vib}$ for $A\sim100$ is, as it should be, twice the $\sim$820-keV 2_1^+ energy of the vibrational even-even Sr nuclei with $50<N<60$. Since $E_\beta\sim E_\gamma\sim E_{\rm vib}$, the equations in Ref. 7 can be simplified to

$$a_{\rm vib}\sim9a_0^2/E_{\rm vib} \quad\text{and}\quad b_{\rm vib}\sim-16a_0^3/E_{\rm vib}^2 ,$$

involving only the deformed core parameter a_0 and the effective $E_{\rm vib}$.

The most deformed even-even nucleus in the $A\sim100$ region is ^{100}Sr, for which a deformation of ~0.3 has been deduced from the enhanced $B(E2)$ value for the 2^+ level.[8] For ^{100}Sr the ratio of 3.23 for E_{4^+}/E_{2^+} is the highest yet found in the $A\sim100$ region. From the 2_1^+ energy of 129.2 keV and the 4_1^+ energy of 417.6 keV, the parameters a and b in the rotor energy expression are $a=21.8$ keV and $b=-0.047$ keV. Using $E_{\rm vib}=1600$ keV and setting $a=a_0+a_{\rm vib}=21.8$ keV, the preceding equations give $a_0=19.6$ keV and $b_{\rm vib}=-0.047$ keV. This exact reproduction of the experimental value of b simply means that the systematically determined value of 1600 keV for $E_{\rm vib}$ is an excellent choice. For less deformed ^{102}Zr, an $E_{\rm vib}$ value of ~1400 keV will reproduce the experimental values of $a=25.9$ keV and $b=-0.098$ keV. (Even for the transitional nucleus ^{100}Zr with an effective a of 38.5 keV, the experimental b of -0.51 keV is reasonably well reproduced with an $E_{\rm vib}$ value of 900 keV, in reasonable agreement with the 830-keV energy recently found[9] for the second-excited 0^+ state in ^{100}Zr.) This agreement indicates that a perturbative treatment of the rotation-vibration interaction is adequate for the lower spins of these nuclei. The significance of this for the odd-A nuclei is that states due to coupling of the odd nucleon to the β or γ bands of the core should occur at energies of $\sim E_{\rm vib}$ and $\sim2E_{\rm vib}$.[7]

The value of the inertial parameter a can be compared with the moment of inertia for a rigid prolate spheroid of mass A,

$$\mathscr{I}_{\rm rigid}=0.4AMR^2(1+0.31\beta) ,$$

where M is the nucleon mass and the nuclear radius $R=1.2A^{1/3}$ fm. [β and the Nilsson deformation δ are related by $\beta=\frac{4}{3}\sqrt{(\pi/5)}\delta(1+\frac{1}{2}\delta+\frac{1}{2}\delta^{2+}\cdots)$.] The effective moment of inertia $\mathscr{I}_{\rm exp}$, obtained from the experimental inertial parameter a, can then be written in terms of $\mathscr{I}_{\rm rigid}$ as

$$\mathscr{I}_{\rm exp}=\{36\ {\rm MeV}/[aA^{5/3}(1+0.31\beta)]\}\mathscr{I}_{\rm rigid} .$$

With $\beta\sim0.3$, $\mathscr{I}_{\rm exp}=0.70\mathscr{I}_{\rm rigid}$ for ^{100}Sr and $\mathscr{I}_{\rm exp}=0.87\mathscr{I}_{\rm rigid}$ for ^{99}Y. These values are much larger than the ratios of 0.5—0.6 found for the most deformed rare earth and actinide nuclei.

For ^{100}Sr, the values of 19.6 keV for the core inertial parameter a_0 implies a core whose moment of inertia is 78% of the rigid moment of inertia for $A=100$ and $\beta=0.3$. Since the effective moment of inertia for ^{100}Sr is 70% of the rigid value, the rotation-vibration interaction causes a relatively small reduction of the moment of inertia. It thus appears that even-even Sr and Zr nuclei in the $A\sim100$ deformed region have "core" values of $\mathscr{I}$ that are remarkably close to $\mathscr{I}_{\rm rigid}$. As noted by Azuma *et al.*,[8] this may be attributed to the unexpectedly low pairing energy deduced from Rb mass measurements.[10] When pairing via the Bardeen-Cooper-Schrieffer (BCS) approximation is taken into account in using a cranking model[11] for the $\mathscr{I}$, the resulting calculated $\mathscr{I}$ is reduced. For deformed rare earth nuclei, the reduction of $\mathscr{I}$ to $\sim 50\%$ of $\mathscr{I}_{\text{rigid}}$ can be well reproduced with such a calculation. The unusually large values of $\mathscr{I}$ for deformed $A \sim 100$ nuclei thus provide additional evidence for an unusually low pairing strength for nuclei in this region.

B. Quasiparticle and bandhead energies

The Nilsson diagram shown in Fig. 1 was calculated with $\kappa=0.067$ and $\mu=0.53$. Values of κ in the range 0.066–0.070 and μ in the range 0.52–0.55 adequately reproduced the experimental single-particle energy ordering and spacing for the spherical Y nuclei near stability. The particular values used in Fig. 1 and in our quasiparticle calculations were selected to also give a good reproduction of the observed bandhead energies of ^{99}Y, following the technique described in the following.

Quasiparticle energies and particle (hole) occupation amplitudes V (U) were obtained by use of the normal BCS approximation[11] with a proton pairing interaction of strength $\Delta_p=690$ keV. The experimentally determined neutron pairing gap energy Δ_n was found[10] to be 570 ± 17 keV for the neutron-rich Rb ($Z=37$) nuclei with $N>50$. Since this is the only experimental information on pairing for nuclei in the $A\sim100$ region, we used this value, multiplied by the factor $1+(N-Z)/A$,[11] to obtain the estimate $\Delta_p=690$ keV for the proton pairing strength for ^{99}Y.

The bandhead energy $E_{K,K}$ contains contributions from the various interactions. Following the notation of Ogle *et al.*,[12] and neglecting the vibrational terms (hence considering only one-quasiparticle states),

$$E_{K,K}=E_K^{qp}+E_K^{rpc}+a\left[K+\gamma(K)-a_1\delta_{K,1/2}\right] .$$

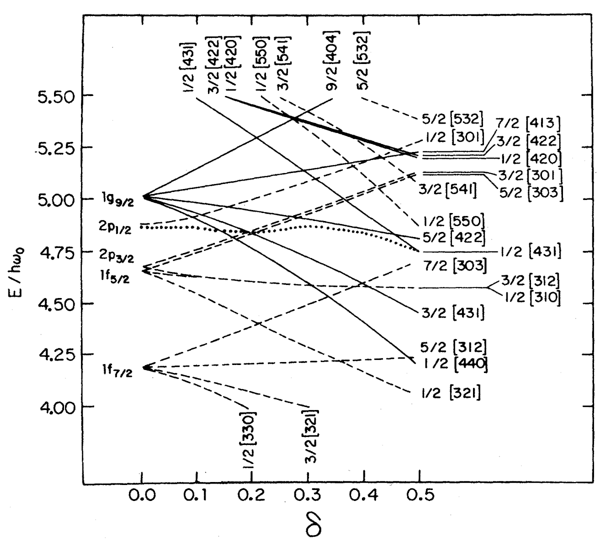

FIG. 1. Nilsson diagram for protons with $\kappa=0.067$ and $\mu=0.534$; BCS model Fermi level for Y ($Z=39$) with $\Delta_p=0.69$ MeV is indicated by dots.

The first term (quasiparticle energy) is obtained by a BCS calculation. The second term is due to Coriolis coupling. The third term (zero-point rotational energy) depends on the intrinsic Nilsson state. However, the simple estimate $\gamma(K)=0.5K^2$ was found by Ogle *et al.*[12] to give reasonable results in the deformed rare-earth region. We used this estimate here. Its significance is discussed in Sec. III. In a first-order treatment with Coriolis mixing ignored, E_K^{rpc} is zero. With Coriolis mixing determined by matrix diagonalization, E_K^{rpc} is the shift in bandhead energy due to the nondiagonal Coriolis matrix elements.

For the inertial parameter a of the deformed core of the odd-A nucleus, we used the value 21.8 keV, which is the same as the experimental value of a for ^{100}Sr. Then the only remaining parameters, κ, μ, and the Nilsson deformation δ, were varied (keeping κ and μ within the narrow ranges consistent with the spherical Y nuclei). The calculated and experimental ^{99}Y bandhead energies were compared. Using this procedure, the values $\kappa=0.067$, $\mu=0.53$, and $\delta=0.34$ gave a remarkably good reproduction of the ^{99}Y bandhead energies, as is evident in Table I.

The best indication of a ^{100}Sr-like core is given by the effective inertial parameters of 19.6 keV for the $\frac{5}{2}[303]$ band and 23.6 keV for the $\frac{3}{2}[301]$ band. If there were no Coriolis mixing, these bands would have the same inertial parameter. With Coriolis mixing, the inertial parameter of the lower band ($K=\frac{5}{2}$) decreases and that of the upper band ($K=\frac{3}{2}$) increases. The required core inertial parameter a must have an intermediate value. The $\frac{5}{2}[303]$ and $\frac{3}{2}[301]$ bands in ^{99}Y thus indicate a core value of about 21.8 keV, which is the effective inertial parameter observed for ^{100}Sr.

To summarize, we used available experimental data to obtain values for the pairing energy Δ_p, the core inertial parameter a, and the Nilsson parameters κ and μ, then adopted the estimate of Ogle *et al.*[12] for the zero-point rotational energy. With these parameters fixed by this procedure, δ (the only remaining parameter) is then determined by the experimental bandhead energies for ^{99}Y. The $\frac{1}{2}[431]$ and $\frac{3}{2}[431]$ bandhead energies play a critical role in fixing the value of δ, as the $\frac{1}{2}[431]$ orbital lies above the Fermi level for Y and its quasiparticle energy falls rapidly as δ increases, whereas the $\frac{3}{2}[431]$ orbital lies below the Fermi level and its quasiparticle energy rises fairly rapidly as δ increases. The quasiparticle energies for the $\frac{1}{2}[431]$ and $\frac{3}{2}[431]$ orbitals are essentially equal for a δ of about 0.34. The δ value of 0.34 best reproduces the five ^{99}Y bandhead energies when the Coriolis contribution to the bandhead energies is included.

C. Level energies and intraband γ transitions

Pure-K rotational bands with the same inertial parameter a are mixed by the Coriolis interaction. In the following the notation used is similar to that of Bunker and Reich.[6] Basis states of given spin parity I^π are labeled for convenience only by their K value. States K and K' of the same I^π are mixed by the Coriolis interaction only if $|K'-K|=1$ or $K'=K=\frac{1}{2}$. The nondiagonal matrix elements are[6]

TABLE I. Experimental and calculated quantities characterizing the observed rotational bands in ^{99}Y.

Nilsson orbital	Bandhead energy	(keV)	$a=\hbar^2/2\mathcal{J}$ (keV)	$2(K)!a_{2K}/a$	Pattern of γ-ray deexcitations
$\frac{5}{2}[422]$	expt.	0	17.89	6.7×10^{-4}	$\lambda(M1)\gg\lambda(E2)$
	cal.[a]	0	17.94	5.8×10^{-4}	
$\frac{5}{2}[303]$	expt.	487	$\sim$19.6		$\lambda(E1)\gg\lambda(E2)\gg\lambda(M1)$[c]
	cal.[b]	496	19.44	3.4×10^{4}	
$\frac{3}{2}[301]$	expt.	536	$\sim$23.6		$\lambda(E1)\sim\lambda(M1)\gg\lambda(E2)$[d]
	cal.[b]	539	23.36	2.4×10^{-4}	
$\frac{1}{2}[431]$	expt.	1012	$\sim$17.8	-1.05	$\lambda(M1)\gg\lambda(E2),\lambda(E1)$
	cal.[a]	1014	22.42	-1.03	
$\frac{3}{2}[431]$	expt.	1120	$\sim$23.6		$\lambda(E1)\gg\lambda(M1),\lambda(E2)$[e]
	cal.[a]	1118	23.36	-6.9×10^{-2}	

[a]For positive-parity bands, $k=0.77$.

[b]For negative-parity bands, $k=1.35$.

[c]$\lambda(M1)\ll\lambda(E2)$ for the $\frac{5}{2}[303]$ band indicates the effective $(g_K-g_R)/Q_0\sim0$.

[d]$\lambda(E1)\sim\lambda(M1)$ for the $\frac{3}{2}[301]$ band implies hinderance of the $\Delta K=1$ $E1$ strength.

[e]$\lambda(E1)\gg\lambda(M1)$ for the $\frac{3}{2}[431]$ band implies a strong enhancement of this $\Delta K=0$ $E1$ strength.

$$H_{K,K+1}=H_{K+1,K}=-aP^{+}_{K,K+1}A_{K,K+1}\times[(I-K)(I+K+1)]^{1/2}\,,$$

$$H_{1/2,1/2'}=-aP^{+}_{1/2,1/2'}A_{1/2,1/2'}(-)^{I-1/2}(I+\tfrac{1}{2})\,,$$

where $A_{K,K'}$ is the matrix element of j_+ between the single-particle states designated by K and K'. The "pairing factor" $P^{+}_{K,K'}$, approximately given by $U_KU_{K'}+V_KV_{K'}$, where U and V are the usual BCS occupation amplitudes, accounts for the fact that quasiparticle states rather than single-particle states are coupled by the Coriolis interaction.[6]

In fitting experimental data by means of a Coriolis-mixing calculation, Bunker and Reich[6] replaced the $H_{K,K'}$ by $kH_{K,K'}$ where k is an adjustable parameter of the order of unity. In well-studied cases involving unhindered Coriolis matrix elements, which involve single-particle orbitals originating from the same spherical shell-model state, typical values of k fall in the range 0.3–0.7, whereas for the asymptotically hindered couplings, which involve relatively small theoretical matrix elements, k can deviate either way from unity.[6]

The eigenstates $|Ia\rangle$ that we obtained from diagonalization using $kH_{K,K'}$ are linear combinations of the basis states $|IK\rangle$,

$$|Ia\rangle=\sum_K D^K_{Ia}\,|IK\rangle\,.$$

The states $|Ia\rangle$ do not have a well-defined value of K. However, if one component of the basis states is dominant, then it is useful to designate the state $|Ia\rangle$ by $|IK_a\rangle$ to indicate this condition. The energies of the basis states are, for $K\neq\frac{1}{2}$,

$$E_{I,K}=E_{K,K}+a[I(I+1)-K(K+1)]\,.$$

For the positive-parity (i.e., unique-parity) orbitals, six bands were included in the matrix diagonalization. With quasiparticle energy in keV in parentheses, the six bands were $\frac{5}{2}[422]$ (0), $\frac{1}{2}[431]$ (1112), $\frac{3}{2}[431]$ (1183), $\frac{7}{2}[413]$ (2162), $\frac{1}{2}[440]$ (2478), and $\frac{9}{2}[404]$ (5088). Except for $\frac{1}{2}[431]$, these orbitals all originate from the spherical $1g_{9/2}$ orbital, and hence have large Coriolis matrix elements. The $\frac{1}{2}[431]$ orbit originates from the spherical $2d_{5/2}$ orbit, and hence has small Coriolis matrix elements with the other orbitals. (For the mixed $\frac{5}{2}[422]$ band, both $\frac{1}{2}[431]$ and $\frac{9}{2}[404]$ bands have negligible effect for all spins from $\frac{5}{2}$ through $\frac{19}{2}$.)

For the negative-parity orbitals, the bands $\frac{5}{2}[303]$ (496), $\frac{3}{2}[301]$ (595), $\frac{1}{2}[301]$ (1863), $\frac{1}{2}[310]$ (1943), $\frac{3}{2}[312]$ (1932), and $\frac{7}{2}[303]$ (2379) were included in the diagonalization. The Coriolis matrix elements are relatively small. The results for the mixed $\frac{5}{2}[303]$ and $\frac{3}{2}[301]$ bands were essentially unchanged if only these two bands were included in the diagonalization.

Figure 2 gives the results of our calculation for $\delta=0.342$. A core inertial parameter of 21.8 keV was used for all five bands. For the unique-parity bands, $k=0.77$ was used, as determined by the best fit to the eight-member $\frac{5}{2}[422]$ band. For the negative-parity bands, $k=1.35$ was used since this value best reproduced the effective inertial parameters of the $\frac{5}{2}[303]$ and $\frac{3}{2}[301]$

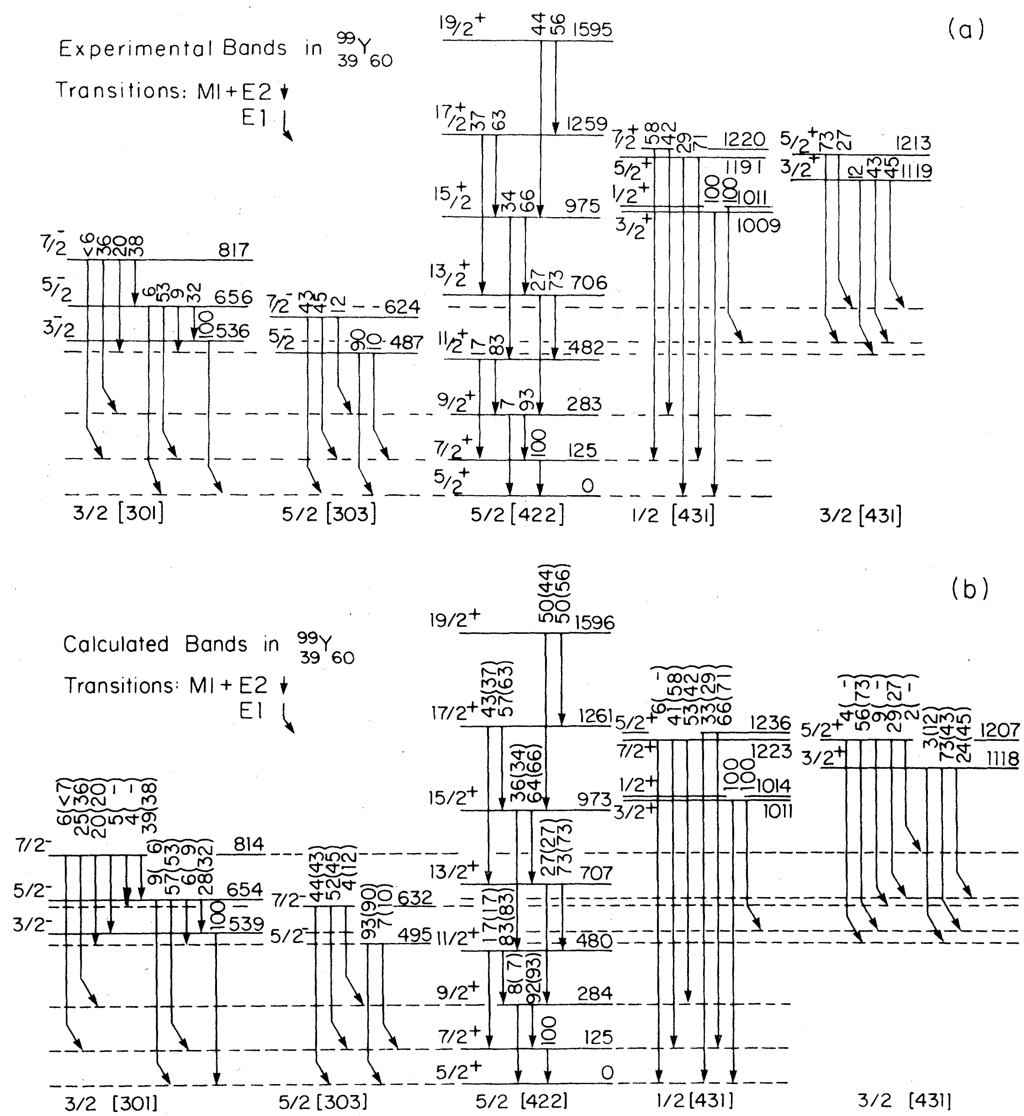

FIG. 2. Nilsson bands and relative transition intensities for ^{99}Y. (a) experiment (from paper I). (b) calculation (one-quasiparticle bands). For ease of comparison the experimental intensities are also given in parentheses in (b).

bands. The effective spin factor g' was determined to be ~ 1.0 by intraband $\frac{5}{2}[422]$ branching ratios. We used $g'=1.0$ for all positive-parity bands. The $\frac{5}{2}[303]$ band is the only band observed to have $M1$ transitions hindered relative to $E2$ transitions. As is shown in Fig. 8 of paper I, the $M1$ strength for the $\frac{5}{2}[303]$ band is calculated to be small due to near cancellation of g_K and g_R. Cancellation occurs for $g' \sim 0.7$. We used $g'=0.7$ for all negative-parity bands. With these two choices of g', unhindered $M1$ transitions are predicted for all bands except for the $\frac{5}{2}[303]$ band. Table I gives a comparison of the band parameters as deduced from experiment and by our one-quasiparticle calculation for the five bands observed.

If only the $\frac{5}{2}[422]$ band were considered, there would be no unique set of values of the parameters δ, a, k, and g' which would reproduce the observed energy levels and intraband transition ratios. With a core similar to ^{98}Sr (i.e., $a=26.5$ keV) and δ in the range 0.30—0.35, we obtained reasonably good reproduction of this band with $0.65 < k < 0.80$ and $0.7 < g' < 0.9$. With a core similar to ^{100}Sr (i.e., $a=21.8$ keV and $\delta=0.34$) the $\frac{5}{2}[422]$ band is well reproduced with $k=0.77$ and $g'=1.00$. This latter set of parameters gives an excellent reproduction of the energy levels of the $\frac{5}{2}[422]$ band, since the calculated signature effect is an order of magnitude too large for the larger a value (i.e., ^{98}Sr-like core) or for the smaller δ value of ~ 0.3. In order to obtain the excellent reproduction of Fig. 2 for the higher-spin levels of the $\frac{5}{2}[422]$ band, it was necessary to "soften" the core by also using a core parameter $b=-8.6$ eV in addition to the core inertial parameter $a = 21.83$ keV.

Table II lists the parameters for intraband $M1/E2$ ratios. Values of $(g_K - g_R)/Q_0$ are given in Table II for both pure-K and Coriolis-mixed bands. For the latter case the effective value of $(g_K - g_R)/Q_0$ depends on the spins I_i and I_f. Values are given in Table II for the lowest intraband transition in each band. As Table II shows, the $\frac{5}{2}$[303] band is calculated to have a small $(g_K - g_R)/Q_0$ that remains small after band mixing. For the bands $\frac{5}{2}$[422], $\frac{5}{2}$[303], and $\frac{3}{2}$[301] the calculated effective $(g_K - g_R)/Q_0$ values of Table II are in good agreement with the experimental[5] effective values of 0.23 ± 0.13, 0.05 ± 0.03, and 0.43 ± 0.20, respectively, for these three bands in ^{103}Nb.

D. Interband γ transition patterns

Table I also gives the experimental γ transition patterns (intraband as well as interband) for the bands. Only $E1$ transitions are observed from the $\frac{5}{2}$[303] band. Branching ratios of these $E1$'s are approximately given by the proportionality of $B(E1)$ to $\langle I1I' \mid KmK' \rangle^2$, as expected for pure $K = \frac{5}{2}$ to $K' = \frac{5}{2}$ transitions. Both $E1$ and $M1 + E2$ transitions are observed for the $\frac{3}{2}$[301] band, but the $E1$ transitions deviate strongly from the $\langle I1I' \mid KmK' \rangle^2$ prediction for pure $K = \frac{3}{2}$ to $K' = \frac{5}{2}$ transitions. (As discussed in the following, the unusual $E1$ patterns observed for the $\frac{3}{2}$[301] band members can only be accounted for by Coriolis mixing of the $\frac{3}{2}$[301] and $\frac{5}{2}$[303] bands.) The hinderance of the $E1$ transitions in the $\frac{3}{2}$[301] band allows the intraband and interband transition to have comparable strengths. The only $E1$ transition observed from the $\frac{1}{2}$[431] band is from the $\frac{1}{2}^+$ state for which no $M1$ transition is possible; this $E1$ is stronger than the (unobserved) $E2$ to the ground state. The $\frac{3}{2}$[431] band seems to be dominated by $E1$ transitions. All of these γ-ray transition patterns are reproduced by our one-quasiparticle calculations, except for the strongly enhanced $E1$ strength of the $\frac{3}{2}$[431] band.

In ^{103}Nb the only $E1$ transitions reported in Ref. 5 are from the $\frac{5}{2}$[303] bandhead (164 keV, $T_{1/2} = 4.7 \pm 0.5$ ns) and the $\frac{3}{2}$[301] bandhead (248 keV, $T_{1/2} = 2.0 \pm 0.6$ ns). Thus the $E1$ pattern in ^{103}Nb is quite different from that of ^{99}Y. In ^{99}Y $E1$'s dominate $M1$'s for the $\frac{5}{2}$[303] band and $E1$'s are comparable in strength to $M1$'s for the $\frac{3}{2}$[301] band, whereas no $E1$'s from these bands in ^{103}Nb can compete with $M1$'s.

$\frac{5}{2}$[303] and $\frac{3}{2}$[301] bands. Comparing the $\frac{5}{2}$[303] and $\frac{3}{2}$[301] bands in the two nuclei ^{99}Y and ^{103}Nb is very informative due to the difference in the strengths of the $E1$ transitions to the $\frac{5}{2}$[422] ground bands in these two nuclei. The unusual $E1$ pattern observed from the $\frac{3}{2}$[301] band in ^{99}Y and the nearly equal strengths of $E1$ and $M1$ transitions tests our understanding of the structure of this band. As mentioned previously, the $E1$ strengths deviate strongly from the $\langle I1I' \mid KmK' \rangle^2$ prediction for pure $K = \frac{3}{2}$ to $K' = \frac{5}{2}$ transitions and can only be reproduced by Coriolis mixing of the $\frac{3}{2}$[301] and $\frac{5}{2}$[303] bands.

The lifetimes of both the $\frac{5}{2}$[303] and $\frac{3}{2}$[301] bandheads in ^{103}Nb, together with $E1$ patterns observed for members of these bands in ^{99}Y, provide sufficient constraints to determine the $E1$ strengths. Tables III and IV give the intrinsic parameters $GE1(K \to K')$ and the pairing factors P^- used to convert Nilsson single-particle $GE1$'s to quasiparticle values $GE1(K \to K')P^-$. The P^- values in Tables III and IV were obtained by the estimate $UU' - VV'$ using the normal BCS theory. For ^{103}Nb a δ value of 0.3 was used.[5] Empirical values of $GE1(K \to K')P^-$ were obtained by fitting the available data for ^{99}Y and ^{103}Nb. Only one parameter was allowed to vary, the enhancement factor H shown in the last column of Tables III and IV. We assume that $\Delta K = 1$ strengths were not enhanced relative to the pairing-corrected Nilsson values. We also assumed that H was the same for both $\Delta K = 0$ values in the two nuclei. Our restriction to only one free $E1$ parameter was motivated in part by our general approach of minimizing the num-

TABLE II. Calculated parameters for intraband $M1 + E2$ transitions among the observed rotational bands in ^{99}Y.

Nilsson orbital	$(g_K - g_R)/Q_0$ for unmixed "pure-K" band	Effective $(g_K - g_R)/Q_0$ after Coriolis mixing of bands
$\frac{5}{2}$[422][a]	0.331	0.277
$\frac{5}{2}$[303][b]	0.0214	0.0370
$\frac{3}{2}$[301][b]	0.380	0.376
$\frac{1}{2}$[431][a,c]	$-0.044 - 0.233(-)^{I_> + (1/2)}$	$-0.019 - 0.426(-)^{I_> + (1/2)}$
$\frac{3}{2}$[431][a]	0.393	0.446

[a]For positive-parity bands the spin factor $g' = 1.0$ and $Q_0 = 4.2$ b.
[b]For negative-parity bands the spin factor $g' = 0.7$ and $Q_0 = 4.2$ b.
[c]Two transitions, $\frac{1}{2} \to \frac{3}{2}$ and $\frac{5}{2} \to \frac{3}{2}$, are used to determine the effective values of $(g_K - g_R)/Q_0$ for this $K = \frac{1}{2}$ band. ($I_>$ is the larger of the initial and final I values.)

TABLE III. Calculated and fitted[a] parameters for $E1$ transitions in ^{99}Y.

Nilsson orbitals initial→final	$GE1(K\rightarrow K')$ Nilsson (without pairing)	P^- (pairing factor)	$GE1(K\rightarrow K')P^-$ (pairing corrected)	$GE1(K\rightarrow K')P^-$ (fitted to data)	H enhancement factor
$\frac{5}{2}[303]\rightarrow\frac{5}{2}[422]$	0.010 77	0.6134	0.006 60	0.0198	3.0
$\frac{3}{2}[301]\rightarrow\frac{3}{2}[431]$	−0.023 29	−0.0887	0.002 07	0.0062	3.0
$\frac{3}{2}[301]\rightarrow\frac{5}{2}[422]$	−0.023 34	0.6329	−0.014 74	−0.0147	1.0
$\frac{3}{2}[301]\rightarrow\frac{1}{2}[431]$	−0.014 22	0.8940	−0.012 71		1.0
$\frac{5}{2}[303]\rightarrow\frac{3}{2}[431]$	−0.001 29	−0.1134	0.000 15		1.0

[a]The fitted values (obtained by assuming an enhancement factor H of 3.0 for both $\Delta K=0$ transitions) give the relative $E1$ and $M1+E2$ rates shown in Fig. 2.

ber of parameters and in part by the general observation for deformed rare-earth nuclei that $\Delta K=0$ $E1$'s tend to be underestimated by $GE1(K\rightarrow K)P^-$, whereas the $\Delta K=1$ $E1$'s are more likely to agree with the $GE1(K\rightarrow K')P^-$ estimates.[13–15]

The results obtained are given in Tables III and IV. For $E1$ and $M1+E2$ transitions in ^{99}Y, relative strengths can be obtained from Fig. 2. Using $H=3$ for both $\Delta K=0$ amplitudes (hence a factor of 9 in $E1$ strengths) explains the observed $E1$ properties of both nuclei, but only if the two bands are Coriolis mixed. Coriolis mixing has no effect on the $\frac{3}{2}[301]$ bandhead, little effect on the $\frac{5}{2}[303]$ band, but a strong effect on $I>\frac{3}{2}$ members of the $\frac{3}{2}[301]$ band in ^{99}Y. The admixed $\frac{5}{2}[303]$ components determine the unusual $E1$ intensity pattern and the comparable strengths of $E1$ and $M1$ transitions for members of the $\frac{3}{2}[301]$ band. (It should be noted that the amount of Coriolis mixing of these two bands was determined by the 1.35 value of k for the negative-parity bands, and we determined k by the level energies without consideration of γ transition characteristics.) Since the quantities $GE1(K\rightarrow K')$ are the same for ^{99}Y and ^{103}Nb, only the quantities P^- and the $E1$ transition energies E_γ determine the $E1$ strengths. These latter quantities account for the greater $E1$ strengths (by a factor of $\sim 10^3$) in ^{99}Y compared to the same bands in ^{103}Nb.

$\frac{1}{2}$[431] band. For the calculations of the $\frac{1}{2}[431]$ band, all parameters were already determined, as we used the $g'=1.0$ value for all unique-parity bands. The $E1$ enhancement factors H of 3.0 and 1.0 was used for all $\Delta K=0$ and 1 transitions, respectively. The resulting transition patterns calculated for members of the $\frac{1}{2}[431]$ band are given in Fig. 2. Comparison of the experimental and calculated patterns for this band shows good agreement. Without any Coriolis mixing (i.e., with pure-K bands) $M1$ transitions would be forbidden for single-particle $M1$ transitions since $\Delta K=2$. Our calculations indicate that the amount of the $\frac{3}{2}[431]$ states admixed into the $\frac{5}{2}[422]$ band is sufficient to allow unhindered single-particle $M1$ transitions to dominate over $E1$ and $E2$ transitions. The most important contribution occurs between the dominant $\frac{1}{2}[431]$ component of the mixed $\frac{1}{2}[431]$ band and the small $\frac{3}{2}[431]$ component of the mixed $\frac{5}{2}[422]$ band. The $\frac{1}{2}[431]$ band is relatively pure since the Coriolis matrix elements are small between this band and the other unique-parity bands, all of which originate from the spherical $1g_{9/2}$ orbital.

The ability of the one-quasiparticle calculations to correctly predict the γ-ray branching ratios for the $\frac{1}{2}[431]$ band indicates that states near the general three-quasiparticle energy of $\sim 2\Delta_p=1380$ keV may remain primarily one-quasiparticle in nature. As was mentioned in paper I, however, near this energy a "particle-vibration" state[6] consisting of the $\frac{5}{2}[422]$ ground state coupled to the 2^+ first γ vibration could also be expected. (As discussed in Sec. II A, $E_\gamma\approx E_{\rm vib}\approx 1600$ keV.) The lower-lying $\frac{1}{2}^+$ coupling of $|\frac{5}{2}^+\times 2^+\rangle$ could admix with the one-

TABLE IV. Calculated and fitted[a] parameters for $E1$ transitions in ^{103}Nb.

Nilsson orbitals initial→final	$GE1(K\rightarrow K')$ Nilsson (without pairing)	P^- (pairing factor)	$GE1(K\rightarrow K')P^-$ (pairing corrected)	$GE1(K\rightarrow K')P^-$ (fitted to data)	H enhancement factor
$\frac{5}{2}[303]\rightarrow\frac{5}{2}[422]$	0.010 77	0.2728	0.002 94	0.0087	3.0
$\frac{3}{2}[301]\rightarrow\frac{3}{2}[431]$	−0.023 29	−0.7057	0.016 44	0.0485	3.0
$\frac{3}{2}[301]\rightarrow\frac{5}{2}[422]$	−0.023 34	−0.1330	0.003 11	0.0031	1.0

[a]The fitted values (obtained by assuming an enhancement factor H of 3.0 for both $\Delta K=0$ transitions) give half-lives of 4.8 ns for the $\frac{5}{2}[303]$ bandhead and 1.9 ns for the $\frac{3}{2}[301]$ bandhead, in excellent agreement with the 4.7 ns and 2.0 ns experimental values of Ref. 5.

quasiparticle $\frac{1}{2}[431]$ band, but such mixing appears to have little effect on the γ-ray branching ratios.

The effective value of -1.0 for the decoupling parameter deduced for the $\frac{1}{2}[431]$ band could be construed as evidence for such a "single-particle-plus vibration" admixture.[6] From the Nilsson model, the calculated value of the decoupling parameter is -1.62. In our calculation, in order to reproduce the experimental value of -1.05 (see Table I), it was necessary to change the Nilsson value to -1.33. If the wave function of such an admixed single-particle-plus-vibrational state is written as $|\mathrm{mix}\rangle = B\,|\mathrm{sp}\rangle + C|\mathrm{vib}\rangle$, then the decoupling parameter for $|\mathrm{mix}\rangle$ would be B^2 times that for $|\mathrm{sp}\rangle$, assuming that $|\mathrm{vib}\rangle$ has a negligible decoupling parameter.[6] In the present case, this gives $B^2 \sim 0.8$, hence $\sim 20\%$ of the band "$\frac{1}{2}[431]$" could be due to the vibrational-coupled component, even though the deexcitation pattern appears consistent with a 100% one-quasiparticle Coriolis-mixed $\frac{1}{2}[431]$ band.

$\frac{3}{2}[431]$ band. For the $\frac{3}{2}[431]$ band, our one-quasiparticle calculations agree with experiment for level energies but disagree for γ-ray transition probabilities. The calculated $E1$ branching ratios shown in Fig. 2 are just ratios of Clebsch-Gordan coefficients, which agree fairly well with experimental ratios, but require an $E1$ strength that is large enough to allow $E1$'s to dominate over $M1$'s. The $E1$ strength required is much larger than we obtain with an enhancement H of 3 for $\frac{3}{2}[431] \rightarrow \frac{3}{2}[301]$ since the $E1$'s are too weak compared to the calculated $M1$'s to the $\frac{5}{2}[422]$ ground band which occur primarily via unhindered $\frac{3}{2}[431] \rightarrow \frac{5}{2}[422]$. Unlike the mixed $\frac{1}{2}[431]$ band (in which $M1$'s dominate $E1$'s even though the $M1$'s occur only via a small admixture of $\frac{3}{2}[431]$ in the mixed $\frac{5}{2}[422]$ band) the $M1$'s calculated for the mixed $\frac{3}{2}[431]$ band connect the major components of the $\frac{3}{2}[431]$ and $\frac{5}{2}[422]$ bands.

In order to have single particle $E1$'s from the mixed $\frac{3}{2}[431]$ band dominate over $M1$'s, an $E1$ enhancement factor H of ~ 50 rather than 3 is required. However, if H were ~ 50 for $\frac{3}{2}[431] \rightarrow \frac{3}{2}[301]$, then the good agreement previously discussed for $E1$'s from the $\frac{3}{2}[301]$ and $\frac{5}{2}[303]$ bands would be destroyed. In particular, neither the unusual $E1$ branching ratios from the $\frac{3}{2}[301]$ band in ^{99}Y nor the lifetimes of the bandheads in ^{103}Nb would be reproduced.

An explanation of this discrepancy requires either that the $\frac{3}{2}[431]$ designation of the band be dropped or that an additional contribution to the $E1$ strength be invoked. The former choice requires that the good agreement between the experimental and the calculated energies be ignored or regarded as merely fortuitous. The latter choice requires that the additional contribution must be something other than an increase in the value of H for the one-quasiparticle $GE1$ for $\frac{3}{2}[431] \rightarrow \frac{3}{2}[301]$. A possible explanation for an additional contribution to $\Delta K = 0$ $E1$ transitions has been given by Faessler, Udagawa, and Sheline,[16] who considered additional contributions from the mixing with octupole vibrational bands due to the particle-vibration interaction. The mixing enhances $\Delta K = 0$ $E1$ transitions, as desired, and does not effect $\Delta K = 1$ $E1$ transitions. Their calculation gave good agreement for the transition probabilities in the Eu isotopes where the parameters needed to calculate the mixing are available from studies of Sm isotopes.

Among the parameters needed for the calculation of Ref. 16 are the energy E_3 and the $B(E3)$ value of the 3^- octupole state in the neighboring even-even core nucleus. An additional parameter enters as a "collective" $E1$ transition amplitude, which is zero unless one assumes a displacement of the center of charge relative to the center of mass.[16] Since none of these three parameters are known for ^{99}Y, it is not possible to adequately constrain a calculation like that of Ref. 16 to verify whether octupole mixing can explain the additional $E1$ strength that can be deduced for the $\frac{3}{2}[431]$ band in ^{99}Y.

E. Summary

As is evident from Fig. 2, the one-quasiparticle calculations reported here reproduce quite well the rotational band structure observed in the odd-Z nuclei ^{99}Y and ^{103}Nb. Only the strongly enhanced $\Delta K = 0$ $E1$ transitions from the $\frac{3}{2}[431]$ band in ^{99}Y cannot be explained by the simple particle-rotor model with one-quasiparticle Nilsson states coupled to an axially symmetric core which is identical for all bands in the nucleus. In these calculations, we followed the general philosophy of minimal parameter variation for two reasons. First, there is insufficient experimental information on the deformation in the neutron-rich $A \sim 100$ region to justify any other approach. Second, since this is the first analysis to go beyond the first-order analyses of previous reports,[2-5] we wanted to provide as stringent a test as possible following the general analysis procedure[6] used in the past to characterize the well-studied deformed rare earth nuclei.

The model parameters used can be placed in two classes, an "energy class" and a "transition class." The energy class of parameters (Δ_p, κ, μ, δ, a, and k) were determined first, then the transition parameters. Section II B explains how we determined values of Δ_p, κ, μ, δ, and a. The relatively small Δ_p value of 690 keV is consistent with the unusually large moments of inertia deduced from the observed rotational bands in this region.[2] The 0.34 value of δ obtained is consistent with the deformations deduced for the even-even deformed nuclei in this region.[2,17]

The parameters a and k of the energy class of parameters are the only parameters which we obtained solely from ^{99}Y data. The core inertial parameter a was determined by the experimental effective inertial parameters of the two Coriolis-mixed bands $\frac{5}{2}[303]$ and $\frac{3}{2}[301]$. This $a = 21.8$ keV value was used for all bands in ^{99}Y. With the other energy parameters fixed, the k value of 0.77 (which was used for all positive-parity bands) was fixed by the eight-member $\frac{5}{2}[422]$ ground band. Finally, the k value of 1.35 for the negative-parity bands was required in order to best reproduce the level energies of the $\frac{5}{2}[303]$ and $\frac{3}{2}[301]$ Coriolis mixed bands. These two values of k are quite reasonable, as $k < 1$ is expected for the "unhindered" Coriolis matrix elements of the unique-parity bands and $k>1$ is consistent with experience for the hindered Coriolis matrix elements of the negative-parity bands.[6]

The "transition" parameters are g' and H. The intrinsic quadrupole moment Q_0 is determined by our choice of δ according to Eq. (A1) of the Appendix. An effective spin factor g' of unity was found to give an excellent reproduction of the intraband $M1/E2$ branching ratios of the $\frac{5}{2}[422]$ band. Since the orbital contribution g_l to the $M1$ amplitude term g_K of Eq. (A8) is much smaller than the spin contribution g_s for this band, and since $g_R=0.39$ is much less than g_K, the parameter $(g_K-g_R)/Q_0$ is nearly proportional to g'/Q_0. Thus, if we decrease our value of Q_0 by 10%, we can obtain the same $M1/E2$ ratios by decreasing g' by $\sim 10\%$. As g' is more likely to be less than unity,[6] we are more likely to have overestimated Q_0 than underestimated it. It is interesting to note that, with the Q_0 of 4.2 b that we obtain from $\delta=0.34$, we obtain $g'=1.0$, which implies that the $\frac{5}{2}[422]$ band is well described by the free proton g_s value, with negligible reduction due to "core polarization" or other effects.

The g' value for the negative-parity bands was determined to be ~ 0.7 by the observation of hindered intraband $M1$ transitions in the $\frac{5}{2}[303]$ bands of both ^{99}Y and ^{103}Nb. This value of g' is quite consistent with the general values of 0.6—0.7 found to best reproduce $M1$ strengths for deformed rare-earth nuclei.[18,19] It will be interesting to determine whether the different values of g' for Nilsson orbitals of unique and nonunique parities is a general characteristic of deformed neutron-rich $A\sim 100$ nuclei.

The $E1$ amplitude enhancement factors H of 3.0 and 1.0 for $\Delta K=0$ and 1, respectively, indicate that the Nilsson $GE1$ values with pairing correction factors P^- can adequately describe the observed $E1$ transitions in ^{99}Y and ^{103}Nb (except for the $\frac{3}{2}[431]$ band). If the parameter H were set to unity (i.e., pure Nilsson one-quasiparticle estimate), the calculated $E1$ strengths would agree with experiment to within a factor of 10, which is better than for the deformed rare-earth nuclei. It should be noted here that the value of 3.0 for $\Delta K=0$ depends on our simple choice of the normal BCS model for the $E1$ pairing factor. If we had used a more sophisticated version of the BCS model (for example with corrections for blocking effects[12]) then the value of H could be different. In essence, we have determined empirically only the product HP^-. Since our single $E1$ parameter ($H=3.0$ for $\Delta K=0$ transitions) worked so well for ^{99}Y and ^{103}Nb, we saw no need to go beyond the normal BCS model. With additional experimental $E1$ information a more sophisticated approach may prove necessary, but with the present sparse data and our acceptable reproduction of it, our simple analysis is sufficient.

III. DISCUSSION

The sensitivity of the calculated rotational bands to certain of the particle-rotor parameters is worth noting. The $M1$ and $E1$ transition rates are not very sensitive to either κ or μ. Indeed, we used $\kappa=0.067$ and $\mu=0.53$, whereas the values $\kappa=0.70$ and $\mu=0.40$ were used in Ref. 5. If both of the bands connected by a γ-ray transition have the same deformation, the values of $GM1$ or $GE1$ are the same (within $\sim 5\%$) with either of these two choices. The $M1$ and $E1$ rates are also quite insensitive to variations of 10—15 % in δ. [The collective $E2$ transitions are, of course, more sensitive to δ since $B(E2)\propto\delta^2$.] Thus all conclusions drawn in the preceding about $E1$, $M1$, and $E2$ transition rates are relatively independent of our choices of Nilsson model parameters.

Our use of the Ogle *et al.*[12] estimate for the zero-point rotational energy was an arbitrary choice. It allowed us to determine all band energies without the ambiguity that results when each bandhead is treated as an independent parameter and it provided us with a means to reduce the range of values for the Nilsson parameters. Since the Coriolis matrix elements (hence the k parameter) are relatively insensitive to the Nilsson parameters, our use of the estimate of Ref. 12 did not significantly affect the resulting values of k or the core inertial parameter a.

The Coriolis interaction plays an important role in the rotational band structure. The first-order approach (in which the Coriolis interaction is ignored) can give reasonable results for the intraband $M1+E2$ transitions of the $\frac{5}{2}[422]$ ground band (if a g' value of ~ 0.6 is used) and for the $E1$ transitions from the $\frac{5}{2}[303]$ band. However, the unusual $E1$ pattern for the $\frac{3}{2}[301]$ band and the dominant $\Delta K=2$ $M1$ transitions from the $\frac{1}{2}[431]$ band cannot be adequately explained without Coriolis mixing.

The rotation-vibration interaction does not appear to contribute to the energies or γ-ray transitions of the $\frac{5}{2}[422]$, $\frac{5}{2}[303]$, and $\frac{3}{2}[301]$ bands. At most a $\sim 20\%$ contribution can be attributed to an admixture of $\frac{5}{2}[422]$ coupled to the 2^+ first γ vibration in the $\frac{1}{2}[431]$ band. This small contribution is reasonable since our simplified analysis of vibrational effects in neighboring even-even nuclei indicate that core quadrupole vibrational bandheads can be expected at ~ 1.6 MeV. The amount of vibrational-coupled states (whether quadrupole or octupole) admixed into the observed $\frac{3}{2}[431]$ band is uncertain, although an additional $E1$ contribution, which is large compared to the one-quasiparticle $E1$ strength, is needed to explain the dominant $E1$ transitions between $\frac{3}{2}[431]$ and $\frac{3}{2}[301]$.

The calculations presented here are an extension beyond the first-order analyses given in Refs. 2—5. As stated in the Introduction, our choice of a "textbook" version of the particle-rotor model was made to facilitate comparison of model parameters for deformed $A\sim 100$ nuclei with those of the well-studied deformed rare-earth nuclei. In this version the adjustable parameters k, g', and H are used in order to obtain better agreement with experimental quantities; if the model were ideal, these parameters would all be unity. Recent improvements on the model have focused on the need for such parameters. The "Coriolis" parameter k is reviewed in Ref. 21. For unique-parity Nilsson orbitals, for which k is an attenuation factor less than unity,[6] inclusion in the Hamiltonian of the two-body part of the recoil term may account for most of the attenuation.[21] It had earlier been shown that the residual pairing interaction, if incorporated other than as a simple BCS pairing factor, can reduce the effect of nondiagonal Coriolis matrix elements.[22] Such improvements on the particle-rotor model may be able to explain the attenuation factor of $k=0.77$ found here for the unique-parity bands in ^{99}Y. However, values of k greater than unity, which occur frequently for nonunique-parity bands in the rare earths[6] and were found here for two nonunique-parity bands in ^{99}Y, appear to require additional modifications to the model.

Extensions of the simple particle-rotor model presented here to incorporate recent developments would also have an effect on the calculated bandhead energies.[21] In the version used here, our choice of the simple empirical estimate of Ogle *et al.*[12] is consistent with the level of our calculations, in which all terms in the general Hamiltonian[7,20] other than the rotational energy, BCS pairing effect, and Coriolis interaction were ignored. (The rotation-vibration interaction was not considered directly but was indirectly included insofar as it contributes $\sim 7\%$, as shown by our perturbative analysis of ^{100}Sr, to the inertial parameter of the even-even rotor.) Inclusion of ignored terms would likely have only small effects on the band structure we calculate for ^{99}Y, since our empirically determined values of κ, μ, and δ may have partially accommodated the effects of the ignored terms.

For slightly deformed ($\delta \sim 0.1$) $A \sim 100$ nuclei with $Z=46$–48 and $N \sim 60$, detailed quasiparticle-plus-rotor calculations have been done.[23] Reference 23 presents such calculations for ^{105,107}Ag nuclei with an axially symmetric deformation δ of 0.12 and a variable moment of inertia. The Coriolis and recoil effects are explicitly included. The energy levels and transition rates are well reproduced by the calculation. Although the structures are quite different for the $N=60$ isotones $^{99}_{39}$Y with $\delta \sim 0.3$ and $^{107}_{47}$Ag with $\delta \sim 0.1$, there are some similarities. In both cases positive-parity bands based on the unique-parity $g_{9/2}$ proton orbital are observed at low excitation energies. Attenuation factors k of 0.7–0.8 are needed for the strong unique-parity nondiagonal Coriolis matrix elements in both cases. However, the effects of Coriolis mixing on the intraband $M1/E2$ mixing ratios within these bands are different. ^{99}Y shows a monotonic variation with I of the (mainly $M1$) mixing ratio. For ^{105,107}Ag, if $n=I-K$, the n-even states are nearly pure $M1$ but the n-odd states are considerably mixed.[23] These differences are natural consequences of the differences in Z, Fermi level, and deformation. The particle-rotor model thus seems to provide a good explanation of the structure of both slightly and highly deformed odd-Z $A \sim 100$ nuclei. It remains to be seen whether this will be true for nuclei with intermediate values of Z and δ in the $A \sim 100$ region.

Our interpretation of the observed rotational structure of ^{99}Y is, of necessity, very model dependent. Neither the spins and parities of the levels nor the multipolarities of the γ-ray transitions have been determined experimentally. Our values for these are deduced indirectly from the available experimental data. Except for the lifetimes of the negative-parity bandheads in ^{103}Nb, the strengths of the γ transitions are not known. Additional measurements on ^{99}Y and other odd-A nuclei in the neutron-rich $A \sim 100$ region are needed to further test our understanding of the structure of these nuclei, whose structural change from spherical to highly deformed is remarkably abrupt.

In spite of the unsolved mystery about the $\frac{3}{2}[431]$ band, the rotational structure of ^{99}Y is well described by our calculations. The values of the parameters k, g', and H reveal that this nucleus comes closer than do the deformed rare earths to being described by the simple particle-rotor model, since these parameters are closer to unity than those found for the rare earths.[6] Our fitted values for these parameters imply less need for their renormalization (to compensate for interaction terms omitted from the Hamiltonian) compared to the rare earths. It is interesting to speculate if this can be simply a consequence of the lower density of single-particle states and lower pairing energy for $A \sim 100$ nuclei. If this were true, then the $A \sim 100$ nuclei, with their stronger dependence of deformation on nucleon number, provide a better testing ground for the underlying causes of deformation in nuclei.

APPENDIX

The following presents expressions for $E1$, $M1$, and $E2$ transition probabilities for rotational bands. The operators involved, including the single-particle $E2$ operator (which we neglected in our calculations), and transition rates are given in Ref. 23.

A. Intraband γ-ray intensity ratios for pure-K bands

The assumption of a rotational band described by a single value of K (or dominated by a single value of K) can be tested by comparing ratios of intensities of intraband γ-ray transitions. The relationships are given in detail in Ref. 20. The intensity ratio within a pure-K band is determined by the absolute value of a single parameter, $(g_K-g_R)/Q_0$, where Q_0 is the intrinsic quadrupole moment of levels within the band, g_R is the collective magnetic-dipole g factor, and g_K is the intrinsic g factor.[20]

For an axially symmetric nucleus of radius R_0, where R_0 is typically taken as $1.2A^{1/3}$ fm,

$$Q_0=0.8ZR^2\delta(1+0.667\delta+0.889\delta^2+\cdots) \quad \text{(A1)}$$

and δ is the Nilsson deformation parameter. For odd-Z deformed rare earth nuclei, the hydrodynamic estimate of Z/A for g_R has been found to be reasonable, whereas smaller values of g_R are in better agreement with experiment for even-even and odd-N rare earth nuclei.[11] The intrinsic g factor g_K depends on the particular Nilsson orbital. Browne and Femenia[18] give the equation for g_K in a form in which the spin and orbital g factors g_s and g_l occur explicitly,

$$2Kg_K=g_s GMS(K\rightarrow K)+g_l GML(K\rightarrow K)\,. \quad \text{(A2)}$$

The quantities GMS and GML are defined by Browne and Femenia and are tabulated for the Nilsson parameters appropriate for the deformed rare earth nuclei. In comparing calculated and experimentally deduced values of g_K, an effective g_s value, given by $g_s^{\text{eff}}=g'g_s^{\text{free}}$, has been frequently used. In Refs. 6 and 18 $g'=0.6$ and in Ref. 19 $g'=0.7$ were used for deformed rare-earth nuclei.

B. γ-ray transition probabilities for mixed-K bands

Letting primes denote final states, the $M1$ reduced transition probability is given by[18]

$$B(M1,Ia\to I'a')=\frac{3}{16\pi}\left[\frac{e\hbar}{2Mc}\right]^2\left|\sum_{K,K'}D_{Ia}^{K}D_{I'a'}^{K'}\times[\langle I1I'\,|\,KmK'\rangle GM1(K\to K')+(-)^{I'+K'}\times\langle I1I'\,|\,K,-1,-K'\rangle GM1(K\to -K')]P_{K,K'}^{+}\right|^2 , \tag{A3}$$

where the $\langle I1I'\,|\,KmK'\rangle$ are Clebsch-Gordan coefficients. $|\,Ia\,\rangle$ denotes a state of spin I in a mixed-K band denoted by the label a. The D_{Ia}^{K} are the linear coefficients of pure-K bands in the initial wave function.

Using a similar notation, the $E1$ reduced transition probability[16] can be written as

$$B(E1,Ia\to I'a')=\frac{3e^2}{4\pi}\frac{\hbar}{M\omega_{00}}\left[\Delta-\frac{Z}{A}\right]^2\left|\sum_{K,K'}D_{Ia}^{K}D_{I'a'}^{K'}[\langle I1I'\,|\,KmK'\rangle GE1(K\to K')+(-)^{I'+K'}\times\langle I1I'\,|\,K,-1,-K'\rangle GE1(K\to -K')]P_{K,K'}^{-}\right|^2 . \tag{A4}$$

The $E1$ pairing factor $P_{K,K'}^{-}$ can be approximated by $(U_K U_{K'}-V_K V_{K'})$.[6] This expression for $B(E1)$ explicitly shows both the oscillator length parameter $\hbar/M\omega_{00}$ and the recoil correction term $(\Delta-Z/A)^2$, where $\Delta=1$ for Z odd and $\Delta=0$ for N odd. (With the usual Nilsson model energy scale of $\hbar\omega_{00}=41A^{-1/3}$ MeV, $\hbar/M\omega_{00}=1.0A^{1/3}$ fm^2.) The intrinsic term $GE1$ is given (for $K\neq\frac{1}{2}$ and $K'\neq\frac{1}{2}$) by

$$GE1(K\to K')=\sum_{l,l'}\langle Nl\,|\,r\,|\,N'l'\rangle[(2l+1)/(2l'+1)]^{1/2}\langle l1l'\,|\,000\rangle\sum_{\Lambda,\Lambda'}\sum_{\Sigma}a_{l\Lambda\Sigma}a_{l'\Lambda'\Sigma'}\langle l1l'\,|\,\Lambda m\Lambda'\rangle , \tag{A5}$$

where $|K-K'|\leq 1$ and $K=\Omega=\Lambda+\Sigma$. The $a_{l\Lambda\Sigma}$ are normalized coefficients of the harmonic oscillator basis states $|\,Nl\Lambda\Sigma\,\rangle$ of the intrinsic Nilsson state with projection quantum number K. With r in oscillator length units, the radial matrix elements are square roots of half integers.

These expressions for $B(M1)$ and $B(E1)$ include the pairing factors $P_{K,K'}^{\pm}$ which "convert" single-particle expressions into quasiparticle expressions.[6] The quantities $GM1$ and $GE1$ depend only upon the intrinsic Nilsson states, hence upon κ, μ, and the deformation δ. The $GM1$ and $GE1$ terms for $K\to -K'$ are nonzero only for $K=\frac{1}{2}$ and $K'=\frac{1}{2}$. With Coriolis mixing, intraband $M1$ transitions are no longer simply proportional to $(g_K-g_R)^2$ as they are for pure-K bands. [For pure-K bands $GM1(K\to K)=2K(g_K-g_R)$.]

For the $E2$ transitions, we assumed that the collective core contributions dominate single-particle contributions, due to observed $B(E2)$ enhancements of ~ 100 for highly deformed $A\sim 100$ nuclei.[17] With only the collective $E2$ contribution, only $K'=K$ terms contribute, hence

$$B(E2,Ia\to I'a')=\frac{5}{16\pi}(eQ_0)^2\times\left|\sum_{K}D_{Ia}^{K}D_{I'a'}^{K}\langle I2I'\,|\,K0K\rangle\right|^2 . \tag{A6}$$

The only parameter involved for collective $B(E2)$ rates is the intrinsic quadrupole moment Q_0 of the even-even core of the deformed odd-A nucleus.

With $B(E1)$ expressed in the "natural units" μ_N^2 (where $\mu_N=e\hbar/2Mc$) generally used for $B(M1)$, the dipole transition rate for either $E1$ or $M1$ is

$$\lambda(\sigma1,Ia\to I'a')=(1.76\times10^{13}/\mathrm{s})B(\sigma1,Ia\to I'a')E_\gamma^3 , \tag{A7}$$

where σ is either E or M and the transition energy E_γ is in MeV. The collective $E2$ transition rate is

$$\lambda(E2,Ia\to I'a')=(1.23\times10^{13}/\mathrm{s})B(E2,Ia\to I'a')E_\gamma^5 , \tag{A8}$$

with E_γ in MeV and $B(E2)$ in units of e^2b^2, where $1b=10^{-28}m^2$.

[1] R. F. Petry, H. Dejbakhsh, J. C. Hill, F. K. Wohn, M. Shmid, and R. L. Gill, Phys. Rev. C 31, 621 (1985) (the preceding paper).

[2] E. Monnand, J. A. Pinston, F. Schussler, B. Pfeiffer, H. Lawin, G. Battistuzzi, K. Shizuma, and K. Sistemich, Z. Phys. A **306**, 183 (1982).

[3] F. K. Wohn, J. C. Hill, R. F. Petry, H. Dejbakhsh, Z. Berant, and R. L. Gill, Phys. Rev. Lett. **51**, 873 (1983).

[4]K. Shizuma, H. Ahrens, J. P. Bocquet N. Kafrell, B. D. Kern, H. Lawin, R. A. Meyer, K. Sistemich, G. Tittel, and N. Trautmann, Z. Phys. A **315**, 65 (1984).

[5]T. Seo, A. M. Schmitt, H. Ahrens, J. P. Bocquet, N. Kafrell, H. Lawin, G. Lhereonneau, R. A. Meyer, K. Shizuma, K. Sistemich, G. Tittel, and N. Trautmann, Z. Phys. A **315**, 251 (1984).

[6]M. E. Bunker and C. W. Reich, Rev. Mod. Phys. **43**, 348 (1971).

[7]J. M. Eisenberg and W. Greiner, *Nuclear Models* (North-Holland, Amsterdam, 1970), Vol. 1, p. 152.

[8]R. E. Azuma, G. L. Borchert, L. C. Carraz, P. G. Hansen, B. Jonson, S. Mattsson, O. B. Nielsen, G. Nyman, I. Ragnarsson, and H. L. Ravn, Phys. Lett. **86B**, 5 (1979).

[9]F. K. Wohn, J. C. Hill, R. F. Petry, K. Sistemich, R. L. Gill, H. Mach, and A. Piotrowski, Bull. Am. Phys. Soc. **29**, 660 (1984).

[10]M. Epherre, G. Audi, C. Thibault, R. Klapisch, G. Huber, F. Touchard, and H. Wollnik, Phys. Rev. C **19**, 1504 (1979).

[11]O. Nathan and S. G. Nilsson, in *Alpha-, Beta- and Gamma-Ray Spectroscopy,* edited by K. Seigbahn (North-Holland, Amsterdam, 1965), Vol. 1, p. 601.

[12]W. Ogle, S. Wahlborn, R. Piepenbring, and S. Fredriksson, Rev. Mod. Phys. **43**, 424 (1971).

[13]G. Monsonego and R. Piepenbring, Nucl. Phys. **58**, 593 (1964).

[14]M. N. Vergnes and J. O. Rasmussen, Nucl. Phys. **62**, 233 (1965).

[15]C. F. Perdrisat, Rev. Mod. Phys. **38**, 41 (1966).

[16]A. Faessler, T. Udagawa, and R. K. Sheline, Nucl. Phys. **85**, 690 (1968).

[17]E. Cheifetz, in *Nuclear Spectroscopy of Fission Products,* IOP Conf. Series Vol. **51**, edited by T. von Egidy (Institute of Physics, London, 1980), p. 193.

[18]E. Browne and F. R. Femenia, Nucl. Data Tables **10**, 81 (1971).

[19]W. Andrejtscheff, K. D. Shilling, and P. Manfrass, At. Data Nucl. Data Tables **16**, 515 (1975).

[20]M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, Mass., 1962), p. 333.

[21]J. Rekstad and T. Engeland, Phys. Lett. **89B**, 316 (1980).

[22]W. Scholz and F. B. Malik, Phys. Rev. **176**, 1355 (1968).

[23]R. Popli, J. A. Grau, S. I. Popik, L. E. Samuelson, F. A. Rickey, and P. C. Simms, Phys. Rev. C **20**, 1350 (1979).