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Microscopic foundations of Dirac phenomenology

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Recently we have seen a series of highly successful fits to data for nucleon-nucleus scattering which are based upon the use of the Dirac equation. In the phenomenological analysis the potential in the Dirac equation is usually limited to two terms, a (Lorentz) scalar and vector potential. In a recent work we have shown that there are eight scalar invariants that are needed to fully specify the relativistic interaction of an off-shell nucleon with an on-shell, spin zero nucleus. It appears that the phenomenological potentials are *effective* potentials in the sense that their values need to be adjusted to compensate for the use of a highly simplified phenomenological form. In this work we present calculations of the complete potential, including estimates of the eight terms noted above. Our preliminary results indicate that the values of the phenomenological potentials can be reproduced in a microscopic calculation and bear out our ideas concerning the significance of the phenomenological potentials. It is found that if one wishes to obtain the correct value for the spin-orbit potential strength, for example, in a *microscopic* calculation, one must consider the optical potential in its most general form. At projectile energies greater than 300 or 400 MeV we expect that only two of the eight terms noted above will be important and therefore the relativistic impulse approximation will provide a satisfactory basis for calculating the optical potential.

I. INTRODUCTION

Recently it has become clear that it is appropriate to use the Dirac equation for the study of nucleon-nucleus scattering. The studies which have been made are of two types: There is an extensive body of work in which one fits the parameters of (Lorentz) scalar and vector optical potentials so that the data on differential cross sections and various spin-dependent observables are reproduced;[1] in addition, we have parameter-free calculations based upon a relativistic impulse approximation.[2–5] These parameter-free calculations appear to be most useful for projectile energies greater than 300—400 MeV, since, at the higher energies, medium corrections and effects due to particle exchange are less important. At these higher energies the calculations based upon the relativistic impulse approximation are able to reproduce the phenomenological strengths of the scalar and vector potentials quite well.[3]

At lower energies the situation is more complex. One can use results of our nuclear matter calculations (which include medium modifications of the scattering amplitude) to obtain estimates of the scalar and vector potentials.[6] For example, we can write for the self-energy of a nucleon in nuclear matter

$$\Sigma(p)=A(p)+\gamma^0 B(p)+\frac{\vec{\gamma}\cdot\vec{p}}{m_{\rm N}}C(p)\ , \tag{1.1}$$

which may be contrasted with the phenomenological form used for the self-energy, in most calculations,[1–5]

$$\Sigma(p)=U_s(p)+\gamma^0 U_v(p)\ . \tag{1.2}$$

[The Dirac equation is usually solved using *local* potentials. In that case, for a uniform system, the potentials $U_s(p)$ and $U_v(p)$ would only depend on the energy of the nucleon.]

Values of $A(p)$, $B(p)$, and $C(p)$ are presented in Ref. 6. In the nonrelativistic domain, one can write

$$\epsilon_{\rm inc}=\frac{\vec{p}_{\rm inc}^{\,2}}{2m_{\rm N}}=\frac{\vec{p}^{\,2}}{2m_{\rm N}}+U_{\rm eff}(p)\ , \tag{1.3}$$

and identify $\epsilon_{\rm inc}$ with the energy of the projectile nucleon. Then, $\vec{p}$ is the momentum of the nucleon in nuclear matter or in the nucleus when the nucleus is approximated by a uniform system. Values of $U_{\rm eff}(p)$ are given in Ref. 6, and Eq. (1.3) can be used to convert tables of $A(p)$, $B(p)$, and $C(p)$ to $A(\epsilon_{\rm inc})$, $B(\epsilon_{\rm inc})$, and $C(\epsilon_{\rm inc})$. When this comparison is made it is found the magnitudes of A and B are about 30 percent less than the magnitudes of U_s and U_v in the energy domain $0<\epsilon_{\rm inc}<200$ MeV.

Now we wish to stress that this discrepancy is not in any way a fundamental problem but has its origin in the fact that we are not comparing the correct expressions. In order to carry out this comparison properly one needs to fully develop the microscopic theory to obtain expressions for the *effective* scalar and vector potentials such as U_s and U_v. One test of the microscopic theory then lies in its ability to reproduce these *effective* potentials.

In Sec. II we review our results for the parametrization of the relativistic optical potential which describes the scattering of a nucleon from a finite nucleus. In Sec. III we indicate how the various terms in this potential may be calculated, and in Sec. IV we compare our results with those of "Dirac phenomenology." Section V contains some concluding remarks and conclusions.

II. PARAMETRIZATION OF THE RELATIVISTIC OPTICAL POTENTIAL

In a previous work we gave the following general expression for the nucleon self-energy for a finite system:[7]

$$\begin{aligned}\langle \vec{k}' | \Sigma(W) | \vec{k} \rangle = A + \gamma^0 B + \vec{\gamma}\cdot\frac{(\vec{k}+\vec{k}')}{2m_N}C \\ + \frac{iD}{m_N}\vec{\gamma}\cdot(\vec{k}'-\vec{k}) - \frac{E}{2m_N}\gamma^0\vec{\gamma}\cdot(\vec{k}'-\vec{k}) \\ - \frac{iF}{2m_N}\gamma^0\vec{\gamma}\cdot(\vec{k}'+\vec{k}) + \frac{iG}{m_N^2}\vec{\Sigma}\cdot(\vec{k}'\times\vec{k}) \\ - \frac{iH}{m_N^2}\gamma^0\vec{\Sigma}\cdot(\vec{k}'\times\vec{k}) \; . \end{aligned} \tag{2.1}$$

In nuclear matter, this expression reduces to that given previously in Eq. (1.1). We also found it useful to define the following quantities:[7]

$$\langle \vec{k}',s' | \Sigma^{++}(W) | \vec{k},s \rangle = \bar{u}(\vec{k}',s')\langle \vec{k}' | \Sigma(W) | \vec{k} \rangle u(\vec{k},s) \; , \tag{2.2}$$

$$\langle \vec{k}',s' | \Sigma^{+-}(W) | \vec{k},s \rangle = \bar{u}(\vec{k}',s')\langle \vec{k}' | \Sigma(W) | \vec{k} \rangle w(\vec{k},s) \; , \tag{2.3}$$

etc. Here, $w(\vec{p},s)=v(-\vec{p},-s)$ of Bjorken and Drell.[8]

In turn, we also introduced functions S_1^{++}, S_2^{++}, S_1^{+-}, S_2^{+-}, etc., defined *via* the relations,[7]

$$\langle \vec{k}',s' | \Sigma^{++}(W) | \vec{k},s \rangle = \left\langle s' \left| \left[S_1^{++} + i\frac{\vec{\sigma}\cdot(\vec{k}'\times\vec{k})}{\epsilon\epsilon'}S_2^{++} \right] \right| s \right\rangle , \tag{2.4}$$

$$\langle \vec{k}',s' | \Sigma^{+-}(W) | \vec{k},s \rangle = \left\langle s' \left| \left[\vec{\sigma}\cdot\left[\frac{\vec{k}}{\epsilon}+\frac{\vec{k}'}{\epsilon'}\right]S_1^{+-} + \vec{\sigma}\cdot\left[\frac{\vec{k}'}{\epsilon'}-\frac{\vec{k}}{\epsilon}\right]S_2^{+-} \right] \right| s \right\rangle , \tag{2.5}$$

etc. Here,

$$\epsilon(\vec{p}) = E_N(\vec{p}) + m_N = (\vec{p}^{\,2}+m_N^2)^{1/2} + m_N \; .$$

Now, comparison of our model with the phenomenological models is best done by calculating S_1^{++}, S_2^{++}, etc., in the two formalisms. In order to reduce the length of the equations we will note at the outset that the quantities D, F, and G appear relatively unimportant. Therefore we will record the approximate expressions [with $N=(\epsilon/2m_N)^{1/2}$, $N'=(\epsilon'/2m_N)^{1/2}$],

$$\begin{aligned} S_1^{++}(\vec{k}',\vec{k}) \simeq NN' \Bigg\{ A\left[1-\frac{\vec{k}\cdot\vec{k}'}{\epsilon\epsilon'}\right] + B\left[1+\frac{\vec{k}\cdot\vec{k}'}{\epsilon\epsilon'}\right] + \frac{C}{2m_N}\left[\frac{\vec{k}'^2}{\epsilon'}+\frac{\vec{k}^2}{\epsilon}+\frac{\vec{k}\cdot\vec{k}'(\epsilon+\epsilon')}{\epsilon\epsilon'}\right] \\ + \frac{E}{2m_N}\left[\frac{\vec{k}'^2}{\epsilon'}+\frac{\vec{k}^2}{\epsilon}-\frac{\vec{k}\cdot\vec{k}'(\epsilon+\epsilon')}{\epsilon\epsilon'}\right] - \frac{H}{m_N^2}\left[\frac{\vec{k}'^2\vec{k}^2-(\vec{k}\cdot\vec{k}')^2}{\epsilon\epsilon'}\right]\Bigg\} \end{aligned} \tag{2.6}$$

$$\simeq (A+B) \; . \tag{2.7}$$

In Eq. (2.7) we have now neglected terms of order $(\vec{k}^2/4m_N^2)$, although we include these terms in our calculations. Using similar approximations, we have

$$S_2^{++}(\vec{k}',\vec{k}) \simeq (-A+B+2C-2E-4H) \; , \tag{2.8}$$

$$S_1^{+-}(\vec{k}',\vec{k}) \simeq (-A+C) \; , \tag{2.9}$$

$$S_2^{+-}(\vec{k}',\vec{k}) \simeq (B-E) \; , \tag{2.10}$$

etc.

In all of these expressions A, B, C, E, and H are functions of $\vec{k}$, $\vec{k}'$, and W, the total energy in the center of mass frame.

Let us now compare these results with the model for the self-energy used in phenomenological studies,[1–5]

$$\Sigma(r) = U_s(r) + \gamma^0 U_v(r) \; . \tag{2.11}$$

In this case we would have

$$S_1^{++}(\vec{k}',\vec{k}) = (U_s+U_v) \; , \tag{2.12}$$

$$S_2^{++}(\vec{k}',\vec{k}) = (-U_s+U_v) \; , \tag{2.13}$$

$$S_1^{+-}(\vec{k}',\vec{k}) = -U_s \; , \tag{2.14}$$

$$S_2^{+-}(\vec{k}',\vec{k}) = U_v \; , \tag{2.15}$$

etc.

We compare Eqs. (2.7)–(2.10) with Eqs. (2.12)–(2.15) and assume, as is borne out in our current calculations, that $C>0$, $E<0$, and $H<0$. (Also $|2C| \sim |2E| \sim 2|4H|$ in a very rough approximation.) We see that the simple model of Eq. (2.11) is really not flexible enough to reproduce the full complexity of the microscopic model, since at this point the microscopic model has five significant parameters. However, one may infer the kind of phenomenology that might lead to a reasonable result. Let us assume that the microscopic theory is correct. Then we could set $U_s = A - |\delta|$ and $U_v = B + |\delta|$, where $2\delta = (2C-2E-4H)$. Then we would have $U_s + U_v = A + B$ and

$$-U_s + U_v = -A + B + 2\delta = -A + B + (2C-2E-4H) \; .$$

Of course this would not give S_1^{+-} and S_2^{+-} correctly, but the general trend would be satisfactory for these quantities. Furthermore, at the low energies which we will consider here, the terms of the optical potential involving S_1^{+-} and S_2^{+-}, while important, are not of major significance. (As one approaches $\epsilon_{\rm inc} \sim 200$ MeV, S_1^{++} becomes small and the relative importance of the relativistic corrections involving S_1^{+-}, S_1^{-+}, S_2^{+-}, and S_2^{-+} become progressively more important.)

We conclude from these observations that the magnitudes of U_s and U_v should not be compared directly with the theoretical values of A and B. In Sec. III, we will report on the results of calculations of the quantities D, E, F, G, and H. We will find it useful to use the results for A, B, and C taken from our nuclear matter calculations,[6] since these calculations include the full effects of correlations, Pauli blocking, etc. Before leaving this section however, we can note that the empirical potentials are usually written as a constant times a local form factor. For example, we may set

$$U_s(r) = U_s f_s(r) , \tag{2.16}$$

$$U_v(r) = U_v f_v(r) , \tag{2.17}$$

where

$$f_s(r) = \frac{(\rho_0/\rho_{\rm NM})}{1+\exp\left[\dfrac{r-R}{c}\right]} . \tag{2.18}$$

Here, $(\rho_0/\rho_{\rm NM})$ is a constant, close to unity, that is introduced in the phenomenological analysis. Therefore, we can write, for the phenomenological model,

$$S_1^{++}(\vec{k}',\vec{k}) = U_s\langle \vec{k}' | f_s | \vec{k}\rangle + U_v \langle \vec{k}' | f_v | \vec{k}\rangle , \tag{2.19}$$

$$S_2^{++}(\vec{k}',\vec{k}) = -U_s\langle \vec{k}' | f_s | \vec{k}\rangle + U_v \langle \vec{k}' | f_v | \vec{k}\rangle , \tag{2.20}$$

etc.

We will consider forward scattering and note that

$$\langle \vec{k} | f_s | \vec{k}\rangle \simeq \langle \vec{k} | f_v | \vec{k}\rangle . \tag{2.21}$$

Thus we have, in an accurate approximation,

$$S_1^{++}(\vec{k},\vec{k}) = (U_s + U_v)\langle \vec{k} | f | \vec{k}\rangle \tag{2.22}$$

and

$$S_2^{++}(\vec{k},\vec{k}) = (-U_s + U_v)\langle \vec{k} | f | \vec{k}\rangle . \tag{2.23}$$

It will therefore be useful to extract a factor of $\langle \vec{k} | f | \vec{k}\rangle$ from our calculations of D, E, F, etc. We set

$$\langle \vec{k} | D | \vec{k}\rangle = \left[\frac{\langle \vec{k} | D | \vec{k}\rangle}{\langle \vec{k} | f | \vec{k}\rangle}\right] \langle \vec{k} | f | \vec{k}\rangle , \tag{2.24}$$

$$\equiv \widetilde{D} \langle \vec{k} | f | \vec{k}\rangle , \tag{2.25}$$

and introduce similar relations for E, F, G, and H. For A, B, and C we can use our nuclear matter results $(A^{\rm NM}, B^{\rm NM}, C^{\rm NM})$ in the following way. We write

$$\langle \vec{k}' | A | \vec{k}'\rangle = A^{\rm NM}\left[\frac{\rho(^{40}{\rm Ca})}{\rho^{\rm NM}}\right] \langle \vec{k}' | g | \vec{k}\rangle \tag{2.26}$$

with

$$g(r) = \frac{1+\exp(-R/c)}{1+\exp\left[\dfrac{r-R}{c}\right]} . \tag{2.27}$$

The ratio $\rho(^{40}{\rm Ca})/\rho^{\rm NM}$ is included to take into account the fact that the central density in ^{40}Ca is not exactly that of nuclear matter. [Note that $g(0) = 1$.] We then set

$$\langle \vec{k} | A | \vec{k}\rangle = \left\{ A^{\rm NM}\left[\frac{\rho(^{40}{\rm Ca})}{\rho^{\rm NM}}\right] \frac{\langle \vec{k} | g | \vec{k}\rangle}{\langle \vec{k} | f | \vec{k}\rangle}\right\} \langle \vec{k} | f | \vec{k}\rangle \tag{2.28}$$

with similar relations relating $\langle \vec{k} | B | \vec{k}\rangle$ and $\langle \vec{k} | C | \vec{k}\rangle$ to $B^{\rm NM}$ and $C^{\rm NM}$. It is useful therefore to write

$$\langle \vec{k} | A | \vec{k}\rangle = \widetilde{A} \langle \vec{k} | f | \vec{k}\rangle , \tag{2.29}$$

with similar definitions of B, C, etc. Using this notation, we have

$$S_1^{++}(\vec{k},\vec{k}) = (\widetilde{A} + \widetilde{B})\langle \vec{k} | f | \vec{k}\rangle , \tag{2.30}$$

$$S_2^{++}(\vec{k},\vec{k}) = (-\widetilde{A} + \widetilde{B} + 2\widetilde{C} - 2\widetilde{E} - 4\widetilde{H})\langle \vec{k} | f | \vec{k}\rangle , \tag{2.31}$$

and so forth.

This formulation allows for a fairly direct comparison with the phenomenological forms given in Eqs. (2.19) and (2.20).

III. CALCULATION OF THE RELATIVISTIC OPTICAL POTENTIAL

As noted in Sec. II, we can use our nuclear matter results to provide values of A, B, and C. However, to obtain estimates of D, E, F, G, and H we must carry out a calculation for a *finite* nucleus. (We will perform our calculations for ^{40}Ca.) The approximation used is depicted in Fig. 1. Here an (off-shell) nucleon of momentum $\vec{k}$ is incident on a nuclear target of momentum $-\vec{k}$. The nucleon is then scattered so that its momentum is $\vec{k}'$. The first part of the figure indicates the calculation of the optical potential in the impulse approximation. Here, we perform the full integration over the spectator nucleus of momentum $\vec{Q}$. (Wave functions which are solutions of an appropriate Dirac equation are used to parametrize the density matrix of the target.) Now in these calculations the nucleon-nucleon scattering amplitude is approximated by the exchange of σ, π, ρ, and ω mesons with coupling constants taken from the potential HEA of Holinde, Erkelenz, and Alzetta.[9] It can easily be inferred that the parameters C, D, E, F, G, and H are nonzero because of the exchange term shown in Fig. 1.

As we remarked earlier, we use our nuclear matter results for A, B, and C. These results include the effect of correlations. We have not as yet included correlation effects in the calculation of D, E, F, G, and H. Probably the greatest uncertainty is introduced at this stage through the neglect of tensor correlations which could effect the pion exchange terms significantly. We keep this feature

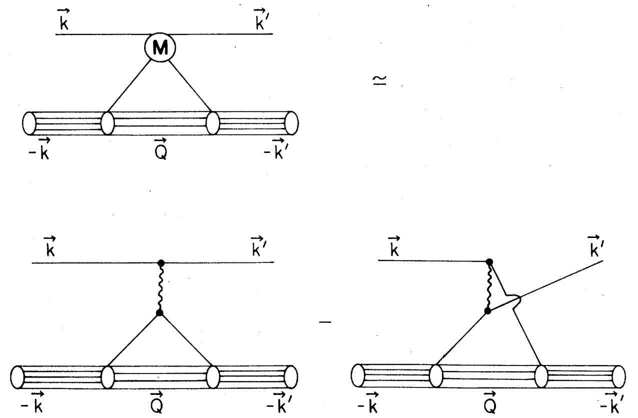

FIG. 1. Direct and exchange terms in the calculation of the relativistic nuclear optical potential in the Born approximation. The wavy line represents the exchange of a σ, π, ρ, and ω meson. In our calculations, a complete integral is performed over the spectator momentum, $\vec{Q}$. (This approximation is used for the construction of the quantities D, E, F, G, and H.)

in mind and consider our results for D, E, F, G, and H to be somewhat uncertain. These uncertainties will be resolved in a future publication where we also provide more details of our calculations. However, considering the current interest in this problem, we feel that our present results, which we believe to be quite instructive, should be presented at this time. Indeed, it is quite possible that the inclusion of correlation effects will not change our present results significantly.

IV. NUMERICAL RESULTS

We consider p-^{40}C scattering at $T_p = 80$ MeV, where T_p is the proton laboratory energy. From a phenomenological study[10] we have the values $U_s = -422.04$ MeV and $U_v = 325.5$ MeV and

$$\langle \vec{k} \,|\, U_s \,|\, \vec{k} \rangle = \frac{1}{(2\pi)^3} \int U_s(r) d\vec{r} \,, \tag{4.1}$$

$$= -408.47 \text{ MeV fm}^3 \,. \tag{4.2}$$

This leads to the value $\langle \vec{k} \,|\, f_s \,|\, \vec{k} \rangle = 1.03$ fm^3 when use is made of the relation

$$\langle \vec{k} \,|\, U_s \,|\, \vec{k} \rangle = U_s \langle \vec{k} \,|\, f_s \,|\, \vec{k} \rangle \,. \tag{4.3}$$

Similarly, we have

$$\langle \vec{k} \,|\, U_v \,|\, \vec{k} \rangle = 317.3 \text{ MeV fm}^3 \,, \tag{4.4}$$

and the value $\langle \vec{k} \,|\, f_v \,|\, \vec{k} \rangle = 1.03$ fm^3. Since $\langle \vec{k} \,|\, f_v \,|\, \vec{k} \rangle = \langle \vec{k} \,|\, f_s \,|\, \vec{k} \rangle$, we see that the approximation made in going from Eqs. (2.19) and (2.20) to Eqs. (2.22) and (2.23) is quite accurate.

Now we note that at 80 MeV, $N^2 = \epsilon/2m_N = 1.04$, and if we set $|\vec{k}| = 1.954$ fm^{-1}, we also have $\vec{k}^2/\epsilon^2 = 0.04$, with $\epsilon = 9.89$ fm^{-1} and $m_N = 4.75$ fm^{-1}. We now use $U_s = -422.04$ MeV and $U_v = 325.5$ MeV to evaluate the phenomenological values of S_1^{++}, S_2^{++}, S_1^{+-}, etc. We keep corrections of order $\vec{k}^2/\epsilon^2$ in this evaluation. The values obtained are listed in Table I under the column labeled phenomenology.

We now turn to our theoretical calculations. From Ref. 6, we find $A = -268$ MeV, $B = 192$ MeV, and $C = 43$ MeV [see Eq. (2.29)]. These numbers include the full effects of correlations and Pauli blocking. From our calculations of the diagrams shown in Fig. 1, we find $i\tilde{D} = 0$ MeV, $\tilde{E} = -33$ MeV, $iF = 0$ MeV, $G = 0$ MeV, and $\tilde{H} = -10.7$ MeV. Thus we see that potentials involving D, F, and G are unimportant, as noted previously. Using these values and keeping terms of order of $\vec{k}^2/\epsilon^2$, we calculate the numbers given in the second column of Table I.

It is important to note that A is -268 MeV and $U_s = -422$ MeV. Furthermore, $B = 192$ MeV and $U_v = 325.5$ MeV. It is clear from the comments made here that direct comparison of A with U_s and B with U_v is inappropriate. One should compare the *effective* central and spin-orbit potentials that appear in the Schrödinger equation. For example, we see that the coefficient of the leading contribution to the spin-orbit potential S_2^{++} is nicely reproduced in our microscopic model (see Table I). However, if we were to compare S_2^{++} to $-A + B = 460$ MeV one would find a large discrepancy. Again, we remark that this would be an inappropriate comparison.

One might be tempted to compare the values of S_1^{++} given in Table I; however, again a more appropriate comparison is at the level of the *effective* central field for use in the Schrödinger equation. We can set

$$p^0 = E_N(\vec{p}) + U_{\text{eff}}(p)$$

and note that[6]

$$p^0 = E_N(\vec{p}) + \frac{m_N}{E_N(\vec{p})} \Sigma^{++}(p^0, \vec{p}) + \left[\frac{m_N}{E_N(\vec{p})} \right]^2 \frac{\Sigma^{+-}(p^0, \vec{p}) \Sigma^{-+}(p^0, \vec{p})}{p^0 + E_N(\vec{p}) - \dfrac{m_N}{E_N(\vec{p})} \Sigma^{--}(p^0, \vec{p})} \,. \tag{4.5}$$

TABLE I. Values of $S_1^{++}(\vec{k}, \vec{k})$, $S_2^{++}(\vec{k}, \vec{k})$, etc., for protons scattering from ^{40}Ca with $T_p = 80$ MeV. The first column gives the phenomenological values and the second column gives the theoretical values (see the text).

	Phenomenology (MeV)	Theory (MeV)
S_1^{++}	-69	-52
S_2^{++}	777	696
S_1^{+-}	439	323
S_1^{-+}	439	323
S_2^{+-}	339	230
S_2^{-+}	-339	-230
S_1^{--}	774	468
S_2^{--}	-100	-49
U_{eff}	-43	-40

We can then obtain $U_{\text{eff}}(p)$ as[6]

$$U_{\text{eff}}(p)=\frac{m_N}{E_N(\vec{p})}\Sigma^{++}(\vec{p})+\left[\frac{m_N}{E_N(\vec{p})}\right]^2 \times\frac{\Sigma^{+-}(\vec{p})\Sigma^{-+}(\vec{p})}{2E_N(\vec{p})+\frac{m_N}{E_N(\vec{p})}[\Sigma^{++}(\vec{p})-\Sigma^{--}(\vec{p})]} . \tag{4.6}$$

For scattering in the forward direction we have

$$\Sigma^{++}(\vec{p})=S_1^{++}(\vec{p},\vec{p}) , \tag{4.7}$$

$$\Sigma^{+-}(\vec{p})=\frac{2\vec{\sigma}\cdot\vec{p}}{\epsilon(\vec{p})}S_1^{+-}(\vec{p},\vec{p}) , \tag{4.8}$$

$$\Sigma^{-+}(\vec{p})=\frac{2\vec{\sigma}\cdot\vec{p}}{\epsilon(\vec{p})}S_1^{-+}(\vec{p},\vec{p}) , \tag{4.9}$$

and

$$\Sigma^{--}(\vec{p})=S_1^{--}(\vec{p},\vec{p}) . \tag{4.10}$$

Therefore, we can write

$$U_{\text{eff}}(\vec{p})=\frac{m_N}{E_N(\vec{p})}S_1^{++}+\left[\frac{m_N}{E_N(\vec{p})}\right]^2\frac{4\vec{p}^{\,2}}{\epsilon^2} \times\frac{S_1^{+-}S_1^{-+}}{2E_N(\vec{p})+\frac{m_N}{E_N(\vec{p})}(S_1^{++}-S_1^{--})} , \tag{4.11}$$

where we have not indicated the arguments of S_1^{++}, S_1^{+-}, etc., for simplicity. Using the values given in Table I, we obtain U_{eff} (phenomenological) $=-43$ MeV and U_{eff} (theory) $=-40$ MeV. Thus the theory agrees quite well with the phenomenological value for the depth of the central potential when the relativistic correction, the second term in Eq. (4.11), is included.

We may also remark that upon comparison of the numbers given in Table I, we see a large difference of the two values of S_1^{--}. It is easy to see that in the phenomenological model one must obtain $S_1^{--}\simeq S_2^{++}$. Since the analysis of the data is sensitive to S_2^{++} and rather insensitive to the value of S_1^{--}, we see that S_1^{--} is largely fixed when fitting the spin-orbit strength in the phenomenological model. In the microscopic analysis, one finds $S_1^{--}\neq S_2^{++}$ as may be seen from Table I.

V. CONCLUSIONS AND SUMMARY

An important conclusion of this work is the general observation that the phenomenological optical potentials which are used in the Dirac equation do not have a direct theoretical significance. As we have seen, there are eight scalar invariants needed to describe the scattering of an off-mass-shell nucleon from an (on-shell) spin zero nucleus. Our calculations indicate that five of these quantities make signficant contributions to the optical potential. We have also shown that with the limited parametrization (scalar plus vector potentials) of the conventional phenomenological model, one needs enhanced magnitudes for the scalar and vector fields to obtain the empirical values of the spin-orbit potential. This enhancement of the magnitudes of the scalar and vector fields, with respect to theoretical estimates of these quantities, is needed to compensate for the contributions to the spin-orbit potential from the quantities C, E, and H.

There are various improvements that can be made in our analysis. We need to study the role of correlations in modifying the values of D, E, F, G, and H since these quantities have been calculated here using the Born approximation. We have also seen that there is excellent agreement for the *effective* central potential, U_{eff}, when a comparison is made between the phenomenological and theoretical values for this quantity. A similar comparison should be made for the *effective* spin-orbit potential; however, this requires a more detailed calculation than that presented here if we are to include the contributions of the relativistic corrections to this potential. (Such contributions involve S_1^{+-} and S_1^{-+}.) The general trend for the spin-orbit interaction may be seen to be given correctly since the theoretical values and phenomenological values of S_2^{++} are in rather good agreement. A more detailed comparison of the effective spin-orbit potential [including the terms involving $(S_1^{+-}S_1^{-+})$, $(S_2^{+-}S_2^{-+})$, etc.] will be presented at a later time.

As a final point we note that in our model the quantities C, D, E, F, G, and H arise when one calculates exchange diagrams. (These terms would be zero in a Hartree approximation.[11]) Now as one increases the energy of the projectile, such exchange terms become progressively less important. The success of the (parameter-free) relativistic impulse approximation for energies greater than about 400 MeV (Refs. 2—5) is then, in part, due to the relative unimportance of exchange effects at the higher energy.

We conclude that any attempt to calculate the relativistic optical potential at low energies requires the calculation of all the pieces of the potential and not only its scalar and vector parts. As we have remarked several times, the phenomenological potentials are best thought of as *effective* potentials, and rather complex microscopic calculations are needed to reproduce the magnitudes of these effective potentials.

ACKNOWLEDGMENTS

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