Spin dependence of the ΛN effective interaction

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A combination of theoretical estimates, based on a ΛN potential model and phenomenological analysis of hypernuclear data, is used to determine a set of four $p_N s_\Lambda$ two-body matrix elements which characterize the spin dependence of the ΛN interaction in the p shell. The central spin-spin and the Λ spin-orbit matrix elements are most strongly constrained by existing data. The spin dependence is weak in the sense that (core)$\times s_\Lambda$ doublet splittings are predicted to be of order 100 keV, except for the special case of $^7_\Lambda$Li where the central spin-spin interaction dominates and the ground state doublet splitting is likely to be of order 600 keV. Detailed consideration is given to those γ ray transitions the observation of which would best serve to further constrain the spin dependence of the ΛN interaction, particularly the tensor interaction.

I. INTRODUCTION

Previous attempts to characterize the spin dependence of the ΛN effective interaction in light hypernuclei have been hampered by a lack of data. In particular, the phenomenological analyses[1–3] of Gal, Soper, and Dalitz (GSD) for the p-shell hypernuclei had available as data 12 ground state binding energies, together with the constraints imposed by a knowledge of three ground state spins and a mixing angle which characterizes the relationship of the ground state wave function of $^8_\Lambda$Li with the two lowest states of the ^{7}Li core nucleus. Fits were performed with some or all of the five $p_N s_\Lambda$ two-body matrix elements as parameters plus a single parameter characterizing a three-body ΛNN interaction which was assumed independent of σ_Λ. A wide variety of minima were found corresponding to very different parameter sets, indicative of the limitations of a data set consisting only of ground state spins. While many of the parameter sets could be ruled out on physical grounds, none were consistent with all the subsidiary data. In addition, the spin-orbit parameters of the favored solution ($\Delta^+ S^+ Q^0_{00}$), which we shall refer to as GSD79, differ considerably from those expected on the basis of estimates using meson exchange models for the ΛN interaction.

The favored solution was also used by Dalitz and Gal in a survey[4] which considered the formation of low-lying excited states in p-shell hypernuclei ($p^n s_\Lambda$ configurations) via (K$^-$,π^-) reactions and the subsequent γ decay of these levels. It was made clear that the most direct information on the spin dependence of the ΛN effective interaction would come from the identification of s_Λ doublets based on core states with nonzero spin. The doublet splittings depend mainly on combinations of three of the four spin-dependent ΛN matrix elements—Δ from the central spin-spin interaction, S_Λ from the Λ-spin dependent spin-orbit interaction, and T from the tensor interaction. The energy separation of states based on different core states depends on S_N, the nucleon-spin dependent spin-orbit interaction, but these separations may be rather more sensitive to contributions from the three-body ΛNN interaction than are the doublet splittings.

In this paper we draw on a number of sources to obtain estimates for Δ, S_Λ, T, and S_N, principally, (i) a ΛN potential model based on fits to two-body data, (ii) the excitation energies of excited 1$^+$ levels in $^4_\Lambda$H and $^4_\Lambda$He, (iii) energy separations deduced from $p_N \rightarrow p_\Lambda$ transitions in (K$^-$,π^-) reactions on p-shell targets, particularly ^{13}C, and (iv) information on doublet splittings in $^7_\Lambda$Li and $^9_\Lambda$Be from (K$^-$,$\pi^-\gamma$) experiments. The resulting interaction reproduces the known ground state spins of p-shell hypernuclei ($^8_\Lambda$Li, $^{11}_\Lambda$B, $^{12}_\Lambda$B) and removes a discrepancy which exists for the GSD79 interaction regarding the ground-state wave function of $^8_\Lambda$Li. We go on to consider γ transitions in other p-shell hypernuclei, particularly those at the end of the shell, which could further constrain the spin dependence of the ΛN interaction. Since most of the detailed treatment of Dalitz and Gal[4] is valid independently of the values of the ΛN matrix elements, our discussion is kept relatively brief.

II. CALCULATIONS

The basic philosophy behind our calculations is very similar to that of Gal, Soper, and Dalitz.[1–3] Our shell-model calculations are performed for $\{s^4p^{A-5}s_\Lambda\}$ configurations using the interaction of Cohen and Kurath[5] for the core wave functions; the formalism follows a recent treatment[6] for $\{s^4p^{A-5}p_\Lambda\}$ configurations. The two-body ΛN interaction can be expressed in terms of five radial integrals, one associated with each term in

$$V_{\Lambda N}(r)=V_0(r)+V_\sigma(r)\mathbf{s}_N\cdot\mathbf{s}_\Lambda+V_\Lambda(r)\mathbf{l}_{N\Lambda}\cdot\mathbf{s}_\Lambda + V_N(r)\mathbf{l}_{N\Lambda}\cdot\mathbf{s}_N+V_T(r)S_{12}\ , \quad (1)$$

where $\mathbf{l}_{N\Lambda}$ is the relative orbital angular momentum and

$$S_{12}=3(\boldsymbol{\sigma}_N\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_\Lambda\cdot\hat{\mathbf{r}})-\boldsymbol{\sigma}_N\cdot\boldsymbol{\sigma}_\Lambda$$

with $r=|\mathbf{r}_N-\mathbf{r}_\Lambda|$. These parameters, denoted by $\overline{V}$, Δ, S_Λ, S_N, and T, are taken to be constant throughout the shell. The assumption of constancy can be justified in several ways, for instance, (i) the measured rms charge radii of all stable p-shell nuclei from ^{6}Li to ^{15}N are very well fitted by the charge radii calculated using A-independent 0s and 0p harmonic oscillator orbitals, for a suitably chosen radial parameter b, and (ii) the same assumption has been successfully used for the NN interaction by Cohen and Kurath.[5]

The doublet splittings are determined mainly by Δ, S_Λ, and T. A simplified guide to the pattern of doublet splittings is given in Fig. 1. The $N^{-1}\Lambda$ combination of parameters applicable for $p_{3/2}^{-1}s_{1/2}$ excitations at the beginning of the p shell is

$$\delta=\tfrac{2}{3}\Delta+\tfrac{4}{3}S_\Lambda-\tfrac{8}{5}T\ , \quad (2a)$$

while for $p_{1/2}^{-1}s_{1/2}$ at the end of the shell it is

$$\delta'=-\tfrac{1}{3}\Delta+\tfrac{4}{3}S_\Lambda+8T\ . \quad (2b)$$

The average central interaction, $\overline{V}$, has no effect on spectra, while S_N behaves as an induced spin-orbit force affecting the core and thus can change the effective spacing of core levels. The actual binding energies, B_Λ, are determined almost entirely by an interplay between $\overline{V}$ and the three-body ΛNN interaction. The B_Λ values can be well fitted by a function quadratic in the number of p-shell nucleons with only small deviations due to the spin-dependent interactions (see Sec. III F).

In the following subsections we consider a number of ways of making estimates for the parameters which characterize the two-body ΛN interaction. Having chosen a set of values we go on to consider the implications for some hypernuclei of interest, particularly in relation to hypernuclear γ transitions.

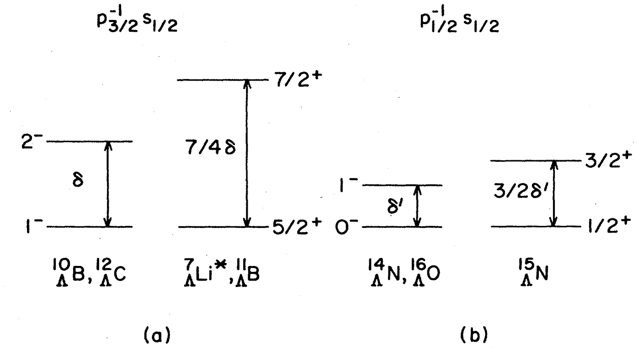

FIG. 1. Hypernuclear doublet splittings typical of the (a) nuclear $p_{3/2}$ subshell and (b) nuclear $p_{1/2}$ subshell. The parameter combinations δ and δ′ are defined in Eq. (2).

A. ΛN potential model

The model D baryon-baryon potential of Nagels *et al.*[7] has been used, following the work of Dover and Gal,[8] to obtain explicit expressions for the radial forms appearing in Eq. (1). In addition to the first-order boson-exchange potentials for the ΛN channel, second-order $\Lambda N\to\Sigma N\to\Lambda N$ coupling terms and second-order tensor terms are included. The resulting ΛN potentials, with hard cores used at short distances, reproduce the Nagels *et al.* free space two-body ΛN phase shifts as a function of energy. The Moszkowski-Scott separation method[9] is used to obtain an effective interaction suitable for nuclear structure calculations; the separation radii, inside which the potential is set to zero, determined for ΛN S waves only, are 1.302 fm for $^1S_0$ and 1.084 fm for $^3S_1$.

To make estimates for the radial matrix elements it is convenient to assume the same harmonic oscillator wave functions, specified by a size parameter b, for the N and Λ. Then the matrix elements can be expressed[10] in terms of the Talmi integrals

$$I_p=\frac{2}{\Gamma(p+3/2)}\int_{x_0}^{\infty}x^{2p}e^{-x^2}V(\sqrt{2}bx)x^2dx\ , \quad (3)$$

where $x=|\mathbf{r}_N-\mathbf{r}_\Lambda|/\sqrt{2}b$ and $x_0=r_0/\sqrt{2}b$ with r_0 a lower limit for the radial integrals, from now on taken to be the same for the $^1S_0$ and $^3S_1$ ΛN potentials as well as all other channels. In terms of the Talmi integrals for the appropriate radial interaction, we have

$$\begin{aligned}\overline{V}&=-\tfrac{1}{2}(I_0+I_1)\ ,\\ \Delta&=\tfrac{1}{2}(I_0+I_1)\ ,\\ S_\Lambda&=\tfrac{1}{2}I_1\ ,\\ S_N&=\tfrac{1}{2}I_1\ ,\\ T&=\tfrac{1}{3}I_1\ .\end{aligned} \quad (4)$$

Note that $\overline{V}$ is defined to be positive for attractive forces so that its contribution to B_Λ is $(A-5)\overline{V}$.

The matrix elements are given in Table I as a function of r_0. Taken literally, the Moszkowski-Scott prescription gives $\overline{V}\approx 2$ MeV, which is a factor of 2 larger than is required to account for the change in B_Λ throughout the p shell. This result reflects the usual overbinding problem,[8] i.e., the Λ-nucleus well depth comes out too large in one-boson exchange models. However, it is possible[11] that a repulsive ΛNN interaction taken together with a ΛN interaction that fits the Λp scattering data can reproduce the observed B_Λ values. Although such considerations are not directly relevant to a determination of the spin dependence of the effective ΛN interaction, we investigate

TABLE I. ΛN matrix elements for a one boson potential with cutoff radius r_0. [Calculated for a harmonic oscillator parameter $b=1.64$ fm, using the Nijmegen (Ref. 7) potential of model D in the equivalent one channel form given in Ref. 8].

Parameter	r_0 (fm)					
(MeV)	0.9	1.0	1.1	1.2	1.3	1.4
$\bar{V}$	3.41	2.78	2.25	1.82	1.46	1.16
Δ	1.28	0.85	0.56	0.36	0.24	0.15
S_Λ	−0.101	−0.073	−0.053	−0.037	−0.026	−0.018
S_N	−0.171	−0.131	−0.100	−0.075	−0.057	−0.043
T	0.038	0.038	0.037	0.035	0.032	0.028

the quality of fit to B_Λ values later in the paper (Sec. III F).

In discussing the values obtained for the remaining matrix elements we choose $r_0=1.2$ fm. The value of $\Delta\approx 0.36$, while only a little smaller than our estimates from data, cannot be taken too seriously, since the use of different separation distances for singlet and triplet channels could lead to widely different values of Δ. In fact, if $r_0\approx 1.1$ fm for the triplet interaction and $r_0\approx 1.3$ fm for the singlet interaction, as suggested by the Moszkowski-Scott prescription, it can be deduced from Table I that $\Delta\approx -0.47$. The negative value for Δ is consistent with the fact that use of the free space ΛN interaction leads[27] to an inversion of the $0^+,1^+$ $A=4$ doublet. For the spin-orbit interactions we find (i) S_N which, like $\bar{V}$, does not involve the Λ spin, is small, and (ii) S_Λ is small and negative and agrees well with values we later deduce from several independent data. However, the choice $r_0\approx 1.2$ fm for the separation distance is probably inappropriate for the spin-orbit interactions that are primarily determined by the ΛN p-wave channels, a more realistic value[8] being about 0.7—0.8 fm. For $r_0=0.8$ fm S_Λ becomes too large in absolute value ($S_\Lambda\approx -0.17$ MeV) compared to the limits placed on it in Sec. II B, while $S_N\approx -0.25$ MeV is consistent with the size and sign of this spin-orbit interaction necessary to fit (Sec. III A) the observed[15] $\frac{5}{2}^+\rightarrow\frac{1}{2}^+$ γ ray energy in ${}^7_\Lambda$Li within a two-body ΛN model. Finally, T is small and positive and is rather stable to the cutoff adopted because of a cancellation between the K and K* exchange contributions at a distance of about 0.5 fm. Of all these estimates, we rely most heavily on that for T, since very little is empirically known about this ΛN interaction. As pointed out by Dalitz and Gal,[4] the magnitude of T is best constrained by measurements of hypernuclear γ rays at the end of the p shell [see, e.g., Eq. (2b)].

B. The spin-orbit interaction

The first suggestion of the smallness of the Λ-nuclear spin-orbit coupling resulted from the observation,[12] via the ^{16}O(K$^-$,π^-)${}^{16}_\Lambda$O reaction, of two 0^+ excitations in the ${}^{16}_\Lambda$O split by 6 MeV. The location and intensity of the two peaks suggested $p_{3/2}^{-1}p_{3/2}$ and $p_{1/2}^{-1}p_{1/2}$ structures which could be accounted for by the nuclear spin orbit splitting alone. This deduction, although correct *a posteriori*, was incomplete because of the neglect of the residual ΛN interaction. In fact, Bouyssy[13] subsequently used a model for this interaction to show that the Λ spin-orbit splitting is indeed small,

$$\epsilon_p(\Lambda)=\epsilon_{p_{1/2}}(\Lambda)-\epsilon_{p_{3/2}}(\Lambda)=0.8\pm0.7 \text{ MeV} , \tag{5}$$

compared to $\epsilon_p(\text{N})=6.2$ MeV in the ^{15}O core.

A tighter constraint on $\epsilon_p(\Lambda)$ comes from a Brookhaven experiment[14] on the ^{13}C(K$^-$,π^-)${}^{13}_\Lambda$C reaction. A shift ΔE for a peak at $E_x\approx 10.4$ MeV,

$$\Delta E=0.36\pm0.3 \text{ MeV} , \tag{6}$$

was observed between the 4° and 15° spectra. The dominant configurations in the two states excited at 4° and 15° are ^{12}C g.s.$\times p_{1/2}$ (upper peak) and ^{12}C g.s.$\times p_{3/2}$ (lower peak), respectively. Since the Λ spin-orbit potential is the only spin-dependent term whose effect is not suppressed by the singlet nature of the lowest ^{12}C core states, ΔE is directly related to the strength of the Λ spin-orbit potential. Small effects from configuration mixing via the central interaction, which give $\epsilon_p(\Lambda)\approx 1.06\,\Delta E$, are canceled by the action of the two-body $p_N p_\Lambda$ spin-orbit interaction with the result that $\epsilon_p(\Lambda)\approx\Delta E$.

For $p^n s_\Lambda$ configurations, the strength, S_Λ, of the effective $\mathbf{l}_N\cdot\mathbf{s}_\Lambda$ interaction can be derived from the two-body spin-orbit interactions $\mathbf{l}_{N\Lambda}\cdot(\mathbf{s}_\Lambda\pm\mathbf{s}_N)$ via Eq. (4), as can $\epsilon_p(\Lambda)$ by the minimal construction of summing over the closed nuclear s shell. We find

$$\epsilon_p(\Lambda)=-6S_\Lambda , \tag{7}$$

and from Eqs. (6) and (7), we obtain

$$S_\Lambda=-0.06\pm0.05 \text{ MeV} . \tag{8}$$

A new estimate of S_Λ comes from the recent observation[15] of a 3.08 MeV γ ray in ${}^9_\Lambda$Be. This line corresponds to the deexcitation of the first excited $(\frac{3}{2}^+,\frac{5}{2}^+)$ doublet to the ${}^9_\Lambda$Be g.s. The two γ rays are expected[4] to be of roughly equal intensity and the failure to resolve the γ line into two components enables[15] one to put an upper limit of 100 keV on the splitting of the doublet. Since the nuclear core is essentially spin singlet, the doublet splitting is given to first approximation entirely by the spin-orbit term. From the shell-model calculation,

$$E(\tfrac{5}{2}^+)-E(\tfrac{3}{2}^+)=2.48S_\Lambda-0.73T+0.02\Delta . \tag{9}$$

Ignoring T and Δ, we have

$$|S_\Lambda|\lesssim 0.04 \text{ MeV} . \tag{10}$$

According to our estimates, we expect $T>0$ and $\Delta>0$, with Δ an order of magnitude larger than T. Thus the last two terms in Eq. (9) tend to cancel, with the term in T dominating unless T is very small indeed. Thus Eq. (10) is a conservative upper limit if $S_\Lambda < 0$. Under the same assumptions as to signs, a constraint is put on the combination $|T| + 3.41|S_\Lambda|$ with an upper limit on T of 0.14 MeV.

In principle there is information to be gained about S_N and the three-body forces from the energy of the observed γ ray in ${}^9_\Lambda$Be, but this is not the case here because the contribution of these terms is expected to be small. Furthermore, the ^{8}Be(2^+) core state is unbound and its energy, $E(2^+)=2.94\pm0.30$ MeV, is poorly defined on the scale of interest.

C. The spin-spin interaction

The 1.1 MeV spacing (an average for ${}^4_\Lambda$H and ${}^4_\Lambda$He) deduced from the observation[15–17] of γ rays between the 1^+ excited state and the 0^+ ground state in the $A=4$ hypernuclei is attributed naively to the spin-spin interaction. The spacing is given by the $s_N s_\Lambda$ equivalent of Δ, namely Δ_s, which in terms of Talmi integrals is just I_0. To obtain an estimate for $\Delta = \frac{1}{2}(I_0+I_1)$, we must allow for a possible difference in size between the $A=4$ hypernuclei and the p shell hypernuclei. A simple estimate can be made by using a Gaussian interaction for $V_\sigma(r)$ in Eq. (1). Then[10]

$$I_p = V_0/(1+2b^2/\mu^2)^{p+3/2} , \tag{11}$$

where V_0 is the strength of the interaction and μ is its range. We take $\mu=1$ fm, not far from the range for 2π exchange, and $b=1.64$ fm for $p_N s_\Lambda$. Then, using $b=1.35$ fm for ^{4}He, we find $\Delta \gtrsim 0.40$, an inequality since the $A=4$ hypernuclei are much less tightly bound than ^{4}He. With $b=1.64$ fm for ^{4}He, i.e., no variation in size, $\Delta=0.64$, so that we might expect a reasonable estimate for Δ to be $0.40 \lesssim \Delta \lesssim 0.64$.

D. The standard interaction

As a starting point for the calculation of hypernuclear spectra in the p shell we choose (in MeV)

$$\Delta = 0.50 , \quad S_\Lambda = -0.04 , \quad S_N = -0.08 , \tag{12a}$$

and

$$T = 0.04 .$$

The value of $|S_\Lambda|$ is at the upper limit as suggested by our analysis, while the choice for S_N and T is motivated mainly by our ΛN potential model, although the actual value of S_N can be additionally affected by three-body ΛNN contributions. For example, the particular model of Ref. 1 [Eq. (5.5)] gave a contribution to S_N of -0.36 MeV. We refer to the preceding choice (12a) as our standard interaction. For comparison, the GSD79 parameters are

$$\Delta = 0.15 , \quad S_\Lambda = 0.57 , \quad S_N = -0.21 , \tag{12b}$$

and

$$T = 0 .$$

III. RESULTS

In this section we investigate whether our estimates for the spin-dependent terms in the ΛN effective interaction are consistent with other data from p-shell hypernuclei. When an energy difference is expressed in terms of the parameters, as in Eq. (9), the proper interpretation of the coefficient of a parameter is as a derivative of the energy difference with respect to that parameter (but we do not write $\delta\Delta$, δS_Λ, etc.). Thus the effect of small changes about the standard solution can be estimated. Except in the case when there is appreciable mixing of configurations based on different core states, the expressions for energy differences can be taken literally instead of as relations involving differentials.

A. The hypernucleus ${}^7_\Lambda$Li

The states of ${}^7_\Lambda$Li which may lie below the ${}^5_\Lambda$He + d threshold at $E=3.94\pm0.04$ MeV are shown in Fig. 2 for the standard interaction. The $\frac{5}{2}^+ \to \frac{1}{2}^+$ γ transition has been observed[15] with $E_\gamma = 2.034\pm0.023$ MeV. In terms of the ΛN parameters

$$E(\tfrac{5}{2}^+) - E(\tfrac{1}{2}^+) = 2.18 + 0.07\Delta - 1.00 S_\Lambda + 0.22T + 0.95 S_N . \tag{13}$$

For our choice of Δ, S_Λ, and T, the only term which lowers the energy of the $\frac{5}{2}^+$ state is S_N, $S_N \approx -0.25$ MeV being required to give the energy of the observed γ ray. This is in agreement with the remark made in Sec. II D on the possible renormalization of S_N by three-body interactions.

An observation of the $\frac{3}{2}^+ \to \frac{1}{2}^+$ g.s. γ-ray line would provide an excellent measure of Δ since ($L=0$ core)

$$E(\tfrac{3}{2}^+) - E(\tfrac{1}{2}^+) = 1.35\Delta + 0.15 S_\Lambda - 1.29T - 0.06 S_N . \tag{14}$$

Unfortunately, direct excitation of the $\frac{3}{2}^+$ level in the (K^-,π^-) reaction requires spin flip which is strongly suppressed at the forward angles where the $p_N \to s_\Lambda$ transition peaks ($\theta_{lab} \approx 10°$ for $p_{K^-} = 800$ MeV/c). Although the $\frac{3}{2}^+$ level can be fed by a weak branch from the $\frac{5}{2}^+$ level and by a stronger branch from the $\frac{1}{2}^+; T=1$ level, if it is particle stable, the Brookhaven National Laboratory (BNL) experiment[15] did not have the sensitivity to detect either of these primary γ rays at the rates predicted, and shown in Fig. 2.

In principle the difference in the Λ binding energies for

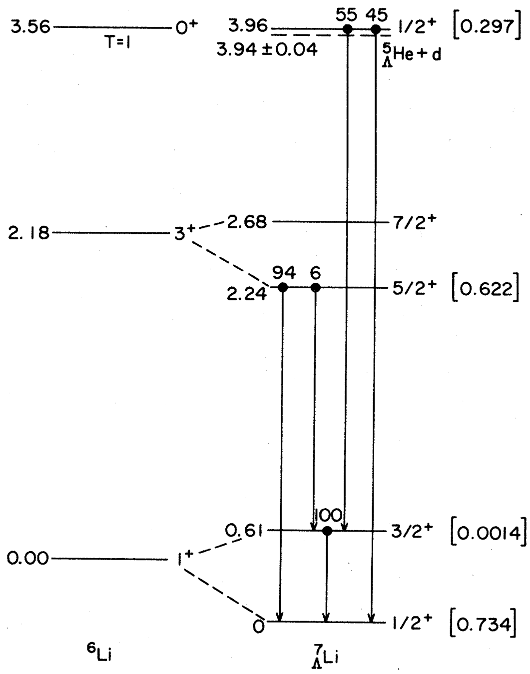

FIG. 2. The energy levels (MeV) for ^{6}Li from experiment and for $^7_\Lambda$Li as calculated from the standard ΛN interaction. The numbers in the square brackets are structure factors [Eq. (4.3) of Ref. 6] governing the formation rates in the reaction ^{7}Li(K$^-$,π^-)$^7_\Lambda$Li. These structure factors, when divided by the number of *p*-shell target neutrons, are just the relative formation rates quoted by Dalitz and Gal in Ref. 4. The measured excitation energy of the $\frac{5}{2}^+$ state is 2.03 MeV (Ref. 15).

$^7_\Lambda$Li and $^7_\Lambda$Be (the analog of the $\frac{1}{2}^+$; $T=1$ level in $^7_\Lambda$Li) contains the same information as the ground state doublet splitting in $^7_\Lambda$Li. Indeed, in terms of the ΛN parameters

$$\begin{aligned}\Delta B_\Lambda &= B_\Lambda(^7_\Lambda\text{Li}) - B_\Lambda(^7_\Lambda\text{Be}) \\ &= 0.913\Delta + 0.087S_\Lambda + 0.085S_N - 0.794T \\ &\approx \tfrac{2}{3}[E(\tfrac{3}{2}^+) - E(\tfrac{1}{2}^+)] \ . \end{aligned} \tag{15}$$

From the measured[18] binding energies one obtains $\Delta B_\Lambda = 0.42\pm0.09$ MeV, which implies $E_\gamma \approx 630\pm140$ keV for the $\frac{3}{2}^+ \to \frac{1}{2}^+$ γ transition in $^7_\Lambda$Li. Assuming charge independence, these binding energies imply that the $\frac{1}{2}^+$; $T=1$ level in $^7_\Lambda$Li lies above the (^{5}He$+^2_\Lambda$H) threshold by ($+0.05\pm0.08$) MeV, so its particle stability is problematical. If it is unbound, the rate predicted for the secondary $\frac{3}{2}^+ \to \frac{1}{2}^+$ γ transition following direct excitation of the $\frac{5}{2}^+$ $^7_\Lambda$Li level would have been too weak to have been seen in the BNL experiment. We note that for $T=0.04$, $S_\Lambda=-0.04$, and $S_N=-0.08$, Eq. (15) yields $\Delta=0.50\pm0.11$ MeV, which is consistent with the estimate based on the $A=4$ system.

However, if the $\frac{1}{2}^+$; $T=1$ level in $^7_\Lambda$Li is particle stable, as appears quite possible, there would be, as a result of its decay, about 0.34 $\frac{3}{2}^+ \to \frac{1}{2}^+$ γ rays per 2.03 MeV $\frac{5}{2}^+ \to \frac{1}{2}^+$ γ ray, which translates into a comparable number of counts for these two γ rays after the efficiency of the BNL detector system as a function of γ ray energy is folded in. Each $\frac{3}{2}^+ \to \frac{1}{2}^+$ γ ray would be accompanied by a $\frac{1}{2}^+$; $T=1 \to \frac{3}{2}^+$ γ ray of energy about 3.3 MeV, and there would be a comparable number of $\frac{1}{2}^+$; $T=1 \to \frac{1}{2}^+$ γ rays of energy about 3.9 MeV, but the detection efficiency for these energetic γ rays is sufficiently low to account for their absence in the BNL data. This possibility for producing the $\frac{3}{2}^+ \to \frac{1}{2}^+$ γ ray is most likely ruled out for the upper range possible for E_γ, say for $E_\gamma > 600$ keV. However, there is some evidence[15] in the BNL γ spectra for an apparent broadening of the 511 keV annihilation peak on the low energy side. If this is attributed to a γ ray peak, it would be compatible with an energy of about 480 keV, with a count rate comparable to that observed for the 2.03 MeV γ ray. If this γ ray can be attributed to $^7_\Lambda$Li, it would imply a quite acceptable value, $\Delta \approx 0.39$ MeV. An alternative source for a γ ray in this energy region may be the 0.48 MeV γ decay of the first excited state of ^{7}Li, formed in the weak decay $^7_\Lambda$Li(g.s.)$\to \pi^0 + {}^7$Li*, but we estimate that this process produces only 0.016 of a γ ray per 2.03 MeV γ ray. It will be important to examine the γ-ray spectrum below 500 keV in more detail, in future (K$^-$,π^-) experiments on ^{7}Li.

The basic structure of $^7_\Lambda$Li which emerges from the present study eliminates many of the possibilities considered by Dalitz and Gal[4,19] and provides some guidance for future searches for the $\frac{5}{2}^+ \to \frac{3}{2}^+$ and $\frac{3}{2}^+ \to \frac{1}{2}^+$ γ transitions, the energies of which should sum to 2.03 MeV.

B. The hypernucleus $^8_\Lambda$Li

The $^8_\Lambda$Li states based on the ^{7}Li ground and first excited states are shown in Fig. 3. The ground state is known to

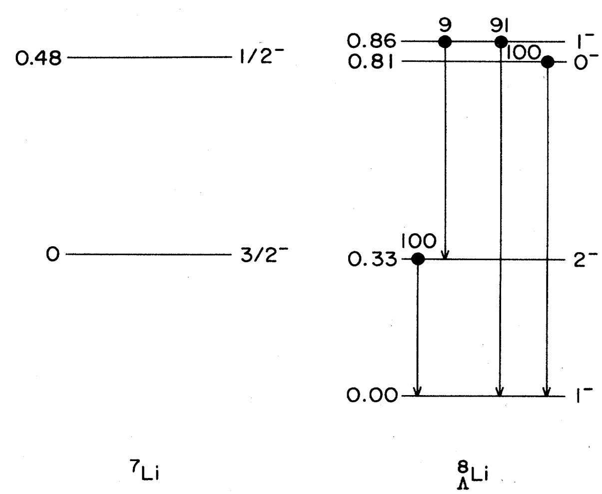

FIG. 3. The energy levels (MeV) of ^{7}Li from experiment and for $^8_\Lambda$Li as calculated from the standard ΛN interaction. The γ-ray branching ratios for the upper 1^- state are a very sensitive function of the mixing angle which determines the wave function (see the text).

have $J^{\pi}=1^-$. In addition, the mixing of 1^- configurations based on the two core states is constrained by an analysis of

$${}^{8}_{\Lambda}\text{Li}\to\pi^- + {}^{8}\text{Be}^*\to\pi^- +\alpha+\alpha$$

data. Specifically, if

$${}^{8}_{\Lambda}\text{Li}(1^-;\text{g.s.})=\cos\epsilon\,|\tfrac{3}{2}^-\times s_\Lambda\rangle+\sin\epsilon\,|\tfrac{1}{2}^-\times s_\Lambda\rangle\,, \qquad (16)$$

the analysis[20] of Zieminska and Dalitz gives (according to our phase convention) $-0.40<\epsilon<-0.13$ or $-1.2<\epsilon<-0.9$, with ϵ in radians. This result has been difficult to reconcile[3] with the solutions found by Gal, Soper, and Dalitz; in particular, their favored solution gives $\epsilon=+0.16$. The present calculations give $\epsilon=-0.35$ for $\Delta=0.5$ and $\epsilon=-0.28$ for $\Delta=0.4$, corresponding to weak mixing, and falling within the range specified previously for small values of $|\epsilon|$. In the LS limit for the ^{7}Li core, which is a very good approximation,

$$|{}^{8}_{\Lambda}\text{Li};L=1\;S=0\;J=1\rangle =\sqrt{2/3}\,|\tfrac{3}{2}^-\times s_\Lambda\rangle-\sqrt{1/3}\,|\tfrac{1}{2}^-\times s_\Lambda\rangle\,, \qquad (17)$$

and $\epsilon=-0.62$. The negative value for ϵ corresponds to the fact that $\Delta>0$ and $S_\Lambda<0$ work to put the $S=0$, 1^- level lowest; for the GSD79 interaction, $S_\Lambda=0.57$ outweighs $\Delta=0.15$ with the result that $\epsilon>0$.

There have been two tentative assignments of γ rays to ${}^{8}_{\Lambda}$Li in the literature. Since there are no stable $A=8$ targets available, these ${}^{8}_{\Lambda}$Li γ rays are secondary products, arising from the breakup of more massive hypernuclear systems resulting from the (K^-,π^-) reaction. They are the following:

(i) $E_\gamma=1.22\pm0.04$ MeV, observed[21] following K$^-$ capture from rest by ^{9}Be, in coincidence with a π^- meson of kinetic energy between 40 and 48 MeV, a range which includes π^- mesons resulting from ${}^{8}_{\Lambda}$Li π^--mesic decay. It appeared natural to identify this with the $1_2^-\to1_1^-$ transition, since the levels shown on Fig. 3 are the only levels of ${}^{8}_{\Lambda}$Li below about 4.5 MeV excitation and this is the most energetic transition possible between them. The parameters of GSD79 predicted 1.28 MeV for this transition, so that this identification did appear rather natural at the time. However, the present calculations give no support for this identification. Furthermore, since this γ ray was not observed in other irradiations which included the same π^- kinetic energies as part of a wider range of π^- detection, it cannot yet be regarded as well established.

(ii) $E_\gamma=0.31\pm0.02$ MeV, observed[22] following K$^-$ capture from rest by ^{9}Be. This γ ray was tentatively identified with the $2^-\to1_1^-$ doublet transition in ${}^{8}_{\Lambda}$Li or ${}^{8}_{\Lambda}$Be. The γ-ray energies for these two transitions, and for the competing transition $1_2^-\to2^-$, are given by

$$E(1_2^-)-E(1_1^-)=0.855+0.829\delta\Delta+0.286\delta S_\Lambda -0.387\delta S_N+1.10\delta T\,, \qquad (18a)$$

$$E(1_2^-)-E(2^-)=0.523-0.077\delta\Delta-0.603\delta S_\Lambda -0.459\delta S_N+1.97\delta T\,, \qquad (18b)$$

$$E(2^-)-E(1_1^-)=0.332+0.931\Delta\delta+0.889\delta S_\Lambda +0.072\delta S_N-0.877\delta T\,, \qquad (18c)$$

for *small* changes in the parameters from the standard values. To account for the mixing in the 1^- levels it is more accurate to diagonalize[3] the 2×2 matrix; the coefficients of Δ, S_Λ, S_N, and T in this matrix are given in Table II.

From Eq. (18a) we note that it is difficult to obtain a γ-ray energy as large as 1.22 MeV with our present estimates for the parameters, and the identification (i) for this γ ray therefore cannot be accepted here. On the other hand Eq. (18c) gives $E_\gamma(2^-\to1_1^-)$ around 300 keV for $\Delta\approx0.5$, as suggested by the ΔB_Λ for ${}^{7}_{\Lambda}$Li and ${}^{7}_{\Lambda}$Be. It is interesting that the branching ratio for the γ decay of the 1_2^- level is an extremely sensitive function of the mixing angle ϵ. Indeed, for ${}^{8}_{\Lambda}$Li and ${}^{8}_{\Lambda}$Be the branching ratios are given by (neglecting energy differences in the mirror core nuclei and the hypernuclei)

$$R({}^{8}_{\Lambda}\text{Li})=\left[\frac{E_{21}}{E_{11}}\right]^3\tfrac{5}{3}(2.115\sin\epsilon+3.049\cos\epsilon)^2(6.230\sin\epsilon\cos\epsilon+1.760\sin^2\epsilon-1.760\cos^2\epsilon)^{-2}\,, \qquad (19a)$$

$$R({}^{8}_{\Lambda}\text{Be})=\left[\frac{E_{21}}{E_{11}}\right]^3\tfrac{5}{3}(0.295\sin\epsilon-2.621\cos\epsilon)^2(-1.700\sin\epsilon\cos\epsilon-1.513\sin^2\epsilon+1.513\cos^2\epsilon)^{-2}\,, \qquad (19b)$$

TABLE II. Coefficients of Δ, S_Λ, S_N, and T in the 2×2 matrix for $J^\pi=1^-$ in ${}^{8}_{\Lambda}$Li.

	Δ	S_Λ	S_N	T
$B_1(3,3)$ coefficients[a]	0.419	0.831	-0.408	-0.835
$B_1(1,1)$ coefficients	0.082	-0.332	0.130	-1.000
$B_1(3,1)$ coefficients[b]	-0.460	0.460	0	-0.755

[a]For notation see Ref. 3.

[b]Our off-diagonal matrix element differs in sign from Ref. 3 since our convention is to couple $\mathbf{L}\times\mathbf{S}\to\mathbf{J}$. In Eq. (3) of Ref. 3 the coefficients of T should be multiplied by $\frac{3}{5}$.

where

$$R=\frac{\Gamma_\gamma(1_2^-\to 2^-)}{\Gamma_\gamma(1_2^-\to 1_1^-)} . \tag{19c}$$

These expressions were obtained using the Cohen and Kurath (6—16) 2BME interaction[5] to calculate the nuclear $M1$ matrix elements (neglecting small $E2$ contributions), which give an excellent account of the ^{7}Li g.s. magnetic moment and the lifetimes of the first excited states in ^{7}Li and ^{7}Be, together[4] with $g_\Lambda=-1.34$. As an example of the sensitivity to ϵ, the coefficient of the energy factor in Eq. (19a) for $^8_\Lambda$Li is 0.92 for $\tan\epsilon=-0.29$ ($\Delta=0.4$) and 0.67 for $\tan\epsilon=-0.37$ ($\Delta=0.5$) compared with 5.00 for $\tan\epsilon=0$. For $^8_\Lambda$Be, Eq. (19b) gives 3.75 for $\tan\epsilon=-0.29$, 3.77 for $\tan\epsilon=-0.37$, and 5.00 for $\tan\epsilon=0$. The $A=8$ hypernuclei are interesting and display a useful sensitivity to Δ, and consequently to ϵ, but unfortunately these hypernuclei can be produced only indirectly from the decay of heavier hypernuclei.

C. The hypernucleus $^{12}_\Lambda$C

The separation of the ground state doublet is approximately given by

$$E(2^-)-E(1^-)=0.51\Delta+1.47S_\Lambda+0.006S_N-2.00T . \tag{20}$$

The ground state spin assignment $J^\pi=1^-$ for $^{12}_\Lambda$B puts a strong constraint on the parameters. For the standard interaction the splitting is ~90 keV, but for Δ near 0.4, at the lower end of the range specified at the end of Sec. II C, the doublet splitting could be very small and sensitive to small variations in Δ, S_Λ, and T. For very small splittings, weak decay of the 2^- level will compete with its γ decay and this possibility needs to be considered in the analysis which leads to the 1^- spin assignment for $^{12}_\Lambda$B.

Another point of interest for $^{12}_\Lambda$C concerns the relative formation strengths of the three lowest 1^- levels via the ^{12}C(K$^-$,π^-)$^{12}_\Lambda$C reaction. The formation strengths given[4] by Dalitz and Gal correspond to a pure weak-coupling approximation for the $^{12}_\Lambda$C states, i.e., the formation strengths are proportional to the neutron pickup spectroscopic factor for the corresponding core state. However, even very small admixtures (intensity less than 1% in our calculation) of the ^{11}C g.s.$\times s_\Lambda$ configuration into the upper 1^- levels can strongly affect the formation rates, particularly if the mixing is destructive as far as the formation amplitudes are concerned. The (K$^-$,π^-) reaction on the predominantly $S=0$ ^{12}C g.s. will populate $^{12}_\Lambda$C states with $L=1$, $S=0$ and it is easy to show that $S_\Lambda<0$ or $\Delta>0$ favor $L=1$, $S=0$ for the lowest 1^- level. Thus, as indicated in Table III, our present interaction reduces the combined strength of the upper two levels relative to the lowest level from 40% in the weak coupling limit to 23%, while the GSD79 interaction increases it to 51.5%. An experimental limit on the strength to the upper two 1^- levels has been given[23] as 6±5 % assuming these levels to be responsible for the total excess of events observed between 2 and 7 MeV excitation energy. Uncertainties in background subtraction, together with the relatively poor resolution of 2.5 MeV, make a more detailed unfolding of the experimental spectrum difficult. However, the theoretical cross section, smeared by a Gaussian resolution function of width 2.5 MeV and gathered in 1 MeV bins (cf. Fig. 11 of Ref. 6), is in excellent agreement with the data presented in Fig. 2 of Ref. 23. This comparison provides a useful consistency check for the current set of ΛN parameters.

D. The hypernuclei $^{11}_\Lambda$B and $^{10}_\Lambda$B

The assignment[24] of $J^\pi=\frac{5}{2}^+$ to the ground state of $^{11}_\Lambda$B provides another consistency check on the ΛN interaction. The ground state doublet splitting is given by

$$E(\tfrac{7}{2}^+)-E(\tfrac{5}{2}^+)=1.03\Delta+2.45S_\Lambda+0.04S_N-3.39T \tag{21}$$

and is 255 keV for the standard interaction. Another quantity of interest[4] is the excitation energy of the first $\frac{1}{2}^+$ level (since weak decay and γ decay may both be important),

$$E(\tfrac{1}{2}^+)-E(\tfrac{5}{2}^+)=0.72-0.26\Delta+1.24S_\Lambda -1.23S_N-0.60T , \tag{22}$$

TABLE III. Excitation energies and formation strengths for $^{12}_\Lambda$C.

J^π_n	Excitation energies E(SI)[a]	E(WC)[b]	E(GSD)[c]	Formation strengths SI[a]	WC[b]	GSD[c,d]
1_1^-	0	0	0	0.810	0.712	0.657
2_1^-	0.086	0	0.95			
0_1^-	2.12	2.00	2.56			
1_2^-	2.19	2.00	3.29	0.113	0.188	0.214
2_2^-	4.26	4.32	4.59			
2_3^-	4.94	4.80	6.36[d]			
1_3^-	5.12	4.80	5.11	0.072	0.094	0.125

[a]Standard ΛN interaction, Eq. (12a).
[b]Weak coupling.
[c]GSD79 interaction, Eq. (12b).
[d]Calculated omitting the ΛNN interaction; the energies from this calculation are very close to those given in column four.

with a value of 0.62 MeV for the standard interaction.

In the case of ${}^{10}_{\Lambda}$B only the members of the ground state doublet will be particle stable among negative parity levels and it is the 2^- member that will be formed in the (K^-,π^-) reaction. The standard interaction places the 2^- level above the 1^- level with a separation of about 170 keV, making ${}^{10}_{\Lambda}$B an attractive case for study via the ^{10}B$(K^-,\pi^-\gamma){}^{10}_{\Lambda}$B reaction. This doublet splitting is approximately given by

$$E(2^-)-E(1^-)=0.62\Delta+1.36S_\Lambda+0.05S_N-1.49T\ . \tag{23}$$

In fact, a candidate for this doublet transition has been reported[25] near $E_\gamma=160$ keV. It is not clear whether or not either member of the $(0^+,1^+)$ doublet based on the lowest $\frac{1}{2}^+$ core level is particle stable, but the direct formation of these levels is probably negligible.[4]

E. The "$p_{1/2}$"-shell hypernuclei

The splitting of the ground state doublet for ${}^{14}_{\Lambda}$C, ${}^{14}_{\Lambda}$N, ${}^{15}_{\Lambda}$N, and ${}^{16}_{\Lambda}$O depends very sensitively on the strength of the tensor interaction. In the simplest model, the doublet separation is given by δ' of Eq. (2b) for the even hypernuclei and by $\frac{3}{2}\delta'$ for ${}^{15}_{\Lambda}$N. The approximate energy separations from the shell-model calculation are

$$A=14:\ E(1^-)-E(0^-)=-0.41\Delta+1.40S_\Lambda-0.01S_N+7.19T\ , \tag{24a}$$

$$A=15:\ E(\tfrac{3}{2}^+)-E(\tfrac{1}{2}^+)=-0.73\Delta+2.23S_\Lambda-0.04S_N+10.66T\ , \tag{24b}$$

$$A=16:\ E(1^-)-E(0^-)=-0.38\Delta+1.38S_\Lambda-0.03S_N+7.85T\ . \tag{24c}$$

The spectra of particle-stable $p^n s_\Lambda$ states for these nuclei are shown in Fig. 4. For the parameters we have chosen, the splittings of the ground state doublets are rather small. In fact, a value for T, which is perfectly reasonable from the point of view of our theoretical estimates, can be chosen in each case to make the members of an individual ground state doublet essentially degenerate.

The experimental situation for measuring a doublet splitting is particularly favorable for ${}^{16}_{\Lambda}$O since both members of the g.s. doublet are fed from the excited $(\frac{3}{2}^-\times s_\Lambda)\ 1^-$ level in such a way that the intensity of the g.s. doublet γ ray is almost independent of the order of the levels (similarly for ${}^{14}_{\Lambda}$C). If the splitting is small, roughly <100 keV, competition from weak decay will be severe. However, an upper limit on the γ ray energy is sufficient to put a tight constraint on T.

The electromagnetic transitions in ${}^{16}_{\Lambda}$O and ${}^{16}_{\Lambda}$N provide another illustration of the sensitivity of certain quantities to very small mixing between weak-coupling states, in this instance a 0.25% intensity admixture of the $\frac{1}{2}^-\times s_\Lambda$ and $\frac{3}{2}^-\times s_\Lambda$ states. This small admixture slows down the $1^-\to 0^-$ doublet transition in ${}^{16}_{\Lambda}$O by 14% and speeds up the corresponding transition in ${}^{16}_{\Lambda}$N by 61% (if core $M1$ matrix elements appropriate to a single hole structure are used). The $1_2^-\to 0^-$ branching ratio is slightly enhanced in both hypernuclei by the admixture. The corresponding transitions in ${}^{14}_{\Lambda}$N and ${}^{14}_{\Lambda}$C, shown in Fig. 4(a), are also sensitive to small admixtures in the hypernuclear wave functions. The common feature of the $A=8$, 14, and 16 γ transitions, which leads to the sensitivity already mentioned, is a large $M1$ matrix element connecting the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ core states. In the case of ${}^{15}_{\Lambda}$N the $M1$ and $E2$ matrix elements connecting the two 1^+; $T=0$ core levels are small, and the branching ratios given in Fig. 4(b) have been evaluated using pure weak-coupling wave functions.

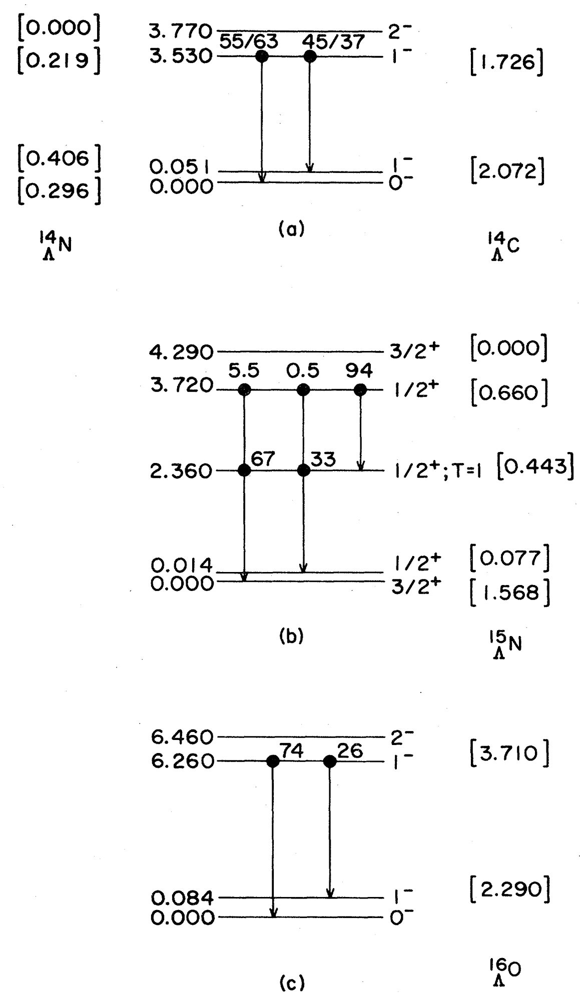

FIG. 4. The energy levels (MeV) of (a) ${}^{14}_{\Lambda}$C, (b) ${}^{15}_{\Lambda}$N, and (c) ${}^{16}_{\Lambda}$O as calculated from the standard ΛN interaction. The numbers in square brackets are the structure factors for the (K^-,π^-) reaction. (See the caption to Fig. 2.)

F. Ground state binding energies

While it is recognized that a fit to the Λ separation energies, B_Λ, for the p-shell hypernuclei has severe limitations as far as the determination of the spin dependence of the ΛN interaction is concerned, it is interesting to fix the spin dependence according to our estimates and ask how well the B_Λ values can be fitted. We take the 12 B_Λ values used by Gal, Soper, and Dalitz and omit two for the following reasons: (i) $B_\Lambda({}^5_\Lambda\mathrm{He})$ on the grounds that ${}^5_\Lambda\mathrm{He}$ is probably more compact than the heavier hypernuclei and is consequently more tightly bound than it would be if ${}^4\mathrm{He}$ were larger, and (ii) $B_\Lambda({}^9_\Lambda\mathrm{Be})$ on the grounds that the core nucleus whose mass enters into B_Λ is unbound (${}^8\mathrm{Be}$ gains binding energy by assuming an extended α-α configuration leading to a lower value of B_Λ than would be obtained for a compact ${}^8\mathrm{Be}$).

We fit the remaining ten B_Λ values with a quadratic in the number of p-shell nucleons. The constant term represents the single particle energy for s_Λ relative to a ${}^4\mathrm{He}$ core [$B_\Lambda({}^5_\Lambda\mathrm{He})$ adjusted for the size effect]; the linear term is $\bar{V}$, and the quadratic term provides a crude representation of a three-body ΛNN interaction (strictly a long range Wigner-type interaction). As can be seen from Fig. 5, the fit to the separation energies, even with no additional terms, is reasonably good ($\chi^2=55$) by the standards[3] of the GSD fits. If the effects of the spin dependent ΛN interaction are now included according to the standard interaction (12a) and the quadratic is refitted to the data, the χ^2 drops by a factor of 2 to 26 (seven degrees of freedom). The B_Λ resulting from this fit are indicated by arrows in Fig. 5. Any attempt to improve χ^2 by varying one of Δ, S_Λ, S_N, or T leads to very small change in the parameter and little change in χ^2; e.g., $T\rightarrow 0.02$ with a change in $\chi^2<0.5$. $B_\Lambda({}^{14}_\Lambda\mathrm{C})$ and $B_\Lambda({}^{15}_\Lambda\mathrm{N})$ are most sensitive to T, but unfortunately these B_Λ values are determined[26] by relatively few events.

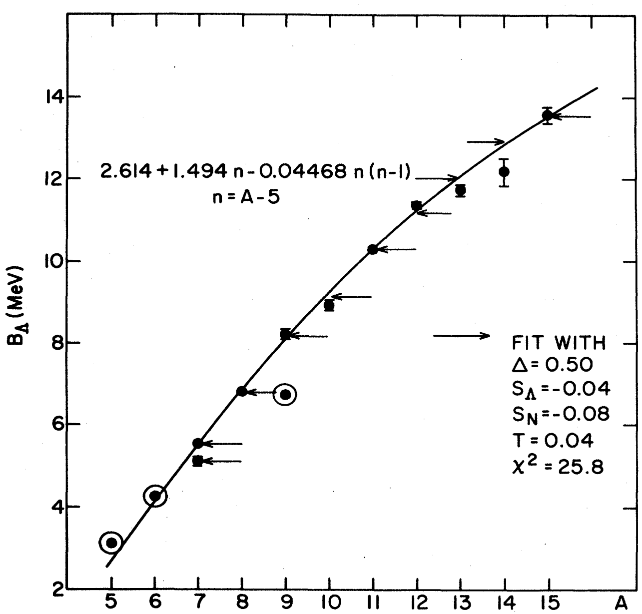

FIG. 5. The Λ separation energies B_Λ for the p-shell hypernuclei. Error bars are not shown when the error is ≤ 50 keV. The solid line is the best fit quadratic in the number of p shell nucleons (see the text). The three B_Λ values encircled were not used in the fit. The arrows indicate the binding energies obtained when the spin dependence of the standard ΛN interaction is incorporated into the fit.

The coefficients of Δ, S_Λ, S_N, and T as they enter into B_Λ are given in Table IV. Since the coefficients of S_Λ for cases included in the GSD79 fit are generally positive, but zero for ${}^9_\Lambda\mathrm{Be}$, a large positive value for S_Λ increases the

TABLE IV. Coefficients of Δ, S_Λ, S_N, and T for B_Λ values. (The first group corresponds to the spin values of the GSD79 fit. The second group includes the alternative spin values for $A=14,15$ and both possibilities for $A=16$.)

(A,T)	J	Δ	S_Λ	S_N	T
(7,0)	$\frac{1}{2}$	0.912	0.088	−0.013	−0.796
(7,1)	$\frac{1}{2}$	0.000	0.000	−0.099	0.000
$(8,\frac{1}{2})$	1	0.680	0.391	−0.346	−0.326
(9,0)	$\frac{1}{2}$	0.000	0.000	−0.375	0.028
(9,1)	$\frac{3}{2}$	0.806	0.627	−1.047	−0.482
$(10,\frac{1}{2})$	1	0.435	0.802	−1.091	−0.824
(11,0)	$\frac{5}{2}$	0.640	1.343	−1.916	−1.816
$(12,\frac{1}{2})$	1	0.376	0.852	−1.799	−1.131
(13,0)	$\frac{1}{2}$	0.011	−0.011	−1.770	0.045
$(14,\frac{1}{2})$	0	−0.240	0.990	−2.108	5.567
(15,0)	$\frac{1}{2}$	−0.423	1.423	−1.944	7.341
$(14,\frac{1}{2})$	1	0.166	−0.407	−2.098	−1.621
(15,0)	$\frac{3}{2}$	0.312	−0.805	−1.903	−3.318
$(16,\frac{1}{2})$	0	−0.250	1.000	−1.000	6.000
$(16,\frac{1}{2})$	1	0.133	−0.378	−0.997	−1.854

binding energy of most hypernuclei relative to $B_\Lambda({}^9_\Lambda\mathrm{Be})$, as is required if ${}^9_\Lambda$Be is included in the fit. This, probably, is the reason for the relatively large positive value of S_Λ that emerged from the GSD79 fit.

IV. CONCLUSIONS

We have proposed a set of matrix elements of the ΛN force, which can account for our present knowledge of p-shell hypernuclear states with the basic configuration $s^4p^ns_\Lambda$. Doublet separations are determined mainly by Δ, S_Λ, and T. As far as S_Λ is concerned, the inferences from p^ns_Λ and p^np_Λ states are nicely consistent and point to a small negative value for S_Λ. Estimates of Δ from the $A=4$ doublet separation and from the difference in Λ separation energies for ${}^7_\Lambda$Be and ${}^7_\Lambda$Li are consistent with each other. Furthermore, the values of S_Λ and Δ, so deduced, give the correct ground state spins for ${}^8_\Lambda$Li, ${}^{11}_\Lambda$B, and ${}^{12}_\Lambda$B, as well as the right sign and magnitude for the mixing angle which characterizes the ground state wave function of ${}^8_\Lambda$Li. The reproduction of the mixing angle in ${}^8_\Lambda$Li, in particular, is a nontrivial result. We conclude that the two-body effective interaction approach works but that the effective interaction differs considerably from estimates based on the free ΛN potential model. The difference is most apparent for the central component while the general weakness and the signs of the noncentral components from the free ΛN interaction do appear to be consistent with the existing data. A resolution of the preceding differences, including perhaps the actual smallness of the Λ-nuclear spin-orbit coupling, presents a challenge to theoretical studies. A mechanism involving ΛN-ΣN coupling has been shown to give[27] a correct ordering of the $0^+,1^+$ doublet splitting in the $A=4$ hypernuclei. However, this mechanism does not lead to a state independent two-body effective interaction. Also ΛNN interactions may have a spin dependence and may play a nonnegligible role, especially in energy separations that are not simply doublet splittings. Nevertheless, it may well be that the effects of such more complex processes can be satisfactorily accounted for in a two-body effective interaction approach.

More data, particularly from γ ray transition energies, are required before a breakdown in the present approach could become apparent. An interesting possibility for studying the spin-spin interaction for $(sd)_Ns_\Lambda$ configurations with a ^{20}Ne target is discussed in the Appendix. As suggested in previous sections, an observation of the $\frac{5}{2}^+\to\frac{3}{2}^+\to\frac{1}{2}^+$ γ cascade in ${}^7_\Lambda$Li, perhaps feasible with a modest increase in K^- intensity, would provide a useful consistency test for the model. Similarly, the γ rays in ${}^8_\Lambda$Li (and ${}^8_\Lambda$Be) would provide an interesting study, in part, because of a strong sensitivity to the mixing angle which determines the structure of the two 1^- levels. A very recent attempt to observe the ground state doublet transition in ${}^{10}_\Lambda$B and ${}^{16}_\Lambda$O has just resulted in the observation[28] of γ rays that closely follow the predictions of the present analysis. These results will permit the placement of significant limits on Δ and T. The ${}^{16}_\Lambda$O splitting, in particular, places the first meaningful constraint on T.

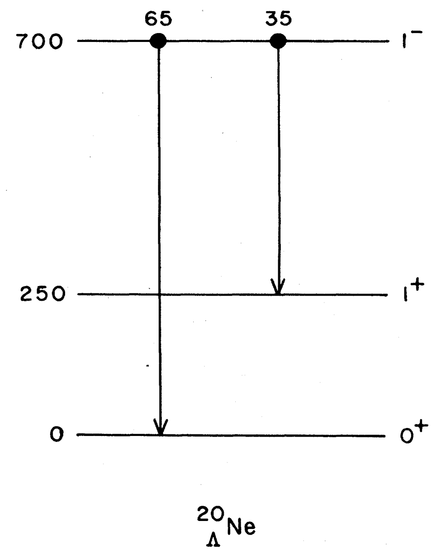

FIG. 6. The energy levels (keV) of ${}^{20}_\Lambda$Ne relevant to a determination of the ground-state doublet splitting via a $(K^-,\pi^-\gamma)$ experiment on a ^{20}Ne target.

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APPENDIX: THE HYPERNUCLEUS ${}^{20}_\Lambda$Ne

The splitting of the $^{19}\mathrm{Ne}(\frac{1}{2}^+)\times s_\Lambda$ ground-state doublet of ${}^{20}_\Lambda$Ne is of interest because it provides an opportunity to study the spin-spin component of the ΛN interaction for the $(sd)_Ns_\Lambda$ configuration. About 90% of the $^{19}\mathrm{Ne}(\frac{1}{2}^+)$ wave function is accounted for by a single $(sd)^3$ configuration with $L=0$ and maximum spatial symmetry. (In jj coupling this state corresponds to a specific but complicated mixture of d^ns^{3-n} configurations.) Thus there is a strong similarity to the $A=4$ hypernuclei, but with the additional ^{16}O core, and the spin-spin interaction should dominate the $(0^+,1^+)$ doublet splitting. Using a model in which $^{19}\mathrm{Ne}(\frac{1}{2}^+)$ is pure $L=0$, we have

$$E(1^+)-E(0^+)=\tfrac{7}{20}(I_0+I_2)+\tfrac{3}{10}I_1\ , \tag{A1}$$

which, according to the arguments based on Eq. (11), gives a splitting of about 250 keV for $b=1.86$ fm. The 1^+ level will not be strongly populated in the (K^-,π^-) reaction. However, ^{19}Ne has a $\frac{1}{2}^-$ level[29] at 275 keV ($\tau=61$ ps) which is essentially a $p_{1/2}$ hole in ^{20}Ne, so that both the resulting hypernuclear $(0^-,1^-)$ doublet splitting and the population of the 1^- member in the (K^-,π^-) reaction will be similar to those for the ground-state doublet in ${}^{16}_\Lambda$O (we estimate a formation cross section of 200 μb/sr at 10° for $p_K=800$ MeV/c). The γ ray branching ratio for the 1^- $E1$ deexcitation is given in the weak-coupling approximation by

$$\frac{\Gamma_\gamma(1^- \to 0^+)}{\Gamma_\gamma(1^- \to 1^+)} = \frac{1}{2} \left[\frac{E_{10}}{E_{11}} \right]^3 , \tag{A2}$$

the expected situation being shown in Fig. 6.

We note that there is a $\frac{5}{2}^+$ state[29] at 238 keV in ^{19}Ne which will give rise to a $(2^+,3^+)$ doublet, the 2^+ member of which can be populated in the (K^-,π^-) reaction, with an estimated cross section of 50 μb/sr at $\theta_\pi = 16°$. However, the $\frac{5}{2}^+ \to \frac{1}{2}^+$ $E2$ transition is slow[29] ($\tau = 26$ ns) and the hypernuclear levels based on the $\frac{5}{2}^+$ level should undergo primarily weak decay.

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